« A Centile Regression Approach for Crisis Analysis » ♣

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Work in Progress

- December 2008 -

Abstract

The dynamics of stocks returns seen as a population are important for asset management and risk control, in particular for options books, or to hedge market neutral positions in long-short equity trading programs. Indeed, market neutrality is usually insured for “typical” days, but is destroyed in high cross-volatile days. The cross-volatility of returns, which has a very intuitive interpretation, is in these cases almost as important to monitor as the volatility.

In this paper, we justify and investigate whether the cross-volatility can be used in international diversification especially during financial crisis contagion episode. Financial crisis contagion is defined as a sharply increase of financial returns co-movements among several markets due to turbulence of one market. It’s also interesting to distinguish financial crisis contagion phenomena and a mere dependence between markets. Lots of empirical works analysed the dependence between returns of financial assets on several markets as those described by Bandt and Hartmann (2002), Dungey *et al.* (2003), Pericoli and Sbracia (2003). Actually, two different approaches can be distinguished: modelling first moments of returns (Forbes and Rigobon, 2002), or estimating the probability of co-exceedance (Longin and Solnik, 2001). Cappiello *et al.* (2005) have developed an econometric framework to investigate the co-dependency structure between random variables.

After having recalled a precise definition, properties and empirical or intuitive interpretations of cross sectional volatility, we theoretically justify the interest to use market return cross-volatility in an international diversification framework. Then we analyse and interpret countries returns cross-volatilities quantiles co-movements during the main crisis and during a more normal behaviour of the market.

Key Words: Crisis, Contagion, Quantile Regression, Dynamic Quantile Model, Extreme Value, Panel Data, Cross-volatility, Higher Moments.
***JEL* Classification:** G11, C13, C14, C22, C32.

♣ We thank Thierry Chauveau and Michael Rockinger for their kind help and advices. The third author thanks the Europlace Institute of Finance for financial support. Preliminary draft: do not quote or diffuse without permission. The usual disclaimer applies.

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« Centile Regression Approaches for Crisis Analysis »

1. Introduction

Recent years showed several periods in which many markets seemed to move more coordinated than before. It has been widely documented that co-movements of prices in financial markets increase significantly during periods of stress, such as the Exchange Rate Mechanism crisis in 1992–93, the Mexican crisis in 1994, the financial crises in East Asia in 1997, in Russia in 1998, in Brazil in 1998, the burst of the internet bubble in march 2000, the junk bonds crisis in 2001, and presently the subprime crisis. Financial crisis contagion can be defined as a sharp increase of financial returns co-movements among several markets due to turbulence of one market. Upward changes in price co-movements or correlations have then been interpreted as evidence of a breakdown of current transmission mechanisms across financial markets, or contagion. During these events, stock markets have fallen jointly and raised concerns about the stability of the financial system and the effectiveness of portfolio diversification. Thus, analyzing joint market movements and crises periods is important for portfolio managers who diversify to minimize risks and also for policymakers who need to assess the functioning of the financial system.

The dynamics of stocks seen as a population are important for asset management and risk control, in particular for options books, or for example to hedge market neutral positions in long-short equity trading programs. Indeed, market neutrality is usually insured for “typical” days, but is destroyed in high cross-volatile days. The cross-volatility of returns, which has a very intuitive interpretation, is in these cases almost as important to monitor as the volatility itself.

We want to justify and investigate whether the cross-volatility can be used in international diversification especially during financial crisis contagion.

In this paper, we use the first cross-sectional moments of asset returns, and especially the cross-sectional dispersion mentioned by Lillo and Mantegna (2000), Solnik and Roulet (2000) to obtain additional information about the association of markets that is not provided by the correlation coefficient.

The cross-sectional dispersion measure can assess the existence of changing market association through time and can also be interpreted as a "herding measure". This is an important feature since flight to quality is one potential explanation for simultaneously falling markets and contagion. The first goal of this paper is to justify and illustrate the use of cross-sectional volatility of returns in an international framework.

After having recalled definitions and properties of cross-sectional measures, we determine how they have been or could be intuitively and empirically interpreted and decomposed (Section 2). Then using a simple theoretical international framework, we justify their use by portfolio and risk managers to reach and measure international diversification (Section 3). Then international cross-moments quantiles co-movements are analysed and interpreted during the main crises and during more normal behaviour of the market to investigate what additional information cross-moments co-movements can bring about international diversification and about flight to quality process during contagion (Section 4).

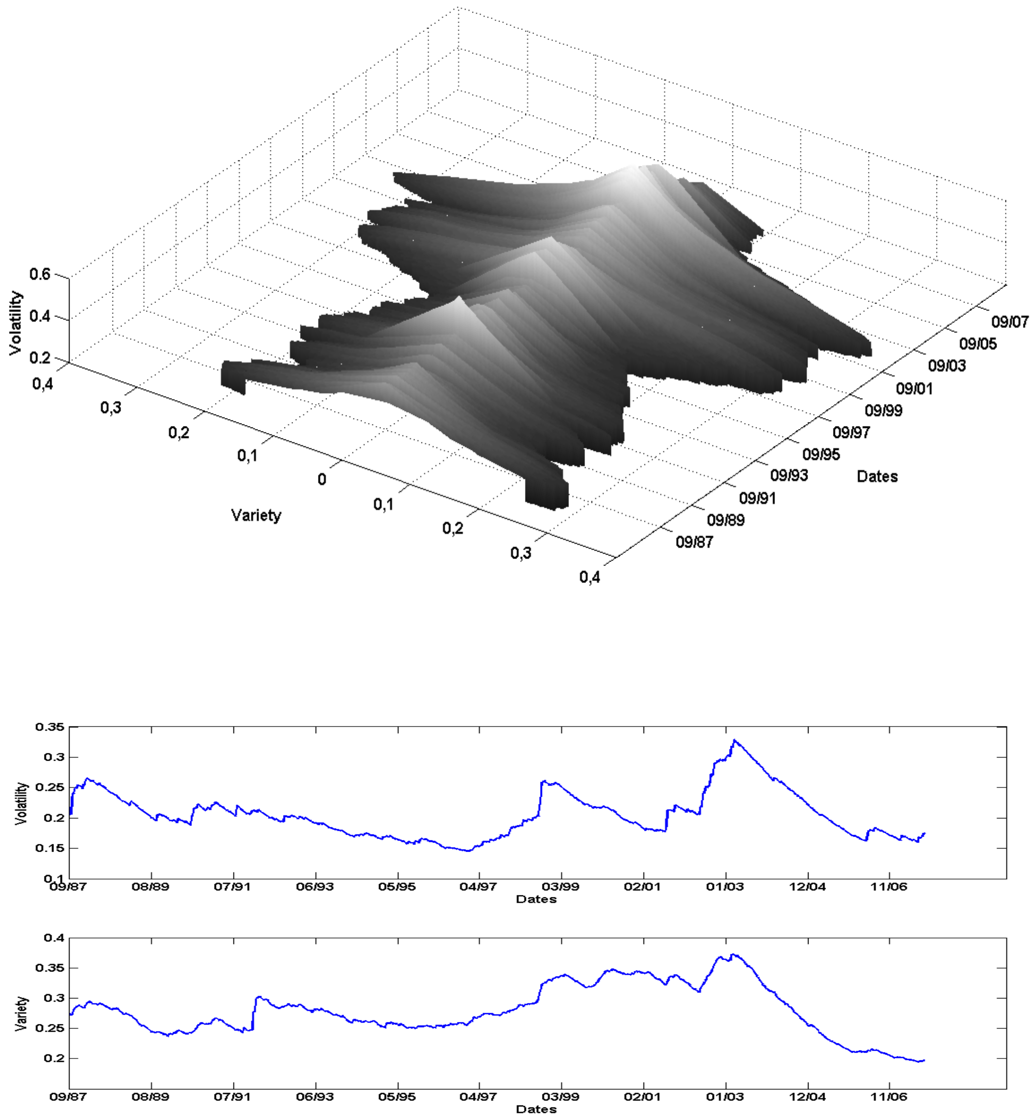
2. Definition, Relevant Literature and Problematic

While the literature on time-varying traditional volatility in stock returns is voluminous, the study of cross-sectional return dispersion is relatively limited. In this paper, we study the cross-sectional dispersion in daily stock returns. The cross-sectional dispersion was used by Ross (1989) to document a positive relationship between return dispersion and trading volume, and mentioned by Lillo and Mantegna (2000) and Solnik and Roulet (2000), through the concept of variety. The cross-sectional volatility (cross-volatility) is defined as the root mean square of the stock returns on a given day:

$$CV_t = \left[\frac{1}{N} \sum_{i=1}^{N} \left(R_{i,t} - R_{m,t} \right)^2 \right]^{1/2} \tag{1}$$

where N is the number of stocks in the portfolio, $R_{i,t}$ is the return of individual firm *i*, and $R_{m,t} = \frac{1}{N}\sum_{i=1}^{N} R_{i,t}$ is the market average.

Figure 1. Cross-volatility (or Variety) *versus* Conventional Volatility



Source: *Datastream*. CAC40 components daily data from September 1987 to September 2007. The chart on the top represents dates on the x-axis, the cross-sectional volatility (smoothed by the Risk Metrics formula) on the y-axis and the Conventional Volatility on the z-axis (also smoothed by the Risk Metrics formula). The chart in the midfield represents the Conventional Volatility whereas on the chart on the bottom, we plot the Variety. Computation by the authors.

The cross-volatility is not the volatility of the index. The volatility refers to the amplitude of the fluctuations of the index from one day to the next, not the dispersion across stocks. For instance, we may have a very volatile day, with a low cross-volatility.

In this definition, the market-level return is an equally-weighted portfolio return. Thus we have a cross-sectional volatility where firm returns are treated with equal weights. A reason for using equally-weighted portfolio returns is that this choice facilitates the statistical decomposition of the cross-sectional standard deviation (see Appendix 1).

One intuition associated with cross-volatility (and its statistical decomposition) is that volatility and cross-volatility are expected to be positively correlated: when the market makes big swings, stocks are expected to be all over the place. Bouchaud *et al.* (2001) obtain a theoretical relation between cross-volatility (variety) and market average return within the framework of the one-factor model. It suggests that the cross-volatility increases when the market volatility increases. Therefore, even if the idiosyncratic cross-volatility is constant, an increase in market volatility (measured by $R_{m,t}^{2}$) predicts an increase of the cross-volatility. The effect is enhanced by the fact that the idiosyncratic cross-volatility increases with market volatility. In its simplest version, the one-factor model assumes that the idiosyncratic part ε_i is independent from the market return. In this case the cross-volatility of idiosyncratic terms is constant in time and independent from the market return $R_{m,t}$. In contrast with these predictions, the empirical results show that a significant correlation between the idiosyncratic cross-volatility and the market average indeed exists. The degree of correlation is different for positive and negative values of the market return. The increase of cross-volatility in highly volatile periods is stronger than expected from the simplest one-factor model, although not as strong for negative (crashes) than for positive (rally) days.

Hwang and Satchell (2001) investigate the properties of cross-volatility, compare those with those of volatility and show that cross-volatility is highly linked to volatility and is more persistent -and hence more informative- than volatility. Very much like the volatility, the cross-volatility is correlated in time: there are long periods where the market volatility is high and where the market cross-volatility is high. The temporal correlation function of these two objects reveals a similar slow decay with time.

Analysing the three largest crashes that occurred at the NYSE in the period from January 1987 to December 1998, Bouchaud *et al.* (2001) observe two characteristics of the cross-volatility which are recurrent during the investigated crashes: the cross-volatility increases substantially starting from the crash day and remains at a level higher than typical for a period of time of the order of sixty trading days, the highest value of the cross-volatility is observed the trading day immediately after the crash.

Despite the lack of direct theoretical work on the issue, intuition and prior empirical work suggest several likely reasons to suspect that cross-volatility may contain supplementary information about the future traditional volatility of stock returns. Thus Hwang and Satchell (2005) introduce GARCHX framework to model the conditional volatility of daily individual stock returns by including a lagged firm-level cross-volatility term in the GARCH framework. To justify their approach they illustrate the fact that conditional volatility of more than 75% of individual stocks composing the FTSE350 and the S&P500 are better specified with the inclusion of cross-volatility measure in the conditional volatility equation.

First from a statistical perspective, Campbell *et al.* (2001) suggest that the time-variation in aggregate firm level volatility does not simply mirror the time-variation in market-level volatility. This implies that a cross-volatility measure should provide supplementary information about the future stock volatility. Another statistical possibility is that cross-volatility may capture information to help identify the unobservable volatility environment.

Moreover, Stivers (2003), finds a sizable and reliable positive relation between monthly dispersion in US large-firm returns and future market-level volatility from 1927 to 1995. He also shows that this relationship remains strong even after controlling for the effects of other economic information variables such as the risk free rate, default spread, government bond return volatility and some widely used common-factor proxies such as those used in Fama and French (1993).

Besides, cross-volatility may have economic interpretations that suggest a positive association with future traditional volatility. For example, one possibility is that cross-volatility reflects firm level information flows which cluster in time. If so, then cross-volatility might contain supplementary information about future volatility including market-level volatility if the future firm information flows are cross-sectionally correlated. Bekaert and Harvey (1997) study cross-country variations in market-level stock volatility and find that a higher return dispersion is associated with higher market volatility for countries where the market capitalization to the GDP ratio is relatively high (typical in more developed markets). They suggest that dispersion may reflect the magnitude of firm-level information flows for these countries.

Another possible economic interpretation is that cross-volatility may be positively associated with the underlying economic uncertainty and/or dispersion in beliefs about the market signal or market state. Connolly and Stivers (2006) find a sizable positive correlation both between the unexpected components of daily cross-volatility and stock turnover. They also find that cross-volatility tends to be relatively low on days when macroeconomic news is announced. If higher turnover is associated with more diverse beliefs and higher uncertainty and macroeconomic news releases tend to resolve uncertainty, then their findings relating cross-volatility to turnover and macroeconomic news provide some support to this economic interpretation.

Low cross-volatility means that it is hard to diversify, because all stocks behave the same way. Correlations are actually less effective than expected using a one-factor model in high volatile periods: the unexpected increase of cross-volatility gives then an additional opportunity for diversification. Other more subtle indicators of correlations, like the exceedance correlation function, actually confirm that the commonly reported increase of correlations during highly volatile periods might only reflect the inadequacy of the indicators that are used to measure them.

Cross-volatility may also have economic interpretations that suggest a positive association with future traditional volatility. For example, return dispersion may reflect firm-level information flows which are persistent over time. If so, then cross-volatility might contain information about future volatility, including index volatility if the firms information flows tends to be cross correlated.

Cross-volatility and returns dispersion measures have also been used in dynamic volatility decomposition of individual asset returns (see Connor *et al.*, 2006) or determining herding behaviour in different markets using low and high frequency data (see Christie and Huang, 1995; Chang *et al.*, 2000, Gleason *et al.*, 2004; Henker *et al.* (2006). Morck *et al.* have also tried to explain the difference in stock price synchronicity measured by returns dispersion between emerging and developed market and Bessembinder *et al.* (1996) determine that that cross-volatility can be considered as a determinant of aggregate trading volume. The cross-volatility is applied to explain global correlation level (Solnik and Roulet, 2000) and correlation asymmetry under different market conditions (Demirer *et al.*, 2004). Others use firm-specific volatility in the predictive regressions of market returns (Goyal and Santa-Clara, 2003; Bali *et al.*, 2005, Jian and Lee, 2006…).

The dynamics of stocks seen as a population are important for risk control, in particular for options books, and for long-short equity trading programs. Cross-volatility is in these cases almost as important to monitor as the volatility. Since this quantity has a very intuitive interpretation and an unambiguous definition, this could become a liquid financial instrument which may be used to hedge market neutral positions. Indeed, market neutrality is usually insured for "typical" days, but is destroyed in high cross-volatile days. Buying the cross-volatility would in this case reduce the risk of these approximate market neutral portfolios.

In this paper, we want to justify whether the cross-volatility can be used in international diversification. In the following section, we express the world return volatility in cross-volatility terms using a simple theoretical international model.

3. Decomposition: Cross-volatility in an International Framework

In an international framework, firm-level dispersion about the world market return can be decomposed into two parts. On the one hand, the dispersion of firm returns about disaggregate portfolios, where the portfolios are formed by grouping firms based on location characteristics. On the other hand, the dispersion of the disaggregate portfolio returns about the world market return. We want to decompose our primary world (market) volatility into country-level cross-volatility and the remaining firm idiosyncratic cross-volatility. Dispersion across country returns might provide better information about the volatility of the underlying common factors than does our broad market firm volatility. If so, and if the volatility information in cross-volatility is attributed to the persistent volatility of multiple common factors, then a country-level dispersion might subsume the information in our primary market cross-volatility.

To begin, we decompose the return on a "typical" stock into three components: the market world wide return, a country-specific residual and a firm-specific residual. Based on this return decomposition, we will be able to construct time-series of volatility measures of the three components for a typical firm, then we will express them about cross-volatilities.

Countries are denoted by a *c* and individual firms are indexed by *i*. The simple excess return[1] of firm *i* that belongs to country *c* in period *t* is denoted R_{ict}. Let w_{ict} be the weight of firm *i* in the country *c*. This methodology is valid for any arbitrary weighting scheme provided that we compute market return using the same weights; in further empirical application, we use country equal weights. The excess return of country *c* in period *t* is given by $R_{ct} = \sum_{i \in c} w_{ict} R_{ict}$. Countries are aggregated correspondingly. The weight of country *c* in the total world market is denoted by w_{ct}, and the excess market return is $R_{mt} = \sum_{c} w_{ct} R_{ct}$.

The next step is the decomposition of firm and country returns into the three components. We first write down a decomposition based on the CAPM and we then modify it to get an expression of the World total Market Risk (world returns volatility) based on firms and countries cross-volatilities.

The CAPM implies that we can set intercepts to zero in the following equations:

$$\begin{cases} R_{ct} = \beta_{cm} R_{mt} + \varepsilon_{ct} \\ R_{ict} = \beta_{ic} \beta_{cm} R_{mt} + \beta_{ic} \varepsilon_{ct} + \eta_{ict} \end{cases} \tag{2}$$

where β_{cm} denotes the beta for country *c* with respect to the world market return R_{mt}, ε_{ct} is the country-specific residual, β_{ic} is the beta of firm *i* in country *c* with respect to its country return, and η_{ict} is the firm-specific residual. The first equation expresses the country returns and the second equation the individual firm returns.[2]

By construction, the firm-specific residual η_{ict} is orthogonal to the country return R_{ct}, we assume that it is also orthogonal to the components R_{mt} and ε_{ct}. Thus, the beta of firm *i* with respect to the world market return, denoted β_{im}, satisfied:

$$\beta_{im} = \beta_{ic} \beta_{cm} \tag{3}$$

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[1] This excess return can be measured over Treasury bill rate.

[2] We could work with the market model, not imposing the mean restrictions of the CAPM and allow free intercepts α_c and α_{ic} in the first and second equations. However we prefer to provide a framework which avoids estimating firm specific parameters, despite the well-known empirical deficiencies of the CAPM, assuming that the zero-intercept restriction is reasonable in this context.

and the weighted sums of the different betas equal unity, we have:

$$\begin{cases} \sum_{c} w_{ct} \beta_{cm} = 1 \\ \sum_{i \in c} w_{ict} \beta_{ic} = 1 \end{cases} \tag{4}$$

The CAPM decomposition (see equation 2) guarantees that the different components of firm's return are orthogonal to one another. Hence it gives the following variance decomposition in which all covariance terms are zero:

$$\begin{cases} V(R_{ct}) = \beta_{cm}^{2} V(R_{mt}) + V(\varepsilon_{ct}) \\ V(R_{ict}) = \beta_{im}^{2} V(R_{mt}) + \beta_{ic}^{2} V(\varepsilon_{ct}) + V(\eta_{ict}) \end{cases} \tag{5}$$

The problem with this decomposition is that it requires knowledge of firm-specific betas that are difficult to estimate and may well be unstable over time. Therefore we work temporally with a simplified model that doesn't require any information about betas. We show that this model permits a variance decomposition similar to equations (5) on an appropriate aggregate level.

First, consider the following simplified country return decomposition that drops the country beta coefficient β_{cm} from equation (2):

$$R_{ct} = R_{mt} + e_{ct} \tag{6}$$

Equation (6) defines e_{ct} as the difference between the country return R_{ct} and the market return R_{mt}.[3] Comparing equations (2) and (6), we get:

$$e_{ct} = \varepsilon_{ct} + (\beta_{cm} - 1) R_{mt} \tag{7}$$

The market adjusted return residual e_{ct} equals the CAPM residual of equation (4) only if the country beta β_{cm} equals one or the world market return R_{mt} is null.

[3] Campbell *et al.* (2001) refer to equation (6) as a "market-adjusted-return model" in contrast to the market model of equation (2).

The apparent drawback of the decomposition (6) is that the world return market, R_{mt}, and the market adjusted return residual, e_{ct}, are not orthogonal, and so one cannot ignore the covariance between them. Computing the variance of the country return yields:

$$V(R_{ct}) = V(R_{mt}) + V(e_{ct}) + 2(\beta_{cm} - 1)V(R_{mt}) \tag{8}$$

Taking account of the covariance term once again introduces the country beta into the variance decomposition. Note, however, that although the variance of an individual country return contains covariance terms, the weighted average of variances across countries is free of the individual covariances:

$$\sum_c w_{ct} V(R_{ct}) = V(R_{mt}) + \sum_c w_{ct} V(e_{ct}) = \sigma_{mt}^2 + \sigma_{et}^2 \tag{9}$$

where $\sigma_{mt}^2 \equiv V(R_{mt})$ and $\sigma_{et}^2 \equiv \sum_c w_{ct} V(e_{ct})$.

The terms involving betas aggregate out because from equation (4) ($\sum_c w_{ct}\beta_{cm} = 1$). Therefore we can use the residual e_{ct} in equation (6) to construct a measure of average country-level volatility that does not require any estimation of betas. The weighted average $\sum_c w_{ct} V(R_{ct})$ can be interpreted as the expected volatility of a randomly drawn country (with the probability of drawing country *c* equal to its weight w_{ct}).

We can proceed in the same fashion for individual firm returns. Consider a firm return decomposition that drops β_{ic} from equation (2):

$$R_{ict} = R_{ct} + \mu_{ict} \tag{10}$$

where μ_{ict} is defined as:

$$\mu_{ict} = \eta_{ict} + (\beta_{ic} - 1) R_{ct} \tag{11}$$

The variance of the firm return is:

$$V\left(R_{ict}\right)=V\left(R_{ct}\right)+V\left(\mu_{ict}\right)+2\left(\beta_{ic}-1\right)V\left(R_{ct}\right) \tag{12}$$

The weighted average of firm variances in country *c* is therefore:

$$\sum_{i\in c} w_{ict}\, V\left(R_{ict}\right)=V\left(R_{ct}\right)+\sigma_{\mu ct}^{2} \tag{13}$$

where $\sigma_{\mu ct}^{2} \equiv \sum_{i\in c} w_{ict}\, V\left(\mu_{ict}\right)$ is the weighted average of firm-level volatilities in the country *c*. Computing the weighted average across countries, using equation (9), yields again a beta-free variance decomposition:

$$V\left(R_{mt}\right)=\sum_{c} w_{ct} \sum_{i\in c} w_{ict}\, V\left(R_{ict}\right)-\sigma_{et}^{2}-\sigma_{\mu t}^{2} \tag{14}$$

where $\sigma_{\mu t}^{2} \equiv \sum_{c} w_{ct}\, \sigma_{\mu ct}^{2} = \sum_{c} w_{ct} \sum_{i\in c} w_{ict}\, V\left(\mu_{ict}\right)$ is the weighted average of firm-level volatility across all firms. As in the case of country returns, the simplified decomposition of firm returns (10) yields a measure of average firm-level volatility that does not require estimation of betas.

We can gain further insight into the relation between the volatility decomposition based on the CAPM if we aggregate the latter (equation (5)) across countries and firms. We find that:

$$V\left(R_{mt}\right)=\sum_{c} w_{ct} \sum_{i\in c} w_{ict}\, V\left(R_{ict}\right)-\sigma_{et}^{2}-\sigma_{\mu t}^{2} \tag{15}$$

with:

$$\begin{cases} \sigma_{et}^{2} = \sum_{c} w_{ct} V\left(\varepsilon_{ct}\right)+V\left(R_{mt}\right)\sum_{c} w_{ct}\left(\beta_{cm}-1\right)^{2} \\ \sigma_{\mu t}^{2} = \sum_{c} w_{ct} \sum_{i\in c} w_{ict}\, V\left(\eta_{ict}\right)+V\left(R_{mt}\right)\sum_{c} w_{ct} \sum_{i} w_{ict}\left(\beta_{im}-1\right)^{2}+\sum_{c} w_{ct}\, V\left(\varepsilon_{ct}\right)\sum_{c} w_{ct} \sum_{i} w_{ict}\left(\beta_{ic}-1\right)^{2} \\ +\sum_{c} w_{ct}\, V\left(\varepsilon_{ct}\right)\sum_{c} w_{ct} \sum_{i} w_{ict}\left(\beta_{ic}-1\right)^{2} \end{cases} \tag{16}$$

where $\sum_{c} w_{ct}\, V\left(\varepsilon_{ct}\right)$ is the average variance of the CAPM country shock e_t, $\sum_{c} w_{ct}\left(\beta_{cm}-1\right)^{2}$ is the cross-sectional variance of countries betas across countries,

$\sum_c w_{ct} \sum_i w_{ict} \left(\beta_{im} - 1\right)^2$ is the cross-sectional variance of firm betas on the market across all firms in all countries, and $\sum_c w_{ct} \sum_i w_{ict} \left(\beta_{ic} - 1\right)^2$ is the cross sectional variance of firm betas on country shocks across all firms in all countries.

Equations (16) highlight that cross-sectional variation in betas can produce common movements in the variance components even if the CAPM variance components corresponding to the country-specific residual and the firm specific residual do not move with the world market return variance.

Using the same assumptions, we express now the world market return (equation (14)) about cross- volatilities. To do so, we substitute betas using the CAPM assumptions and equation (2). We get:

$$V\left(R_{mt}\right) = \left[\psi - \gamma\right] \mathrm{M}\left(\varphi\right) \tag{17}$$

with:

$$\begin{cases}
\psi \equiv \left\{ \sum_c w_{ct} \sum_{i \in c} w_{ict} V\left(R_{ict}\right) - w_{ct} \sum_c V\left(\varepsilon_{ct}\right) - \sum_c w_{ct} \sum_{i \in c} w_{ict} V\left(\eta_{ict}\right) \right\} \\
\gamma \equiv \sum_c w_{ct} V\left(\varepsilon_{ct}\right) \dfrac{\sum_c w_{ct} \left[\sum_i w_{ict} \left(R_{ict} - R_{ct}\right)^2 + \sum_i w_{ict} \left(\eta_{ict} - 0\right)^2 \right]}{\sum_c w_{ct} \left(R_{ct}\right)^2} \\
\mathrm{M}\left(\varphi\right) = \left[\dfrac{\left(R_{mt}\right)^2}{\left(R_{mt}\right)^2 + \left\{ \sum_c w_{ct} \left[\left(R_{ct} - R_{mt}\right)^2 \right] + \sum_c w_{ct} \left[\left(\varepsilon_{ct} - 0\right)^2 \right] \right\} + \varphi} \right]^2 \\
\varphi \equiv \left(\sum_c w_{ct} \sum_i w_{ict} \left(R_{ict} - R_{mt}\right)^2 + \theta \right) \\
\theta \equiv \left(\dfrac{\sum_c w_{ct} \sum_i w_{ict} \left(\eta_{ict}\, \varepsilon_{ct}\right)^2}{\sum_c w_{ct} \sum_i w_{ict} \left(R_{ct}\right)^2} \right) - 2 \dfrac{\sum_c w_{ct} \sum_i w_{ict} \left(\eta_{ict}^{\ 2}\, \varepsilon_{ct}\right)}{\sum_c w_{ct} \sum_i w_{ict} \left(R_{ct}\right)} + \sum_c w_{ct} \sum_i w_{ict}\, \eta_{ict}
\end{cases}$$

Thus the aggregate world market return variance can be expressed as the average of disaggregated firms variances less the averages variances of firm-specific and country-specific residuals and terms about cross-volatilities. Cross-volatilities are expressed through the average of firms cross-volatility.

Equation (17) shows that in this international framework, the world return variance is the average of countries variances and a term about firms, countries, and idiosyncratic cross-volatilities.

This theoretical decomposition of world market volatility allows us to conclude that cross-volatilities are linked with the correlation structure between markets returns.

4. First Empirical Evidences

In the previous section we have linked international cross-volatilities and the correlation structure between markets returns.

Earlier empirical work has analysed the dependence between returns of financial assets on several markets. See Bandt and Hartmann (2002), Dungey *et al*. (2003), Pericoli and Sbracia (2003). Actually, two different approaches can be distinguished: modelling first moments of returns (Forbes and Rigobon, 2002), or estimating the probability of co-exceedance (Longin and Solnik, 2001). Cappiello *et al.* (2005) have developed an econometric framework to investigate the co-dependency structure between random variables.

We analyse international cross-moments quantiles co-movements during the main crisis and during more normal behaviour of the market. It is an alternative way to study the evolution of correlation structures between market returns (see Section 3). The commonly reported increase of correlations during highly volatile periods could reflect the inadequacy of the indicators that are used to measure them. Thus, we investigate whether cross moments co-movements can bring additional information about international diversification or about flight-to-quality process during contagion.

The sample period used in our study consists in 16 years of daily data of the constituents of the NASDAQ100, the DJSTOXX 50, the NIKKEI225 and the FTSE100, from 28 December 1991 to 21 September 2007. This period delivers 3,933 returns for each component of these markets. Cross-volatility and cross-mean are estimated at each date using equation (1).

Volatility and cross-volatility are positively correlated (see Table 1), which is in line with the classical volatility decomposition. Analysing correlations between cross-means and cross-volatilities on European, Japanese, and UK markets, we observe that cross-volatility is, on average, larger when the amplitude of the market return is larger (Table 1). The cross-volatility is serially correlated (like the volatility), thus there are long periods where the markets cross-volatilities are high (see figures 2, 3, 5 and 7), cross-volatility seem to be persistent as volatility.

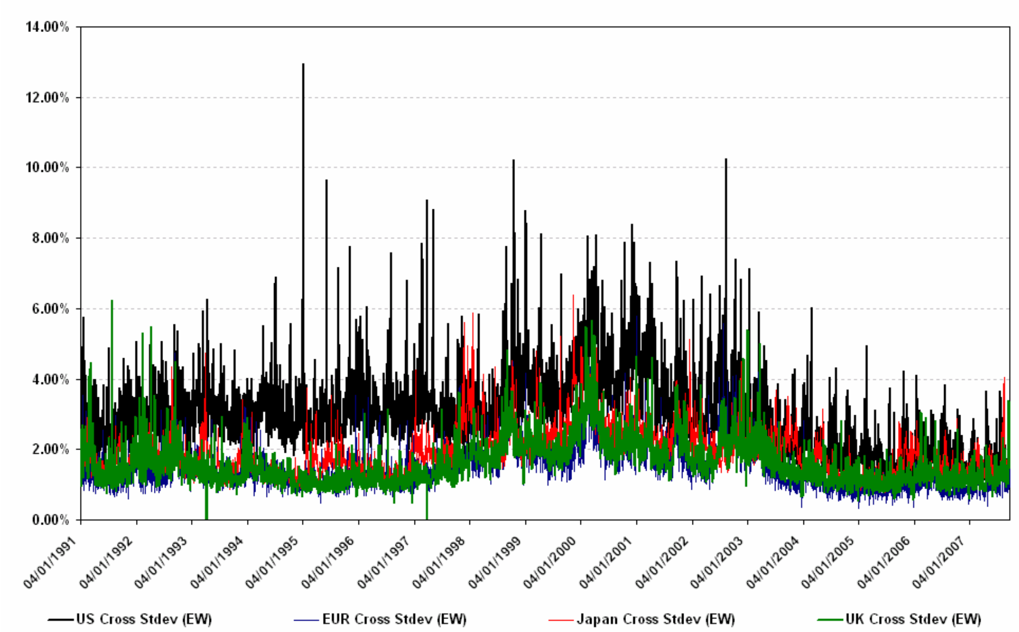
Analysing the seven largest crashes from 1991 to 2007, we observe that on each market the cross-volatility increases substantially starting from the crash day and remains at a high level for months, the highest value of the cross-volatility is often observed the trading day immediately after the crash. But the four equities market returns cross-volatility quantile co-movement analysis (see figure 10, 11 and Table 2) highlight that the extreme cross-volatilities through these four main equity markets are most of the time not exactly synchronised. This fact is confirmed by the weaker centile correlation of cross-volatility over European, Japanese, and UK stock markets than for more central quantiles. The cross-volatility may thus be considered as an international indicator of crashes. Actually, market extreme cross-volatility quantiles should maybe be used as a diversification leading indicator.

Table 1. Correlations between Cross-Volatility and the Absolute Value of Countries Returns

US	Europe	Japan	UK
0.45	0.45	0.37	0.39

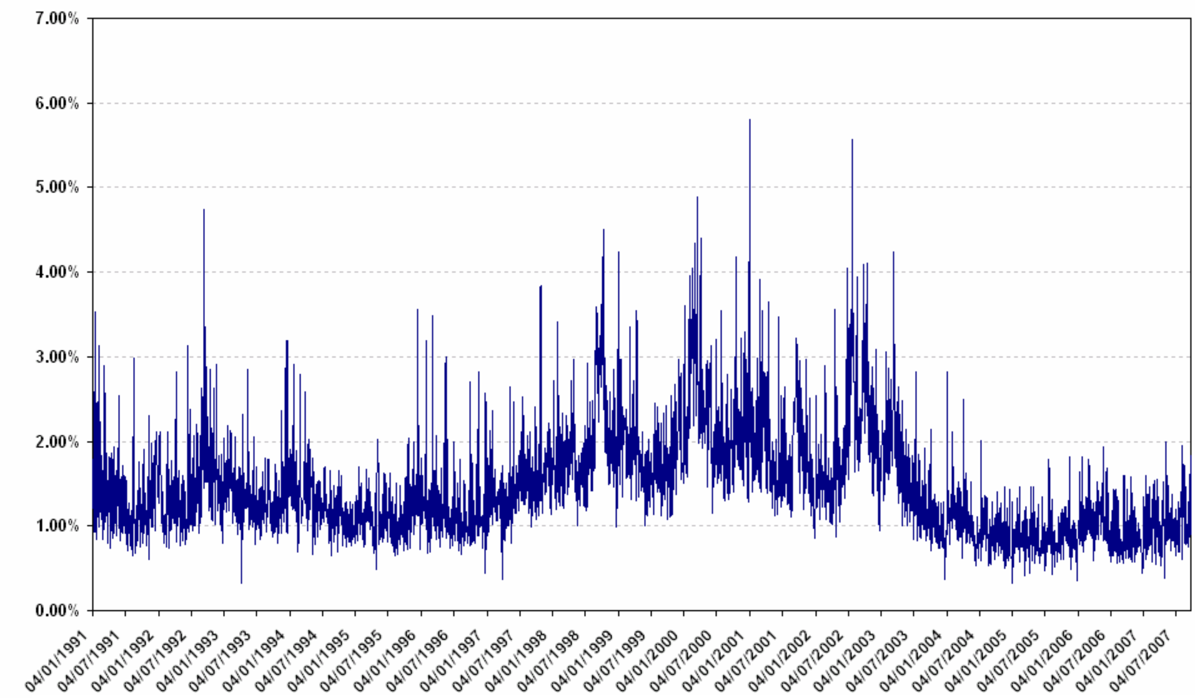
Source: *Datastream*, daily data, Index underlying prices (NASDAQ100, DJSTOXX 50, NIKKEI225, FTSE 100) from 28/12/1991 to 21/09/07 ; computation by the authors.

Figure 2. Cross-volatility (EW)



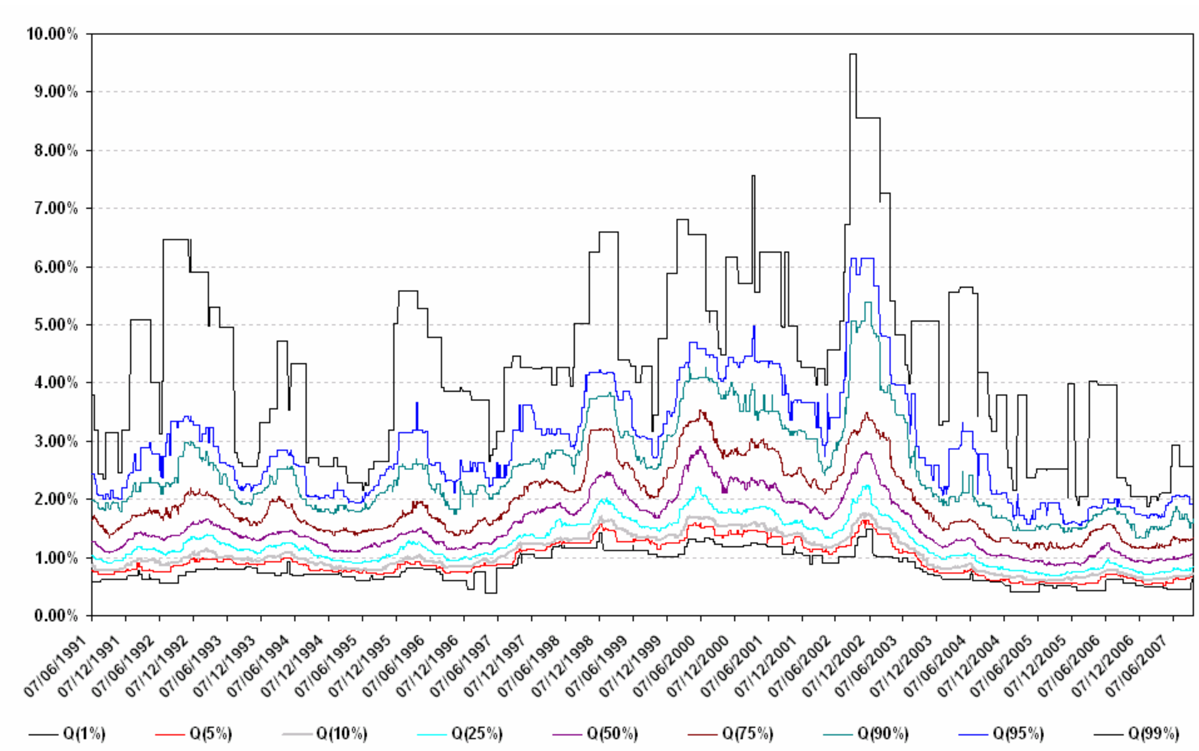
Source: *Datastream*, daily data, Index underlying prices (NASDAQ100, DJSTOXX 50, NIKKEI225, FTSE 100) from 28/12/1991 to 21/09/07 ; computation by the authors.

Figure 3. Europe Cross-Volatility (EW)



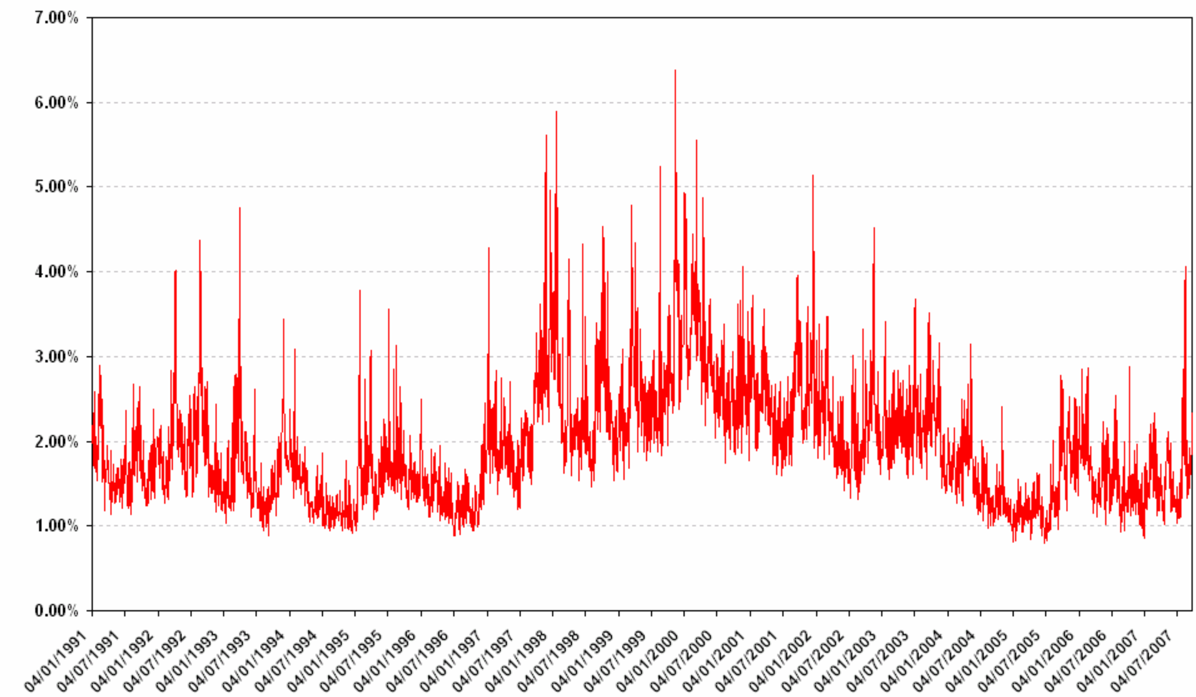
Source: *Datastream*, daily data, Index underlying prices (NASDAQ100, DJSTOXX 50, NIKKEI225, FTSE 100) from 28/12/1991 to 21/09/07 ; computation by the authors.

Figure 4. Europe Cross-volatility Quantiles (EW)



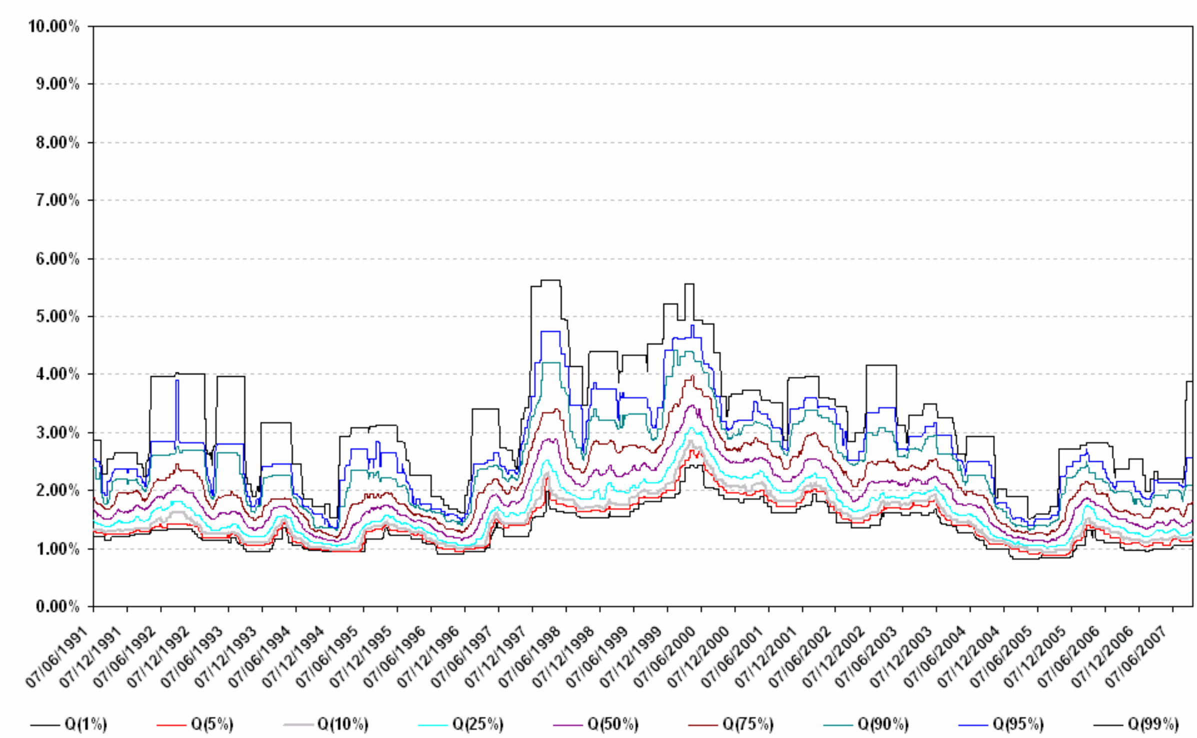
Source: *Datastream*, daily data, Index underlying prices (NASDAQ100, DJSTOXX 50, NIKKEI225, FTSE 100) from 28/12/1991 to 21/09/07 ; computation by the authors.

<u>Figure 5.</u> Japan Cross-Volatility (EW)



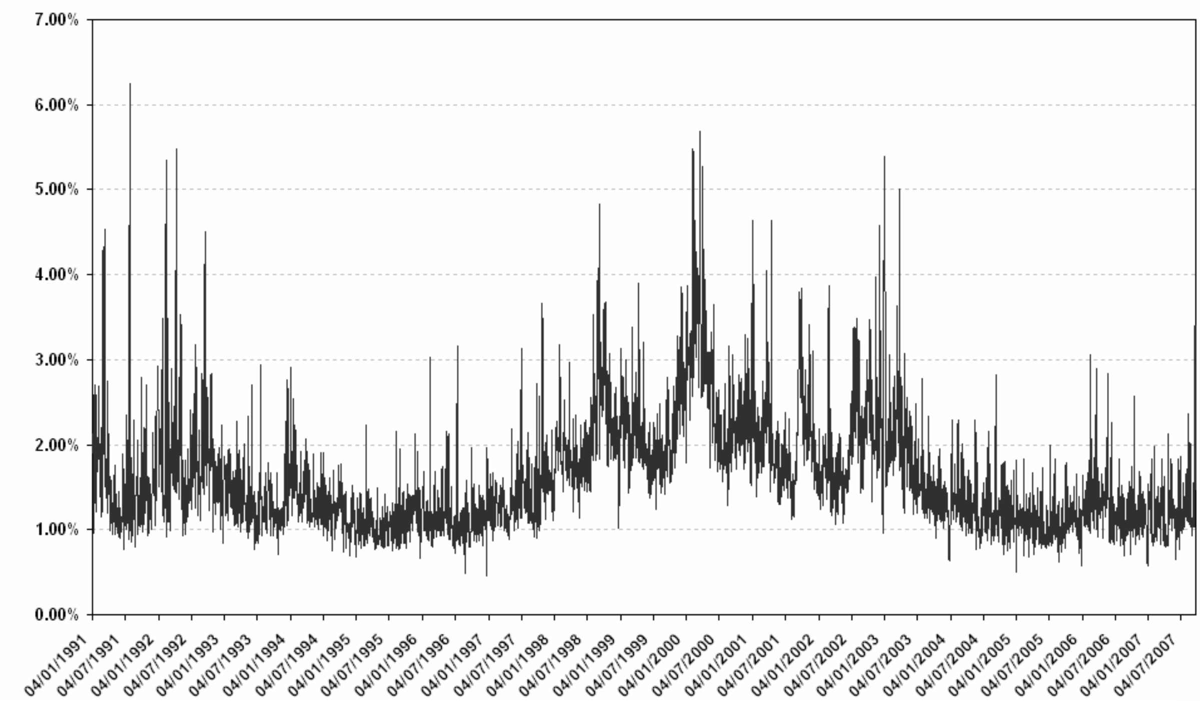
Source: *Datastream*, daily data, Index underlying prices (NASDAQ100, DJSTOXX 50, NIKKEI225, FTSE 100) from 28/12/1991 to 21/09/07 ; computation by the authors.

<u>Figure 6.</u> Japan Cross-Volatility Quantiles (EW)



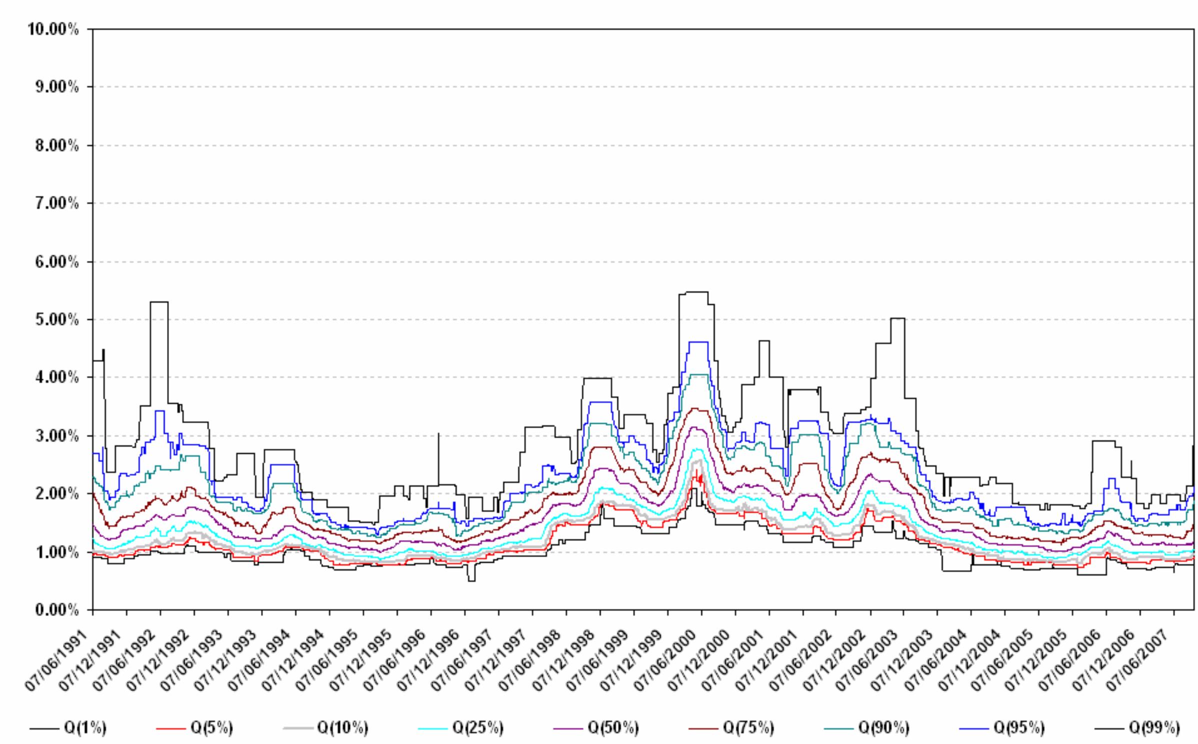
Source: *Datastream*, daily data, Index underlying prices (NASDAQ100, DJSTOXX 50, NIKKEI225, FTSE 100) from 28/12/1991 to 21/09/07 ; computation by the authors.

Figure 7. UK Cross-Volatility (EW)



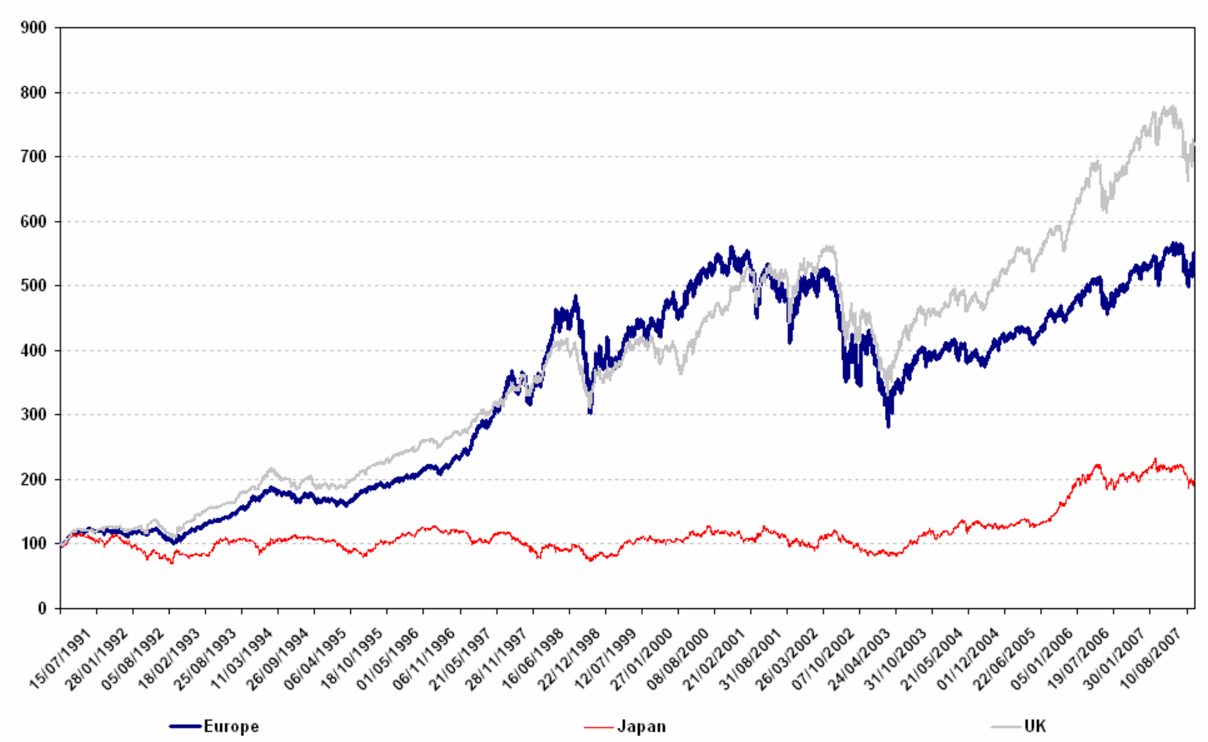
Source: *Datastream*, daily data, Index underlying prices (NASDAQ100, DJSTOXX 50, NIKKEI225, FTSE 100) from 28/12/1991 to 21/09/07 ; computation by the authors.

Figure 8. UK Cross-Volatility Quantiles (EW)



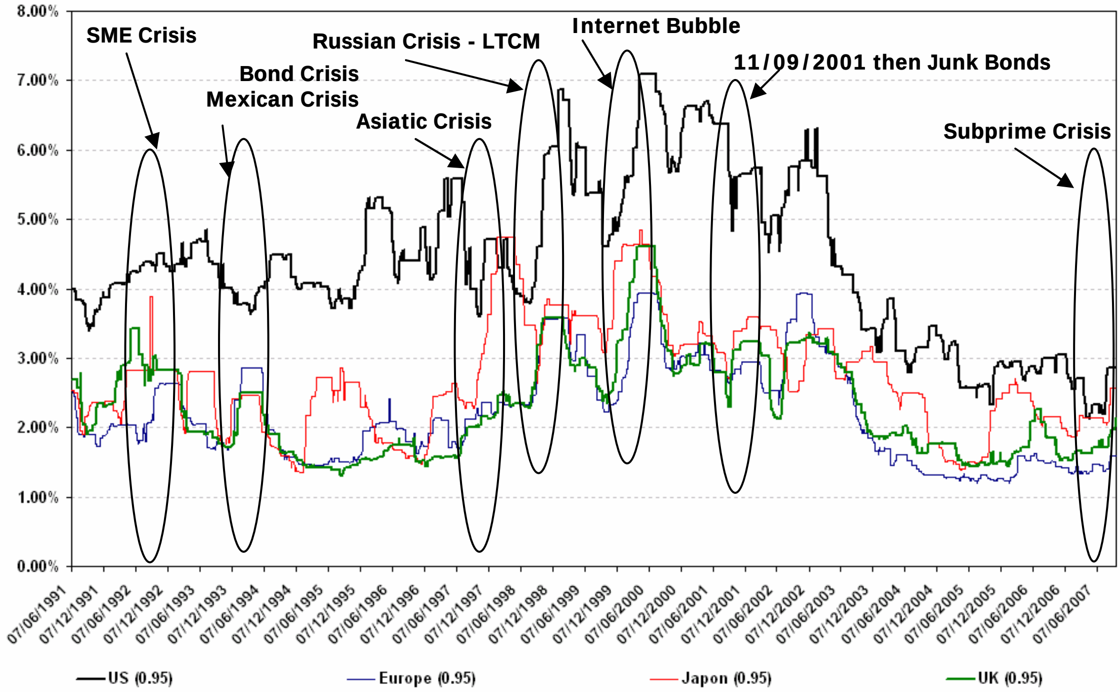
Source: *Datastream*, daily data, Index underlying prices (NASDAQ100, DJSTOXX 50, NIKKEI225, FTSE 100) from 28/12/1991 to 21/09/07 ; computation by the authors.

Figure 9. Market Average Prices



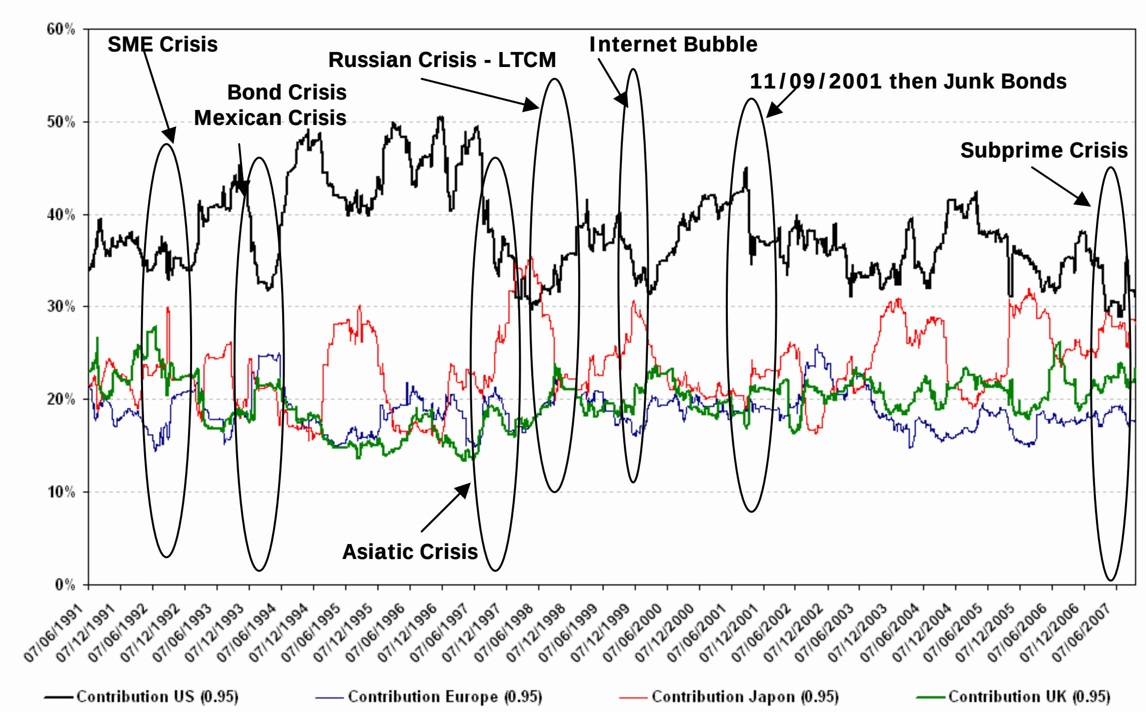
Source: *Datastream*, daily data, Index underlying prices (NASDAQ100, DJSTOXX 50, NIKKEI225, FTSE 100) from 28/12/1991 to 21/09/07 ; computation by the authors.

Figure 10. Cross-Volatility 0.95-quantile (EW)



Source: *Datastream*, daily data, Index underlying prices and traded volume (NASDAQ100, DJSTOXX 50, NIKKEI225, FTSE 100) from 12/28/1991 to 09/21/07 ; computation by the authors.

Figure 11. Contribution to the Cross-volatility 0.95-quantile (VW)



Source: *Datastream*, daily data, Index underlying prices and traded volume (NASDAQ100, DJSTOXX 50, NIKKEI225, FTSE 100) from 12/28/1991 to 09/21/07 ; computation by the authors.

Table 2. Correlations between Cross-Volatility Quantiles

	Europe / Japan	Europe / UK	Japan / UK
Q(1%)	0.78	0.88	0.87
Q(5%)	0.80	0.91	0.88
Q(10%)	0.80	0.91	0.87
Q(25%)	0.79	0.92	0.87
Q(50%)	0.81	0.93	0.87
Q(75%)	0.80	0.93	0.85
Q(90%)	0.68	0.87	0.79
Q(95%)	0.61	0.79	0.72
Q(99%)	0.48	0.62	0.64

Source: *Datastream*, daily data, Index underlying prices and traded volume (NASDAQ100, DJSTOXX 50, NIKKEI225, FTSE 100) from 12/28/1991 to 09/21/07 ; computation by the authors.

5. Preliminary Conclusion

Cross-volatility appears to be almost as important to monitor as the volatility. In this paper, using a simple theoretical international framework, we justify the use of cross-volatility of returns in an international framework to increase the effectiveness of portfolio international diversification. Indeed, we show that international cross-volatilities and the correlation structure between markets returns are linked.

We also provide a descriptive analysis of cross-volatilities behaviour during “typical” days and crisis on the main equity market (US, Europe, Japan, and UK) from 1991 to 2007.

As shown by the classical volatility statistical decomposition, we find a positive correlation between volatility and cross-volatility through these four equity markets. Analysing co-movements between cross-means and cross-volatilities on these equity markets, we observe that cross-volatility is, on average, larger when the amplitude of the market return is larger. The cross-volatility is serially correlated (like the volatility), thus there are long periods where the market volatility is high and where the market cross-volatility is high.

Cross-volatility can be considered as an indicator of crashes. Indeed, analysing the seven largest crashes from 1991 to 2007, we observe that on each market the cross-volatility increases substantially starting from the crash day and remains at a higher level for months, the highest value of the cross-volatility is often observed the trading day immediately after the crash. But the four equity market returns, cross-volatility quantile co-movement analysis highlights that the extreme cross-volatilities through these four main equity markets are, most of the time, not exactly synchronised. Markets extreme cross-volatility quantiles may be used as a diversification leading indicator.

In an advanced version of this paper, we will complete our analysis with Asian and South American data, test empirically the theoretical relation of Section 3, and try to propose an international asset management program based on it.

Since cross-volatility has a very intuitive interpretation and an unambiguous definition, this could also become a liquid financial instrument which may be used for example to hedge market neutral positions or to lead international diversification.

References

Bae K., G. Karolyi, and R. Stulz, (2003), « A New Approach to Measuring Financial Contagion », *Review of Financial Studies* 16, 717-763.

Bali T., N. Cakici, X. Yan, and Z. Zhang, (2005), « Does Idiosyncratic Risk Really Matter? », *Journal of Finance* 60, 905-929.

Bandt O. and P. Hartmann, (2002), « Systematic Risks in Banking »,in *Financial Crisis Contagion and the Lender of Last Resort, Oxford University Press*, 578 pages.

Bessembinder H., K. Chan and P. Seguin, (1996), « An Empirical Examination of Information, Differences of Opinion and Trading Activities », *Journal of Financial Economics* 40, 105-134.

Bouchaud J., F. Lillo, R. Mantegna and M. Potters, (2001), «Introducing Variety in Risk Management », *eprint arXiv:cond-mat* 0107208,12 pages.

Beirlant J., Y. Goegebeur, J. Sergers, and J. Teugels, (2004), *Statistics of Extremes*, *John Wiley,* 478 pages.

Buchinsky M. and J. Hahn, (1998), « An Alternative Estimator for the Censored Regression Model », *Econometrica* 66, 653-671.

Buchinsky M., (1998), « Recent Advances in Quantile Regression Models: A Practical Guideline for Empirical Research », *Journal of Human Resources* 33, 88-126.

Bryan M. and G. Cecchetti, (1999), « Inflation and the Distribution of Prices Changes », *Review of Economics and Statistics* 81, 188-196.

Campbell J., M. Lettau, B. Malkiel, and Y. Xu (2001), « Have Individual Stocks Become More Volatile? An Empirical Exploration of Idiosyncratic Risk », *Journal of Finance* 61, 88-126.

Cappiello L., B. Gérard, and S. Manganelli, (2005), « Measuring Comovements by Regression Quantiles », *ECB Working Paper 501*, 48 pages.

Chaudhuri P., K. Doksum, and A. Samarov, (1997), « On Average Derivative Quantile Regression », *The Annals of Statistics* 25 (2), 715-744.

Chang E., J. Cheng and A. Khorana, (2000), « An Examination of Herd Behavior in Equity Markets: An International Perspective », *Journal of Banking & Finance* 24, 1651-1679.

Chernozhukov V., (2005), « Extremal Quantile Regression », *Annals of Statistics* 33 (2), 806-839.

Christie W. and R. Huang, (1995), « Following the Pier Piper: Do Individual Returns Herd around the Market? », *Financial Analysts Journal* 51, 31-37.

Connolly R. and C. Stivers, (2006), « Information Content and other Characteristics of the Daily Cross-sectional Dispersion in Stock Returns », *Journal of Empirical Finance* 13, 79-112.

Connor G., R. Korajczyk and O. Linton, (2006), « The Common and Specific Components of Dynamic Volatility », *Journal of Econometrics* 132, 231-255.

Corsetti G., M. Pericolli, and M. Sbracia, (2005), « Some Contagion, Some Interdependence: More Pittfalls in tests of Financial Contagion », *Journal of International Money and Finance* 24, 1177-1199.

Demirer R. and D. Lien, (2004), « Firm-level Return Dispersion and Correlation Asymmetry: Challenges for Portfolio Diversification », *Applied Financial Economics* 14, 447-456.

Donald S. and H. Paarsch (1993), « Piecewise Pseudo-Maximum Likelihood Estimation in Empirical Models of Auctions », *International Economic Review* 34, 121-148.

Dungey D., R. Fry, B. Gonzalez-Hermosillo and V. Martin, (2004), « Empirical Modeling of Contagion: A Review of Methodologies », *IMF Working Papers 04/78*, 33 pages.

Embrechts P., C. Kluppelberg, and T. Mikosch, (2000), *Modelling Extremal Events*, *Springer-Verlag*, 645 pages.

Fama E.and K. French, (1993), « Common Risk Factors in the Returns on Stocks and Bonds », *Journal of Financial Economics* 33, 3-56.

Forbes K., and R. Rigobon, (2001), « Contagion in Latin America: Definitions, Measurement, and Policy Implications », *Economia* 1, 1-46.

Forbes K. and R. Rigobon, (2001), « Measuring Contagion: Conceptual and Empirical Issues », *in International Financial Contagion*, Claessens-Forbes (Eds), Kluwer Academic Publishers, 43-66.

Forbes K. and R. Rigobon, (2002), « No Contagion, only Interdependence: Measuring Stock Market Comovements », *Journal of Finance* 57, 2223-2261.

Gleason K., I. Mathur and M. Peterson, (2004), « Analysis of Intraday Herding Behavior among the Sector ETFs », *Journal of Empirical Finance* 11, 681-694.

Goyal A. and P. Santa-Clara, (2003), « Idiosyncratic Risk Matters!», *Journal of Finance* 63, 975-1007.

Gropp R. and G. Moerman, (2004), « Measurement of Contagion in Banks' Equity Prices », *Journal of International Money and Finance* 23, 405-459.

Gleason K., I. Mathur and M. Peterson, (2004), « Analysis of Intraday Herding Behavior among the Sector ETFs », *Journal of Empirical Finance* 11, 681-694.

Hartman P., S. Straetmans, and C. de Vries, (2004), « Asset Market Linkages in Crisis Period », R*eview of Economics and Statistics* 86, 405-459.

Henker J., T. Henker and A. Mitsios, (2006), « Do Investors Herd Intraday in Australian Equities? », *International Journal of Managerial Finance* 2, 196-219.

Hou K. and T. Moskowitz, (2005), « Market Frictions, Price Delay, and the Cross-Section of Expected Returns », *Review of Financial Studies* 18, 981-1020.

Hwang S. and S. Satchell, (2005), « GARCH Model with Cross-sectional Volatility; GARCHX Model », *Applied Financial Economics* 15, 203-216.

Jiang W., ans B.-S. Lee, (2006), « On the Dynamic Relation between Returns and Idiosyncratic Volatility », *Financial Management* 35, 43-65.

Judd K., (1999), *Numerical Methods in Economics*, MIT Press, 633 pages.

Karolyi G. and R. Stulz, (1996), « Why do Markets Move Together? An Investigation of U.S.-Japan Stock Return Comovements », *Journal of Finance* 51, 951-986.

Karolyi G., (2003), « Does International Financial Contagion Really Exist? », *International Finance* 6, 179-199.

King M., and S. Wadhwani, (1990), « Transmission of Volatility between Stock Markets», *Review of Financial Studies* 3, 5-33.

King M., E. Sentana, and S. Wadhwani, (1994), « Volatility and Links between National Stock Markets », *Econometrica* 46, 33-50.

Koenker R. and G. Basset, (1978), « Regression Quantiles », *Econometrica* 46, 33-50.

Koenker R. and O. Geiling, (2001), « Reappraising Medfly Longevity: A Quantile Regression Approach », *Journal of American Statistic Association* 96, 458-468.

Koenker R. and K. Hallock, (2001), « Quantile Regression », *Journal of Economic Perspectives* 15 (4), 143-156.

Koenker R. and J. Machado, (1999), « Goodness-of-Fit and Related Inferences Processes for Quantile Regression », *Journal of the American Statistical Association* 94, 1296-1310.

Koenker R. and S. Portnoy, (1997), « The Gaussian Hare and the Laplacian Tortoise: Computability of Squared-Error *versus* Absolute-Error Estimators », *Statistical Science* 12 (4), 279-296.

Kotz S. and S. Nadarajah, (2000), *Extreme Value Distribution*s, *Imperial College Press*, 187 pages.

Leadbetter M., R. Lindgren, and H. Rootzen, (1983), *Extremes and Related Properties of Random Sequences and Processes*, *Springer-Verlag*, 336 pages.

Lillo F. and R. Mantegna, (2000), « Variety and Volatility in Financial Markets », *Physical Review* E62, 6126-6134.

Longin F. and B. Solnik, (1995), « Is the Correlation in International Equity Return Constant: 1970-1990? », *Journal of International Money and Finance* 14, 3-26.

Longin F. and B. Solnik, (2001), « Extreme Correlation of International Equity Markets », *Journal of Finance* 56, 649-676.

Longin F., (1996), « The Asymptotic Distribution of Extreme Stock Market Returns », *Journal of Business* 69, 383-408.

Ma L. and R. Koenker, (2006), « Quantile Regression Methods for Recursive Structural Equation Models », *Journal of Econometrics* 134, 471–506.

Maio P., (2007), « What Does the Cross-section Tell about Itself? An Asset Pricing Model with Cross-sectional Moment », *Working Paper*, Bilkent University, 68 pages.

Martens M. and S.-H. Poon, (2001), « Returns Synchronization and Daily Correlation Dynamics between International Stock Markets », *Journal of Banking and Finance* 25, 1805-1827.

Mitton T. and K. Vorkink, (2007), « Equilibrium Under-diversification and the Preference for Skewness », *Review of Financial Studies* 20, 1255-1288.

Mukherjee K., (1999), « Asymptotics of Quantiles and Rank Scores in Non Linear Time Series », *Journal of Time Series Analysis* 20, 173-192.

Morck R., B. Yeung and W. Yu, (2000), « The Information Content of Stock Markets: Why do Emerging Market have Synchronous Stock Prices Movements? », *Journal of Financial Economics* 58, 215-260.

Pericoli M. and M. Sbracia, (2003), « A Primer on Financial Contagion », *Journal of Economic Surveys* 17, 571-608.

Rahman M., (2007), « The Information Content of Cross-sectional Volatility for Future Market Volatility: Evidence from Australian Equity Returns », *Frontiers in Finance and Economics* 4(1), 91-124.

Rigobon R., (2002), « Contagion: How to Measure It? », in *Preventing Currency Crises in Emerging Markets*, Edwards-Frankel (Eds), University Chicago Press, 269-334.

Ross S., (1989), « Information and Volatility: The No-arbitrage Approach to Timing and Resolution of Irrelevancy », *Journal of Finance* 44, 541-566.

Sander H. and S. Kleimeier, (2003), « Contagion and Causality: An Empirical Investigation of Four Asian Crisis Episodes », *Journal of International Financial Markets, Institutions and Money* 13, 171-186.

Solnik B. and J. Roulet, (2000), « Dispersion as Cross-sectional Correlation », *Financial Analysts Journal* 56(1), 54-61.

Stivers C., (2003), « Firm-level Return Dispersion and the Future Volatility of Aggregate Stock Market Returns », *Journal of Financial Markets* 6, 389-411.

Tsay R., (2002), *Analysis of Financial Time Series*, John Wiley and Sons, 443 pages.

Yuan K., (2005), « Asymmetric Price Movements and Borrowing Constraints: a Rational Expectations Equilibrium Model of Crises, Contagion and Confusion», *Journal of Finance* 60, 379-411.

Appendix 1. Statistical Decomposition between Volatility and Cross-volatility

A market M with *n* assets is set, at time *t* cross-volatility can be written as:

$$CV_t = \frac{1}{(n-1)} \times \sum_{i=1}^{n} \left(r_{i,t} - \frac{1}{n}\sum_{i=1}^{n} r_{i,t} \right)^2$$

$$CV_t = \left[\frac{1}{n}\sum_{i=1}^{n} \left(r_{i,t}{}^2 \right) \right] - \left[\frac{1}{n}\sum_{i=1}^{n} \left(r_{i,t} \right) \right]^2$$

Where *i* is the set of assets at time *t*.

$$\frac{1}{T}\sum_{t=1}^{T} CV_t = \frac{1}{T}\sum_{t=1}^{T} \left[\frac{1}{n}\sum_{i=1}^{n} \left(r_{i,t}{}^2 \right) \right] - \frac{1}{T}\sum_{t=1}^{T} \left\{ \left[\frac{1}{n}\sum_{i=1}^{n} \left(r_{i,t} \right) \right]^2 \right\}$$

with $R_{M,t} = \frac{1}{n}\sum_{i=1}^{n} r_{i,t}$.

Thus $R_{M,t}$ is the average return of the Market assets at time *t*.

$$E\left(R_{M,t}{}^2\right) = \frac{1}{T}\left\{ \sum_{t=1}^{T} \left[\frac{1}{n}\sum_{i=1}^{n} \left(r_{i,t}{}^2 \right) \right] - \sum_{t=1}^{T} CV_t \right\}$$

$$E\left(R_{M,t}{}^2\right) - E\left(R_{M,t}\right)^2 = \frac{1}{T}\left\{ \sum_{t=1}^{T} \left[\frac{1}{n}\sum_{i=1}^{n} \left(r_{i,t}{}^2 \right) \right] - \sum_{t=1}^{T} CV_t \right\} - E\left(R_{M,t}\right)^2$$

$$V\left(R_{M,t}\right) = -E\left[CV_t\right] + \frac{1}{n}\sum_{i=1}^{n} \left[E\left(r_{i,t}{}^2 \right) \right] - \left\{ \frac{1}{n}\sum_{i=1}^{n} \left[E\left(r_{i,t} \right) \right] \right\}^2$$

$$V\left(R_{M,t}\right) = \frac{1}{n}\sum_{i=1}^{n} \left[V\left(r_{i,t} \right) \right] + \frac{1}{(n-1)} \times \sum_{i=1}^{n} \left(E\left(r_{i,t}\right) - \frac{1}{n}\sum_{i=1}^{n} E\left(r_{i,t}\right) \right)^2 - E\left[CV_t\right]$$

$$E\left[CV_t\right] = \frac{(n-1)}{n^2}\sum_{i=1}^{n} V\left(r_{i,t}\right) + \frac{1}{(n-1)} \times \sum_{i=1}^{n} \left(E\left(r_{i,t}\right) - \frac{1}{n}\sum_{i=1}^{n} E\left(r_{i,t}\right) \right)^2 - \left(\frac{2}{n^2}\right) \sum_{i \in N} \sum_{\substack{j \in N \\ i \neq j}} \operatorname{cov}\left(R_{i,t}, R_{j,y}\right)$$

At time *t*, the traditional longitudinal Market return variance can be expressed as the average at time *t* of the assets variances through time augmented by the cross sectional variance at time *t* of the longitudinal averages of return, less the average cross sectional variance through time.