We show that in the presence of idiosyncratic risk, the public revelation of information about risky aggregate outcomes such as policy choices can have a welfare-reducing effect. By announcing information on non-insurable aggregate risk, the policy maker distorts households’ incentives for insurance of idiosyncratic risk and increases the riskiness of the optimal self-enforceable allocation. The negative effect of distorted insurance incentives can be quantitatively important for a monetary authority that reveals changes in its short-run inflation target. We characterise parameters for which the effect dominates conventional effects that favour releasing better information.

Nowadays central banks all over the world provide more information and release it to the public earlier than ever before in their history (Blinder et al., 2008; Bernanke, 2010; Plosser, 2012). Though not falling back to secrecy after the financial crisis of 2008, central banks have been more cautious when it comes to the release of information. We develop a novel argument in support for the cautiousness. In particular, we show that by providing better information on future aggregate risk (e.g. by announcing future policies or by revealing economic forecasts) policy makers may decrease social welfare by distorting private insurance incentives.

We consider risk-averse households that face idiosyncratic and aggregate income risk. The households voluntarily participate in insurance arrangements to reduce their consumption risk. Such arrangements are self-enforceable or are compatible with voluntary participation incentives if in any period following the realisation of idiosyncratic risk households choose not to walk away from the arrangement and live in autarky from that period on. The outside option is tempting for households with a high current income because the insurance arrangements prescribe transfers from these households to the others in the current period. The lack of commitment thus creates a tension for high-income households between higher current consumption and the future benefits of the insurance promised in the arrangements.

Information plays a crucial role in households’ trade-off between future insurance and current incentives. Here, we study disclosure polices by introducing a public signal through which the future aggregate state can be revealed. The signal is common to all agents and does not resolve households’ idiosyncratic risk. After the realisation of current-period idiosyncratic income and given the public signal on future aggregate risks, households decide to participate in the social insurance arrangement.

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As our main result, we show formally that less precise public information about the future aggregate state can be preferable to perfect public information when incentive constraints matter. Under the socially optimal insurance arrangement, the future benefits of the insurance relative to the outside option are reflected by the amount of the consumption good that the households with high income in the current period are willing to transfer. The key point is that households value the insurance arrangement conditionally not only on their idiosyncratic realisation but also on the public signal about the aggregate state. In particular, if the signal indicates that the future aggregate state with relatively large benefits of the arrangement is likely, then households are willing to give up a larger share of current-period consumption goods for these future benefits. Similarly, if the signal informs of a future aggregate state in which the gains of the risk-sharing agreement are relatively low, then households with a high current income are less willing to share their good fortune. The more informative the signal, the more dispersed and riskier \textit{ex ante} is the optimal consumption allocation of the high-income households to account for all realisations of the signal. Nonetheless, for high-income households with binding participation constraints, the expected utility before the signal realisation does not depend on signal precision. Therefore, when the signals are more informative, risk-averse high-income households are less willing to transfer goods to low-income households. Correspondingly, with smaller transfers and more dispersed consumption, low-income households are worse off under better information and from the \textit{ex ante} perspective, households prefer uninformative policy announcements.

Transparency and secrecy are of particular interest for central banks. Not only financial specialists but also the general public pay attention to their monetary policy announcements. As our main application, we thus develop a stochastic equilibrium model that integrates the risk-sharing mechanism into a monetary production economy in which households are subject to cash-in-advance constraints and face idiosyncratic employment opportunities. To insure against the idiosyncratic risk, households may engage in risk-sharing arrangements consistent with voluntary participation incentives. The monetary authority is assumed to pursue a stochastic inflation target. The authority knows the target in advance and may choose to release that information with a certain precision.\footnote{The stochastic inflation target can be interpreted as a shortcut for a situation in which the policy target varies in response to a fundamental shock over which the public authority is better informed than the private agents.} Our novel finding in this environment is that more precise announcements on monetary policy reduce social welfare. Furthermore, we show that the level of patience needed to sustain perfect risk sharing as the first best allocation is strictly increasing in the precision of the monetary policy announcements.

To evaluate the welfare-reducing effect of policy announcements, we allow for a conventional motive to provide better information about future inflation. We thus introduce a fraction of firms that set prices one period in advance. More precise announcements allow the sticky-price firms to preset their prices more accurately, thereby resulting in fewer price distortions and better allocation of resources. We calibrate the monetary production economy to match inflation, cross-sectional income and cross-sectional consumption characteristics of the US economy on an annual basis. We find that the negative effect of information on aggregate risk is quantitatively

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important. For degrees of relative risk aversion below five, the cost of information disclosure can account for up to 14% of the benefit from complete removal of aggregate fluctuations. Employing the evidence on the frequency of price adjustments reported by Bils and Klenow (2004), the negative effect of information dominates the positive effect for coefficients of relative risk aversion as low as 2.5. Furthermore, the recent increase in income inequality in the US, as documented by Krueger and Perri (2006) among others, amplifies the negative rather than the positive effect of public information.

In times of financial instability, financial markets are less efficient in channelling funds to those desiring them most, thereby boosting undesirable fluctuations in private consumption. According to our argument, central banks’ information policies should be cautious because frequent release of information can amplify the undesirable fluctuations leading to higher consumption inequality.

To the best of our knowledge, we are the first to shed light on the negative welfare effects of announcements on risks that are common to all agents under the realistic assumption that the idiosyncratic risk is not completely but only partially insurable. Our study builds a bridge between two distinct strands of literature: the literature describing global games that focuses on aggregate risk and the literature analysing efficient risk sharing that concentrates on the insurance of idiosyncratic risk.

In a global games framework, Morris and Shin (2002) show that when the coordination of agents is driven by strategic complementarities in their actions, better public information on aggregate risks may be undesirable in the presence of private information on these risks. This result is due to the inefficiently high weight that agents assign to public information relative to private information. Svensson (2006) and Woodford (2005) point out that the conditions for a welfare-decreasing effect of more precise public information are rather special and violated for economic models with standard preferences. Angeletos and Pavan (2007) clarify that coordination incentives in beauty-contest economies as considered by Morris and Shin (2002) are inefficiently strong. Public information thus reduces welfare in these economies only because coordination is socially undesirable. In contrast, we show that the social value of information can be negative even when individual preferences and social welfare coincide. Further, the main focus in global games is on aggregate risk, while idiosyncratic risk to fundamentals is either absent or is assumed to be completely insurable due to the existence of complete financial markets.

In the literature on efficient risk sharing, Hirshleifer (1971) is among the first to point out that perfect information makes risk-averse agents worse off ex ante if such information leads to evaporation of risks that otherwise could have been shared in a competitive equilibrium. Schlee (2001) provides the general conditions under which better public information about idiosyncratic risk is undesirable. Unlike Hirshleifer (1971) and his successors, we focus on the welfare effects of more precise signals on aggregate risk, not on idiosyncratic risk. This difference is substantial. In our model, there are aggregate states in which more precise signals lead to better risk sharing, which cannot happen in the case of signals on idiosyncratic risk. In these states, the value of the arrangement relative to the outside option is high; thus, better informed high-income households are willing to share more but the overall effect of information is negative.

The remainder of the article is organised as follows: in the next Section, we start with a simple two-period example to highlight the basic voluntary risk-sharing mechanism.
involved and we state our main result in that simple environment. In Section 2, we set up a model that integrates the mechanism into a monetary production economy with an infinite horizon and flexible prices. In Section 3, we state the main results for that application. In Section 4, we evaluate the importance of the distortions of risk-sharing possibilities caused by policy announcements. The last Section concludes.

1. Simplified Two-period Real Economy

We set up an illustrative example that captures the interaction between individual incentives for sharing idiosyncratic risk and the precision of public signals on aggregate risk. When participation in a risk-sharing arrangement is voluntary, we show that risk-averse agents prefer completely uninformative public signals over perfectly informative signals on the aggregate risk.

Consider a two-period, pure-exchange economy with a continuum of \textit{ex ante} identical agents and a single perishable consumption good. In each period, an agent obtains either a high endowment, \(y^h\), or a low endowment, \(y^l\), with equal probability and independently across time and agents. Furthermore, in the second period, the agents’ income is affected by taxes.\(^2\) To ease the exposition, we assume that the government can, with equal probability, either tax away all goods (type-\(b\) policy) or impose a tax of zero (type-\(g\) policy); furthermore, we assume that tax revenues are spent by the government and that this spending does not affect agents’ preferences.

The preferences of agents are given by the following expected utility function:

\[
E[u(c_1) + \beta u(c_2)],
\]

(1)

where \(c_1\) and \(c_2\) are consumption in the first and in the second period, respectively; \(\beta\) is the time discount factor and the period utility function, \(u(c)\), is increasing and strictly concave. We measure social welfare according to (1), that is, as agents’ expected utility before any risk has been resolved.\(^3\)

If the agents are able to commit before their endowments are realised in the first period, the optimal risk-sharing arrangement is perfect risk sharing. The commitment requirement is crucial. After observing current endowments, an agent with a high income may have an incentive to deviate from the perfect risk-sharing agreement, thus making such an agreement unsustainable.

To capture this rational incentive, we analyse risk-sharing possibilities under two-sided lack of commitment by introducing voluntary participation constraints. In the two-period model, the voluntary participation constraints apply only for the first period. The constraints characterise the trade-off between the first-period consumption and the value of risk sharing provided by the arrangement in the second period. A risk-sharing arrangement is sustainable if each agent, after observing his first-period endowment, at least weakly prefers to follow the arrangement rather than to defect

\(^2\) The tax is a convenient and general way to introduce aggregate risks associated with government policies. It also captures the inflation tax that we consider in our main application in the next Section.

\(^3\) We consider equal Pareto weights across \textit{ex ante} identical agents. If we were to allow for non-equal Pareto weights, the social welfare would still be higher under imperfect information than under perfect information about aggregate risk.

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into autarchy. In other words, it is in the best rational interest of each agent to support the agreement. For the second period, we assume that agents respect the commitments made in the first period. Otherwise, if voluntary participation were allowed in both periods, there would be no room for social insurance because agents would always choose to consume their endowments. While commitment for the second period is necessary for the existence of insurance in the two-period model, we do not need to impose any commitment in the infinite horizon model, which represents our main application and is provided in the next Section.

We compare two environments with different information precision about the future government policy. In the environment of perfect information, agents know the second-period government policy when they decide to sustain the risk-sharing agreement or to deviate to autarchy in the first period. In the environment of completely imperfect information, agents are left uninformed about the future government policy. In the first environment, when the future government policy is known, the participation constraints are given by the following:

\[
\begin{align*}
    u(c_{1g}^h) + \frac{\beta}{2} \left[ u(c_{2g}^{hh}) + u(c_{2g}^{hl}) \right] & \geq u(y^h) + \frac{\beta}{2} [u(y^h) + u(y')], \\
    u(c_{1b}^h) + \beta u(0) & \geq u(y^h) + \beta u(0), \\
    u(c_{1g}^l) + \frac{\beta}{2} \left[ u(c_{2g}^{hl}) + u(c_{2g}^{ll}) \right] & \geq u(y^l) + \frac{\beta}{2} [u(y^h) + u(y')], \\
    u(c_{1b}^l) + \beta u(0) & \geq u(y^l) + \beta u(0),
\end{align*}
\]

where \( c_{1k} \) is the first-period consumption of an agent with a \( y^l \) first-period endowment under a type-\( k \) government policy. The term \( c_{2k}^j \) is the second-period consumption of an agent with a \( y^l \) endowment in the first period and a \( y^j \) endowment in the second period. In the constraints, we explicitly substituted a consumption of zero for the type-\( b \) policy. The first two constraints are relevant for agents with a high first-period income and the other two describe the incentives of agents with a low first-period income. The left-hand side of each constraint constitutes the expected utility of the arrangement and the right-hand side is the value of living in autarky as the outside option. The resource feasibility constraints are the following:

\[
\begin{align*}
    \frac{1}{2} (c_{1g}^h + c_{1g}^l) = \frac{1}{2} (c_{1b}^h + c_{1b}^l) = \frac{1}{4} (c_{2g}^{hh} + c_{2g}^{hl} + c_{2g}^{ll}) = \frac{1}{2} (y^h + y^l).
\end{align*}
\]

The optimal risk-sharing arrangement in the perfect-information environment is a consumption allocation, \( \{ c_{1k}^j, c_{2k}^j \} \), that maximises \( \text{ex ante} \) utility (1), subject to the participation constraints (2)–(5) and the resource constraints (6).

The second environment is set to represent completely imperfect information. In the first period, after observing their current endowments and without knowing the

---

4 In the language of the literature on randomisation in policymaking (Polemarchakis and Weiss, 1977, Arnott and Stiglitz, 1988), the perfect information environment corresponds to \( \text{ex ante} \) randomisation and the imperfect information environment to \( \text{ex post} \) randomisation.

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government policy in the second period, agents decide about participation in the risk-sharing agreement. Correspondingly, the voluntary participation constraints are the following:

$$u(c^h_1) + \beta \left[ u(c^{hh}_{2g}) + u(c^{hl}_{2g}) + 2u(0) \right] \geq u(y^h) + \beta \left[ u(y^h) + u(y^l) + 2u(0) \right],$$  

$$u(c^i_1) + \beta \left[ u(c^{ih}_{2g}) + u(c^{il}_{2g}) + 2u(0) \right] \geq u(y^i) + \beta \left[ u(y^h) + u(y^l) + 2u(0) \right],$$

where $c^i_1$ is first-period consumption of an agent with a $y^i$ first-period endowment. Resource feasibility requires the following:

$$\frac{1}{2} (c^h_1 + c^i_1) = \frac{1}{4} (c^{hh}_{2g} + c^{hl}_{2g} + c^{ih}_{2g} + c^{il}_{2g}) = \frac{1}{2} (y^h + y^l).$$  

The optimal risk-sharing arrangement under completely imperfect information is a consumption allocation, $\{c^i_1, c^g_{2g}\}$, that maximises ex ante utility (1) subject to participation constraints (7)–(8) and resource constraints (9).

Our goal is to highlight that information about aggregate risk can be harmful for social welfare because it distorts the insurance possibilities of idiosyncratic risk under voluntary participation. The result is formally stated in Theorem 1. The intuition is the following. From an ex ante perspective, the agents wish to share their idiosyncratic endowment risk. The optimal insurance scheme prescribes transfers from high-income agents to low-income agents in all states. While agents with a low income are never worse off in the agreement, living alternatively in autarchy may be an attractive outside option for agents with a high income. The better informed high-income agents are about the future tax policy the less willing they are to transfer resources to the less fortunate agents.

**Theorem 1.** Under completely imperfect information, social welfare is strictly higher than under perfect information.

**Proof.** Here, we consider that all participation constraints for high-endowment agents under perfect and imperfect information are binding.\(^5\) The proof that applies when the constraints are not binding can be found in the Appendix along with the algebraic details.

For both information environments, it follows from the first-order conditions of the optimal risk-sharing problem that consumption of the agents under the type-$g$ policy should be perfectly smoothed over time. In the perfect-information environment, this condition reads as follows:

$$c^h_{1g} = c^{hh}_{2g} = c^{hl}_{2g},$$

Similarly, under imperfect information the following condition is true:

$$c^i_1 = c^{ih}_{2g} = c^{il}_{2g}.$$

\(^5\) We say that a constraint is binding when the associated Lagrange multiplier is positive.

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We thus compare the information environments in terms of the first-period allocations. From the binding participation constraints (2), (3) and (7), it follows that the first-period allocations under the two informational environments are characterised by the following inequality: \( c_{1g}^h < c_{1b}^h < c_{1b}^l \), which is further illustrated in Figure 1.

From the binding participation constraints (2), (3) and (7), it also follows that agents with a high first-period endowment obtain the same expected utility under perfect and imperfect information

\[
\left(\frac{1}{2} + \frac{\beta}{2}\right) u(c_{1g}^h) + \frac{1}{2} u(c_{1b}^h) = \left(1 + \frac{\beta}{2}\right) u(c_1^l). \tag{10}
\]

Therefore, the consumption allocation for the high-income agents under perfect information spreads out and becomes riskier from an \textit{ex ante} perspective. Due to strictly concave preferences, (10) implies the following

\[
\left(\frac{1}{2} + \frac{\beta}{2}\right) c_{1g}^h + \frac{1}{2} c_{1b}^h > \left(1 + \frac{\beta}{2}\right) c_1^h. \tag{11}\]

For the expected utility of agents with a low income in the first period under perfect and imperfect information, the following is implied:

\[
\left(\frac{1}{2} + \frac{\beta}{2}\right) u(c_{1g}^l) + \frac{1}{2} u(c_{1b}^l) < \left(1 + \frac{\beta}{2}\right) u\left(\frac{1 + \beta}{2 + \beta} c_{1g}^l + \frac{1}{2 + \beta} c_{1b}^l\right)
= \left(1 + \frac{\beta}{2}\right) u\left(\frac{y^h + y^l}{2 + \beta} c_{1g}^h - \frac{1}{2 + \beta} c_{1b}^h\right)
< \left(1 + \frac{\beta}{2}\right) u(y^h + y^l - c_1^l) = \left(1 + \frac{\beta}{2}\right) u(c_1^l), \tag{12}\]

where the first inequality is due to strict concavity and the second one is implied by (11). Thus, agents with low first-period endowments are strictly better off under completely imperfect information. Taking the unconditional expectation by adding (10) and (12), we conclude that imperfect information is strictly preferable.

![Fig. 1. Optimal Allocations for Perfect and Imperfect Information under Binding Participation Constraints](image_url)

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Compared with the literature on efficient risk sharing and public information (Hirshleifer, 1971; Schlee, 2001), we show that not only public information on idiosyncratic risk but also on non-insurable aggregate risk can be harmful to social welfare. Unlike the situations described in the literature, there are aggregate states in which perfectly informative signals improve risk sharing. This situation occurs when the government reveals a type-g policy. Because the expected utility of the arrangement is high relative to the outside option, high-income agents in this state are willing to share more with low-income agents (see Figure 1).

The result of the negative social value of public information about the second-period government policy is robust to any policies that lead to a non-identical dispersion of agents’ disposable income. For example, if the tax were lump-sum or if the government were to redistribute the tax revenues equally among agents, then better information on the taxes would still be undesirable. Also, it is not crucial for the finding in Theorem 1 to require a policy under which the idiosyncratic risk vanishes completely. Even if taxation were not as extreme as a 100% tax, the result on the negative value of information is valid.

The negative effect of public information is also robust to the possibility of self-insurance in the outside option. In the first period, after the realisation of the endowment and the announcement of the future tax, an agent could improve his situation in autarchy by purchasing non-negative amounts of a risk-free asset. Compared with the lack of self-insurance, the value of the outside option increases in both information environments. Thus, in the case of self-insurance, the consumption of agents with a high first-period income is higher and the social welfare is lower. However, when the future tax is known, the agents come up with a better savings plan than if the announcement were uninformative, that is, the possibility of self-insurance is a fundamental motive for acquiring better information. The superior self-insurance under perfect public information leads to a relatively higher value of the outside option, the high-income agents require more from the arrangement than under imperfect information and the optimal allocation further diverges from perfect risk sharing. Therefore, imperfect information is preferable ex ante and the negative effect of information is preserved.

The decision on policy itself and on the optimal announcement about the policy cannot always be disentangled. When the two can be disentangled, then the negative effect of the announcements that we specify applies. For example, when the government is allowed to tax agents in some states of nature and distributes the tax revenues back to the agents equally, then the benevolent government would impose a 100% tax whenever it can, regardless of the policy announcements. In line with Theorem 1, it is desirable for the benevolent government not to inform agents whether the state of nature is one in which the government is allowed to tax or not.

In the next Section, we embed the risk-sharing mechanism into a richer environment with a monetary authority that announces a signal on its future inflation target. Relative to the illustrative example, the model is extended in several dimensions. First, we do not impose any commitment and we consider an economy with an infinite number of periods. Second, we allow for continuity in information precision.

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2. Monetary Policy and Infinite Horizon

We proceed by integrating the voluntary risk-sharing mechanism into a monetary production economy. In this Section, we introduce an economy and describe the notion of equilibrium. In the economy, households’ consumption expenditures are linked to nominal balances from the previous period with a cash-in-advance constraint originated by Clower (1967). As in Lucas (1980), each household consists of a worker–shopper pair. The production part comprises two sectors. Following Dixit and Stiglitz (1977) and Blanchard and Kiyotaki (1987), each sector is populated by a continuum of monopolistically competitive firms. Sectors differ in productivity of the monopolistic firms. The productivity is not known to workers at the moment they choose a firm and this constitutes the idiosyncratic risk. The aggregate risk is given by stochastic monetary policy and the monetary authority can reveal the policy early with certain precision. The notion of equilibrium is introduced in two steps. First, we define an incomplete markets equilibrium for the given risk-sharing transfers among households. Second, we introduce the possibility for households to insure the idiosyncratic risk in arrangements that are consistent with their rational participation incentives as in Kocherlakota (1996b). The exchange of consumption goods prescribed by the arrangements is reflected in risk-sharing transfers among households. Similar to a Ramsey planner – and this is our main contribution here – we define the problem to find the optimal risk-sharing transfers for given precision of the public signal on the monetary policy.

We consider an infinite-period production economy with a continuum of households of measure one. The time is indexed by $t$ from 0 onwards.

Households are identical ex ante and their preferences over the stream of consumption are given by the following utility function:

$$E \left[ \sum_{t=0}^{\infty} \beta^t u(c_t) \right],$$

where $c_t$ is consumption of household $i$ in period $t$, $0 < \beta < 1$ is the time discount factor, and $u(c)$ is the period utility function. We assume the period utility function to be twice continuously differentiable, increasing and strictly concave.

Each household consists of two members: a shopper and a worker. In each period, the worker earns idiosyncratic income by supplying one unit of labour to one of two production sectors inelastically. Meanwhile, the shopper buys consumption goods. Money is the only means for facilitating transactions and transferring wealth across periods. The period budget constraint of household $i$ is given by the following:

$$M_t^i + p_t c_t^i = M_{t-1}^i + p_t w_t^i + d_t + p_t \tau_t^i,$$

where $M_t^i$ are nominal money holdings at the end of period $t$, $d_t$ are shares of nominal profits of monopolistically competitive firms equally distributed among households, $\tau_t^i$ are real transfers prescribed by a risk-sharing arrangement, $w_t^i$ is the real wage in the production sector in which the worker is employed in period $t$ and $p_t$ is the aggregate price level.
A shopper and a worker are distinguished by activities. In each period, while a worker works and earns money, a shopper exchanges the money earned by the worker in the previous period for consumption goods:

\[ p_t x_t^i = M_{t-1}^i, \]  

where \( x_t^i = c_t^i - e_t^i \) is the amount of the consumption good bought directly in the market.\(^6\)

The production part of the economy is represented by two production sectors of equal size. Both sectors include a final good firm and a continuum of intermediate good firms. In each period the final good firms in both sectors produce an identical consumption good by aggregating over sector-specific differentiated intermediate goods. The intermediate goods are aggregated into the final good with a constant elasticity of substitution:

\[ y_t^n = \left[ \int_0^1 (y_t^{nm})^{(\theta-1)/\theta} \frac{\partial m}{\partial m} \right]^{\theta/(\theta-1)}, \]  

where \( y_t^n \) is the amount of the consumption good produced by the final good firm in sector \( n \), \( y_t^{nm} \) is an intermediate good produced by differentiated good firm \( m \) in sector \( n \) and \( \theta > 1 \) is the elasticity of substitution between the differentiated goods. The production technology of the differentiated good firms is given by the following:

\[ y_t^{nm} = a_t^n l_t^{nm}, \]  

where \( l_t^{nm} \) is the labour input. The productivity of the differentiated good firms, \( a_t^n \), is the same for all intermediate good firms within a production sector but it is different across the sectors.

Acting under perfect competition, final-good firms minimise costs by choosing the factor demand for each intermediate good to satisfy aggregate demand. The cost minimisation problem is given by the following:

\[ \min \int p_t^{nm} y_t^{nm} \, dm, \]  

subject to technology constraint (16), where \( p_t^{nm} \) is the price of the intermediate good \( m \) that the final-good firm in sector \( n \) takes as given.

The intermediate good producers operate under monopolistic competition. A measure, \( \lambda \), of monopolistically competitive firms set prices in period \( t \) knowing the actual demand for their product. The profit maximisation problem of the monopolistically competitive firms with flexible price setting is the following:

---

\(^6\) Alternatively, the cash-in-advance constraints can be stated with inequalities. However, allowing for inequalities (and therefore for the possibility of self-insurance) does not qualitatively affect our main result regarding the negative value of information. In Section 4, we conduct the latter exercise as a robustness check.

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given the demand of the final good firm, nominal sector wages and the production technology (17). The other \((1-\lambda)\) firms preset prices one period ahead based on a public signal regarding the inflation by solving the expected profit maximisation problem
\[
\max_{p_t^{nm}} \mathbb{E}_{t-1}\left( p_t^{nm} y_t^{nm} - p_t w_t^{nm} | s_{t-1} \right),
\]
where \(s_{t-1}\) is the signal released in period \(t-1\) about the inflation target in period \(t\). The aggregate profits in sector \(n\) in period \(t\) are denoted by \(\Pi_t^{nm}\).

In each period, a worker is employed in the sector of either high productivity \((a^h)\) or low productivity \((a^l)\) with equal probability. The worker is not aware of the firm’s productivity at the moment of the employment decision. Once the final goods are sold to the shoppers, a worker obtains labour income and an equal share of profits.

Monetary policy is characterised by a stochastic inflation target.\(^7\) All agents in the economy are rational and know the stochastic properties of the inflation target process. In addition, the monetary authority knows the inflation target one period in advance and provides a public signal on the future inflation target with a certain precision. The exogenous process for the gross inflation target, \(\pi_t\), is given by an i.i.d. process with two states of equal probability: high inflation \((\pi_{h})\) and low inflation \((\pi_{l})\).\(^8\) Similarly, the public signal on the next-period inflation target, \(s_t\), takes two values: a high realisation \((s_h)\) and a low realisation \((s_l)\). The precision of the public signal is given by \(\kappa = \text{Prob}(\pi_{h}|s_h) = \text{Prob}(\pi_{l}|s_l)\) with \(1/2 \leq \kappa \leq 1\). Other than the public signal, households do not obtain any other information. The information asymmetry between the central bank and the households captures the particular attention the public pays to announcements by central banks.

The actual inflation coincides with the inflation target by appropriate money injections in all states. Because the seigniorage is spent on government expenditures, the government budget constraint reads as follows:
\[
p_t g_t = M_t - M_{t-1},
\]
where \(g_t\) denotes real government expenditures and \(M_t\) is the aggregate money supply.\(^9\)

We combine the optimal behaviour of households and firms in the presence of the government into the definition of an incomplete markets equilibrium. In this stochastic model, there are two sources of risk. First, there is idiosyncratic income risk faced by each household. Second, there is aggregate risk that results from stochastic inflation and stochastic inflation announcements.

\(^{7}\) The mean of the inflation target process can be interpreted as the long-run inflation target of the central bank and realisations of it as inevitable short-run deviations from the long-run target.

\(^{8}\) The inflation target process and productivity are assumed to be non-degenerate: \(\pi_{l} < \pi_h\) and \(a^l < a^h\).

\(^{9}\) Alternatively, when seigniorage is equally distributed back to households, our main results stated in Theorem 2 remain valid.

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DEFINITION 1. An incomplete markets equilibrium is a household allocation \( \{c^i_t, x^i_t, M^i_t \} \), a production allocation \( \{y^n_t, y^{nm}_t, d^n_t \} \), a government allocation \( \{g_t, M^g_t \} \) and a price system \( \{p_t, p^n_t, w^{nm}_t \} \) such that the initial distribution of nominal money balances \( \{M^i_{-1} \} \), and the initial price setting of sticky-price firms \( \{p^{nm}_0 \} \) hold the following conditions:

(i) for each household \( i \) and given prices \( \{p_t, w^i_t \} \) and profits \( \{d^i_t \} \), the allocation \( \{c^i_t, x^i_t, M^i_t \} \) maximises household’s utility \( (13) \) subject to the budget constraint \( (14) \) and the cash-in-advance constraint \( (15) \),

(ii) for each production sector \( n \) and given prices \( \{p_t, w^n_t \} \) and prices \( \{\{p^{nm}_t \} \} \), solve the cost minimisation problem of the final goods firms \( (18) \) and the profit maximisation problems of the differentiated goods firms \( (19) \) and \( (20) \),

(iii) monetary injections are consistent with the inflation target:

\[ p_t = \pi_t p_{t-1} \quad \forall t, \]

(iv) the government budget constraint \( (21) \) is fulfilled in each period \( t \) and

(v) all feasibility constraints are satisfied:

\[ \int c^i_t \, di + g_t = \int y^n_t \, dn, \quad \int M^i_t \, di = M^i_t, \quad \int \tau^i_t \, di = 0, \quad \int t^{nm}_t \, dm = \frac{1}{2} \quad \forall t. \]

We assume that the low realisation of the inflation target is high enough to satisfy the resource feasibility with non-negative government expenditures. When we refer to social welfare derived from a certain allocation, we mean the ex ante utility \( (13) \), which is evaluated before any risk has been resolved.

The main element of our model is households’ risk-sharing arrangement under voluntary participation. Without risk-sharing transfers, the consumption allocation that results from the incomplete markets equilibrium is not efficient from an ex ante perspective due to market incompleteness that prevents households from optimal borrowing and lending. However, the efficient use of a complete set of securities requires commitment or enforceability of the arrangements. In the absence of commitment, the consumption allocation can still be improved by risk-sharing transfers consistent with voluntary participation incentives. We determine the socially optimal transfer scheme under voluntary participation in the incomplete markets equilibrium. Voluntary participation in social insurance provided by risk-sharing transfers implies that in each period, households may decline the risk-sharing arrangement offered. In such a case, they live forever in an economy with no transfers and consume only the goods bought directly in the market.

With this mechanism, we seek to capture financial market imperfections, either through incompleteness of the financial markets themselves or through private agents’ limited access to it. When participation incentives matter, the resulting equilibrium consumption allocations share key properties with individual consumption patterns in the data (Krueger and Perri, 2006). In particular, lack of commitment results in a
positive correlation between current income and current consumption. This is a stylised fact that cannot be explained in models with complete financial markets (Kocherlakota, 1996b).

The timing of events is illustrated in Figure 2. In each period, agents first obtain a public signal on the next period’s inflation target and observe the current period’s inflation target. Second, households decide whether to sustain the risk-sharing arrangement. Third, workers and shoppers separate and the workers inelastically supply their labour services into the production process. Fourth, market exchange takes place. Flexible-price monopolistic firms set intermediate good prices, shoppers acquire consumption goods in exchange for cash held from the previous period, workers receive wages and shares of profits and the government collects seigniorage from money injections. Fifth, an exchange according to the risk-sharing arrangement takes place among shoppers. Finally, members of each household meet again and consume. Then, money balances are passed from workers to shoppers for next-period consumption purchases, and sticky-price firms preset prices for the next period based on the public signal.

Formally, the risk-sharing arrangement is built upon the consumption allocation of the incomplete markets equilibrium with no transfers as the outside option \((\tau_i^t = 0, \forall i, t)\). This ‘off-equilibrium’ allocation coincides with the equilibrium amount of consumption goods directly bought in the market, \(\{(x_i^t), \}_{t=0}^{\infty}\), because the amount of money the agents hold from one period to the next (and therefore the amount of their market purchases) does not depend on the risk-sharing transfers. Moreover, because the equilibrium of the goods’ market is not linked to the distribution of consumption goods across households, prices in the equilibrium are independent of the transfers.

Let the individual state of household \(i\) at time \(t\) be \(h_i^t = (x_i^t, H_t)\), where \(H_t = (X_t, s_t)\) characterises the aggregate state, \(X_t\) denotes period-\(t\) per capita disposable income in real terms and \(s_t\) is the public signal in period \(t\) about inflation in period \(t + 1\). The individual state is public knowledge. We restrict our analysis to pure insurance arrangements as emphasised by Kimball (1988), Coate and Ravallion (1993) and Ligon et al. (2002). These arrangements imply that the current risk-sharing transfers do not depend on transfers received in the past.\(^{11}\) With pure insurance transfers, we can characterise the effect of information on social welfare analytically.

![Fig. 2. Timing of Events in the Monetary Production Economy](image-url)
DEFINITION 2. A consumption allocation \( \{c_t^i\}_{t=0}^{\infty} \) is sustainable if there exist transfers \( \{\tau(h_t^i)\}_{t=0}^{\infty} \) such that the following are true:

(i) for each household \( i \) the consumption allocation \( \{c_t^i\}_{t=0}^{\infty} \) solves the incomplete markets equilibrium with the transfers \( \{\tau(h_t^i)\}_{t=0}^{\infty} \), and

(ii) for each household \( i \) and state \( h_t^i \), the consumption allocation \( \{c_t^i\}_{t=0}^{\infty} \) is weakly preferable to the outside option \( (x_t^i)^{\infty}_{t=0} \)

\[
E \left[ \sum_{j=0}^{\infty} \beta^{t+j} u(c_{t+j}^i|h_t^i) \right] \geq E \left[ \sum_{j=0}^{\infty} \beta^{t+j} u(x_{t+j}^i|h_t^i) \right]. \tag{22}
\]

DEFINITION 3. A socially optimal arrangement under voluntary participation is a consumption allocation \( \{c_t^i\}_{t=0}^{\infty} \) that provides the highest expected utility among the set of sustainable allocations.

Our ultimate objective is to find the socially desirable precision of policy announcements. This problem resembles a problem of a Ramsey planner. The planner chooses the optimal precision of the public signal and the optimal transfer scheme and is restricted to transfer schemes that sustain voluntary participation of households and that are consistent with the incomplete markets equilibrium.

It is natural to compare the optimal arrangement under voluntary participation to an optimal arrangement under commitment. We define the optimal commitment allocation as a consumption allocation that provides the highest expected utility among the set of consumption-feasible allocations. An allocation is consumption-feasible if it solves the incomplete markets equilibrium with resource-feasible transfers, \( \{\tau_t^i\}_{t=0}^{\infty} \).

3. Negative Social Value of Information

In this Section, we deliver our main result that policy announcements about future monetary policy can have a negative effect on social welfare. We show that more precise information on the future inflation target can be welfare reducing because it harms individual risk-sharing possibilities when rational participation incentives matter. In addition, we show that under more informative signals perfect risk sharing requires a higher degree of patience to be supported as a sustainable allocation.

To highlight the main effect, we abstain from examining the effect of public signals on optimal pricing decisions of firms in this Section. We avoid the pricing friction on the firm side by assuming that all intermediate firms are flexible-price firms. In the next Section, we extend the main result by illustrating a trade-off in public signal precision when a fraction of firms has to preset prices one period in advance.

3.1. Optimal Risk Sharing under Voluntary Participation

In the following paragraphs, we characterise the incomplete markets equilibrium under flexible prices. We proceed to state the problem to design the socially optimal arrangement in recursive form and derive general properties of the optimal solution.
As a starting point of our analysis, we characterise households’ direct market purchases, which is the outside option for households when they decide to sustain the risk-sharing scheme. For the initial distribution of money balances, we consider the stationary distribution. From (15) to (19), monopolistically competitive firms set their prices with a fixed markup $\mu = \theta/(\theta - 1)$ above marginal costs. When the current inflation is $\pi_j$ and the worker was employed in sector $n$ in the previous period, the direct market purchases of the household in the current period are given by the following

$$x^n(\pi_j) = \frac{1}{\pi_j} \left( \frac{1}{\mu} a^n + \frac{\mu - 1}{\mu} \frac{a^h + a^l}{2} \right). \tag{23}$$

The first term in the parentheses on the right-hand side of (23) is labour income and the second term are profits equally distributed among households. Due to constant labour supply and because all firms are flexible in their price setting, the labour income depends only on worker’s productivity. We refer to high-income households when the worker of the household was employed in the sector of high-productivity firms in the previous period. Combining the goods’ market clearing condition with the government budget constraint (21) and the cash-in-advance constraint (15), the government expenditures are given by the following

$$g(\pi_j) = \frac{a^h + a^l \pi_j - 1}{2 \pi_j}. \tag{24}$$

From (23) and (24), it follows that both direct market purchases and government expenditures are independent of the precision of the inflation target signal.

The incomplete markets equilibrium consumption comes from direct market purchases, given by (23), and risk-sharing transfers. With pure insurance transfers, the current period consumption depends only on the current period direct purchases $x^n(\pi_j)$, current inflation $\pi_j$ and the signal $s_k$ on the next period inflation target. Thus, the equilibrium consumption of a household in the current period is given by the following

$$c^n(\pi_j, s_k) = \frac{1}{\pi_j} \left( \frac{1}{\mu} a^n + \frac{\mu - 1}{\mu} \frac{a^h + a^l}{2} \right) + \tau(a^n, \pi_j, s_k).$$

For two inflation states and two signals on the next-period inflation, the optimal contract problem is the following

$$\max_{\{x^n(\pi_j, s_k)\}} \frac{1}{1 - \beta} V_n,$$ \tag{25}

subject to participation constraints for high- and low-inflation signals

$$u[c^n(\pi_j, s_k)] + \beta \kappa V_n(\pi_h) + \beta (1 - \kappa) V_n(\pi_l) + \frac{\beta^2}{1 - \beta} V_n \geq 0,$$

$$u[x^n(\pi_j)] + \beta \kappa V_{out}(\pi_h) + \beta (1 - \kappa) V_{out}(\pi_l) + \frac{\beta^2}{1 - \beta} V_{out} \quad \forall j, n \tag{26}$$
\[ u[c^n(\pi_j, s_t)] + \beta \kappa V_{rs}(\pi_t) + \beta(1 - \kappa) V_{rs}(\pi_h) + \frac{\beta^2}{1 - \beta} V_{rs} \geq \]
\[ u[x^n(\pi_j)] + \beta \kappa V_{out}(\pi_t) + \beta(1 - \kappa) V_{out}(\pi_h) + \frac{\beta^2}{1 - \beta} V_{out} \quad \forall j, n \] (27)

and consumption-feasibility constraints:
\[ \sum_n c^n(\pi_j, s_k) = \sum_n x^n(\pi_j) \quad \forall j, k, \] (28)

with the expected values of the arrangement and of the outside option given by the following

\[ V_{rs}(\pi_j) \equiv E\left\{ u[c^n(\pi_j, s_k)]|\pi_j \right\}, \quad V_{rs} \equiv E[V_{rs}(\pi_j)], \]
\[ V_{out}(\pi_j) \equiv E\left\{ u[x^n(\pi_j)]|\pi_j \right\}, \quad V_{out} \equiv E[V_{out}(\pi_j)]. \]

Given that all firms are flexible in their price setting, the consumption allocation in the current period does not depend on the signal from the previous period. Correspondingly, the expected values of the arrangement, \( V_{rs}(\pi_j) \), and of the outside option, \( V_{out}(\pi_j) \), do not depend on today’s realisation of the signal.

As the first point in characterising socially optimal arrangements, we show that the optimal arrangement exists and is unique and that social welfare is continuous in signal precision. Among the participation constraints (26) and (27), only restrictions for high-income households can potentially be binding because a risk-sharing arrangement prescribes transfers to the low-income households. These properties are summarised in the following Lemma:

**Lemma 1.** Consider the maximisation problem to design a socially optimal arrangement given in (25)–(28).

1. The socially optimal arrangement exists and is unique. The arrangement and the social welfare are continuous functions in the precision of the public signal.
2. The socially optimal arrangement satisfies the following
\[ x^l(\pi_j) \leq c^l(\pi_j, s_k) \leq c^h(\pi_j, s_k) \leq x^h(\pi_j). \]

The proof is provided in the Appendix. The first part of Lemma 1 is based upon the maximum theorem under convexity. The second part reflects households’ preferences to insure the idiosyncratic risk efficiently.

As an immediate corollary from Lemma 1, the socially optimal arrangement satisfies \( V_{rs}(\pi_j) - V_{out}(\pi_j) \geq 0 \) for all inflation states \( \pi_j \). In other words, in any inflation state the value of the optimal arrangement cannot be lower than the expected value of the allocation in the equilibrium without transfers.
3.2. Information, Patience and Folk Theorems

There are cases when the information precision does not affect social welfare. Before we proceed to our main result, we consider these cases, which appear if the households are very patient or too impatient. We then show that perfect risk sharing is less likely to be sustainable when the precision of public announcements increases.

First, even without commitment the optimal commitment allocation is sustainable if the discount factor ($b$) is high enough. This result is a well-known implication of the folk theorem. Because all households are \textit{ex ante} the same, the optimal commitment allocation is perfect risk sharing $c^\nu(p_j, s_k) = [x^h(p_j) + x^l(p_j)]/2$, $\forall j,k,n$.

Second, if the level of patience is relatively low, the set of sustainable allocations may shrink to one point, which is the equilibrium allocation in the absence of transfers. If this allocation is the only sustainable allocation for a certain level of patience then it is also the socially optimal allocation if households are even less patient. These results are stated in the following Lemma:

**Lemma 2.** Consider the maximisation problem to design a socially optimal arrangement given in (25)–(28).

1. There exists $\bar{b} < 1$ such that for any discount factor $b \geq \bar{b}$ the socially optimal arrangement for any signal precision is perfect risk sharing.
2. If for $\bar{b} > 0$ the equilibrium allocation in the absence of transfers is the socially optimal arrangement for any signal precision, then for any discount factor $b \leq \bar{b}$ the socially optimal arrangement for any signal precision is the equilibrium allocation in the absence of transfers.

The proof is provided in the Appendix. Both parts of the Lemma follow from the observation that the set of sustainable allocation is increasing in $\beta$. For any arrangement, an increase in patience increases future gains from the arrangement relative to the outside option and does not affect current period gains.

There is one additional case when information precision has no effect on social welfare. In general, the relative gain of the optimal arrangement $V_{rs}(\pi_j) - V_{out}(\pi_j)$, is different across inflation states $\pi_j$. However, there is a degenerate possibility that the gain is exactly the same in all states. 12 Throughout the following analysis we exclude this possibility. 13

We can now analyse how informative policy announcements influence the outcome of the optimal insurance arrangement under voluntary participation. Signal precision plays an important role for the sustainability of perfect risk sharing. In the following Proposition, we show that the level of patience that is needed to sustain perfect risk sharing increases with the precision of the signal.

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12 For homogenous preferences the relative gains of the insurance arrangement are the same across inflation states when the degree of homogeneity is zero.

13 This exclusion allows us to state strict monotonicity in Proposition 1 and Theorem 2 rather than weak monotonicity.

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PROPOSITION 1. Let $\tilde{\beta}(\kappa)$ be the cutoff point such that for each $\beta \geq \tilde{\beta}(\kappa)$, perfect risk sharing is the socially optimal arrangement. The cutoff point, $\tilde{\beta}(\kappa)$, is strictly increasing in the precision of the public signal.

The proof is provided in Appendix A.1. The cutoff point is determined by a participation constraint for high-income households that imposes the tightest restriction. Which particular constraint is the tightest depends on the gains that the perfect risk-sharing arrangement offers relative to the equilibrium in the absence of transfers, as can be seen from (26) and (27). Without loss of generality, suppose that the perfect risk-sharing arrangement provides higher expected value relative to the equilibrium allocation without transfers under high inflation than under low inflation. While for high-income households the current-period loss of staying in the arrangement is independent of signal precision, the expected future gain of insurance is lower for informative signals than for uninformative signals when the next-period inflation signal is low. Therefore, the level of patience needed to sustain the perfect risk-sharing allocation is higher under an informative signal.

3.3. Information and Welfare under Partial Risk Sharing

A mounting number of studies indicate that the realistic case is one where risk sharing is neither perfect nor absent but instead is partial (Blundell et al., 2008) This case is analysed below. We show that the transfers prescribed by the arrangement depend on signal precision and the signal can shape the resulting consumption allocation significantly. As our main result, we provide conditions for social welfare to be decreasing in the precision of the public signal. We exclude the cases when the optimal arrangement is either perfect risk sharing or the outside option and signal precision does not directly affect the arrangement and social welfare. Lemmas 3 and 4 provide sufficient conditions for a socially optimal arrangement that is neither perfect risk sharing nor the outside option.

If perfect risk sharing is not sustainable, a number of participation constraints of high-income households are binding. Which constraints are binding depends on the current loss relative to the outside option and the future value of the arrangement. We focus on the case when all constraints are binding – which is the empirically relevant case as we show in the next Section – and state sufficient conditions for this case to apply.

**LEMMA 3.** If all participation constraints for high-income households are violated under the arrangement that prescribes perfect risk sharing in all states, then all the constraints are binding under the optimal arrangement.

The proof of this Lemma is provided in the Appendix. First, under the conditions of the lemma, we show that for all states, the consumption allocation in the optimal arrangement satisfies strict inequalities $c^l(p_j, s_k) < c^h(p_j, s_k)$. Second, by contradiction, we show that a Lagrange multiplier on any participation constraint of a high-income household cannot be zero because otherwise the inequalities do not hold.

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The optimal arrangement may be given by another extreme, which is the outside option. In the following Lemma, we provide conditions under which there exists a socially optimal arrangement different from the consumption allocation in the absence of transfers. In particular, we consider a situation when the signal is uninformative.

**LEMMA 4.** Consider the case of an uninformative public signal with all participation constraints for high-income households binding in the optimal arrangement. If and only if

$$
\frac{1}{2} \left\{ u'[x^l(\pi_h)] + u'[x^h(\pi_h)] \right\} > \frac{2 - \beta}{\beta},
$$

the socially optimal arrangement is characterised by non-zero transfers.

The proof is provided in the Appendix. The intuition is the following. From the perspective of a household with a high current-period income, risk sharing in future periods is attractive if the household values the future significantly enough and if the household is subject to high enough consumption risk in the equilibrium without transfers. Both aspects are reflected in condition (29) of Lemma 4. By taking this situation to an extreme, consider that the future consumption is worthless for households (i.e. \( \beta = 0 \)). It follows that the outside option is the only sustainable arrangement. Therefore, the threshold for \( \beta \) implied by condition (29) is strictly positive. On the other hand, if the consumption risk in the equilibrium without transfers is significant, then the marginal utility for consuming the low income relative to the high income, \( u'[x^l(\pi_j)]/u'[x^h(\pi_j)] \), may become substantial. Thus, the required level of patience for engaging in the social insurance arrangement is low.

In the following theorem, we establish our main result that social welfare is strictly decreasing in the precision of the public signal.

**THEOREM 2.** If all participation constraints for high-income households are binding and the equilibrium allocation in the absence of transfers is not the only sustainable arrangement, then social welfare is strictly decreasing with the precision of the public signal on future inflation.

The proof is provided in Appendix A.2. For any two values of signal precision such that \( \kappa_1 < \kappa_2 \), we construct a consumption allocation for \( \kappa_1 \) based on the optimal allocation for \( \kappa_2 \) as follows. The allocation is constructed to satisfy the participation constraints for \( \kappa_1 \) with equality, whereas the value of the arrangement in future periods corresponds to the optimal arrangement for \( \kappa_2 \). We show that this allocation delivers strictly higher welfare than the optimal allocation for \( \kappa_2 \) and that it is also sustainable for signal precision \( \kappa_1 \). Thus, because the optimal allocation for \( \kappa_1 \) must be at least as good as the one constructed, welfare is strictly higher for lower signal precision.

The detrimental effect of informative signals on social welfare can be illustrated as follows. Assume that the expected gain of the optimal arrangement relative to the outside option is larger under high inflation. Suppose further that the realised signal
indicates that the next-period inflation is more likely to be low. From the signal, households infer that the future value of the arrangement relative to the outside option is lower, which is an unfavourable outcome for all households. Therefore, the high-income households require higher current-period consumption. In contrast, under the high-inflation signal, which indicates a higher value of the arrangement relative to the outside option, the high-income households can be satisfied with lower current-period consumption. Compared with uninformative signals, the consumption allocation prescribed by the optimal arrangement diverges as precision increases, that is, the consumption allocation of high-income households becomes riskier \textit{ex ante}. Binding participation constraints imply that the expected utility of the high-income households before the signal realisation is independent of signal precision. Because households are risk averse, high-income agents are less willing to share their good fortune with low-income agents when information gets more precise. Correspondingly, from the resource constraint, it follows that low-income households are better off under imperfect information. \textit{Ex ante}, risk-averse households thus prefer uninformative policy announcements.

The negative value of information does not depend on whether the gain from the arrangement relative to the outside option is larger under low or high inflation. If the relative gain is larger under high inflation, the high-income households require lower current-period consumption following a low-inflation signal and they demand higher current-period consumption following a high signal. Nonetheless, from an \textit{ex ante} perspective such divergences are still welfare reducing for risk-averse households.

The negative effect of information in Theorem 2 is amplified when the value of the outside option increases, that is, when the endogenous enforcement mechanism becomes weaker. In the next Section, we consider the possibility of re-engaging in social insurance and the possibility of self-insurance that both illustrate the case. If the value of the outside option is higher, the high-income households are less willing to share the risk with the other households under the optimal arrangement. Then, the marginal utility of low-income households is higher, the marginal gains of redistribution are higher and public information has thus a stronger negative effect on social welfare.

In this Section, we have characterised how the precision of public signals on future inflation affects optimal insurance under voluntary participation when prices are flexible. If the optimal arrangement is partial risk sharing, then the precision of the signal effectively influences the distribution of consumption in the risk-sharing arrangement. We show that higher precision in signals is socially undesirable because it decreases the willingness of high-income households to transfer resources to less-fortunate agents. In addition, we find that the level of patience needed to sustain the perfect risk-sharing allocation is strictly increasing in the precision of the signal. One reason for this negative effect is that the public information provided by the monetary authority does not help households to make better decisions for the future. In the next Section, we extend our framework to allow for a beneficial role of public information and we thereby assess the importance of the detrimental effect of policy announcements on risk sharing.
4. Assessment of Risk-sharing Distortions

The main purpose of this Section is to evaluate the risk-sharing effect. To meet this goal, we introduce a positive effect of information by considering imperfectly flexible prices. As in Woodford (2003), we assume that a positive fraction of intermediate good producers preset their prices one period in advance, which results in increasing aggregate resources with better public information. We proceed to assess the importance of the negative and positive effects of information quantitatively by setting up a numerical example that shares some salient features with the US economy. We find that the negative effect of information prevails for degrees of risk aversion that are not unreasonably high. Furthermore, the increase of income inequality in the US over the recent decades tends to amplify the negative role of public information about aggregate risk on social welfare. As robustness checks, we subsequently allow for self-insurance, a weaker penalty for default, private information and a persistent idiosyncratic income process. As an alternative means of measuring the negative effect of information without the introduction of a particular positive effect of information, we follow Lucas (2003) in considering the elimination of all aggregate fluctuations as a benchmark. The welfare gain of uninformative signals is then compared relative to the elimination of all inflation fluctuations.

4.1. Imperfectly Flexible Prices

When some monopolistically competitive firms have to preset prices, firms’ problems become non-trivial. Solving the cost-minimisation problem of the perfectly competitive final good firms (18), we obtain the demand for each of the variety goods:

\[ y_{nm}^n = \left( \frac{p_{nm}}{p_t} \right)^{-\theta} y_t^n, \]  

(30)

where the aggregate price level is defined by the following expression

\[ p_t = \left[ \int_0^1 (p_{nm})^{1-\theta} \, dm \right]^{1/(1-\theta)}. \]  

(31)

Using the production technology (17), the demand function (30) and integrating over all monopolistically competitive firms within a sector, production per worker in sector \( n \) is given by the following

\[ y_t^n = \frac{a^n}{\Delta_t}, \]  

(32)

where the price dispersion, \( \Delta_t \equiv \int_0^1 (p_{nm}^n / p_t)^{-\theta} \, dm \), is measurable with respect to current inflation as well as with respect to the previous-period inflation signal.\(^{14}\) Price dispersion satisfies \( \Delta_t \geq 1 \) by Jensen’s inequality. The highest level of production is

\(^{14}\) The price dispersion is the same in both production sectors.
achieved when both flexible and sticky-price firms set the same price, $p_t^m = p_t$, as in the case of a perfectly informative public signal.

Signal precision under imperfectly flexible prices affects the outcome of the optimal insurance arrangement in two different ways. First, it influences the willingness of high-income households to share with low-income households, as highlighted in the previous Section. Second, it affects the amount of resources that can be shared among the households. The influence of the latter effect can be illustrated by a particular participation constraint. With a positive fraction of prices preset, the participation constraint under a high current-period inflation signal (26) is modified to the following

$$u\left\{ e^{\beta (1 - \kappa) V_{rs} [\Delta (\pi_t, s_k), \pi_k]} + \frac{\beta^2}{1 - \beta} V_{rs} \right\} + u\left\{ e^{\beta V_{out} [\Delta (\pi_t, s_k), \pi_k]} + \frac{\beta^2}{1 - \beta} V_{out} \right\}$$

(33)

where $s_k$ is the previous-period inflation signal, $\pi_j$ is the current-period inflation and $V_{rs}$ and $V_{out}$ are the expected value of the arrangement and the expected value of the outside option, respectively. An increase in precision distorts risk-sharing opportunities when risk sharing is partial. On the other hand, it allows sticky-price firms to set their prices more accurately, thereby resulting in less relative price distortions and a better allocation of resources. As an extreme, if the socially optimal arrangement is either the outside option or perfect risk sharing, then the expected utility of households is increasing in signal precision.

We compute social welfare in two steps. First, for any given precision we calculate prices and production by solving the problems of final good firms (18) and monopolistically competitive firms (19) and (20). Second, taking the resources available for consumption as given, we derive the value of the outside option and compute the optimal consumption allocation according to (25)–(28).

4.2. Quantitative Assessment: Imperfectly Flexible Prices

We set up a numerical example to assess the effect of public announcements quantitatively. The baseline is constructed to match stylised facts for the US economy on an annual basis. We calibrate the inflation process to the US consumer price index from 1980 to 2007, which results in two states with 1.3% and 6.4% inflation rates, respectively. We normalise the mean of the logarithm of the productivity process to zero and set its variance, $\sigma_y^2$, equal to 0.1849, which is the idiosyncratic variance of the transitory component of log-labour income before taxes for the US between 2000 and 2007. This figure was estimated by Krueger and Perri (2011) using micropanel data provided by the US Consumer Expenditure Survey. Throughout the example, we employ standard preferences that feature constant relative risk aversion and we calibrate the elasticity of substitution between differentiated goods to a value of six, following Woodford (2003). The fraction of firms that do not adjust their prices within
one year is set to 13%, which is the value found by Klenow (2004) using US data for 1995–7 that were collected by the Bureau of Labor Statistics. For any combination of parameters, we adjust the discount factor to keep the insurance coefficient, \(1 - \text{var}[\log(c^*)]/\sigma^2\), equal to 0.61 under informative signals, which is the value calculated for the US in 2003 by Krueger and Perri (2006). Using this calibration strategy, we find that all participation constraints of high-income households are binding at the optimal arrangement.

We measure the social value of policy announcements as the percentage difference in certainty-equivalent consumption between uninformative and perfectly informative signals. In other words, this measure captures the percentage amount of annual consumption the households are willing to give up under uninformative announcements to be indifferent between uninformative and perfectly informative announcements.

We find that the optimal announcements are either no announcement (\(\kappa = 1/2\)) or perfect announcements (\(\kappa = 1\)). The negative effect of information dominates for any coefficient of relative risk aversion that exceeds 4.53, which is not an unreasonably high value of the coefficient.\(^{15}\) For this degree of risk aversion, if prices were flexible, households would be willing to give up 0.026% of their annual consumption to be indifferent between uninformative and informative signals.

Fig. 3. The Welfare Gain of Uninformative Signals Relative to Perfectly Informative Signals Expressed in Percentage of Certainty-equivalent Consumption as a Function of Risk Aversion

\(^{15}\) There is quite a controversy about the magnitude of the constant risk aversion coefficient (Mehra and Prescott, 1985; Kocherlakota, 1996a; Campbell, 2003). Kocherlakota (1996a) summarised the prevailing view ‘that a vast majority of economists believe that values for (the coefficient of relative risk aversion) above ten (or, for that matter above five) imply highly implausible behaviour on part of the individuals’.
The result is illustrated in Figure 3 where the social value of information is shown as a function of risk aversion for three different fractions of preset prices \((1 - \lambda)\), including the fraction of 13%, which is our baseline value. When a larger fraction of prices is adjusted more frequently, the social value of information becomes negative for even lower degrees of risk aversion (see the dashed line for \(1 - \lambda = 0.05\) in Figure 3).

Over the last three decades, the US has experienced a substantial increase in income inequality (Gottschalk and Moffitt, 2002; Krueger and Perri, 2006). We capture this evidence by an increase in the variance of the income process, \(\sigma^2_y\), which results from the employment in sectors of different productivity. How does this increase in income inequality affect the trade-off between the destruction of insurance possibilities on one hand and the better allocation of resources on the other hand when policy announcements become more precise? For this exercise, we set the coefficient of relative risk aversion to 4.53 to imply that the positive and negative effect of more precise information cancel out for the average of the idiosyncratic variance in the US between 2000 and 2007. Employing our baseline calibration, we find that for the variance of the idiosyncratic income shocks of \(\sigma^2_y = 0.15\), the social value of information is negative for a fraction of preset prices smaller than 10%. This result is illustrated in Figure 4. For an income inequality of \(\sigma^2_y = 0.22\), a secretive inflation target is desirable unless the fraction of prices preset for one year exceeds 16%. In short, a further increase in income inequality in the US strengthens the negative effect rather than the positive effect of information.

We proceed further by conducting and discussing three robustness checks: allowing for self-insurance, lowering the penalty for default and enriching the information structure.

Fig. 4. The Welfare Gain of Uninformative Signals Relative to Perfectly Informative Signals Expressed in Percentage of Certainty-equivalent Consumption as a Function of the Fraction of Prices Preset

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4.2.1. Self-insurance
The negative effect of information on social welfare is amplified when we allow for self-insurance. We introduce this possibility with the cash-in-advance constraints written as inequalities, that is, $p_t x_t \leq M_t$ instead of (15). With these constraints, households can save money for purchases in future periods. However, we find that the households optimally choose not to save in the optimal arrangement. Both self-insurance and voluntary transfers facilitate consumption insurance but self-insurance is associated with the burden of inflation costs, and therefore, the households find it inferior. Nonetheless, the cash-in-advance constraints written as inequalities do influence the optimal arrangement through the outside option. When deciding about participation in a risk-sharing arrangement, households take into account that self-insurance increases the value of their outside option.16

Quantitatively, self-insurance significantly amplifies the negative effect of information. Employing our baseline calibration, the negative effect of information now dominates the positive effect for degrees of relative aversion higher than 2.51 (compared with 4.53 in the baseline model). After fixing the degree of relative risk aversion to the latter value, the effect of distorted insurance arrangements prevails unless the fraction of preset prices exceeds 39%. For any given discount factor, the value of the outside option is higher under self-insurance and the high-income households have smaller incentives to share with the low-income households. Consequently, the optimal arrangement is worse from an \textit{ex ante} perspective. For our baseline calibration with flexible prices and coefficients of risk aversion ranging from one to five, the utility loss can add up to 6% measured in consumption equivalents. This result implies a larger degree of consumption dispersion between high and low-income households and the marginal gain of redistribution that can be achieved by uninformative signals is now higher than is possible in the absence of self-insurance.

4.2.2. Weaker penalty for default
Qualitatively similar results hold when the penalty for default is decreased, that is, the value of the outside option is higher. To illustrate this property, we compute the social value of information when households are allowed to reengage in social insurance after one period instead of living forever in the equilibrium without transfers. The corresponding participation constraint for a high-inflation signal (33) is modified to the following

\[
\begin{align*}
\min & \quad c^n[M(\pi_j, s_h), \pi_j] + \beta \kappa V_{nx} [M(\pi_j, s_h), \pi_j] + \beta (1 - \kappa) V_{nx} [\Delta(\pi_j, s_h), \pi_j] \\
\text{subject to} & \quad c^n[M(\pi_j, s_h), \pi_j] + \beta \kappa V_{out} [\Delta(\pi_j, s_h), \pi_j] + \beta (1 - \kappa) V_{out} [\Delta(\pi_j, s_h), \pi_j].
\end{align*}
\]

Though qualitatively similar to our standard case in which agents are not allowed to reengage in risk-sharing arrangements, the results differ quantitatively. Under a lower penalty for default, the negative aspect of information dominates the positive effect,

\[\text{16 The value of the outside option follows from an optimisation problem. To get accurate solutions for this optimal self-insurance problem we iterate on the value function subject to the cash-in-advance constraints formulated as inequalities.}\]
even for lower degrees of risk aversion and even when the idiosyncratic income risk is lower (see Figures B.1 and B.2 in the Appendix). For example, when the fraction of preset prices equals the value found by Bils and Klenow (2004), the negative effect of information outperforms the positive effect for degrees of risk aversion greater than 4.24 or for a fraction of preset prices below 15%.

4.2.3. Information structure
The welfare gain of uninformative announcements is robust to changes in information structure, in particular, in which public information affects precision of heterogeneous estimates devised by agents. In the model considered, households are homogeneous in their estimates of the future value of the arrangement and of the outside option. Here, we exploit one possibility that leads to heterogeneous estimates: a private source of information on changes in the inflation target.

Suppose that in addition to the public signal, each household has access to a private source of information on the future inflation target. Similar to the public signal, the private signal can take one of two values and it has a certain precision. Following the standard practice in the literature on global games (Angeletos and Pavan, 2007), we consider the decentralised information optimum, that is, the optimal arrangement contingent on both private and public signals. Evidently, if the private signals are completely uninformative the value of public information is the one found in the baseline. As in the case of a public signal, the more informative is the private signal, the less will ing are high-income households to share with the others, resulting in a riskier allocation ex ante. The informative private signals thus reduce the risk-sharing possibilities. For private signals with precision 0.6 and 0.7, we obtain that the cut-point for the coefficient of relative risk-aversion that renders the value of public information negative increases from 4.53 to 4.57 and 4.71, respectively.

4.3. Quantitative Assessment: Aggregate Fluctuations
As an alternative to the introduction of a positive effect of information due to price stickiness, the negative effect of policy announcements on future inflation targets can be evaluated relative to a benchmark. Following Lucas (2003), we choose the well-studied welfare gain from complete removal of aggregate fluctuations as a benchmark. For the baseline calibration with flexible prices, we first compute social welfare with stochastic inflation under perfectly informative and uninformative signals. Second, we calculate the socially optimal arrangement with constant inflation, which is set to the mean of the stochastic inflation process in the baseline. We find that the welfare gain of completely uninformative signals relative to perfectly informative signals (measured in certainty equivalent consumption) is in the range of 4–62% of the welfare gain of constant inflation relative to stochastic inflation with perfectly informative signals. The results are displayed in Table 1 for a number of values for the coefficient of relative risk aversion up to 50, which is the value implied by the risk-premium puzzle (Mehra and Prescott, 1985). For a degree of relative risk aversion of five, providing uninformative signals on aggregate fluctuations reduces the welfare difference to the complete removal of aggregate fluctuations by 14%.

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5. Conclusion

Our primary message is that more precise public information on risks, which are common to all agents, may harm social welfare. By revealing information on future realisations of the aggregate risk, the policy maker distorts households’ insurance incentives and thereby increases the riskiness of the optimal consumption allocation that is consistent with rational participation incentives. In contrast to Morris and Shin (2002), we show that the social value of information can be negative even when individual preferences and social welfare coincide.

We show that more informative policy announcements can lead to a less diversified consumption allocation. This theoretical result is in line with recent developments in the US economy: the dramatic shift of the Fed’s disclosure policies towards greater transparency and the increase in consumption inequality as, for example, documented by Krueger and Perri (2006). Moreover, standard models with voluntary participation systematically understate the rise in consumption inequality in response to the increase in income inequality. If the change in the information policy is taken into account as in our model, the rise in consumption inequality is less puzzling.

The effect we describe is one additional channel through which public announcements affect social welfare. While we focus on monetary policy announcements, the effect is relevant for any announcements of public policy and should be considered by policymakers. The size of the effect for each particular application is the subject of assessment and further research.

Appendix A

A.1. Proof of Proposition 1

The cutoff point for $\beta$ is characterised by the tightest participation constraint, which is the one that becomes binding for any level of patience below the cutoff point. Among the participation constraints, only constraints for high-income households can be binding, which limits our consideration to four cases.

There are two factors that determine the tightest participation constraint: the relative gain of deviation from perfect risk sharing to the outside option for high-income households and the expected future gain of the perfect risk-sharing arrangement relative to the outside option. Without loss of generality, assume that for any precision of the signal, the participation constraint of high-income households under high current inflation and a low future inflation signal is the one that imposes the tightest restriction. This is the case when the following conditions hold

$$u[x^h(\pi_i)] - u[\bar{x}(\pi_i)] \leq u[x^h(\pi_h)] - u[\bar{x}(\pi_h)],$$  \hspace{1cm} (A.1)
where \(\bar{x}(\pi_j) \equiv [(x^h(\pi_j) + x^l(\pi_j))/2 \) is the perfect risk-sharing allocation. The first inequality (A.1) states that the current-period deviation for a high-income household is more attractive in the high-inflation state. The second inequality (A.2) implies that the perfect risk-sharing arrangement provides a higher expected value when compared with the outside option under high inflation. The tightest participation constraint holds with equality at the cutoff point:

\[
\begin{align*}
&u[\bar{x}(\pi_h)] - u[x^h(\pi_h)] + \beta k[V_{n}(\pi_j) - V_{out}(\pi_j)] \\
&+ \frac{\beta}{1 - \beta} (V_{r}(\pi_h) - V_{out}(\pi_h)) = 0,
\end{align*}
\]

(A.3)

where \(V_{r}(\pi_j) = u[\bar{x}(\pi_j)]\) and \(V_{r} = \{u[\bar{x}(\pi_h)] + u[\bar{x}(\pi_l)]\}/2\).

Solving (A.3), there exists a unique solution for \(\beta\) in (0,1) because \(u[x^h(\pi_h)] > u[\bar{x}(\pi_h)]\).

Employing the implicit function theorem, from (A.3) we get the following expression

\[
\begin{align*}
\frac{d\beta}{dk} &= \frac{\beta(1 - \beta)\bar{V}_{r}(\pi_h) - V_{out}(\pi_h) - \bar{V}_{r}(\pi_j) + V_{out}(\pi_j)}{u[x^h(\pi_h)] - u[\bar{x}(\pi_h)] + dV(k) + 2\beta(dV(1/2) - dV(k))},
\end{align*}
\]

where \(dV(k) = \kappa[V_{r}(\pi_j) - V_{out}(\pi_j)] + (1 - \kappa)\bar{V}_{r}(\pi_h) - V_{out}(\pi_h)\) and satisfies \(0 \leq dV(k) \leq dV(1/2)\).

### A.2. Proof of Theorem 2

Suppose \(V_{r}(\kappa_1) \leq V_{r}(\kappa_2)\) for some \(\kappa_1 < \kappa_2\). Consider an alternative consumption allocation \(\{\bar{c}^i(\pi_j, s_k, \kappa_0)\}\) for signal precision \(\kappa_1\) constructed on the basis of the optimal allocation \(\{c^i(\pi_j, s_k, \kappa_1)\}\) for \(\kappa_2\) according to the following

\[
\begin{align*}
&u[\bar{c}^h(\pi_j, s_k, \kappa)] = -\beta\{k[V_{r}(\pi_h, \kappa_2) - V_{out}(\pi_h)] + (1 - \kappa)[V_{r}(\pi_l, \kappa_2) - V_{out}(\pi_l)]\} \\
&\quad + u[x^h(\pi_j)] - \frac{\beta^2}{1 - \beta} [V_{r}(\kappa_2) - V_{out}],
\end{align*}
\]

(A.4)

\[
\begin{align*}
&u[\bar{c}^l(\pi_j, s_k, \kappa)] = -\beta\{(1 - \kappa)[V_{r}(\pi_h, \kappa_2) - V_{out}(\pi_h)] + k[V_{r}(\pi_l, \kappa_2) - V_{out}(\pi_l)]\} \\
&\quad + u[x^l(\pi_j)] - \frac{\beta^2}{1 - \beta} [V_{r}(\kappa_2) - V_{out}]
\end{align*}
\]

(A.5)

and the corresponding allocation for low-income households given by consumption feasibility. \(V_{r}(\pi_j, \kappa_2)\) and \(V_{r}(\kappa_2)\) characterise the optimal allocation for \(\kappa_2\).

First, the alternative allocation \(\{\bar{c}^i(\pi_j, s_k, \kappa_1)\}\) is consumption feasible by construction. Second, the alternative allocation \(\{\bar{c}^i(\pi_j, s_k, \kappa_1)\}\) delivers strictly higher expected utility than the optimal allocation for signal precision \(\kappa_2\), that is, \(V_{r}(\kappa_1) > V_{r}(\kappa_2)\), where \(V_{r}(\kappa) \equiv \sum_{s_k, h} u[\bar{c}^i(\pi_j, s_k, \kappa)]\).

We prove this result by showing that high-income households are indifferent between the optimal allocation and the alternative allocation; low-income households strictly prefer the alternative allocation.

For signal precision \(\kappa_2\), the outside option is not the only sustainable arrangement. Without loss of generality, suppose that the relative expected value of the arrangement is higher under high inflation, that is, \(V_{r}(\pi_1, \kappa_2) - V_{out}(\pi_1) < V_{r}(\pi_1, \kappa_2) - V_{out}(\pi_h)\). Subtracting (A.5) from (A.4), we obtain the following

\[
\begin{align*}
&u[\bar{c}^h(\pi_j, s_k, \kappa)] - u[\bar{c}^h(\pi_j, s_k, \kappa)] = (2\kappa - 1)\beta\{[V_{r}(\pi_1, \kappa_2) - V_{out}(\pi_1)] - [V_{r}(\pi_h, \kappa_2) - V_{out}(\pi_h)]\}.
\end{align*}
\]

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Therefore, for any \( \kappa < \kappa_2 \), the following is true:

\[
u[\epsilon^h(\pi_j, s_k, \kappa_2)] < V_l(\pi_j, s_k, \kappa) \leq u[\epsilon^h(\pi_j, s_l, \kappa)] < u[\epsilon^h(\pi_j, s_1, \kappa_2)].\] (A.6)

For high-income households, the alternative allocation for \( \kappa_1 \) and the optimal allocation for \( \kappa_2 \) deliver the same expected utility in any state \( \pi_j \), as demonstrated by adding (A.4) and (A.5). For low-income households, the expected utility in state \( \pi_j \), defined by the following

\[
V_l(\pi_j, \kappa) \equiv \frac{1}{2} \sum_k u[\epsilon^h(\pi_j, s_k, \kappa)]
\]

is strictly decreasing in precision over \( \kappa \leq \kappa_2 \). This result follows from (A.4)–(A.6), the consumption feasibility and the risk aversion of agents:

\[
\frac{\partial \hat{V}^l(\pi_j, \kappa)}{\partial \kappa} = -\frac{1}{2} \left\{ u'[\hat{\epsilon}^h(\pi_j, s_k, \kappa)] - u'[\epsilon^h(\pi_j, s_l, \kappa)] \right\} \times 
\beta \left\{ V^l_n(\pi_j, \kappa_2) - V^l_{out}(\pi_j) \right\} < 0 \quad \forall \kappa > 1/2,
\]

and \( \partial \hat{V}^l(\pi_j, 1/2)/\partial \kappa = 0 \). In particular, this expression implies that \( V^l_n(\pi_j, \kappa_1) > V^l_n(\pi_j, \kappa_2) \) and, therefore, \( V^l_n(\kappa_1) > V^l_n(\kappa_2) \).

Third, the alternative allocation \( \{\epsilon^r(\pi_j, s_k, \kappa)\} \) satisfies the participation constraints for signal precision \( \kappa_1 \). This result follows immediately from construction of the alternative allocation and from the finding that the alternative allocation for \( \kappa_1 < \kappa_2 \) provides strictly higher utility in all inflation states than the optimal allocation for \( \kappa_2 \).

Finally, the social value of the optimal allocation for \( \kappa_1 \) is at least as large as for any other feasible allocation compatible with the participation constraints \( V^l_n(\kappa_1) \geq V^l_n(\kappa_1) \). Therefore, \( V^l_n(\kappa_1) > V^l_n(\kappa_2) \) is a contradiction to \( V^l_n(\kappa_1) \leq V^l_n(\kappa_2) \).

**Appendix B**

**B.1. Additional Details for the Proof of Theorem 1**

First, consider the perfect information environment in which all participation constraints for high-endowment agents are binding. The Lagrangian of the optimal risk-sharing problem can be written as the following expression

\[
\mathcal{L} = \frac{1}{4} \left( u[\epsilon^h_{1g}] + u[\epsilon^h_{2g}] + u(y^h) + u(y^l) \right) \]

\[
+ \frac{\beta}{8} \left[ u(\epsilon^h_{1g}) + u(\epsilon^h_{2g}) + u(\epsilon^l_{2g}) + u(\epsilon^l_{2g}) + 4u(0) \right] \]

\[
+ \mu_1 \left( \bar{y} - \frac{1}{2}(\epsilon^h_{1g} + \epsilon^h_{2g}) \right) + \mu_2 \left( \bar{y} - \frac{1}{4} (\epsilon^h_{2g} + \epsilon^l_{2g} + \epsilon^l_{2g}) \right) \]

\[
+ \lambda \left\{ u(\epsilon^h_{1g}) + \frac{\beta}{2} \left[ u(\epsilon^h_{2g}) + u(\epsilon^l_{2g}) \right] - u(y^h) - \frac{\beta}{2} [u(y^h) + u(y^l)] \right\},
\]

where \( \mu_1, \mu_2 \) and \( \lambda \) denote the Lagrange multipliers, \( \bar{y} \equiv (y^h + y^l)/2 \), and with consumption under type-\( \beta \) policy already substituted in.

The first-order conditions with respect to \( \epsilon^h_{1g}, \epsilon^h_{2g} \) and \( \epsilon^l_{2g} \) are given by the following

\[
\frac{1}{4} u'(\epsilon^h_{1g}) - \mu_1 \frac{1}{2} + \lambda u'(\epsilon^h_{1g}) = 0, \quad (B.1)
\]
\[ \frac{\beta}{8} u'(c_{1g}^h) - \mu_2 \frac{1}{4} + \lambda \frac{\beta}{2} u'(c_{1g}^h) = 0, \quad (B.2) \]

\[ \frac{\beta}{8} u'(c_{2g}^h) - \mu_2 \frac{1}{4} + \lambda \frac{\beta}{2} u'(c_{2g}^h) = 0, \quad (B.3) \]

and with respect to \( c_{1g}^h, c_{2g}^h \) and \( c_{2g}^h \) are the following equations

\[ \frac{1}{4} u'(c_{1g}^h) - \mu_1 \frac{1}{2} = 0, \quad (B.4) \]

\[ \frac{\beta}{8} u'(c_{2g}^h) - \mu_2 \frac{1}{4} = 0, \quad (B.5) \]

\[ \frac{\beta}{8} u'(c_{2g}^h) - \mu_2 \frac{1}{4} = 0. \quad (B.6) \]

The first-order conditions imply the following. First, the optimal consumption in the second period is independent of the second period endowment realisation. From (B.2) and (B.3), we immediately obtain \( c_{1g}^h = c_{2g}^h = c_{2g}^h \). Similarly, from (B.5) and (B.6) we get \( c_{2g}^h = c_{2g}^h = c_{2g}^h \).

Second, agents prefer to smooth consumption between the first and the second period. Substituting the Lagrange multipliers from (B.4) and (B.5) into (B.1) and (B.2), and combining with the resource constraints we obtain the following

\[ \frac{u'(c_{1g}^h)}{u'(y^h + y^l - c_{1g}^h)} = \frac{u'(c_{2g}^h)}{u'(y^h + y^l - c_{2g}^h)} \]

From the two steps we conclude that the socially optimal consumption of the high-endowment agents is constant over time:

\[ c_{1g}^h = c_{2g}^h = c_{2g}^h \]

except for type-\( b \) policy when in the second period all goods are taxed away. When the high-endowment agent participation constraints (2) and (3) are binding, from the participation constraints we obtain the following expression for the optimal consumption

\[ u(c_{1g}^h) = \frac{1 + \beta/2}{1 + \beta} u(y^h) + \frac{\beta/2}{1 + \beta} u(y^l). \quad (B.7) \]

Second, in the imperfect information environment the Lagrangian is given by the following

\[ \mathcal{L} = \frac{1}{2} \left( u(c_{1g}^h) + u(c_{1g}^l) \right) \]

\[ + \frac{\beta}{8} \left[ u(c_{1g}^h) + u(c_{2g}^h) + u(c_{2g}^l) + u(c_{2g}^l) + 4u(0) \right] \]

\[ + \mu_1 \left[ \tilde{y} - \frac{1}{2} (c_{1g}^h + c_{1g}^l) \right] + \mu_2 \left[ \tilde{y} - \frac{1}{4} (c_{1g}^h + c_{2g}^h + c_{1g}^l + c_{2g}^l) \right] \]

\[ + \lambda \left\{ u(c_{1g}^l) + \frac{\beta}{4} \left[ u(c_{1g}^l) + u(c_{2g}^l) + 2u(0) \right] - u(y^h) - \frac{\beta}{4} \left[ u(y^h) + u(y^l) + 2u(0) \right] \right\} \]

Similarly to perfect information, it follows from the first-order conditions that

\[ c_{1g}^h = c_{2g}^h = c_{2g}^h \]

and when participation constraint (7) is binding the optimal consumption is expressed by the following

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\[
\begin{align*}
\tilde{u}(\tilde{c}_i^h) &= \frac{1 + \beta/4}{1 + \beta/2} u(y^h) + \frac{\beta/4}{1 + \beta/2} u(y').
\end{align*}
\] (B.8)

Comparing (B.7) and (B.8), reveals that \(u(\tilde{c}_i^h) < u(c_i^h) < u(y^h)\), and taking into account \(c_i^h = y^h\) we obtain \(\tilde{c}_i^h < c_i^h < c_i^h\).

We are left to consider two cases where not all participation constraints for high-endowment agents are binding.

When the participation constraints in the environment of imperfect information are not binding, the optimal allocation in this environment is perfect risk sharing. This outcome is preferable to the one under perfect information where the first best is not incentive compatible because the participation constraints for type-\(b\) policy (3) and (5) always hold with equality.

In an intermediate case when the participation constraints for high first period endowment agents under type-\(g\) policy (2) are not binding but the participation constraints for high-income agents in the completely uninformative environment (7) do bind, imperfect information is still preferable. As agents become more patient the first period allocation for perfect information cannot be improved upon, but under imperfect information social welfare is still increasing towards the first best.

**B.2. Proof of Lemma 1**

Consider the optimal arrangement design problem in recursive formulation (25)–(28) and let \(S(\kappa) \subset \mathbb{R}_+^n\) be the set of sustainable consumption allocations. The outside option is always in the set of sustainable allocations, and the restrictions imposed by the participation constraints (26), (27) and consumption feasibility (28) define a bounded and closed set. Therefore, for any precision of the public signal, \(S(\kappa)\) is non-empty and compact-valued. Furthermore, the constraints are linear in \(\kappa\) as well as continuous in consumption, therefore the correspondence that maps \(\kappa \mapsto S(\kappa)\) is continuous. Given that the objective function (25) is also continuous, by the theorem of the maximum there exists a solution to the optimal arrangement problem for any public signal precision and the expected utility of the socially optimal arrangement is continuous in signal precision. In addition, the set of sustainable allocations is convex-valued due to concavity of the utility function, and the objective function is strictly concave. By the maximum theorem under convexity the optimal arrangement is unique and continuous in signal precision.

Next, we show that the optimal arrangement satisfies \(c_i^h(\pi_j, s_k) \geq c_i^h(\pi_j, s_k)\) in all states. By contradiction, assume there exists a state \((\pi_j, s_k)\) such \(c_i^h(\pi_j, s_k) < c_i^h(\pi_j, s_k)\). Let the perfect risk sharing allocation be defined as \(\tilde{\pi}(\pi_j) \equiv [x^h(\pi_j) + x' \pi_j])/2\), and consider an arrangement \(\{\tilde{c}^i(\pi_j, s_k)\}\) defined by

\[
\begin{align*}
\tilde{c}^h(\pi_j, s_k) &= \tilde{c}^i(\pi_j, s_k) = \tilde{\pi}(\pi_j), \\
\tilde{c}^i(\tilde{H}) &= c_i(\tilde{H}),
\end{align*}
\]

where \(\tilde{H}\) is the set of all states excluding \((\pi_j, s_k)\). By construction the arrangement \(\{\tilde{c}^i(\pi_j, s_k)\}\) provides strictly higher utility for risk-averse households than the optimal arrangement \(\{c_i(\pi_j, s_k)\}\). In order to claim a contradiction, we are left to prove that the arrangement \(\{\tilde{c}^i(\pi_j, s_k)\}\) is sustainable. High-income households undoubtedly accept the arrangement because it delivers both higher current period consumption and higher future arrangement utility. In state \((\pi_j, s_k)\), low-income households accept the arrangement because high-income households accept it. In this state, the outside option is worse for low-income households, and the arrangement prescribes the same utility to them as for the high-income households. In other states, the arrangement is also sustainable for low-income households because of higher future utility. In short, the arrangement \(\{\tilde{c}^i(\pi_j, s_k)\}\) is sustainable and socially preferable over \(\{c_i(\pi_j, s_k)\}\), which contradicts that \(\{c_i(\pi_j, s_k)\}\) is the socially optimal arrangement.
Finally, we show that for any state $c^k(\pi_j, s_k) \leq x^k(\pi_j)$. Again, by contradiction, assume that there is a state such that $c^k(\pi_j, s_k) > x^k(\pi_j)$. If the participation constraint for the high-income household under $\pi_j$ inflation and $s_k$ signal holds with equality, then the future value of the arrangement is lower than the outside option value, and taking into account that for the low-income households from the resource constraint $c(\pi_j, s_k) < x^k(\pi_j)$ the participation constraints for the low-income households are violated. Therefore, the considered participation constraints for the high-income households can only hold with a strict inequality. Then, consider a consumption allocation $\{\tilde{c}(\pi_j, s_k)\}$ given by the following

$$\tilde{c}^k(\pi_j, s_k) = c^k(\pi_j, s_k) - \epsilon, \quad \tilde{c}(\pi_j, s_k) = c(\pi_j, s_k) + \epsilon, \quad \tilde{c}(\tilde{H}) = c'(\tilde{H}).$$

By continuity there exists $\epsilon > 0$ such that the consumption allocation $\{\tilde{c}(\pi_j, s_k)\}$ is sustainable. By concavity, the constructed allocation provides higher utility then the allocation $\{c'(\pi_j, s_k)\}$ which contradicts that $\{c'(\pi_j, s_k)\}$ is the socially optimal arrangement.

### B.3. Proof of Lemma 2

First, perfect risk sharing provides the highest ex ante utility among the consumption-feasible allocations. The existence of $\beta$ follows from monotonicity of participation constraints in $\beta$ and $\tilde{V}_n > V_{out}$, where $\tilde{V}_n$ is the value of the perfect risk-sharing arrangement. In the participation constraints (26) and (27), a higher $\beta$ increases the future value of perfect risk sharing relative to the allocation in the equilibrium without transfers, leaving the current incentives to deviate unaffected. Therefore, if the participation constraints are not binding for $\tilde{\beta}$, they are not binding for any $\beta \geq \tilde{\beta}$.

Second, assume that for some $\beta < \tilde{\beta}$, there exists an optimal arrangement different from the equilibrium allocation with no transfers. The arrangement allocation is sustainable. By Lemma 1, the value of this arrangement is at least as high as the value of defecting into the outside option for any inflation state. Then for $\tilde{\beta}$ the allocation is also sustainable since the value of the arrangement other than the outside option gets an even higher weight in the participation constraints. This contradicts that for $\tilde{\beta}$ the optimal arrangement is the no-transfer equilibrium allocation.

### B.4. Proof of Lemma 3

First, we show that if participation constraints are violated under perfect risk sharing for any state then the optimal consumption allocation satisfies the following

$$c^k(\pi_j, s_k) > \tilde{x}(\pi_j) > x^k(\pi_j, s_k),$$

where $\tilde{x}(\pi_j)$ again is the perfect risk-sharing allocation in inflation state $\pi_j$. Without loss of generality, consider the participation constraint for high-income households under currently high inflation that receive a high signal on future inflation. The constraint holds for the socially optimal arrangement and is given by the following expression

$$u[c^k(\pi_h, s_h)] + \beta[kV_n(\pi_h) + (1 - k)V_n(\pi_l)] + \frac{\beta^2}{1 - \beta}V_n \geq u[x^k(\pi_h)] + \beta[kV_{out}(\pi_h) + (1 - k)V_{out}(\pi_l)] + \frac{\beta^2}{1 - \beta}V_{out},$$

but the constraint is violated by assumption under perfect risk sharing. Formally, the following holds

$$u[\tilde{x}(\pi_h)] + \beta[ku[\tilde{x}(\pi_h)] + (1 - k)u[\tilde{x}(\pi_l)] + \frac{\beta^2}{1 - \beta}V_n < u[x^h(\pi_h)] + \beta[kV_{out}(\pi_h) + (1 - k)V_{out}(\pi_l)] + \frac{\beta^2}{1 - \beta}V_{out},$$

where $\bar{V}_n \equiv u[\tilde{x}(\pi_h)] + u[\tilde{x}(\pi_l)]/2$ denotes the expected value of the perfect risk-sharing arrangement. The right-hand side of (B.10) or (B.11) represents the total value of the outside option. Combining (B.10) and (B.11), we obtain the following

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\[ u[\tilde{e}(\pi_h, s_h)] + \beta \{ \kappa V_n(\pi_h) + (1 - \kappa) V_n(\pi_i) \} + \frac{\beta^2}{1 - \beta} V_n > u[\tilde{x}(\pi_h)] + \beta \{ \kappa u[\tilde{x}(\pi_h)] + (1 - \kappa) u[\tilde{x}(\pi_i)] \} + \frac{\beta^2}{1 - \beta} \tilde{V}_n. \] (B.12)

Taking into account that the optimal arrangement delivers a value no larger than the value of perfect risk sharing, \( V_n(\pi_j) \leq u[\tilde{x}(\pi_j)] \forall \pi_j \) and \( V_n \leq \tilde{V}_n \), from (B.12) we obtain the following inequality

\[ u[\tilde{e}(\pi_h, s_h)] > u[\tilde{x}(\pi_h)]. \]

Combining this finding with resource feasibility results in the following

\[ \tilde{e}(\pi_h, s_h) > \tilde{x}(\pi_h) > \tilde{e}(\pi_h, s_h). \]

Similarly, we can show that the same inequalities hold for the other public states.

Second, by contradiction, assume that there is one participation constraint for high-income households that is not binding. The Lagrangian of the optimal contract problem (25)–(28) can be written as the following

\[ \mathcal{L} = \left[ 1 + \sum_{(\pi_j, s_k) \in \tilde{H}} \lambda(\pi_j, s_k) \right] \left\{ u[\tilde{e}(\pi_j, s_k)] + u[\tilde{e}(\pi_j, s_k)] \right\} + \mu(\pi_j, s_k) \left[ \tilde{e}(\pi_j, s_k) + \tilde{e}(\pi_j, s_k) - 2 \tilde{x}(\pi_j) \right] + \zeta(H), \] (B.13)

where \((\pi_j, s_k)\) is the state for which the participation constraint is not binding, \( \tilde{H} \) is the set of all possible states, excluding \((\pi_j, s_k)\), \( \lambda(\pi_j, s_k) \) are the normalised Lagrange multipliers for the participation constraints, \( \mu(\pi_j, s_k) \) is the Lagrange multiplier for the resource constraint and \( \zeta(H) \) collects consumption and resources for states in \( \tilde{H} \), and respective multipliers. The Lagrange multiplier for the participation constraint for state \((\pi_j, s_k)\) is zero and is explicitly excluded from the Lagrangian. Employing the Lagrangian (B.13), the socially optimal arrangement is characterized by the following optimality condition

\[ \tilde{e}(\pi_j, s_k) = \tilde{e}(\pi_j, s_k) = \tilde{x}(\pi_j) \]

for the state \((\pi_j, s_k)\), which contradicts the partial risk-sharing condition (B.13) stated above.

**B.5. Proof of Lemma 4**

The socially optimal risk sharing arrangement under uninformative signals can be analysed as a fixed-point problem expressed in terms of the value of arrangement.

The fixed-point problem is constructed as follows. Let \( w \) be the expected value of an arrangement. We restrict attention to \( w \in [V_{out}, \tilde{V}_n] \) because per assumption all participation constraints for high-income households are binding. The binding participation constraints are given by the following

\[ u[\tilde{e}(\pi_j)] + \frac{\beta}{1 - \beta} w = u[\tilde{x}(\pi_j)] + \frac{\beta}{1 - \beta} V_{out} \quad \forall j, \] (B.14)

and consumption feasibility is given by the following

\[ \tilde{e}(\pi_j) + \tilde{e}(\pi_j) = \tilde{x}(\pi_h) + x(\pi_h) \quad \forall j. \] (B.15)

The objective function of the optimal arrangement problem is given by the following expression

\[ V(w) \equiv \frac{1}{4} \sum_{j, h} u[\tilde{e}(\pi_j)]. \]

The optimal arrangement should necessarily solve the fixed-point problem \( w = V(w) \).
We show that $V(w)$ is strictly increasing and strictly concave, therefore there are at most two solutions to the fixed-point problem. From the participation constraints (B.14) and from the consumption feasibility constraints (B.15), it follows that $V(w)$ is strictly increasing with $w$:

$$V'(w) = \frac{\beta}{1 - \beta} \left\{ -2 + \frac{u'[c'(\pi_h)]}{u'[c'(\pi_l)]} + \frac{u'[c'(\pi_l)]}{u'[c'(\pi_0)]} \right\} > 0,$$

because perfect risk sharing is not sustainable by assumption. Strict concavity of $V(w)$ is implied by the following:

$$\frac{d}{dw} \left\{ \frac{u'[c'(\pi_j)]}{u'[c'(\pi_j)]} \right\} = \frac{\beta}{1 - \beta} \left\{ \frac{1}{u'[c'(\pi_j)]} \right\} < 0.$$

By construction, one solution to the fixed-point problem is $V_{out}$. The concavity of $V(w)$ implies that the derivative of $V(w)$ at $V_{out}$ is higher than at any partial risk-sharing allocation. Therefore, the derivative of $V'(w)$ at $V_{out}$ must be greater than 1 which implies

$$\frac{1}{2} \left\{ \frac{u'[c'(\pi_h)]}{u'[c'(\pi_h)]} + \frac{u'[c'(\pi_l)]}{u'[c'(\pi_l)]} \right\} > \frac{2 - \beta}{\beta}.$$

Then, the optimal arrangement is different from the outside option.

From the other end, suppose there exists a socially optimal arrangement different from the allocation in the absence of transfers and participation constraints are binding. Then, the value of this arrangement must be a solution to the fixed-point problem. This requires that the slope of $V(w)$ at $V_{out}$ must be necessarily larger than unity, due to the concavity of $V(w)$ and because the allocation in the absence of transfers must always be one solution of the constructed fixed-point problem.

B.6. Figures in Case of Re-engagement in Social Insurance

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![Figure B.1](image-url)

Fig. B.1. *The Welfare Gain of Uninformative Signals Relative to Perfectly Informative Signals as a Function of Risk Aversion when Households are Allowed to Re-engage in Social Insurance After One Period*
Fig. B.2. The Welfare Gain of Uninformative Signals Relative to Perfectly Informative Signals as a Function of the Fraction of Prices Preset when Households are Allowed to Re-engage in Social Insurance After One Period

References


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