Search costs, demand-side economies, and the incentives to merge under Bertrand competition

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We study the incentives to merge and the aggregate implications of mergers in a Bertrand competition model where firms sell differentiated products and consumers search sequentially for satisfactory deals. When search frictions are substantial, firms have an incentive to merge and to retail their products within a single store, which induces consumers to begin their search there. Such a merger lowers the profits of the outsiders and may benefit consumers due to more efficient search. Overall welfare may even increase. If the merged entity limits itself to coordinating the prices of the constituent firms, merging may not be profitable.

1. Introduction

Although no one would deny that searching for price and product fit can be quite costly in real-world markets—think, for example, about the time we spend test-driving new cars, acquiring new furniture, touching digital tablets, trying on new clothes, etc.—there has been little work in the industrial organization literature about the influence of search costs on the incentives to merge and on the aggregate implications of mergers. In this article, we demonstrate that when search costs are sizable, the predictions obtained in an otherwise standard model of price competition and differentiated products about the effects of mergers differ markedly from the current state of knowledge.

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We study mergers in a model that could well be referred to as the workhorse model of consumer search for differentiated products. The model was introduced by Wolinsky (1986) and was further studied by Anderson and Renault (1999). A finite number of firms sell horizontally differentiated products and compete in prices. Consumers search for satisfactory deals sequentially and can recall the offers at previously visited firms costlessly. In the equilibrium of the premerger market, all firms are equally attractive and consumers randomly pick a first shop to visit. Those consumers who fail to find a satisfactory product at the first shop they visit continue searching and randomly pick a second shop to visit, and so on.

When a number of firms merge, consumers no longer search for good deals in a random way. The order in which consumers visit merging and nonmerging firms depends on the prices these firms are expected to charge and on the amount of variety they carry. We distinguish between the short-run and the long-run effects of mergers. In the short-run, firms that merge coordinate their prices and everything else is kept constant. In the long-run, by contrast, the merged entity may choose to undertake a business reorganization consisting in retailing all the products of the parent firms within a single store. We show that when search frictions are substantial, firms have an incentive to merge and to stock the merged entity with all the products of the merging firms, which induces consumers to begin their search there. Such a merger lowers the profits of the outsiders and may benefit consumers due to more efficient search. If the merged entity limits itself to just coordinating the prices of the parent firms, merging may not be profitable.

In the short-run, what happens is that price coordination among the merging firms leads them to charge higher prices than the nonmerging firms. Given this, consumers optimally begin their search for satisfactory products at the nonmerging firms and then, in the event they fail to find a product to their liking at those firms, they continue searching at the merging stores. Thus, the merging firms, by internalizing the pricing externalities they impose on one another, confer the nonmerging firms a prominent position in the marketplace. This puts the merging firms at a market disadvantage vis-à-vis the nonmerging firms. In fact, in equilibrium, as search costs increase, consumer traffic from the nonmerging stores to the shops of the merged entity diminishes, which makes merging less profitable. We show that any two firm merger is unprofitable if search costs are sufficiently high. Moreover, any arbitrary k-firm merger becomes unprofitable if search costs and the number of nonmerging firms are sufficiently high. With these results we establish a new merger paradox. What is interesting about this paradox is that it arises when firms sell horizontally differentiated products and compete in prices.

In the long-run, when the merged entity chooses to retail all the products of the parent firms within a single store, the merged entity effectively lowers the consumers’ costs of searching for satisfactory goods at the merging firms. This generates demand-side economies which, when significant, may confer the merged entity a prominent position in the marketplace. We show that, when search costs are sufficiently high, in equilibrium the merged entity gains prominence in the marketplace and attracts all consumer first visits. If unsatisfied with the products available at the merged entity, consumers continue searching at the nonmerging stores. We show that when search costs are sufficiently high and the merged entity offers all the products of the parent firms, merging becomes profitable. In addition, and in contrast to most papers on mergers, we find that the outsiders’ profits decrease after a merger takes place, which helps us understand why the outsiders to a merger sometimes oppose consolidation processes of rival firms. We finally show

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1 More recently, this model has been used to explain incentives to invest in quality (Wolinsky, 2005), product-design differentiation (Bar-Isaac et al., 2011), and the emergence and effects of market prominence (Armstrong et al., 2009; Armstrong and Zhou, 2011; Haan and Moraga-González, 2011; Zhou, 2009).

2 Hewlett Packard and Compaq, whose merger was cleared in 2002, soon started selling each other’s products in their separate online shops; in recent days, the merged entity has chosen to downplay the Compaq name in its products. KLM, which merged with Air France in a transaction that was cleared subject to conditions in 2004, sells flights operated by Air France in its online shop, and vice versa. Daimler-Benz and Chrysler merged in 1998, but their retail sales largely remained separate. This likely hindered Chrysler’s market penetration in Europe and added to the difficulties experienced by the automobile giant shortly after merging.
that in the long-run, consumers may even benefit from consolidation in the marketplace because of the benefits arising from lower search frictions.

The literature on the incentives to merge and the aggregate implications of mergers is quite extensive. For a recent survey of the main theoretical and empirical insights see Whinston (2006). A seminal paper in the literature is Salant et al. (1983), which demonstrated that mergers are often not profitable when firms compete in quantities and offer similar products. This result is referred to as the merger paradox. Deneckere and Davidson (1985) showed that price-setting firms selling horizontally differentiated products, other things equal, always have an incentive to merge. In contrast to the Cournot case analyzed by Salant et al. (1983), this result arises because price increases of the merging firms, by strategic complementarity, are accompanied by price increases of the nonmerging firms. Our article puts forward a new merger paradox, which surprisingly arises under price competition with differentiated products. The underlying reason is based on the impact of price coordination on optimal consumer search, something quite different from the merger paradox of Salant et al. (1983), which concerns competition with decision variables that are strategic substitutes.

Since the seminal paper of Williamson (1968), the role of mergers at generating supply-side economies (or cost-synergies) that more than offset the market power effects of consolidation has been the focus of a considerable amount of research. Perry and Porter (1985), Farrell and Shapiro (1990), and McAfee and Williams (1992) explicitly modelled the cost efficiencies that arise from economies of sharing assets in product markets and stated conditions for the so-called efficiency defense of mergers. Our article brings out a new efficiency argument in favor of mergers, but based on demand-side rather than on supply-side economies. We show that the economies of search that unfold when the merging firms stock a wider range of products can result in the merging firms becoming prominent in the marketplace, thereby weakening (and sometimes even more than offsetting) their incentives to raise prices above the outsider firms. Altogether, these effects may make a merger welfare improving.

To the best of our knowledge, the US and EU guidelines do not mention demand-side economies arising from merger activity. By contrast, Section 5.7 of the 2010 Merger Assessment Guidelines of the UK Competition Commission and the Office of Fair Trading acknowledges the importance of demand-side efficiencies in merger control. However, the guidelines mainly focus on cases where consumers buy multiple items and product complementarities are significant: “Demand-side efficiencies arise if the attractiveness to customers of the merged firm’s products increases as a result of the merger. Common examples of demand-side efficiencies include: network effects, pricing effects, and ‘one-stop shopping’.” The argument in our article is clearly different. Sizable demand-side efficiencies can also arise when products are substitutes and consumers buy a single product, savings in search costs being at the heart of such efficiencies. We hope that this will add to the design of future merger guidelines.

Because in the postmerger market consumers visit merging and nonmerging firms in an order that maximizes expected utility, our article is also related to the recent literature on ordered search. Arbatskaya (2007) studies a market for homogeneous products where the order in which firms are visited is exogenously given. In equilibrium prices must fall as the consumer walks away from the firms visited first. Zhou (2011) considers the case of differentiated products and finds the opposite result. Armstrong et al. (2009) study the implications of prominence in consumer search markets. In their model, there is a firm that is always visited first and this firm charges lower prices and derives greater profits than the rest of the firms, which are visited randomly after consumers have visited the prominent firm. Zhou (2009) studies the case in which a set of firms, rather than just one, is prominent. In Haan and Moraga-González (2011) firms gain

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3 Merger Assessment Guidelines (OFT1254, p. 57, 2010). When network effects are present, users value a product more highly when it is used by a greater number of other customers. A merger may make the networks of the merging firms compatible with one another and in this way the welfare of consumers will rise. Pricing effects arise when complementary products are brought under common ownership, which may result in a decrease in the prices of all products. Gains from one-stop shopping arise when consumers have a strong preference for buying a range of products from a single supplier.
prominence by investing in advertising; they find that an increase in consumer search costs may result in higher advertising efforts and lower firm profits. Armstrong and Zhou (2011) present alternative ways in which firms can become prominent. Our article shows one other way to gain prominence: merging and stocking the shelves of the merged entity with all the products of the parent firms. Interestingly, we show that this business reorganization is only profitable if search costs are relatively high.

Our article is also related to a strand of the consumer search literature dealing with firm entry and choice of location, where consumer search economies also play a central role. In Stahl (1982) and Wolinsky (1983), savings in search costs can explain the observed geographical concentration of stores selling differentiated products. Fischer and Harrington (1996) investigate the role of product heterogeneity in explaining interindustry variation in firm agglomeration. Schulz and Stahl (1996) show that economies of scope in search costs can lead to excessive (price-increasing) entry.

Finally, our article also contributes to the literature on the nature of multiproduct firm pricing in the presence of search frictions. Although our article focuses on situations where consumers buy one of the products only, this literature has centered around models where consumers buy various products and prefer to concentrate their purchases within a single supplier. Klemperer (1992) shows that in these situations, firms may prefer head-to-head competition over product-line differentiation and Klemperer and Padilla (1997) demonstrate that search cost economies can lead to excessive product-line variety. Rhodes (2012) studies the pricing strategy of a monopolist selling an array of independent products. He demonstrates that when a retailer sells enough products, the Diamond’s (1971) hold-up problem disappears. Zhou (2012) also examines pricing by multiproduct firms selling independent products. Interestingly, he finds that equilibrium prices are lower than the prices that single-product firms would set; moreover, he shows that prices can decrease with search costs.\footnote{See also Shelegia (2012), which studies a model where some consumers exogenously visit all shops and others visit only one. In equilibrium, prices are dispersed and when the products are substitutes, like in our model, their prices are uncorrelated.}

The remainder of the article is organized as follows. Section 2 describes the consumer search model and the benchmark premerger market equilibrium. Section 3 focuses on the effects of price coordination between the merging parties. Section 4 extends the analysis by allowing the merged entity to stock all the products of the constituent firms. Section 5 discusses the main results and studies conditions under which the merged entity prefers to keep selling its products in separate shops. Section 6 concludes. The main proofs are placed in the Appendix to ease the reading of the article.\footnote{For the rest of the proofs, as well as for additional details of derivations, we refer the reader to our working paper Moraga-González and Petrikaitė (2013).}

2. The model and the premerger symmetric equilibrium

We study mergers in Wolinsky’s (1986) model of consumer search for differentiated products. On the supply side of the market, there are $n \geq 3$ firms selling horizontally differentiated products. All firms use the same constant returns to scale technology of production, and we normalize unit production costs to zero. Firms compete in prices, and they choose them simultaneously. On the demand side of the market, there is a unit mass of consumers. A consumer has tastes described by the following indirect utility function: $u_i = \varepsilon_i - p_i$, if she buys product $i$ at price $p_i$. The parameter $\varepsilon_i$ can be thought of as a match value between the consumer and product $i$. We assume that the match value $\varepsilon_i$ is the realization of a random variable uniformly distributed on $[0, 1]$.\footnote{The uniform distribution is adopted for simplicity, especially in the subsequent analysis of mergers. In an earlier working paper that contains part of the analysis here, Moraga-González and Petrikaitė (2011), we show that alternative distributions give similar results.} Match values are independently distributed across consumers and products. Moreover, they are private information of consumers so personalized pricing is not possible. For
later reference, it is useful to calculate the optimal pricing policy of a multiproduct monopolist selling \( k \) varieties. Let \( \{p_1, p_2, \ldots, p_k\} \) be the vector of prices. This vector must maximize the expression \( \sum_{i=1}^{k} p_i (\Pr[\varepsilon_i - p_i \geq \max_{j \neq i}(\varepsilon_j - p_j, 0)]) \). The symmetric solution to this problem is \( p_i = p_i^{\infty} \equiv (1 + k)^{-1}, \forall i \). Setting \( k = 1, \) we get the price of the single-product monopolist, which we simply denote by \( p^\infty \) and is equal to \( 1/2 \). Consumers search sequentially with costless recall. At all times, consumers have correct beliefs about the equilibrium prices.\(^8\) Each search costs the consumer \( s \). To avoid that a market equilibrium fails to exist (Diamond, 1971), we assume throughout that search costs are not too high. When \( s = 0, \) the model is similar to Perloff and Salop (1985) and the effects of mergers in that case are similar to Deneckere and Davidson (1985).

\[ p - p^* \]

**The premerger market equilibrium.** In the premerger market, firms are *ex ante* alike and correspondingly consumers search for satisfactory products randomly. The symmetric equilibrium is then the same as that in Wolinsky (1986). For completeness, we provide here the characterization of such a symmetric equilibrium.\(^9\) Assume that search cost \( s \in [0, 1/8] \).\(^{10}\) Let \( p^* \) denote the symmetric equilibrium price charged by firms other than firm \( i \) and consider the (expected) payoff to a firm \( i \) that deviates by charging a price \( p_i \neq p^* \). In order to compute firm \( i \)'s demand, we need to characterize consumer search behavior. Because consumers do not observe deviations before searching, we can rely on Kohn and Shavell (1974), who study the search problem of a consumer who faces a set of independently and identically distributed options with known distribution. Kohn and Shavell (1974) show that the optimal search rule is static in nature and has the stationary reservation utility property. Accordingly, denote the solution to

\[ \int_{x}^{1} (\varepsilon - x) d\varepsilon = s \]

by \( \pi(1 - \sqrt{2} s) \). The left-hand-side (LHS) of (1), which is equal to \( (1 - x)^2/2 \), is the expected benefit in symmetric equilibrium from searching one more time for a consumer whose best option so far is \( x \). Its right-hand-side (RHS) is the cost of search. Therefore, \( \pi \) represents the threshold match value above which a consumer will optimally decide not to continue searching for another product. The number \( \pi - p^* \) is referred to as the reservation utility for visiting a firm. Because \( s \in [0, 1/8] \), we have that \( \pi \in [1/2, 1] \), and, correspondingly, \( \pi - p^* \geq 0 \).

In order to compute firm \( i \)'s demand, consider a consumer who visits firm \( i \) in her \( h \)-th search (after having walked away from \( h - 1 \) other firms), \( h = 1, 2, \ldots, n \). Because consumers expect all firms to charge the same price \( p^* \), the probability firm \( i \) is in \( h \)-th position is \( 1/n \). Let \( \varepsilon_i - p_i \) denote the utility the consumer derives from the product of firm \( i \). Let \( z_k \equiv \max(\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_k) \). Suppose \( \varepsilon_i - p_i \geq \max(z_{k-1}, p^*, 0) \) for otherwise the consumer would not buy product \( i \). The expected gains from searching one more firm, say \( j \), are equal to

\[ \int_{x}^{1} \varepsilon_j - (\varepsilon_i - p_i + p^*) d\varepsilon_j \]

Comparing this to (1) and assuming that \( p_i < 1 - \pi + p^* \), it follows that, conditional on the

\[^7\]When \( k = 2 \), this problem can be shown to be globally concave and the symmetric solution to the FOCs therefore gives the global maximum. However, for arbitrary \( k \) this problem seems nontrivial because it is very hard to prove payoff quasiconcavity. We have been able to prove analytically that the Hessian matrix at the symmetric solution is negative definite and we have then numerically checked that deviations from the symmetric solution lead to a lower payoff.

\(^8\) Although we can view this assumption as a standard Nash assumption, its rationalization requires some sophistication on the part of consumers. In fact, we shall assume that consumers do know the ownership structure of the firms, their equilibrium prices, and the number of products sold in each of the establishments.

\[^9\]We note that asymmetric equilibria may be sustained in this model. The idea is that if consumers believe that firms’ prices follow a given ranking, say, \( p_1 < p_2 < \cdots < p_n \), then it is optimal for consumers to start their search at firm 1, continue at firm 2, and so on, and for firms to price in such a way as to make consumer beliefs consistent with equilibrium. The unattractive feature of these equilibria is that they are not determined by the underlying characteristics of the market, but by an indeterminacy of consumer beliefs. In general, we will ignore asymmetric equilibria in our article.

\[^{10}\]In order to make sure that the first search is always worthwhile, we take the worst case scenario where consumers expect the firms to charge the monopoly price. Therefore, we require that \( s \leq \Pr[\varepsilon \geq p^*] E[\varepsilon \geq p^*] \), where \( E \) denotes the expectation operator. This is equivalent to requiring \( s \leq \pi \equiv (1 - p^*)^2/2 \). Since \( p^w = 1/2, s \leq 1/8 \) suffices.
deviant firm being in $h^{th}$ position, the probability that the buyer visits firm $i$ and stops searching at firm $i$ is equal to $\Pr[\epsilon_i - p_i > \bar{x} - p^* > z_{h-1} - p^*] = \bar{x}^{h-1}(1 - \bar{x} - p_i + p^*)$. Summing the unconditional probability for all $h$, we obtain a demand equal to $\frac{1 - \bar{x}}{n(1 - \bar{x})}(1 - \bar{x} - p_i + p^*)$.

The consumer also buys the product of firm $i$ when she walks away from it, walks away from the rest of the firms in the market, and happens to return to firm $i$ because such a firm offers her the best deal after all. Conditional on the deviant firm being in $h^{th}$ position, this occurs with probability $\Pr[\max\{0, z_{n-1} - p^*\} < \epsilon_i - p_i < \bar{x} - p^*]$. Summing the unconditional probability for all $h$, we obtain a demand from returning consumers equal to

$$r_d(p_i; p^*) = \int_{p_i}^{(\bar{x} + p^*)(1 - \bar{x} - p_i + p^*)} (\epsilon - p_i + p^*)^{n-1} d\epsilon = \frac{1}{n}(\bar{x}^n - p^*^n),$$

where the notation $r_d(p_i; p^*)$ is to indicate that these sales originate from consumers who buy from firm $i$ after having visited all the firms in the market.

We can now write firm $i$’s expected profits as

$$\pi_i(p_i; p^*) = p_i \left[ \frac{1 - \bar{x}^n}{n(1 - \bar{x})}(1 - \bar{x} - p_i + p^*) + r_d(p_i; p^*) \right].$$

We look for a symmetric Nash equilibrium in prices. Because the payoff in (3) is strictly concave, the first-order condition (FOC) suffices for a maximum. After applying symmetry, that is, $p_i = p^*$, the FOC is

$$1 - p^* - p^* \frac{1 - \bar{x}^n}{1 - \bar{x}} = 0.$$  

(4)

It is easy to check that (4) has a unique solution that satisfies $\bar{x} \geq p^* \geq 1 - \bar{x}$. To ensure that $p^*$ is indeed an equilibrium, we need to check that firms do not have an incentive to deviate from it. Because of the strict concavity of (3), deviations such that $p_i < 1 - \bar{x} + p^*$ are clearly not profitable. Suppose now that the deviant firm charges a price $p_i \geq 1 - \bar{x} + p^*$. In such a case, consumers always walk away from the deviant firm no matter the position in which they visit it for the first time. As a result, firm $i$ only sells to those consumers who have visited all firms. The deviant profits become $\pi_i(p_i; p^*) = p_i \int_{p_i}^{(\bar{x} + p^*)(1 - \bar{x} - p_i + p^*)} (\epsilon - p_i + p^*)^{n-1} d\epsilon$ and the deviation is not profitable either.\footnote{Because the reservation utility is stationary, no consumer who walks away from a firm will return to such a firm without first having visited all the firms in the market.}

The profits of a typical firm in the premerger situation are

$$\pi^* = \frac{1}{n} p^*(1 - p^*).$$

(5)

It is readily seen that the equilibrium price and profits increase in search cost $s$.

Next, we study the impact of a merger of an arbitrary number of firms. We first focus on the effects of a merger on the prices of insiders and outsiders and optimal consumer search. Then we examine whether the merging firms have indeed an incentive to merge and the welfare implications of mergers. The analysis is divided into two parts. In Section 3, we study the effects of mergers abstracting from any source of efficiency gains. In this sense, the focus in Section 3 is on the effects of joint (price) decision making, exactly as in Deneckere and Davidson (1985). In Section 4, we study the effects of mergers from a medium- to long-run perspective; there we let the merged entity stock the shelves of its shops with all the products of the parent firms. Finally, in Section 5, we examine the merged entity’s incentives to undertake such a business reorganization.

\footnote{Because of log-concavity of the uniform density function, this profits expression is quasiconcave in own price (Caplin and Nalebuff, 1991). Taking the derivative of the deviating profits with respect to $p_i$ and setting $p_i = p^*$, we get $d\pi_d/dp_i|_{p_i=p^*} = (1 - p^* - np^*)/n < 0$, where the inequality follows from the fact that $p^*$ solves (4). Since deviating profits are quasiconcave and decrease at $p_i = p^*$, we conclude they are even lower at prices $p_i$ such that $p_i \geq 1 - \bar{x} + p^*$.}
3. Effects of mergers in the short-run

Consider that \( k \) firms merge, with \( 2 \leq k \leq n - 1 \). Let us continue to focus on symmetric equilibria in the sense that all nonmerging firms charge a price denoted by \( \hat{p}^* \) and all merging firms a price denoted by \( \hat{p}^\ast \). Because a nonmerging firm controls the price of a single variety, we let \( \hat{p}^* \in [0, p_m^*] \); the merged entity, by contrast, controls the price of \( k \) varieties and, correspondingly, we let \( \hat{p}^\ast \in [0, p_m^{\ast}] \). In this section, we assume that \( s \in [0, (1 - (1 + k)^{-1/k})^2/2] \); note that the upper bound on the search cost becomes tighter as more firms merge.\(^{\text{13}}\)

We start by assuming that the merging firms charge a higher price than the nonmerging firms, that is, \( \hat{p}^* < \hat{p}^\ast \). A priori, this is a reasonable conjecture because the merging firms internalize the pricing externalities they impose upon one another. In Section 5, however, we study whether an alternative symmetric equilibrium exists where the merging firms charge a lower price than the nonmerging firms.

Given that consumers correctly expect equilibrium prices to satisfy the inequality \( \hat{p}^* < \hat{p}^\ast \), they now face a problem of search where the set of available options have known, independent but nonidentical utility distributions. Weitzman (1979) shows that also in such a case the optimal decision rule is static in nature and has the reservation utility property. At every step of the optimal search process, a consumer should consider visiting next the (not-yet-visited) shop for which her reservation utility is the highest; moreover, a consumer should terminate her search whenever the maximum utility obtained so far is higher than the reservation utility at the shop to be visited next. Because the reservation utility at the nonmerging firms is higher than at the merging ones, consumers should first check the products of the nonmerging firms; after having checked all the products of the nonmerging firms, consumers should decide whether to return to one of the previously visited firms or else continue searching among the merging firms.

Let \( \bar{x} \) be given by (1). The number \( \bar{x} - \hat{p}^* \) defines the reservation utility for searching the product of a merging firm. Likewise, \( \bar{x} - \hat{p}^\ast \) is the reservation utility for searching the product of a nonmerging store. Because \( \hat{p}^* < \hat{p}^\ast \), Weitzman’s (1979) results prescribe consumers to start searching for a satisfactory product at the nonmerging firms. If no alternative is found to be good enough in those firms, buyers should continue searching for a fine product at the merging firms. This implies that the postmerger demands of the two types of stores (merging and nonmerging) are related to the demands derived by Zhou (2009) in his paper on prominent and nonprominent firms.\(^\circ\)

To calculate the postmerger equilibrium prices, we proceed by computing the payoff that merging and nonmerging firms would obtain when deviating from the equilibrium prices. While deriving the payoffs of the two types of (deviating) firms, we require consumer expectations about the prices charged by firms not yet visited to be equal to the equilibrium prices.

\[ \text{Payoff to a deviant nonmerging store.} \]

Consider a nonmerging store \( j \) that deviates by charging a price \( \tilde{p} \neq \hat{p}^* \), with \( \tilde{p} < 1 - \bar{x} + \hat{p}^* \). As consumers expect all nonmerging firms to charge \( \hat{p}^* \), they visit them randomly. The deviant firm may thus be visited in first place, second place, and so on until the \( (n - k)^{th} \) place, each position occurring with probability \( 1/(n - k) \). Because of the nonstationarity of the reservation utility, it is convenient to distinguish among consumers who visit the deviant in their first, second, \ldots, or \( (n - k)^{th} \) search and consumers who visit it in their \( (n - k)^{th} \) search.

Consider then first the case in which a consumer visits the deviant nonmerging firm \( j \) in her \( h^{th} \) search, with \( h = 1, 2, \ldots, n - k - 1 \). Suppose the deal the consumer observes upon entering

\[ \text{\footnotesize{\( ^{\text{13}} \)}} \]

Again, we assume that consumers find it worthwhile to make a first search, even if they expect firms to charge the monopoly price. Since \( p_m^* > p_m^\ast \) for all \( k \), in order to ensure that the merged entity has a positive market share, we require that \( s \leq P\{s \geq p_m^*\}E[s - p_m^*/|s - p_m^*|] \).

\[ \text{\footnotesize{\( ^{\text{14}} \)}} \]

In Zhou’s (2009) paper, consumers search for a satisfactory product first at the prominent firms; if they do not find there a product they like enough, they proceed by searching at the nonprominent firms. The payoff of a nonmerging firm here is exactly identical to the payoff of a prominent firm in his paper. However, the payoff of the merged entity is clearly different from the payoff of a nonprominent firm in Zhou’s paper.

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the deviant’s shop is $\varepsilon_j - \tilde{p}$. There are three circumstances in which the consumer will buy the product of the deviant. First, the consumer may stop searching at this shop and buy there right away. Conditional on the deviant being in the $h^{th}$ position, this occurs with probability $\Pr[z_{n-k-1} - \tilde{p}^* < x - \tilde{p}^* < \varepsilon_j - \tilde{p}]$, which is equal to $\tilde{x}^{n-k-1} (1 - \tilde{x} + \tilde{p}^* - \tilde{p})$. Summing the unconditional probability for all $h$, we get a demand equal to $\frac{1 - \tilde{x}^{n-k-1}}{(n-k)(1-\tilde{x})} (1 - \tilde{x} + \tilde{p}^* - \tilde{p})$.

Second, the consumer may walk away from the deviant nonmerging firm and come back to it after having visited the rest of the nonmerging stores. Conditional on the deviant being in $h^{th}$ place, this occurs with probability $\Pr[max\{z_{n-k-1} - \tilde{p}^*, x - \tilde{p}^*\} < \varepsilon_j - \tilde{p} < x - \tilde{p}]$. To indicate that these sales originate from consumers who buy from the deviant nonmerging firm $j$ after having visited all the nonmerging stores, we denote this conditional probability by

$$\tilde{\mu}_m(p; \tilde{p}^*, \tilde{p}^*) \equiv \int_{\tilde{p}^* - \tilde{p}}^{\varepsilon_j - \tilde{p}} (\varepsilon - \tilde{p} + \tilde{p}^*)^{n-k-1} d\varepsilon = \frac{1}{n-k} \left( \tilde{x}^{n-k} - (x - \tilde{p}^* + \tilde{p}^*)^{n-k} \right).$$

Summing the unconditional probability for all $h$, we get a demand equal to $\frac{n-k-1}{n-k} \tilde{\mu}_m(p; \tilde{p}^*, \tilde{p}^*)$.

Finally, the consumer may walk away from the deviant nonmerging firm and come back to it after having visited the rest of the firms in the market. Conditional on the deviant being in $h^{th}$ position, this occurs with probability $\Pr[max\{z_{n-k-1} - \tilde{p}^*, z_{n-k} - \tilde{p}^*\} < \varepsilon_j - \tilde{p} < x - \tilde{p}^*]$. To indicate that these are the sales to consumers who buy from the deviant nonmerging firm $j$ after having visited all the stores in the market, we denote this conditional probability by

$$\tilde{\mu}_a(p; \tilde{p}^*, \tilde{p}^*) \equiv \int_{\tilde{p}^* - \tilde{p}}^{\varepsilon_j - \tilde{p}} (\varepsilon + \tilde{p}^* - \tilde{p})^{n-k-1} (\varepsilon + \tilde{p}^* - \tilde{p})^k d\varepsilon = \int_{\tilde{p}^* - \tilde{p}}^{\varepsilon_j - \tilde{p}} (\varepsilon + \tilde{p}^*)^{n-k-1} (\varepsilon + \tilde{p}^*)^k d\varepsilon.$$

Summing the unconditional probability for all $h$, we get a demand equal to $\frac{n-k-1}{n-k} \tilde{\mu}_a(p; \tilde{p}^*, \tilde{p}^*)$.

We now consider the case in which the consumer visits the deviant nonmerging firm in her $(n-k)^{th}$ search. There are two situations in which the consumer will buy the deviant’s product. First, the consumer may stop searching at the deviant’s shop and buy there right away. Conditional on visiting the deviant in $(n-k)^{th}$ place, this occurs with probability $\Pr[z_{n-k-1} - \tilde{p}^*, x - \tilde{p}^*] > \varepsilon_j - \tilde{p} < x - \tilde{p}^*]$. This gives a demand equal to

$$\frac{1}{n-k} \left( \tilde{x}^{n-k} - (x - \tilde{p}^* + \tilde{p}^*)^{n-k} \right).$$

Second, the consumer may walk away from the deviant firm and come back to it after having visited the rest of the firms in the market. In this second case, we have exactly the same expression for returning consumers as in (6).

Adding the demands above for $h = 1, 2, \ldots, n-k-1$ to the demand for the case in which the consumer visits the deviant in her $(n-k)^{th}$ search and simplifying, we get the profits of a deviant nonmerging firm:

$$\tilde{\pi}(p; \tilde{p}^*, \tilde{p}^*) = \tilde{p} \left[ \frac{1}{n-k} \frac{1 - \tilde{x}^{n-k}}{1 - x} (1 - \tilde{x} + \tilde{p}^* - \tilde{p}) + \tilde{\mu}_m(p; \tilde{p}^*, \tilde{p}^*) + \tilde{\mu}_a(p; \tilde{p}^*, \tilde{p}^*) \right].$$

□

**Payoff to a deviant merged entity.** The merged entity chooses its prices to maximize the joint profit of the $k$ partner firms. Therefore, the vector of prices $(\tilde{p}^*, \tilde{p}^*, \ldots, \tilde{p}^*)$ is part of a symmetric equilibrium if the merged entity does not have an incentive to deviate by choosing a different set of prices $(p_1, p_2, \ldots, p_k)$. It can readily be seen that for the purpose of writing the FOC at a symmetric equilibrium, it is enough to write down the payoff of a merged entity that deviates by charging a different price for one of its products only. We next compute such a payoff and relegated the issue of existence and uniqueness of symmetric equilibrium to the Appendix.

Consider that the merged entity deviates by charging a price $\tilde{p} \neq \tilde{p}^*$ for its product $i$, with $\tilde{p} < 1 - \tilde{x} + \tilde{p}^*$. The deviation affects not only the demand for product $i$ but also the demand for the other $k - 1$ products controlled by the merged entity. Let us start by computing the demand for product $i$. Because $\tilde{p}^* < \tilde{p}^*$ in equilibrium, consumers only contemplate visiting the stores
of the merged entity after having visited all the nonmerging firms. The probability consumers search product \( i \) in position \( h = 1, 2, \ldots, k \) (after the nonmerging firms) is \( 1/k \). Take now a consumer who searches product \( i \) in her \( h^{th} \) search and denote the deal she gets there by \( \hat{p}_i - \hat{p} \).

There are two cases in which the consumer will buy product \( i \). First, the consumer may stop searching and buy product \( i \) right away. Conditional on firm \( i \) being in \( h^{th} \) position, this occurs with probability \( \Pr[max\{z_{n-k} - \tilde{p}^*, z_{k-1} - \tilde{p}^*\} < x - \tilde{p}^* < \varepsilon_i - \hat{p}] \), which gives a demand equal to \( (x - \tilde{p}^* + \tilde{p}^{*})^{h-1} (1 - x + \tilde{p}^* - \hat{p}) \). Summing the unconditional probability for all \( h \), we obtain a demand equal to \( \frac{1}{k} \int_{\varepsilon_i - \hat{p}}^{x - \tilde{p}^*} (x - \tilde{p}^* + \tilde{p}^{*})^{h-1} (1 - x + \tilde{p}^* - \hat{p}) \).

Second, the consumer may walk away from the firm selling product \( i \) and come back to it after visiting the rest of the merging firms. Conditional on visiting firm \( i \) in \( h^{th} \) place, this happens with probability \( \Pr[max\{z_{n-k} - \tilde{p}^*, z_{k-1} - \tilde{p}^*, 0\} < \varepsilon_i - \hat{p} < x - \tilde{p}^*] \). Using a similar notation as above, we denote this probability by

\[
\hat{r}_{ia}(\hat{p}; \tilde{p}^*, \tilde{p}^*) = \int_{\hat{p}}^{x - \tilde{p}^* + \tilde{p}^{*}} (\varepsilon - \hat{p} + \tilde{p}^{*})^{h-1} (\varepsilon + \tilde{p}^*)^{k-1} d\varepsilon.
\]

Summing the unconditional probability for all \( h \), we get a demand equal to \( \hat{r}_{ia}(\hat{p}; \tilde{p}^*, \tilde{p}^*) \).

Putting terms together and simplifying, we obtain the demand for product \( i \):

\[
d_{ia}(\hat{p}; \tilde{p}^*, \tilde{p}^*) = \frac{(1 - \tilde{x})}{k(1 - \bar{x})} (x - \tilde{p}^* + \tilde{p}^{*})^{h-1} (1 - \bar{x} + \tilde{p}^* - \hat{p}) + \hat{r}_{ia}(\tilde{p}^*, \tilde{p}^*).
\]

The deviation also affects the merged entity’s demand for products other than \( i \). Let us compute next the demand for one of the other products, say product \( m \). Suppose firm \( m \) is visited by a consumer in her \( h^{th} \) search, \( h = 1, 2, \ldots, k \), which happens with probability \( 1/k \). Note that the probability that the deviant’s product \( i \) has not yet been inspected by the consumer is \( (k - h)/(k - 1) \). Conditional on the consumer visiting firm \( m \) in \( h^{th} \) place and on not having yet visited firm \( i \), the consumer will stop searching at firm \( m \) and buy there right away with probability \( \Pr[\varepsilon_{m-k} - \tilde{p}^* > x - \tilde{p}^* > \max\{z_{n-k} - \tilde{p}^*, z_{k-1} - \tilde{p}^*\}] \), which is equal to \( \frac{k-h}{k} (x - \tilde{p}^* + \tilde{p}^{*})^{h-1} (1 - \tilde{p}) \).

Summing the unconditional probability for \( h = 1, 2, \ldots, k \) we obtain a demand for product \( m \) equal to

\[
(x - \tilde{p}^* + \tilde{p}^{*})^{h-1} (1 - \tilde{x}) \sum_{h=1}^{k-1} \frac{k-h}{k(k-1)} \tilde{x}^{h-1}.
\]

With probability \( (h - 1)/(k - 1) \), the deviant’s product \( i \) has already been checked by the consumer. Conditional on visiting firm \( m \) in \( h^{th} \) place and on having already inspected product \( i \), the consumer will stop searching at firm \( m \) and buy there right away with probability \( \Pr[\varepsilon_{m-k} - \tilde{p}^* > \bar{x} - \tilde{p}^* > \max\{z_{n-k} - \tilde{p}^*, z_{k-2} - \tilde{p}^* - \varepsilon_i - \hat{p}\}] \), which equals \( (x - \tilde{p}^* + \tilde{p}^{*})^{h-1} (x - \tilde{p}^* + \hat{p})(1 - \bar{x}) \). Summing the unconditional probability for \( h = 1, 2, \ldots, k - 1 \) we obtain a demand for product \( m \) equal to

\[
(x - \tilde{p}^* + \tilde{p}^{*})^{h-1} (\bar{x} - \tilde{p}^* + \hat{p}) (1 - \bar{x}) \sum_{h=1}^{k-1} \frac{k-h}{k(k-1)} \bar{x}^{h-2}.
\]

The consumer may also buy at firm \( m \) if she walks away from it and happens to return to it after having visited the rest of the merging firms. Conditional on firm \( m \) being visited in \( h^{th} \) position, this happens with probability \( \Pr[max\{z_{n-k} - \tilde{p}^*, z_{k-2} - \tilde{p}^*, \varepsilon_i - \hat{p}, 0\} < \varepsilon_m - \tilde{p}^* < \bar{x} - \tilde{p}^*] \).
Merging firms charge a price that satisfies the FOCs (9)–(10). We then show that these prices satisfy the optimal. We finally check that no firm gains by deviating from the symmetric equilibrium prices.

Summing the unconditional probability for \( h = 1, 2, \ldots, k \) gives \( \tilde{r}_{ma}(\tilde{p}; \tilde{p}^*, \tilde{p}^*) \).

Putting together the above demands gives the following total demand for product \( m \):

\[
d_m(\tilde{p}; \tilde{p}^*, \tilde{p}^*) = \frac{k(1 - \tilde{x}) - (1 - \tilde{x}^k)}{k(k - 1)(1 - \tilde{x})} (\tilde{x} - \tilde{p}^* + \tilde{p}^*)^{n-k} + \frac{1 - \tilde{x}^k - k\tilde{x}^{k-1}(1 - \tilde{x})}{k(k - 1)(1 - \tilde{x})} (\tilde{x} - \tilde{p}^* + \tilde{p}^*)^{n-k} (\tilde{x} - \tilde{p}^* + \tilde{p}) + \tilde{r}_{ma}(\tilde{p}; \tilde{p}^*, \tilde{p}^*). \tag{10}
\]

Because the demands for the other products of the merged entity are the same, the payoff function of the deviant merged entity is

\[
\hat{\pi}(\tilde{p}; \tilde{p}^*, \tilde{p}^*) = \hat{p}d_m(\tilde{p}; \tilde{p}^*, \tilde{p}^*) + (k - 1) \hat{p}^* d_m(\tilde{p}; \tilde{p}^*, \tilde{p}^*). \tag{8}
\]

\[\square\] Results. Taking the first order derivative of the payoff in (7) with respect to the deviation price \( \hat{p} \) and applying symmetry yields the following FOC for the nonmerging firms:

\[
1 - \frac{1 - \tilde{x}^{n-k}}{1 - \tilde{x}} \hat{p}^* - (\tilde{x} - \hat{p}^* + \tilde{p}^*)^{n-k} + (n - k) \int_{\tilde{p}^*}^{\tilde{p}^*} (\tilde{x} - \hat{p}^* + \tilde{p}^*)^{n-k} (\tilde{x} - \hat{p}^* + \tilde{p}) d\epsilon = 0. \tag{9}
\]

Likewise, the FOC for the merged entity is

\[
(\tilde{x} - \hat{p}^* + \tilde{p}^*)^{n-k} (1 - \tilde{x}^k - k\hat{p}^* \tilde{x}^{k-1}) + k \int_{\tilde{p}^*}^{\tilde{p}^*} (\tilde{x} - \hat{p}^* + \tilde{p}^*)^{n-k} (\tilde{x} - \hat{p}^* + \tilde{p}) d\epsilon = 0. \tag{10}
\]

Proposition 1. Assume that \( k \) firms merge. Then, for any \( s \), there exists a unique symmetric Nash equilibrium in the short-run postmerger market where:

- Consumers start searching at the nonmerging stores and then, if they wish so, they proceed by searching at the merged ones.
- Merging firms charge a price \( \hat{p}^* \) and the nonmerging stores charge a price \( \tilde{p}^* \); these prices are given by the unique solution to the system of FOCs (9)–(10) and the price ranking is consistent with consumer search behavior, that is, \( \hat{p}^* > \tilde{p}^* \).

The proof of this proposition has the following steps. We first show that there exists a unique pair of prices \( (\hat{p}^*, \tilde{p}^*) \) that satisfies the FOCs (9)–(10). We then show that these prices satisfy the inequality \( \hat{p}^* > \tilde{p}^* \), which immediately implies that the prescribed consumer search behavior is optimal. We finally check that no firm gains by deviating from the symmetric equilibrium prices.

When some firms merge, two effects take place. On the one hand, because consumers expect the insiders to charge higher prices than the outsiders, consumers place the merging firms all the way back in the queue they follow when they search for satisfactory products. On the other hand, as usual when firms merge, there is an internalization-of-pricing-externalities effect. These two effects take place simultaneously and it is illustrative to separate them in a graph.

Following Deneckere and Davidson (1985), because the prices of similar firms are identical, the effects of a merger can be illustrated in a two-dimensional graph. In Figure 1a, the crossing point between the solid curves gives the premerger equilibrium. The line \( r_k^{pre} (r_{n-k}^{pre}) \) is the joint reaction of the potential insiders (outsiders) to a price \( \hat{p} (\tilde{p}) \) charged by the outsiders (insiders),
given consumer beliefs that the equilibrium price is $p^*$. The curves cross the 45 degrees line at $p^*$ so both types of firms charge $p^*$ and consumers’ expectations are fulfilled.

When a merger takes place, by the search-order effect, consumers start their search for satisfactory products at the shops of the outsiders. Equilibrium pricing in a similar market where some firms are visited first by consumers has been studied by Zhou (2009). In his paper, consumers search first the products of the so-called prominent firms and if they do not find a satisfactory product there, they continue by searching the products of the so-called nonprominent firms. The line $r_k(r_{n-k})$ in Figure 1a is the joint reaction of the nonprominent (prominent) firms to a price $\tilde{p}$ ($\hat{p}$) charged by the prominent (nonprominent) ones, given consumer beliefs that the equilibrium prices are Zhou’s equilibrium ones. As we can see, by the search-order effect, the joint reaction curve of the insiders (outsiders) shifts upward (leftward) from $r_{pre}^k(r_{pre}^{n-k})$ to $r_{post}^k(r_{post}^{n-k})$. These moves capture the fact that, relative to the premerger situation, the insiders’ demand becomes more inelastic while the outsiders’ demand turns more elastic. The crossing point between the dashed curves $r_k$ and $r_{n-k}$, denoted $\{\tilde{p}^*_1, \hat{p}^*_1\}$, gives Zhou’s equilibrium.

The effects of price coordination among the insiders are shown in Figure 1b. The line $r_{post}^k(r_{post}^{n-k})$ is the joint reaction of the insiders (outsiders) to a price $\tilde{p}$ ($\hat{p}$) charged by the outsiders (insiders), given consumer beliefs that the equilibrium prices are $\tilde{p}^*$ and $\hat{p}^*$. The internalization-of-pricing-externalities effect is captured by the shift from $r_k(r_{n-k})$ to $r_{post}^k(r_{post}^{n-k})$. The postmerger equilibrium is given by the crossing point of the two dotted-dashed curves, where consumer expectations are also fulfilled.15

Whether the postmerger prices are higher or lower than the premerger price is a priori ambiguous. Because the merging firms internalize the pricing externalities they impose on one another, they raise their prices. This, by strategic complementarity, pushes the prices of the nonmerging firms up, too. The merging firms, however, raise their prices more than the nonmerging firms, which confers the nonmerging stores a prominent position in the marketplace. This provides the nonmerging firms with incentives to lower their prices because when a firm becomes prominent, its pool of consumers becomes more elastic. The merging firms, by contrast, tend to raise their prices even further because consumers postpone visiting them and, correspondingly,

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15 Normally, the internalization-of-pricing-externalities effect only changes the reaction curve of the insiders. Here, the reaction function of the outsiders also moves because of the change in consumer expectations.
their demands become more inelastic. Our next proposition shows that when the search-order effects are not very strong, then all prices increase after a merger.\(^{16}\)

**Proposition 2.** In the short-run postmerger equilibrium of Proposition 1, the ranking of pre- and postmerger equilibrium prices is \(p^* < \hat{p}^* < \tilde{p}^*\) whenever one of the following conditions holds: (i) the search cost is sufficiently low, (ii) the search cost is sufficiently high, (iii) the number of firms \(n = 3\).

As expected, the case in which search cost is small reproduces naturally the situation in Deneckere and Davidson (1985). However, as search costs increase, fewer consumers walk away from the nonmerging stores and visit the merged ones. This fall in consumer traffic from the outsiders to the insiders has important consequences for merger profitability.

**Proposition 3.** In the short-run postmerger equilibrium of Proposition 1: (i) Any 2-firm merger is not profitable if the search cost is sufficiently high. (ii) Any \(k\)-firm merger is not profitable if the search cost and the number of competitors are sufficiently high. (iii) If the search cost is sufficiently small, any \(k\)-firm merger is profitable.

Proposition 3 shows that, unless there are many firms in the industry and the merger comprises almost all of them, eventually as the search cost becomes relatively high, merging is not profitable for the merging firms. The interest of this proposition is that it puts forward a new merger paradox, which arises under price competition with differentiated products. The underlying reason is based on consumer search costs, something quite different from the merger paradox of Salant et al. (1983), which concerns competition with decision variables that are strategic substitutes.\(^{17}\)

Propositions 2 and 3 are illustrated in Figure 2, where we plot the postmerger prices and profits against search costs. For comparison purposes, we also plot the premerger prices and profits. Figure 2a shows that all prices are increasing in search costs because when searching becomes more costly, an individual firm has more market power over the consumers who visit it.\(^{18}\) As the graph reveals, postmerger prices, whether from merging or nonmerging firms, are higher than the premerger prices.

\(^{16}\) We note, however, that solving numerically the model we have found no instance in which this does not happen.

\(^{17}\) We are implicitly assuming that the nonmerging firms can absorb the (possibly large) postmerger increase in consumer traffic toward their stores. If firms were capacity constrained, our result would have to be qualified. Moreover, we are abstracting from cost-synergies. If, as in Farrell and Shapiro (1990), the costs of the insiders decreased and their prices fell below those of the outsiders as a result of the merger, then the situation would be quite the opposite.

\(^{18}\) For a proof of this fact, see our earlier working paper, Moraga-González and Petrikaitė (2011).
Figure 2b shows that the profits of a typical merging firm, $\hat{\pi}^*/k$, decline as search cost goes up. The reason is that, as the search cost increases, fewer consumers walk away from the nonmerging firm in order to check the products of the merged entity. This has a major implication on merger profitability: for search costs approximately above 0.019 (about 3.8% of the average value of a firm’s product), merging is not profitable. The graph also reveals that the nonmerging firm “gets a free ride” and that “this ride is freer” the higher the search cost.\(^\text{19}\)

4. Effects of mergers in the long-run

In this section, we take a medium- to long-term view of mergers and assume that, after a more or less complex business reorganization, the merged entity starts selling the $k$ products of the mother firms in a single shop.\(^\text{20}\) For the moment, we take this reorganization as exogenous and study its effects. Later in Section 5, we examine the circumstances under which such a reorganization is profitable for the merging firms.

We assume throughout that consumers incur a search cost\(^\text{21}\)

\[
s \in \left[0, \min \left\{ \frac{1}{8}, \frac{k}{k+1} \left[ 1 - (k+2)(k+1)^{-\frac{1}{k}} \right] \right\} \right]
\]

(11)

when they visit a store. Note that for reasonable levels of $k$, the relevant upper bound on the search cost is $1/8$. We also assume that consumers do not incur any “intrastore” search cost, that is, once consumers arrive at the merged entity they can inspect all its $k$ products at no additional cost. As before, let $\hat{p}^*$ and $\hat{p}^*$ denote the symmetric equilibrium prices of the nonmerging and merging firms, respectively. Because consumers can try $k$ products at the merged entity, the trade-off they face is clear: relative to the deal offered by a nonmerging firm, at the merged entity, consumers encounter more variety but probably, though not surely as we will see later, at higher prices.\(^\text{22}\)

To characterize the order of search, we invoke again Weitzman’s (1979) results. Let $\bar{x}$ be the solution to

\[
\int_x^{1} (\epsilon - x) k \epsilon^{k-1} d\epsilon - s = 0.
\]

As in (1), $\bar{x}$ represents a threshold match value above which a consumer will decide not to continue searching at the merged entity. Correspondingly, the number $\bar{x} - \hat{p}^*$ defines the reservation utility for searching the $k$ products of the merged entity. As before, $x - \hat{p}^*$ is the reservation utility for searching the product of a nonmerging store.

Momentarily, assume $\bar{x} - \hat{p}^* > \bar{x} - \hat{p}^*$. Given this assumption, which according to Weitzman’s optimal search rule prescribes consumers to start searching at the merged entity, we next calculate the postmerger equilibrium prices. To do this, we proceed by computing the payoffs the merging and nonmerging firms would obtain when deviating from the equilibrium prices. After taking the FOCs and applying symmetry, we check if the inequality $\bar{x} - \hat{p}^* > \bar{x} - \hat{p}^*$

\(^{19}\)The reader may think that our result that mergers are unprofitable for high search costs is driven by the fact that search is random in the premerger market while it is directed in the postmerger market. This is not so. If we assumed, for example, that the potentially merging firms are searched last in the premerger market, our result would also hold. For further details, see our working paper.

\(^{20}\)Alternatively, we can assume that the merged entity keeps all the shops open but stocks each of them with the $k$ varieties stemming from the $k$ original merging firms. If there are positive fixed costs of keeping shops open, the first type of business reorganization is more economical. Nevertheless, up to the fixed costs, it is easy to see that both alternatives yield exactly the same equilibrium payoff.

\(^{21}\)Once again we need to make sure that both types of firm obtain a positive market share. For a consumer to visit a nonmerging firm, the condition $s \leq 1/8$ suffices. However, if a consumer contemplates to visit the merged entity, it must be the case $s \leq \Pr[z_i \geq p^*_k]E[z_i - p^*_k | z_i \geq p^*_k]$. Using the facts that $p^*_k = (k+1)^{-1/k}$ and the distribution of $z_i$ is $\tilde{s}$, we obtain the RHS expression inside the curly brackets in (11).

\(^{22}\)The prices set by the merged entity need not be higher than the prices of the nonmerging firms. While the merged entity internalizes the pricing externalities between its products and this tends to raise its prices, the fact that this firm is being visited first in the marketplace tends to lower them.

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consider that the merged entity deviates from equilibrium by setting a vector of prices \((p_1, p_2, \ldots, p_k) \neq (\hat{p}^*, \hat{p}^*, \ldots, \hat{p}^*)\). As before, for the purpose of writing out the FOC in a symmetric equilibrium, it is sufficient to consider the payoff of a merged entity that deviates by picking a different price for just one of its varieties, say \(i\). We now write such a payoff for the case where the deviation price \(\hat{p} < \min(\hat{p}^*, 1 - \bar{x} + \tilde{p}^*)\).  

Take a consumer who visits the merged entity. The consumer stops searching and buys product \(i\) right away with probability \(Pr[\varepsilon_i - \hat{p} > \max\{z_{k-1} - \hat{p}^*, \bar{x} - \tilde{p}^*\}]\). This gives the merged entity a demand for product \(i\) equal to

\[
\hat{p}^* - \hat{p} + \int_{\hat{p} - \hat{p}^* + \hat{p}}^{1 - \hat{p}^* + \hat{p}} (\varepsilon - \hat{p} + \tilde{p}^*)^k d\varepsilon = \hat{p}^* - \hat{p} + \int_{\hat{p} - \hat{p}^*}^{1 - \hat{p}^*} (\varepsilon + \hat{p}^*)^{k-1} d\varepsilon.
\]

The merged entity also receives demand for product \(i\) from consumers who decide to walk away from it, visit all the nonmerging firms, and finally return to it because product \(i\) turns out to be their best match in the market. This happens with probability \(Pr[\max\{z_{n-k} - \hat{p}^*, z_{k-1} - \hat{p}^*, 0\} < \varepsilon_i - \hat{p} < \bar{x} - \tilde{p}^*]\), which gives the merged entity an additional demand for product \(i\) equal to

\[
\hat{c}_{ia}(\hat{p}; \hat{p}^*, \tilde{p}^*) \equiv \int_{\hat{p}}^{\hat{p} - \hat{p}^* + \tilde{p}^*} (\varepsilon - \hat{p} + \tilde{p}^*)^{n-k} (\varepsilon - \hat{p} + \tilde{p}^*)^{k-1} d\varepsilon = \int_{\hat{p} - \hat{p}^*}^{\hat{p} - \hat{p}^*} (\varepsilon + \hat{p}^*)^{n-k} (\varepsilon + \hat{p}^*)^{k-1} d\varepsilon, \tag{13}
\]

where we use a notation similar to that above. The deviation also affects the merged entity’s demand for products other than \(i\). Let us compute now the demand for one of the other products, say \(m\). A consumer who visits the merged entity stops searching and buys product \(m\) right away with probability \(Pr[\varepsilon_m - \hat{p}^* > \max\{z_{k-2} - \hat{p}^*, \varepsilon_i - \hat{p}, \bar{x} - \tilde{p}^*\}]\), which gives the merged entity a demand for product \(m\) equal to

\[
\hat{c}_{ma}(\hat{p}; \hat{p}^*, \tilde{p}^*) \equiv \int_{\hat{p} - \hat{p}^* + \tilde{p}^*}^{1} (\varepsilon - \hat{p}^* + \tilde{p}) \varepsilon^{k-2} d\varepsilon.
\]

Product \(m\) is also bought by consumers who walk away from the merged entity, visit all the nonmerging firms, and return to the former because product \(m\) is the best for them. This occurs with probability \(Pr[\max\{z_{n-k} - \hat{p}^*, z_{k-2} - \hat{p}^*, \varepsilon_i - \hat{p}, 0\} < \varepsilon_m - \hat{p}^* < \bar{x} - \tilde{p}^*]\), which gives the merged entity a demand for product \(m\) equal to

\[
\hat{c}_{ma}(\hat{p}; \hat{p}^*, \tilde{p}^*) \equiv \int_{\hat{p} - \hat{p}^* + \tilde{p}^*}^{\hat{p} - \hat{p}^* + \tilde{p}^*} (\varepsilon - \hat{p}^* + \tilde{p}^*)^{n-k} (\varepsilon - \hat{p}^* + \tilde{p}^*)^{k-2} d\varepsilon = \int_{\hat{p} - \hat{p}^*}^{\hat{p} - \hat{p}^*} (\varepsilon + \hat{p}^*)^{n-k} (\varepsilon + \hat{p}^*)^{k-2} d\varepsilon
\]

The total profit of the deviant merged entity therefore equals

\[
\hat{\pi}(\hat{p}; \hat{p}^*, \tilde{p}^*) = \hat{p} \left[ \hat{p}^* - \hat{p} + \int_{\hat{p} - \hat{p}^* + \tilde{p}^*}^{1 - \hat{p}^* + \tilde{p}^*} (\varepsilon + \hat{p}^*)^{k-1} d\varepsilon + \hat{c}_{ia}(\hat{p}; \hat{p}^*, \tilde{p}^*) \right]
\]

\[
+ (k - 1) \hat{p}^* \left[ \int_{\hat{p} - \hat{p}^* + \tilde{p}^*}^{1} (\varepsilon - \hat{p}^* + \tilde{p}) \varepsilon^{k-2} d\varepsilon + \hat{c}_{ma}(\hat{p}; \hat{p}^*, \tilde{p}^*) \right]. \tag{14}
\]

\(^{23}\) When the deviation price \(\hat{p} > \hat{p}^*\) (and still \(\hat{p} < 1 - \bar{x} + \tilde{p}^*\)), the payoff function is slightly different but the FOC in symmetric equilibrium is exactly the same.
Payoff to a deviant nonmerging store. Consider a nonmerging firm $j$ that deviates to a price $\hat{p} \neq \hat{p}^*$, with $\hat{p} < 1 - \bar{x} + \bar{p}^*$. As all nonmerging firms are expected to charge $\hat{p}^*$, consumers visit them randomly. The deviant nonmerging firm $j$ may be visited in first place (after the merged entity), second place, and so on until the $(n - k)^{th}$ place. Each of these positions occurs with probability $1/(n - k)$.

Consider that the deviant’s firm is visited by a consumer in her $h^{th}$ search, with $h = 1, 2, \ldots, n - k$. Denote the deal the consumer observes upon entering its shop by $\varepsilon_j - \hat{p}$. There are two situations in which the deviant’s firm sells to this consumer. First, the consumer may stop searching at the deviant’s shop and buy there right away. Conditional on firm $j$ being in $h^{th}$ place, this occurs with probability $\Pr[\max[z_{k-1} - \hat{p}*, z_k - \hat{p}^*] < \bar{x} - \bar{p}^* < \varepsilon_j - \hat{p}]$, which gives the deviant’s firm a demand equal to $\bar{x}^{-k} (\bar{x} - \bar{p}^* + \hat{p}^*)^k (1 - \bar{x} + \bar{p}^* - \hat{p})$. Summing the unconditional probability for all $h$ gives a demand equal to

$$\frac{1}{n - k} \frac{1 - \bar{x}^{n-k}}{1 - \bar{x}} (\bar{x} - \bar{p}^* + \hat{p}^*)^k (1 - \bar{x} + \bar{p}^* - \hat{p}).$$ (15)

Second, the consumer may walk away from the deviant’s firm and come back to it after checking the products of the rest of the nonmerging stores. Conditional on the consumer visiting firm $j$ in her $h^{th}$ search, this occurs with probability $\Pr[\max[z_{k-1} - \hat{p}^*, z_{n-k-1} - \hat{p}^*] < \varepsilon_j - \hat{p} < \bar{x} - \bar{p}^*]$, which gives a demand from returning consumers equal to

$$\bar{c}_a(\hat{p}; \bar{p}^*, \hat{p}^*) = \int_0^{\bar{x} - \bar{p}^* + \hat{p}^*} (\varepsilon + \hat{p}^* - \hat{p})^k (\varepsilon + \bar{p}^* - \hat{p})^{n-k-1} d\varepsilon = \int_0^{\bar{x} - \bar{p}^* + \hat{p}^*} (\varepsilon + \hat{p}^* + \hat{p}^*)^k (\varepsilon + \bar{p}^* - \hat{p})^{n-k-1} d\varepsilon.$$ (16)

Summing the unconditional probability for all $h$ gives a demand equal to $\bar{c}_a(\hat{p}; \bar{p}^*, \hat{p}^*)$.

The total profits of a deviating nonmerging firm are

$$\tilde{\pi}(\hat{p}; \bar{p}^*, \hat{p}^*) = \hat{p} \left[ \frac{1}{n - k} \frac{1 - \bar{x}^{n-k}}{1 - \bar{x}} (\bar{x} - \bar{p}^* + \hat{p}^*)^k (1 - \bar{x} + \bar{p}^* - \hat{p}) + \bar{c}_a(\hat{p}; \bar{p}^*, \hat{p}^*) \right].$$ (17)

Results. Taking the first order derivative of (14) with respect to $\hat{p}$ and setting $\hat{p} = \hat{p}^*$, we obtain the following FOC:

$$1 - (\bar{x} - \bar{p}^* + \hat{p}^*)^{k-1} (\bar{x} - \bar{p}^* + (k + 1) \hat{p}^*) + k \int_0^{\bar{x} - \bar{p}^* + \hat{p}^*} (\varepsilon + \bar{p}^*)^{n-k} (\varepsilon + \hat{p}^* + k \hat{p}^*) d\varepsilon = 0.$$ (18)

Likewise, taking the FOC in (17) and imposing symmetry among the prices of the nonmerging firms gives

$$\frac{1}{n - k} (\bar{x} - \bar{p}^* + \hat{p}^*) \frac{1 - \bar{x}^{n-k}}{1 - \bar{x}} (1 - \bar{x} - \bar{p}^*) + \int_0^{\bar{x} - \bar{p}^* + \hat{p}^*} (\varepsilon + \hat{p}^* + \hat{p}^*)^{n-k-1} (\varepsilon + \bar{p}^*) d\varepsilon = 0.$$ (19)

Proposition 4. Assume that $k < 10$ firms merge. Then, in the long-run, after the merged entity stocks all the products of the parent firms, there exists a Nash equilibrium in the postmerger market where:

- Consumers prefer to start searching at the merged entity and then, if they wish so, continue searching at the nonmerging firms, i.e. $\bar{x} - \bar{p}^* > \bar{x} - \hat{p}^*$.
- The merged entity charges a price $\hat{p}^*$ and the nonmerging stores charge a price $\hat{p}^*$; these prices solve the system of FOCs (18)–(19).

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24 We note that when the consumer visits the deviant immediately after leaving the merged firm, $h = 1$, the consumer, even if surprised by a deviation, will never return to the merged entity without searching further. In fact, this event has probability $\Pr[\varepsilon - \hat{p} < z_k - \hat{p}^* < \bar{x} - \hat{p}^* \text{and } \varepsilon - \hat{p} > \bar{x} - \hat{p}^*] = 0$. 

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This equilibrium exists if the search cost $s$ is sufficiently large, in which case $\hat{p}^* > \tilde{p}^*$. 

The proof is organized in the same way as the proof of Proposition 1. As we did in the previous section, it is illustrative to look at the behavior of the reaction functions of the different types of firms once a merger occurs. We illustrate the main effects in Figure 3. As in Figure 1, the crossing point between the two solid reaction functions gives the premerger equilibrium. When the potentially merging firms merge, a search-order effect and an internalization-of-pricing-externalities effect take place.

The search-order effect stems from the demand-side economies that unfold after the potentially merging stores merge and start carrying all the products of the parent firms. By this effect, the reaction curve of the outsiders (insiders) shifts rightward (downward) from $r_{n-k}^{\text{pre}}$ ($r_k^{\text{pre}}$) to $r_{n-k}^{\text{post}}$ ($r_k^{\text{post}}$). These moves are driven by the changes in the demand elasticity of nonmerging and merging firms after the merger affects the search-order. The crossing point between the dashed curves $r_{n-k}$ and $r_k$ determines the price implications of the search-order effect. The usual internalization-of-pricing-externalities effect shifts the joint reaction function of the insiders (outsiders) further from $r_k$ ($r_{n-k}$) to $r_k^{\text{post}}$ ($r_{n-k}^{\text{post}}$). At the postmerger equilibrium (crossing point between the dotted-dashed reaction functions), all prices, whether from outsiders or insiders, increase. Proposition 4 shows that when search costs are sufficiently large, the price of the merged entity is higher than the price of the nonmerging firms. This means that the internalization-of-pricing-externalities effect dominates the search-order effect. Still, the trade-off consumers face turns out to be favorable for the merging firms: consumers prefer to start searching at the merged entity despite the fact that this firm has a higher price. Economies of search are at the heart of this result.

Before turning to a discussion of the aggregate implications of mergers in the long-run, we make two remarks in connection with Proposition 4. The first observation is that, even though the proposition is proven for the case where the search cost converges to its maximum value, the result is true for much lower search costs. This can be seen in Figure 4a, where we plot the

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25 The restriction $k < 10$, (or, alternatively, $n \leq 10$) is adopted for convenience. If $k \geq 10$, the search cost bound in (11) is a complicated function of $k$ and this makes the calculations cumbersome. Since mergers are relevant in relatively concentrated markets and often take place between two firms at most, the restriction $k < 10$ implies little loss of generality.
reservation utilities $\bar{x} - \hat{p}^*$ and $x - \tilde{p}^*$ against search costs for the $n = 3$ case. The equilibrium of Proposition 4 exists for search costs to the right of the point where the two reservation utility curves intersect (approximately 0.015, i.e., 3% of the average value of a firm’s good). In Figure 4b, we see that the price of the merged entity is higher than the price of the nonmerging firm no matter the level of search costs.

However, our second observation is that the ranking of merging and nonmerging firm prices given in Proposition 4 need not hold for all parameters. In fact, it is possible that the search-order effect more than offsets the internalization-of-pricing-externalities effect, in which case the price of the merged entity is lower than the price of the nonmerging stores. This occurs when the search cost is relatively small and the number of merging firms relative to the total number of firms in the market is also small. In the graphs of Figure 5, the number of merging firms is set equal to 2 and the search cost is very small ($s = 0.005$). Figure 5a plots the postmerger equilibrium prices and shows that the merged entity charges a price lower than that of the nonmerging firms when $n \geq 7$. Figure 5b plots consumer reservation utilities for searching the two types of firms and shows that consumer search-order is consistent with equilibrium pricing for all $n \geq 4$.

We now turn to the impact of merging activity on the profits of insiders and outsiders.

**Proposition 5.** In the long-run postmerger equilibrium of Proposition 4: (i) Any $k$-firm merger is profitable for the merging firms, that is, $\hat{\pi}^*/k > \pi^*$. (ii) If search cost is sufficiently large,
in any $k$-firm merger the nonmerging firms obtain lower profits than the merging firms, that is, $\hat{\pi}^*/k > \tilde{\pi}^*$.

In the short-run equilibrium of Proposition 1, firms did not have an incentive to merge when the search costs are high. The reason is that, everything else equal, the merging firms are placed in the end of consumers’ search-order. When search costs are high, in the long-run, firms that merge gain a prominent position in the marketplace because their shops are stocked with a larger array of products. This clearly makes merging profitable and, in addition, it has a serious impact on the profits of the nonmerging firms. In fact, Proposition 5 shows that, when search frictions are high, the nonmerging firms obtain lower profits than the merged entity. The nonmerging firms, being relegated to the end of the optimal consumer search-order, receive little demand and, correspondingly, lose out relative to the merging firms. This result is in contrast with the standard “free-riding effect” by which outsiders to a merger benefit more than the insiders. To the extent that the free-riding effect is at odds with observed merger waves, our result is more comforting.

Figure 6 illustrates the results in Proposition 5. The merged entity’s profits (dotted-dashed curve) are clearly above premerger levels (solid curve). This is the outcome of two forces: on the one hand, the merged entity benefits from the market prominence it gains by stocking all the products of the parent firms; on the other hand, the merged entity profits from increased market power. The figure also shows that, unless search costs are very low, outsiders lose out (dashed curve). Finally, it is also worth mentioning the asymmetry in the way search costs affect the profits of the different firms after a merger. As search costs increase, the profits of the merged entity go up while the profits of the nonmerging firms fall. This is due to the fact that consumer traffic from the merged entity to the nonmerging firms decreases as search costs rise.

Our final result in this section pertains to the aggregate implications of mergers. As usual, we evaluate the effects of a merger on welfare grounds by comparing the pre- and postmerger sum of consumer surplus and firms’ profits minus search costs. We now compute the expected surplus consumers derive in the postmerger market. Consider first those consumers who buy from

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26 Numerical calculations for markets with up to nine firms reveal that the critical search cost value above which the long-run equilibrium of Proposition 4 exists is lower than the search cost value above which mergers in the short-run equilibrium of Proposition 1 are unprofitable. This suggests that, if the merged entity can reorganize its business costlessly, mergers are always profitable.
the merged entity. These consumers either buy there directly upon arrival or after having visited all the nonmerging firms. Their expected consumer surplus is

$$\hat{CS} = \int_{\pi - \hat{p}^*}^{1} (\varepsilon - \hat{p}^*) \, d\varepsilon + \int_{\pi - \hat{p}^*}^{\pi - \tilde{p}^*} (\varepsilon - \hat{p}^* + \tilde{p}^*)^{(1-k)} (\varepsilon - \hat{p}^*) \, d\varepsilon.$$  (20)

Consider now those consumers who buy from the nonmerging firms. Again, these consumers may buy directly upon arrival or after visiting all the firms in the market. Their expected consumer surplus is

$$\tilde{CS} = (x - \tilde{p}^* + \hat{p}^*)^k \frac{1 - \tilde{x}}{1 - x} \int_{\pi - \hat{p}^*}^{1} (\varepsilon - \hat{p}^*) \, d\varepsilon + (n - k) \int_{\pi - \hat{p}^*}^{x} \varepsilon^{n-k-1} (\varepsilon - \hat{p}^* + \tilde{p}^*)^{(1-k)} (\varepsilon - \hat{p}^*) \, d\varepsilon.$$  (21)

In the long-run postmerger equilibrium, search economies play a crucial role. Consumers who buy directly at the merged entity search only one time. Consumers who walk away from the merged entity and stop searching at the first nonmerging store they enter search only two times, and so on and so forth. The total number of searches is denoted

$$NS = 1 - (\bar{x} - \tilde{p}^* + \hat{p}^*)^k + (\bar{x} - \tilde{p}^* + \hat{p}^*)^k (1 - \bar{x}) \sum_{j=1}^{n-k} (j + 1)(x - \tilde{x})^{j-1} + (\bar{x} - \tilde{p}^* + \hat{p}^*)^k \bar{x}^{n-k}(n - k + 1).$$

Let $Sc \equiv s \ast NS$ be the total search costs incurred by consumers. After simplification,

$$Sc = \frac{1}{2} (1 - \bar{x}) \left[ 1 - \bar{x} + (\bar{x} - \tilde{p}^* + \hat{p}^*)^k (1 - \bar{x})^{n-k} \right].$$  (22)

which, keeping prices fixed, clearly decreases in $k$. Taking into account the costs of searching, net consumer surplus is therefore $CS = \hat{CS} + \tilde{CS} - Sc$. Adding the profits of the firms, we obtain a measure of expected social welfare $SW = CS + \hat{\pi} + (n - k)\tilde{\pi}.$

**Proposition 6.** In the long-run postmerger equilibrium of Proposition 4, if search cost is high enough: (i) Any $k$-firm merger results in an increase in industry profits. (ii) Consumer surplus increases after a $k$-firm merger. As a result, a merger increases social welfare.

The aggregate implications of a merger are illustrated in Figure 7. In Figure 7a we compare pre- and postmerger industry profits. Collectively firms obtain greater profits postmerger (dotted-dashed curve) than premerger (solid curve).

Figure 7b depicts pre- and postmerger consumer surplus and social welfare. The graph illustrates our result in Proposition 6 that when search cost is relatively high, the search economies consumers experience after a merger takes place more than offset the negative price effects of consolidation. When search costs are intermediate, the price effects are stronger than the search economies and consumers lose out; however, their loss is not so large because of the savings in search costs and, therefore, overall welfare increases. When search costs are small, the negative price effects associated to consolidation have a dominating influence and a merger results in a welfare loss, as in Deneckere and Davidson (1985).

5. **Alternative symmetric equilibria and the decision to stock all the products of the parent firms**

The most important results of the article arise in situations where search costs are relatively high, for otherwise the model is similar to the perfect information, case of Deneckere and

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27 In fact, numerical calculations show that the total search costs also decrease in $k$ when we take into account how prices change with $k$. © RAND 2013.
Opposite consumer search-order in the short-run. The discussion in Section 3 centered around a symmetric equilibrium with the merging firms charging a higher price than the nonmerging firms and, correspondingly, with consumers starting their search for satisfactory products at the nonmerging stores. This equilibrium was portrayed as a natural extension of the equilibrium that arises under perfect information, and we showed in Proposition 1 that it exists for all admissible levels of the search cost.

However, another symmetric equilibrium can be proposed. Suppose consumers hold the belief that the merging stores charge lower prices than the nonmerging firms and, consequently, they start searching for satisfactory products at the former; given this, firms respond by setting prices in such a way that consumer beliefs are fulfilled. This could very well occur if the merger process looms so large in consumer minds that the merging firms capture consumer attention and become prominent in the marketplace. In that case, the power of consumer beliefs at dictating the prices of the firms must be sufficiently strong so as to more than offset the internalization-of-pricing externalities effect. In this section, we characterize such an equilibrium and prove that it does not exist when search costs are high.

To do this, we compute the payoff functions of (deviating) merging and nonmerging firms, derive the FOCs and study whether the solution to the system of FOCs satisfies the above mentioned price ranking. Consider first the merged entity’s problem. It is a matter of proceeding similarly as in Sections 3 and 4 (see our working paper for the details) to arrive to the following payoff for a deviating merged firm that changes the price of one of its products, say \(i\), to \(\hat{p} \neq \hat{p}^*\), with \(\hat{p} < 1 - \bar{x} + \hat{p}^*\):

\[
\hat{\pi}(\hat{p}; \hat{p}^*, \hat{p}^*) = p_i h_i(\hat{p}; \hat{p}^*, \hat{p}^*) + (k - 1) \hat{p}^* h_m(\hat{p}; \hat{p}^*, \hat{p}^*).
\]
where
\[ h_1(\hat{p}; \hat{p}^*, \hat{p}^*) = \frac{1}{k} \frac{1 - \bar{x}}{1 - x} (\hat{p}^* - \hat{p}) + \int_{x = \hat{p}^*}^{1 - \hat{p}^*} (\varepsilon + \hat{p}^*)^{k-1} d\varepsilon + \int_0^{\tau - \hat{p}^*} (\varepsilon + \hat{p}^*)^{n-k-1} \varepsilon^{k-1} d\varepsilon \]
\[ h_n(\hat{p}; \hat{p}^*, \hat{p}^*) = \frac{1}{n-1} \frac{1 - \bar{x}}{1 - x} (\hat{p}^* - \hat{p}) + \int_{x = \hat{p}^*}^{1 - \hat{p}^*} (\varepsilon + \hat{p}^*)^{k-2} (\varepsilon + \hat{p}) d\varepsilon + \int_0^{\tau - \hat{p}^*} (\varepsilon + \hat{p}^*)^{n-k-2} \varepsilon^{k-1} d\varepsilon. \]

Consider now the problem of a nonmerging firm \( j \). Its demand is made of consumers who walk away from all the merging stores, happen to stop by firm \( j \) and buy right away there; in addition, some consumers return to firm \( j \) after having visited all shops. The decision to walk away from the last merging store is based on the comparison between \( z_k - \hat{p}^* \) and \( \bar{x} - \hat{p}^* \). This condition is exactly the same as that in Section 4 for consumers to leave the merged entity and visit one of the nonmerging stores. As a result, the payoff of a (deviant) nonmerging firm here is exactly the same as (17).

**Proposition 7.** Assume that \( k \) firms merge. Then, in the short-run, a symmetric equilibrium where \( \hat{p}^* < \hat{p}^* \) so that consumers start searching at the stores of the merged entity and then proceed by searching at the nonmerging stores does not exist whenever one of the following conditions holds: (i) the search cost is sufficiently low, (ii) the search cost is sufficiently high, (iii) \( n = 3 \), (iv) the number of competitors is sufficiently large.

The intuition behind this result is as follows. The price ranking of the firms is the outcome of the tension between the search-order effect, which, being visited first, gives merging firms incentives to lower prices, and the internalization-of-pricing-externalities effect, which works in the opposite direction. The magnitude of the search cost and the number of nonmerging firms affect the outcome of this tension. For example, we know that when the search cost is exactly equal to zero, the price ranking of Proposition 7 is impossible. By continuity, we expect this alternative equilibrium to fail to exist when the search cost is positive but small, and this is what the first part of Proposition 7 shows. What happens is that when the search cost is arbitrarily close to zero, the search-order effect is practically nonexistent and the internalization-of-pricing-externalities effect is the strongest. When the search cost increases, the search-order effect gains importance, while the internalization-of-pricing-externalities effect weakens. For intermediate levels of the search cost, the alternative equilibrium where the merging firms charge lower prices and are visited first may exist (though not necessarily as demonstrated for the case \( n = 3 \)). Finally, when search costs are very high, prices, whether from merging or nonmerging firms, are close to monopoly prices and the search-order effect is again weaker than the internalization-of-pricing-externalities effect. The number of firms affects the tension between the search-order effect and the internalization-of-pricing-externalities effect in a similar way.

Proposition 7 implies that the alternative equilibrium where merging firms charge lower prices and consumers start their search for fine products at the merging firms can only exist for intermediate levels of the search cost and the number of firms. This casts doubts about the appeal of such an equilibrium. At the very least, taking such an alternative equilibrium seriously requires consumer beliefs to be discontinuous in parameters such as the search cost and the number of firms. We find such a requirement on beliefs difficult to justify.

\[ \text{\footnotesize{28 We have explored alternative ways to affect the trade-off between the search-order effect and the internalization-of-pricing-externalities effect in order to rule out the alternative equilibrium of Proposition 7. What is important is to weaken the power consumer beliefs have at dictating equilibrium prices, and this happens, for example, when there is a sufficiently large number of consumers who have perfect information. The equilibrium in Proposition 1 as well as our merger paradox result in Proposition 3, by contrast, survive this modification.}} \]

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Opposite consumer search-order in the long-run. In Section 4, we have characterized an equilibrium where the potentially merging firms gain market prominence if they indeed merge. This gain in prominence arises because the merged entity is assumed to put all its products on display at each of its stores. If this is so, and provided search costs are sufficiently large, consumers find it optimal to first search for a satisfactory product at the merged entity and, if desired, continue searching later at the nonmerging firms.

In this section, we argue that the equilibrium in Proposition 4 is the unique symmetric equilibrium when search costs are relatively high. We prove this by contradiction. Suppose that consumers find it optimal to start searching for a satisfactory good at the nonmerging firms. If this is so, then the reservation utility at the merged entity, \( \bar{x} - \hat{p}^* \), must be lower than the reservation utility at the nonmerging firms, \( x - \hat{p}^* \) (where \( \bar{x} \) and \( x \), as before, solve (1) and (12), respectively). We now show that this is not possible when search costs are sufficiently large.

To do this, we compute the payoff functions of (deviating) merging and nonmerging firms, derive the FOCs, and show that the reservation utility ranking mentioned above is impossible when the search cost is sufficiently high. Consider first the merged entity’s problem. The payoff to a merged entity that deviates by charging a price \( \hat{p} < \hat{p}^* \) for its product \( i \) can be shown to be (for details, see our working paper)\(^{29}\)

\[
\hat{\pi}(\hat{p}; \hat{p}^*, \hat{p}^*) = \hat{p}\ell_i(\hat{p}; \hat{p}^*, \hat{p}^*) + (k - 1)\hat{p}^*\ell_m(\hat{p}; \hat{p}^*, \hat{p}^*),
\]

where

\[
\ell_i(\hat{p}; \hat{p}^*, \hat{p}^*) = \frac{(\bar{x} - \hat{p}^* + \hat{p})}{k} - \frac{k\hat{p}^* - k\hat{p} + 1 - \bar{x}}{k} + \int_0^{\bar{x} - \hat{p}^*} (\epsilon + \hat{p}^*)^{k-1} (\epsilon + \hat{p}^*)^{n-k} \, d\epsilon,
\]

and

\[
\ell_m(\hat{p}; \hat{p}^*, \hat{p}^*) = (\bar{x} - \hat{p}^* + \hat{p})^{n-k}\left(\int_0^{1} (\epsilon - \hat{p}^* + \hat{p})\epsilon^{k-2} \, d\epsilon \right)
\]

\[
+ \int_0^{\bar{x} - \hat{p}^*} (\epsilon + \hat{p}^*)^{k-2} (\epsilon + \hat{p}) (\epsilon + \hat{p}^*)^{n-k} \, d\epsilon.
\]

To compute the payoff of a (deviant) nonmerging firm, we only need to modify the payoff in (7) by properly taking into account that the decision to walk away from the last nonmerging firm and visit the merged entity depends on whether the best of the nonmerging firms’ deals gives a lower or higher utility than \( \bar{x} - \hat{p}^* \). The total payoff of a deviating nonmerging firm equals

\[
\hat{\pi}(\hat{p}; \hat{p}^*, \hat{p}^*) = \hat{p}\left\{ \frac{1}{n-k} \left[ \frac{1 - x^{n-k}}{1 - \bar{x}} (1 - \bar{x} + \hat{p}^* - \hat{p}) + \bar{x}^{n-k} - (\bar{x} - \hat{p}^* + \hat{p})^{n-k} \right] \right\},
\]

\[
+ \hat{p} \int_0^{\bar{x} - \hat{p}^*} (\epsilon + \hat{p}^*)^{n-k-1} (\epsilon + \hat{p}^*) \, d\epsilon.
\]

Proposition 8. Assume that \( k < 10 \) firms merge and that the search cost is sufficiently high. Then, in the long-run, after the merged entity stocks all the products of the parent firms, an equilibrium where consumers prefer to search for a satisfactory product first at the nonmerging firms and continue, if they wish so, at the merged entity does not exist.

The decision to sell all products under one roof. Next, we study whether the merged entity prefers to sell all products in a single store over selling them in separate stores. We restrict ourselves to the extreme cases of high and low search costs because, for those two cases, there exists a unique symmetric equilibrium in each of the business organizations we compare.

For a given consumer search-order, a merged entity that puts on display all its products becomes more attractive for consumers since they probably find a product they like without

\(^{29}\) When \( \hat{p} > \hat{p}^* \) instead, the payoff function is slightly different but the FOC is exactly the same.
incurs additional search costs. By this effect, we expect the merged entity to prefer to stock each of its shops with all the products of the parent firms. At the same time, stocking all products together increases competition with the nonmerging firms, which tends to reduce (merging and nonmerging) firm prices. By this second effect, we expect the merged entity to prefer to sell the products in separate stores. Finally, putting all products together on display may lead to a change in the order consumers visit the firms (cf. Propositions 1 and 4).

Consider the case in which search costs are high. In such a case, as shown in Propositions 4 and 8, if the merged entity sells its products together, there exists a unique symmetric equilibrium where consumers search first the products of the merged entity. If the products of the merged entity are instead sold in separate shops, Propositions 1 and 7 show that there exists a unique symmetric equilibrium where consumers search first the products of the nonmerging firms. Therefore:

**Corollary 1.** (of Propositions 3 and 5). Assume that \( k \) firms merge and that \( s \) is high enough. Then, the merged entity prefers to sell all its products in a single shop over selling them in separate stores. By doing so, the merged entity gains market prominence and its profits rise.

Consider now the case of low search costs. If the merged entity continues to sell its products in separate shops, then, by Propositions 1 and 7, we know the only equilibrium has consumers visiting first the nonmerging firms. Likewise, when the merged entity puts all the products under the same roof, the equilibrium of Proposition 4 does not exist, and the only symmetric equilibrium that can exist has also consumers visiting first the nonmerging firms. A comparison between these two equilibria leads to the result that selling the products in separate shops may generate higher profits than selling them in a single shop.\(^{30}\)

The graphs of Figure 8 illustrate these observations. Figure 8a shows the case of low search costs. When \( s \) is very low, both the demand effect and the price effect are quite small but the price effect has a dominating influence. As a result, the profits of the single-shop merged entity are lower than the profits of the multishop one. For higher search costs, the demand effect is stronger than the price effect, and profits when selling products together are higher than profits when selling them separately. Figure 8b shows the case of high search costs. Here, because the merged entity gains prominence in the marketplace, the merger is highly profitable.

\(^{30}\) In fact, in our working paper, we prove that when \( k = n - 1 \) and the search cost is sufficiently small, then the merged entity prefers to sell its products in separate shops.
6. Concluding remarks

This article has studied the aggregate consequences of mergers in markets where consumers have to search in order to find satisfactory goods. We have used a model where firms compete in prices to sell differentiated products and consumers search sequentially to find product fit information. The order in which consumers visit merging and nonmerging firms depends on the prices firms are expected to charge and on the amount of variety they carry.

We have distinguished between the short-run and the long-run effects of mergers. In the short-run, the merging firms just coordinate their prices, and we have shown that mergers may not be profitable when search costs are high. In the long-run, however, the merging firms can also choose to undertake a business reorganization consisting of retailing all the products of the parent firms within a single store. We have shown that this business reorganization generates substantial demand-side economies because, everything else equal, consumers do not need to search very intensively to find satisfactory products. In contrast to a large literature on cost synergies and supply-side economies, this article has emphasized the importance of these demand-side economies for the aggregate implications of merger activity. We have shown that firms that merge may gain a prominent position in the marketplace, which induces consumers to begin their search for satisfactory products at the merged entity. In equilibrium, insider firms gain customers and increase their profits, while outsider firms lose out because they are placed all the way back in the optimal order consumers follow when they search for products. Importantly, we have shown that consolidation may create sufficiently large search economies so as to generate rents for consumers, too.

We believe the arguments in this article are novel and useful to further understand the effects of consolidation processes. We have shown that mergers may not be profitable in a market where strategic variables are complements and our merger defense result is based on demand-side economies arising from sources other than complementarities (network externalities, complement products, one-stop shopping of an array of products, etc.). Moreover, because the main mechanisms at play are intuitive and powerful, they are expected to play a role in more general market settings, provided search costs are significant. Ultimately, we hope this article adds to a finer design of merger policy.

Efficiency gains arising from mergers may take a relatively long time to materialize. Our theory points out that after-merger business reorganization may lead to important search economies that, in the long-run, may even result in price decreases relative to the short-run. Whether supply- or demand-side economies are at the heart of after-merger potential welfare gains remains an empirical question. Developing methods to quantify the importance of economies of search and cost synergies seems a fascinating area for future empirical research.

Appendix

This Appendix contains the proofs of Propositions 1 through 6. For the proofs of Propositions 7 and 8, please see our working paper.

Proof of Proposition 1. The proof is organized in three Claims. Claim 1 shows that there is a pair of prices \( \hat{p}^*, \tilde{p}^* \) that satisfies the FOCs (9) and (10). Claim 2 shows that such a pair of prices is unique. Claim 3 demonstrates that \( \tilde{p}^* < \hat{p}^* \). Finally, we need to check that firms do not gain by deviating from the prices that solve the FOCs. For arbitrary \( k \), unfortunately, this is extremely complicated. In our working paper, we show it for \( k = 2 \). In what follows, we drop the “*” super-indexes to shorten the expressions.

Claim 1. There is at least one pair of prices \( \{ p^*, \hat{p}^* \} \) that satisfies (9) and (10).

Proof. We first rewrite the FOC (10) as \( G(\hat{p}, \tilde{p}) = 0 \), where

\[
G(\hat{p}, \tilde{p}) = \frac{1 - x}{\hat{p}^{k-1}} - \hat{p} + g(\hat{p}, \tilde{p})
\]  

(A1)
FIGURE A1
EXISTENCE AND UNIQUENESS OF SYMMETRIC EQUILIBRIUM

\[ g(\hat{p}, \tilde{p}) = \int_{\tau - \hat{p}}^{\tau - \tilde{p}} \frac{(\tau + \hat{p})^{k - 1} (\tau + \tilde{p})^{n - k} (\tau + k \hat{p}) d\epsilon}{(\tau - \hat{p} + \tilde{p})^{n - 1}}. \]

Because \( G \) is continuously differentiable, the FOC \( G(\hat{p}, \tilde{p}) = 0 \) defines an implicit relationship between \( \hat{p} \) and \( \tilde{p} \). Let the function \( \eta_1(\tilde{p}) \) define this relationship. This function is represented in Figure A1. By the implicit function theorem we have

\[ \frac{\partial \eta_1(\tilde{p})}{\partial \tilde{p}} = -\frac{\partial G/\partial \hat{p}}{\partial G/\partial \tilde{p}} = -\frac{\partial g/\partial \tilde{p}}{\partial g/\partial \hat{p}} - 1. \] (A2)

The numerator of (A2) is positive. This is because

\[ \frac{\partial g}{\partial \tilde{p}} = \int_{\tau - \hat{p}}^{\tau - \tilde{p}} \frac{(\tau + \hat{p})^{k - 2} (\tau + \tilde{p})^{n - k} (\tau + k \hat{p}) d\epsilon}{(\tau - \hat{p} + \tilde{p})^{n - 1}} > 0. \]

The denominator of (A2) is, however, negative. To see this, we note first that

\[ \tau^{k-1} (\tau - \hat{p} + \tilde{p})^{n-k+1} \left[ \frac{\partial g}{\partial \hat{p}} - 1 \right] = (n-k) \int_{\tau - \hat{p}}^{\tau - \tilde{p}} (\tau + \hat{p})^{k-2} (\tau + \tilde{p})^{n-k} (\tau + k \hat{p}) d\epsilon \]

\[ + (\tau - \hat{p} + \tilde{p}) (k-1) \int_{\tau - \hat{p}}^{\tau - \tilde{p}} (\tau + \hat{p})^{k-3} (\tau + \tilde{p})^{n-k} (2s + k \hat{p}) d\epsilon \]

\[ - (\tau - \hat{p} + \tilde{p})^{n-k+1} \tau^{k-2} [2\tau + (k-1) \hat{p}] \] (A3)

Assuming \( k > 2,31 \) let us take the derivative of the RHS of (A3) with respect to \( \tau \). After simplifying it, we obtain

\[ - \tau^{k-2} (\tau - \hat{p} + \tilde{p})^{n-k} [\hat{p} (n-k+2) + (k-1) \tilde{p}] + (k-1) \int_{\tau - \hat{p}}^{\tau - \tilde{p}} (\tau + \hat{p})^{k-3} (\tau + \tilde{p})^{n-k} (2s + k \hat{p}) d\epsilon \] (A4)

If we now take the derivative of (A4) with respect to \( \tau \) and simplify it we get

\[ - (n-k) \tau^{k-2} (\tau - \hat{p} + \tilde{p})^{n-k+1} [\tau (n+1) + \tilde{p} (k-1)] < 0. \]

This implies that the derivative of the RHS of (A3) with respect to \( \tau \), given in equation (A4), is decreasing in \( \tau \). Setting \( \tau \) equal to its lowest value, \( \hat{p} \), in (A4) gives

\[ - \hat{p}^{k-2} \hat{p}^{n-k} [\hat{p} (n-k+2) + (k-1) \hat{p}] < 0. \]

31 When \( k = 2 \), equation (A3) changes slightly, but the same can be proven (see our working paper).
As a result, the RHS of (A3) is also decreasing in $\tau$. If we set now $\tau = \hat{\rho}$ in the RHS of (A3), we obtain $-\hat{\rho} m^{-k+1} \hat{\rho}^{k-1} (k+1) < 0$. From this, we conclude that (A3) is negative. As a result, because the numerator of $\partial \eta_1(\hat{\rho}) / \partial \hat{\rho}$ is positive and the denominator is negative, we infer that the function $\eta_1(\hat{\rho})$ increases in $\hat{\rho}$.

Now consider the other equilibrium condition. Let us denote the LHS of (9) as $H(\hat{\rho}, \hat{\rho})$. The condition $H(\hat{\rho}, \hat{\rho}) = 0$ also defines an implicit relationship between $\hat{\rho}$ and $\hat{\rho}$. Let the function $\eta_1(\hat{\rho})$ define such a relationship. This function is represented in Figure A1. By the implicit function theorem, we have

$$\frac{\partial \eta_1(\hat{\rho})}{\partial \hat{\rho}} = -\frac{\partial H(\hat{\rho}, \hat{\rho})}{\partial H(\hat{\rho}, \hat{\rho})}.$$  
(A5)

We note that $H$ increases in $\hat{\rho}$. In fact,

$$\frac{\partial H}{\partial \hat{\rho}} = (n-k)(\tau - \hat{\rho} + \hat{\rho})^{k-1} (1-\tau) + (n-k)k \int_0^{\tau^p} (\varepsilon + \hat{\rho})^{k-1} (\varepsilon + \hat{\rho})^{k-1} d\varepsilon > 0.$$  
Moreover, $H$ decreases in $\hat{\rho}$. In fact, for $k < n - 1$ we have

$$\frac{\partial H}{\partial \hat{\rho}} = \frac{1}{1-\tau} - \left[ (n-k)(\tau - \hat{\rho} + \hat{\rho})^{k-1} + (n-k)k \int_0^{\tau^p} (\varepsilon + \hat{\rho})^{k-1} d\varepsilon \right] < 0,$$

while for $k = n - 1$ we get $\partial H / \partial \hat{\rho} = -2 < 0$. As a result, we conclude that the function $\eta_1$ is increasing in $\hat{\rho}$.

Therefore, both $\eta_1$ and $\eta_2$ increase in $\hat{\rho}$. To show that at least one pair of prices $\{\hat{\rho}, \hat{\rho}'\}$ exists that satisfies the system of FOCs (10) and (9), we need to show that the functions $\eta_1$ and $\eta_2$ cross at least once in the space $[0; 1/2] \times [0; \bar{p}_n^\tau]$. As shown in Figure A1 we observe that $\eta_1(0) > 0$. To demonstrate this, note that

$$G(\hat{\rho}, 0) = \frac{1 - \tau^p}{k \tau - \hat{\rho}} - \hat{\rho} + \frac{1}{\tau^p - \hat{\rho}} \int_0^{\tau^p} (\varepsilon + \hat{\rho})^{k-1} (\varepsilon + \hat{\rho}) d\varepsilon > 0,$$

Because $G$ decreases in $\hat{\rho}$ and because

$$G(0, 0) = \frac{1 - \tau^p}{k \tau - \hat{\rho}} + \frac{1}{\tau^p - \hat{\rho}} \int_0^{\tau^p} \varepsilon^{k-1} d\varepsilon > 0,$$

we conclude that $\eta_1(0) > 0$.

On the contrary, we now observe that $\eta_2(0) < 0$ (see Figure A1). This is because

$$H(\hat{\rho}, 0) = 1 - (\tau - \hat{\rho})^{k-1} + (n-k) \int_0^{\tau^p} (\varepsilon + \hat{\rho})^{k-1} (\varepsilon + \hat{\rho})^{k-1} d\varepsilon$$

is increasing in $\hat{\rho}$ and $H(0, 0) = 1 - \tau^p + (n-k) \int_0^{\tau^p} \varepsilon^{k-1} d\varepsilon > 0$.

Secondly, as depicted in Figure A1, we show that $\eta_1(1/2) < \bar{p}_n^\tau < \eta_2(1/2)$, which ensures that the functions $\eta_1$ and $\eta_2$ cross at least once in the area $[0; 1/2] \times [0; \bar{p}_n^\tau]$. To see that $\eta_1(1/2) > \bar{p}_n^\tau$, we show that $H(\bar{p}_n^\tau, 1/2) < 0$ where

$$H\left(\bar{p}_n^\tau, \frac{1}{2}\right) = 1 - \frac{1 - \tau^p}{1 - \tau} - \left( \tau - \bar{p}_n^\tau + \frac{1}{2} \right)^{k-1} + (n-k) \int_0^{\tau^p} \left( \varepsilon + \frac{1}{2} \right)^{k-1} d\varepsilon.$$

Taking the derivative of $H(\bar{p}_n^\tau, 1/2)$ with respect to $\tau$ gives

$$-1 - \frac{(n-k) \tau^{k-1} + (n-k-1) \tau^{k-1}}{2 (1-\tau)^2} - (n-k) \left( \tau - \bar{p}_n^\tau + \frac{1}{2} \right)^{k-1} (1-\tau) < 0,$$

so $H(\bar{p}_n^\tau, 1/2)$ is decreasing in $\tau$. Setting $\tau$ equal to its lowest possible value, $\bar{p}_n^\tau$, we get

$$H\left(\bar{p}_n^\tau, \frac{1}{2}\right)_{\tau = \bar{p}_n^\tau} = 1 - \frac{1}{2^{k-1}} - \frac{1 - (\bar{p}_n^\tau)^{k-1}}{2 (1-\bar{p}_n^\tau)}.$$  
(A6)

This expression is decreasing in $n$. In fact, its derivative with respect to $n$ can be written as

$$\frac{2^{k-1}(\bar{p}_n^\tau)^{k-1} \ln \bar{p}_n^\tau + (1 - \bar{p}_n^\tau) \ln 2}{2^{k-1} (1-\bar{p}_n^\tau)} < \frac{1}{2^{k-1} (1-\bar{p}_n^\tau)} \left[ \bar{p}_n^\tau \ln \bar{p}_n^\tau + (1 - \bar{p}_n^\tau) \ln 2 \right] < 0.$$  

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The last inequality follows from the fact that \( p^n_0 \ln p^n_0 + (1 - p^n_0) \ln 2 < 0 \).
Because \( H(p^n_0, 1/2) \) is decreasing in \( n \), if we set \( n \) equal to its lowest possible value, \( k + 1 \), in (A6) we obtain

\[
H \left( p^n_0 \cdot \frac{1}{2} \right) \leq H \left( p^n_0 \cdot \frac{1}{2} \right) = 1 - \frac{1}{2(k+1)} - \frac{1 - (p^n_0)^{k+1}}{2(1 - p^n_0)} = 1 - \frac{1}{2(k+1)} = 0.
\]

Therefore, because \( H(p^n_0, 1/2) \) is decreasing in \( \hat{p} \), we conclude that \( H(p^n_0, 1/2) \) is always negative. Because \( H \) is increasing in \( \hat{p} \), we obtain the result that \( n_1(1/2) > p^n_0 \).

We now show that \( n_1(1/2) < p^n_0 \). Because \( G \) is decreasing in \( \hat{p} \), it suffices to demonstrate that

\[
G \left( p^n_0 \cdot \frac{1}{2} \right) = \frac{1}{k^{n+1}} - p^n_0 + \frac{k \ln(kp^n_0)}{(k^{n+1} - 1)} \frac{\epsilon}{\epsilon + k p^n_0} < 0.
\]

Taking the derivative of \( G(p^n_0, 1/2) \) with respect to \( n \) gives

\[
\left( \tilde{\pi} - p^n_0 + \frac{1}{2} \right) \left( \tilde{\pi}^{-k} \frac{\partial G(p^n_0, 1/2)}{\partial n} \right) = \int_0^{\tilde{\pi}^{-k}} \left( \epsilon + k p^n_0 \right) \ln \left( \frac{\epsilon + 1/2}{\epsilon + k p^n_0} \right) d\epsilon < 0.
\]

Because \( G(p^n_0, 1/2) \) decreases in \( n \), we can set \( n = 1 \) to its lowest value and write

\[
G \left( p^n_0 \cdot \frac{1}{2} \right) < G \left( p^n_0 - \frac{1}{2} \right) = \frac{1}{k^{n+1}} \left( \tilde{\pi} - p^n_0 + \frac{1}{2} \right) T(\tilde{\pi}),
\]

where

\[
T(\tilde{\pi}) = \left( \tilde{\pi} - p^n_0 + \frac{1}{2} \right) \left( \tilde{\pi} - k \tilde{\pi}^{-k} p^n_0 \right) + k \int_0^{\tilde{\pi}^{-k}} \left( \epsilon + k p^n_0 \right) \ln \left( \frac{\epsilon + 1/2}{\epsilon + k p^n_0} \right) d\epsilon.
\]

Note that \( [k^{n+1}(\tilde{\pi} - p^n_0 + 1/2)]^k > 0 \). Thus, \( G(p^n_0, 1/2) \) is negative if \( T(\tilde{\pi}) < 0 \). \( T(\tilde{\pi}) \) decreases in \( \tilde{\pi} \) because \( \partial T(\tilde{\pi}) / \partial \tilde{\pi} = 1 - k \tilde{\pi}^{-k} p^n_0 \) and this expression decreases in \( \tilde{\pi} \). Therefore, using \( \tilde{\pi} = p^n_0 \), we can write

\[
\frac{\partial T(\tilde{\pi})}{\partial \tilde{\pi}} < \frac{\partial T(\tilde{\pi})}{\partial \tilde{\pi}} \bigg|_{\tilde{\pi} = p^n_0} = 1 - (p^n_0)^{k+1} - k \left( p^n_0 \right)^{k+1} p^n_0 = 0.
\]

Because \( T(\tilde{\pi}) \) decreases in \( \tilde{\pi} \), we conclude that \( T(\tilde{\pi}) = T(p^n_0) = 0 \). As a result, the functions \( n_1 \) and \( n_2 \) cross at least once in the domain \([0, 1/2] \times [0, p^n_0] \).

**Claim 2.** The pair of prices \( \{\hat{p}, \hat{p}^*\} \) that satisfies (9) and (10) is unique.

**Proof.** To show this, it is enough to show that \( n_1 \) increases in \( \hat{p} \) at a rate less than 1, while \( n_2 \) does so at a rate greater than 1. From (A2), because \( \partial G / \partial \hat{p} < 0 \), we know that \( n_1 \) increases in \( \hat{p} \) at a rate less than 1 if and only if \( \partial G / \partial \hat{p} + \partial G / \partial \hat{p} < 0 \).

For the case \( k > 2 \),\(^{34}\) we can then write

\[
\tilde{\pi}^{-k} \left( \tilde{\pi} - \hat{p} + \hat{p} \right)^{k-1} \left[ \frac{\partial G(\hat{p}, \hat{p})}{\partial \hat{p}} + \frac{\partial G(\hat{p}, \hat{p})}{\partial \hat{p}} \right] = (n - k) \int_0^{\tilde{\pi}-\hat{p}} \left( \epsilon + \epsilon + k \hat{p} \right) \ln \left( \frac{\epsilon + 1/2}{\epsilon + k \hat{p}} \right) d\epsilon
\]

\[
+ (k - 1) \int_0^{\tilde{\pi}-\hat{p}} \left( \epsilon + \epsilon + k \hat{p} \right) \ln \left( \frac{\epsilon + 1/2}{\epsilon + k \hat{p}} \right) d\epsilon
\]

\[
- \left( \tilde{\pi} - \hat{p} + \hat{p} \right)^{k-1} \left( 2 \tilde{\pi} - (k - 1) \hat{p} \right).
\]

(A7)

We now notice that the RHS of (A7) decreases in \( \tilde{\pi} \). In fact, its derivative, after rearranging, is equal to

\[
-(n - k) \tilde{\pi}^{-k} \left( \tilde{\pi} - \hat{p} + \hat{p} \right)^{k-1} < 0.
\]

Therefore, if (A7) is negative when setting \( \tilde{\pi} = \hat{p} \), then it is always negative. Checking this, we obtain

\[
\tilde{\pi}^{-k} \left( \tilde{\pi} - \hat{p} + \hat{p} \right)^{k-1} \left[ \frac{\partial G(\hat{p}, \hat{p})}{\partial \hat{p}} + \frac{\partial G(\hat{p}, \hat{p})}{\partial \hat{p}} \right] < -\hat{p}^{k-1} \hat{p}^{k-1} (k + 1) < 0.
\]

\(^{33}\) Taking the derivative of \( p^n_0 \ln p^n_0 + (1 - p^n_0) \ln 2 \) with respect to \( k \) gives \( \partial p^n_0 / \partial k \ln 2 \). The sign of this depends on the sign of \( 1 - \ln 2 \ln p^n_0 \), which is monotonically increasing in \( k \), first negative and then positive. As a result, \( p^n_0 \) first decreases and then increases in \( k \). At \( k = 2 \), it takes on a negative value while at \( k \to \infty \), it is equal to zero. Therefore, it is always negative.

\(^{34}\) Though the derivations are slightly different, the same holds for the case when \( k = 2 \) (see Moraga-González and Petrikaitė, 2013).
Similarly, using (A5), since $\partial H / \partial \hat{p} > 0$, we know that $\partial \eta_1 / \partial \hat{p} < 1$ if and only if $\partial H / \partial \hat{p} + \partial H / \partial \check{p} < 0$. For the case $k < n - 1$, using the expressions above, we then compute

$$
\frac{\partial H}{\partial \hat{p}} + \frac{\partial H}{\partial \check{p}} = -\frac{1 - \tilde{x}^{-k}}{1 - \tilde{x}} + (n - k)(n - k - 1) \int_0^{\tilde{x}^{-k}} (e + \hat{p})^{-1} (e + \check{p})^d d \xi
$$

$$
- (n - k)(\tilde{x} - \hat{p} + \check{p})^{-1} \tilde{x}^{-k} + (n - k) k \int_0^{\tilde{x}^{-k}} (e + \hat{p})^{-1} (e + \check{p})^d d \xi. \quad \text{(A8)}
$$

This expression decreases in $\tilde{x}$ because its partial derivative with respect to $\tilde{x}$, after rearranging, is equal to

$$
-1 - (n - k)\tilde{x}^{-k-1} + (n - k - 1)\tilde{x}^{-k},
$$

and we have already shown above that the numerator of this expression is positive. Thus, using $\tilde{x} = \hat{p}$ in (A8), we can write

$$
\frac{\partial H}{\partial \hat{p}} + \frac{\partial H}{\partial \check{p}} < -\frac{1 - \hat{p}^{-k}}{1 - \hat{p}} - (n - k) \hat{p}^{-k-1} \hat{p}^d < 0.
$$

The result then follows.

**Claim 3.** The price of the merging stores is higher than the price of the nonmerging ones, that is, $\hat{p}^* > \check{p}^*$.

**Proof.** Let $\hat{p}_1$ be the price at which the function $\eta_1$ crosses the 45 degrees line, that is, $\eta_1 (\hat{p}_1) = \hat{p}_1$; likewise, let $\check{p}_2$ be such that $\eta_1 (\check{p}_2) = \check{p}_2$ ($\hat{p}_1$ and $\check{p}_2$ are represented in Figure A1). Given the properties of $\eta_1$ and $\eta_2$, if we show that $\hat{p}_1 > \check{p}_2$, then we can conclude that $\hat{p}^* > \check{p}^*$.

If we set $\hat{p} = \hat{p}_1$ in the FOC $G (\hat{p}, \check{p}) = 0$, we obtain

$$
\hat{p}_1 = 1 - \tilde{x}^{-k} + \frac{\int_0^{\tilde{x}^{-k}} (e + \hat{p}_1)^{-1} (e + k \hat{p}_1) d \xi}{\tilde{x}^{-k}}. \quad \text{(A9)}
$$

Similarly, when $\hat{p} = \hat{p}_2$, the FOC $H (\hat{p}, \check{p}) = 0$ gives

$$
\check{p}_2 = 1 - \tilde{x} + \frac{1 - \tilde{x}^{-k}}{1 - \tilde{x}^{-k}} (n - k) \int_0^{\tilde{x}^{-k}} (e + \check{p}_2)^d d \xi. \quad \text{(A10)}
$$

For a contradiction, suppose that $\check{p}_2 > \hat{p}_1$. Then the difference between the RHS of (A9) and the RHS of (A10) must be negative. Let us denote this difference as $V$ and note that

$$
V \equiv \frac{\int_0^{\tilde{x}^{-k}} (e + \hat{p}_1)^{-1} (e + k \hat{p}_1) d \xi}{\tilde{x}^{-k}} + \frac{1 + (k - 1)\tilde{x}^{-k} - k\tilde{x}^{-k-1}}{k\tilde{x}^{-k-1}} - \frac{1 - \tilde{x}}{1 - \tilde{x}^{-k}} (n - k) \int_0^{\tilde{x}^{-k}} (e + \hat{p}_2)^d d \xi
$$

$$
\geq \frac{\int_0^{\tilde{x}^{-k}} (e + \hat{p}_1)^{-1} d \xi + 1 + (k - 1)\tilde{x}^{-k} - k\tilde{x}^{-k-1}}{k\tilde{x}^{-k-1}} - \frac{1 - \tilde{x}}{1 - \tilde{x}^{-k}} (n - k) \int_0^{\tilde{x}^{-k}} (e + \check{p}_2)^d d \xi,
$$

where the inequality follows from replacing $e + k \hat{p}_1$ by $e + \hat{p}_1$ in the first integral.

Because the second integral in (A11) is equal to $[\tilde{x}^{-k} (\hat{p}_2^*)]/n$, the whole expression in (A11) increases in $\check{p}_2$. Therefore, (A11) must be higher than when we replace $\check{p}_2$ by $\hat{p}_1$. That is, (A11) is higher than

$$
\frac{1}{\tilde{x}^{-k}} \int_0^{\tilde{x}^{-k}} (e + \hat{p}_1)^{-1} d \xi + \frac{1 - (k - 1)\tilde{x}^{-k} - k\tilde{x}^{-k-1}}{k\tilde{x}^{-k-1}} - \frac{1 - \tilde{x}}{1 - \tilde{x}^{-k}} (n - k) \int_0^{\tilde{x}^{-k}} (e + \hat{p}_1)^{-1} d \xi
$$

$$
= \frac{\tilde{x}^{-k} - \hat{p}_1^* n}{(1 - \tilde{x}^{-k})^2} \left[ 1 - \tilde{x}^{-k} - (n - k)\tilde{x}^{-k} (1 - \tilde{x}) \right] + \frac{1 + (k - 1)\tilde{x}^{-k} - k\tilde{x}^{-k-1}}{k\tilde{x}^{-k-1}}. \quad \text{(A12)}
$$

This last expression is positive, which establishes a contradiction. As a result, $\hat{p}^* > \check{p}^*$.

**Proof of Proposition 2.** From Proposition 1, $\hat{p}^* > \check{p}^*$. Let us now show that $\hat{p}^* > p^*$. In what follows we drop the "super-indexes.

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35 The case $k = n - 1$ is similar (see Moraga-González and Petrikaitė, 2013).

36 If $k = n - 1$, then $\frac{\partial H}{\partial \hat{p}} + \frac{\partial H}{\partial \check{p}} = -2 + (1 - \tilde{x}^{-1}) + (n - 1) \int_0^{\tilde{x}^{-1}} (e + \hat{p})^{-1} d \xi = -1 - \hat{p}^{-1} < 0$.

37 The term in squared brackets is positive. To see this, note that it is concave in $k$. Therefore, if it is positive for $k = 2$ and $k = n - 1$, then it is positive for all $k$. Setting $k = 2$ gives $1 - \tilde{x}^{-2} - (n - 2)\tilde{x}^{-1} (1 - \tilde{x})$, which decreases in $\tilde{x}$ since its derivative is $- (n - 2)\tilde{x}^{-3} (1 - \tilde{x}) (1 + \tilde{x}) + (n - 1) \tilde{x}^{-1} < 0$. If we set $\tilde{x} = 1$ in the value for $k = 2$, gives zero. Therefore, it is positive for all $\tilde{x}$ and $k = 2$. Setting now $k = n - 1$ gives $(1 - \tilde{x})(\tilde{x}^{-1}) > 0$. 

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(i) For a contradiction, assume that \( \hat{p} < p \) when \( \mathbf{T} \to 1 \). Denote the equilibrium quantity sold by a nonmerged firm by \( \hat{q} \), that sold by all the merging firms together by \( \hat{q} \), and the aggregate quantity sold in the market by all firms by \( Q \). We note that \( Q = 1 - \hat{p}^k \hat{p}^{n-k} \).38 Using the FOCs we can write that

\[
Y(\hat{p}, \hat{p}) \equiv Q - (n - k)\hat{q} - \hat{q} = 0,
\]

where

\[
(n - k)\hat{q} = \frac{1 - x^{\hat{p}}}{1 - \hat{p}} \hat{p}
\]

\[
\hat{q} = k\hat{p}x^{\hat{p}-1}(\mathbf{T} - \hat{p} + \hat{p})^{\hat{p}^{n-k}} - k(1 - k\hat{p})\int_{0}^{\mathbf{T}} (\epsilon + \hat{p})^{\hat{p}^{n-k}} d\epsilon.
\]

We now argue that \( Y(\hat{p}, \hat{p}) \) is decreasing in \( \hat{p} \). This is because \( \partial Q/\partial \hat{p} < 0 \), \( \partial \hat{q}/\partial \hat{p} > 0 \) and

\[
\frac{1}{k(n - k)\hat{p}} \frac{\partial \hat{q}}{\partial \hat{p}} = x^{\hat{p}-1}(\mathbf{T} - \hat{p} + \hat{p})^{\hat{p}^{n-k}-1} - (k - 1) \int_{0}^{\mathbf{T}} (\epsilon + \hat{p})^{\hat{p}^{n-k}-1} d\epsilon
\]

\[
> x^{\hat{p}-1}(\mathbf{T} - \hat{p} + \hat{p})^{\hat{p}^{n-k}-1} - (k - 1)(\mathbf{T} - \hat{p} + \hat{p})^{\hat{p}^{n-k}-1} \int_{0}^{\mathbf{T}} (\epsilon + \hat{p})^{\hat{p}^{n-k}-1} d\epsilon = (\mathbf{T} - \hat{p} + \hat{p})^{\hat{p}^{n-k}-1} \hat{p}^{\hat{p}^{n-k}-1} > 0.
\]

Next, because \( Y \) is decreasing in \( \hat{p} \) and by assumption \( \hat{p} = p \), we must have \( Y(p, \hat{p}) < 0 \). In other words, using the notation \( \lim_{\mathbf{T} \to 1} p = p \), and \( \lim_{\mathbf{T} \to 1} \hat{p} = \hat{p} \), it must be the case that

\[
\lim_{\mathbf{T} \to 1} Y(p, \hat{p}) = 1 - \hat{p}^{1/k} \hat{p}^{n-k} - (n - k) p_1 - k\hat{p}_1 (1 - \hat{p}_1 + \hat{p}_1)^{\hat{p}^{n-k}}
\]

\[
+ (k - 1) \hat{p}_1 \int_{0}^{1/k} (\epsilon + \hat{p}_1)^{\hat{p}^{n-k}} (\epsilon + p_1)^{\hat{p}^{n-k}} d\epsilon < 0. \quad (A13)
\]

Now we invoke the FOC of the merged entity, denoted above by \( G(\hat{p}, \hat{p}) \). The function \( G(\hat{p}, \hat{p}) \) was shown to be increasing in \( \hat{p} \), so when \( \hat{p} < p \), we must have \( G(\hat{p}, \hat{p}) > G(\hat{p}, \hat{p}) = 0 \). Therefore

\[
\lim_{\mathbf{T} \to 1} G(p, \hat{p}) = -\hat{p}_1 + \frac{(k - 1)}{(1 - \hat{p}_1 + \hat{p}_1)^{\hat{p}^{n-k}}} \hat{p}_1 \int_{0}^{1/k} (\epsilon + \hat{p}_1)^{\hat{p}^{n-k}} (\epsilon + p_1)^{\hat{p}^{n-k}} d\epsilon
\]

\[
+ \frac{1}{(1 - \hat{p}_1 + \hat{p}_1)^{\hat{p}^{n-k}}} \int_{0}^{1/k} (\epsilon + \hat{p}_1)^{\hat{p}^{n-k}} (\epsilon + p_1)^{\hat{p}^{n-k}} d\epsilon
\]

must be positive, which implies that it must be the case that

\[
-\hat{p}_1 (1 - \hat{p}_1 + p_1)^{\hat{p}^{n-k}} + (k - 1) \hat{p}_1 \int_{0}^{1/k} (\epsilon + \hat{p}_1)^{\hat{p}^{n-k}} d\epsilon > -\int_{0}^{1/k} (\epsilon + \hat{p}_1)^{\hat{p}^{n-k}} (\epsilon + p_1)^{\hat{p}^{n-k}} d\epsilon.
\]

Using this inequality in (A13), we get that

\[
\lim_{\mathbf{T} \to 1} Y(p, \hat{p}) > 1 - \hat{p}^{1/k} \hat{p}^{n-k} - (n - k) p_1 - k \int_{0}^{1/k} (\epsilon + \hat{p}_1)^{\hat{p}^{n-k}} (\epsilon + p_1)^{\hat{p}^{n-k}} d\epsilon. \quad (A14)
\]

This last expression is increasing in \( \hat{p}_1 \). This is because the sign of its derivative with respect to \( \hat{p}_1 \) is the same as the sign of the following expression

\[
-\hat{p}_1^{\hat{p}^{n-k}} - (k - 1) \int_{0}^{1/k} (\epsilon + \hat{p}_1)^{\hat{p}^{n-k}} (\epsilon + p_1)^{\hat{p}^{n-k}} d\epsilon + (1 - \hat{p}_1 + p_1)^{\hat{p}^{n-k}}
\]

\[
> -\hat{p}_1^{\hat{p}^{n-k}} - (k - 1)(1 - \hat{p}_1 + p_1)^{\hat{p}^{n-k}} \int_{0}^{1/k} (\epsilon + \hat{p}_1)^{\hat{p}^{n-k}} d\epsilon + (1 - \hat{p}_1 + p_1)^{\hat{p}^{n-k}} = -\hat{p}_1^{\hat{p}^{n-k}} (1 - \hat{p}_1 + p_1)^{\hat{p}^{n-k}} \hat{p}_1^{\hat{p}^{n-k}} > 0.
\]

We now argue that \( \hat{p}_1 \geq p_1 \). To show this, we first invoke the result in Proposition 3 of Zhou (2009) that the equilibrium price of firms visited last is higher than \( p \). In our model, in addition to the search-order effect of Zhou, the firms visited last internalize the pricing externalities they confer on one another, and this leads the firms to raise further their prices. As a result, here it must also be the case that \( \hat{p}_1 \geq p_1 \). Given this, (A14) is greater than after setting \( \hat{p}_1 = p_1 \), that is, \( \lim_{\mathbf{T} \to 1} Y(p, \hat{p}) \) is larger than

\[
1 - (p_1)^{y} - (n - k) p_1 - k \left[ \frac{1}{n} - \frac{1}{n} (p_1)^{y} \right] = np_1 - (n - k) p_1 - kp_1 = 0.
\]

\[38\] A consumer does not buy at all when the match value drawn at every firm is lower than its corresponding price.
where for the first equality we have used the FOC of a typical firm in the premerger market (when \( \bar{\pi} \to 1 \), the FOC of a firm in a premerger market becomes \( 1 - n p_1 - (p_1)'' = 0 \)). Consequently, if \( \bar{p}_1 > p_1 \), then we have \( \lim_{\pi \to 1} Y(p, \bar{p}) > 0 \), which establishes a contradiction.

(ii) Let us take the limit of the LHS of (9) and (10) when \( \bar{\pi} \to p^n_1 \) and let \( \bar{p}_n = \lim_{\pi \to 1} \bar{p} \) and \( p^n = \lim_{\pi \to 1} p^n \) (which is the monopoly price). Then we get the following expressions

\[
\begin{align*}
(p_\hat{n})^- = & \left[ 1 - (k + 1) \left( p^n \right)^k \right] = 0 \\
\left( 1 - p^n \right) (1 - (p^n)^- - \bar{p}_1 \left[ 1 - (p^n)^- \right] = 0.
\end{align*}
\]

The first equation is indeed zero, given the definition of \( p^n \), and the second equation therefore gives the value of \( \bar{p} \) when \( \bar{\pi} \to p^n_1 \). We note that \( \bar{p}_1 < p^n = 1/2 \) because, as shown in the proof of Proposition 1, \( H(p^n, 1/2) \leq 0 \).

Let \( \bar{p}_n = \lim_{\pi \to 1} \bar{p} \). We now argue that \( \bar{p}_1 > p_1 \). To show this, we take the limit when \( \bar{\pi} \to p^n_1 \) of the FOC that determines \( p_1 \). This gives \( (1 - p^n_1)(1 - (p^n_1)') - p_1[1 - (p^n_1)'] = 0 \). The solution of this equation, \( p_1 \), decreases in \( n \). Comparing this equation with (A15), because \( n - k < n \), it is immediately clear that \( \bar{p}_1 > p_1 \).

(iii) If \( n = 3 \), then the FOC of a merging firm may be rearranged as

\[
\bar{p} = \left[ 1 + \bar{p}^3 - \bar{p}X^2 - \bar{p} X (3 \bar{p}^2 - 1) = \frac{\pi^{\bar{p}}}{3} - \frac{\bar{p}^3}{3} - \bar{X} + \bar{\pi} \right. \]  

The FOC of a nonmerging firm gives us the relation \( \bar{\pi}^2 / 3 - \bar{p}^2 / 3 - \bar{\pi} + \bar{p} = 2 \bar{p} - 1 \). Using this expression in (A16), we have \( \bar{p}^3 - \bar{p}X^2 - 3 \bar{p} X^2 - \bar{p} + 1 = 0 \), or

\[
\bar{p} = \frac{1 + \bar{p}^3 - \bar{p}X^2}{1 + 3 \bar{p}^2}.
\]

From the FOC in the premerger market, we know that

\[
p = \frac{1 - p^3}{1 + \bar{X} + \bar{X}}.
\]

Because, by strategic complementarity, \( \bar{p} \) increases in \( \bar{p} \) and because \( \bar{p} > p \), the difference \( \bar{p} - p \) is greater than when we replace \( \bar{p} \) by \( p \). Therefore,

\[
\bar{p} - p = \frac{1 + \bar{p}^3 - \bar{p}X^2}{1 + 3 \bar{p}^2} - \frac{1 - p^3}{1 + \bar{X} + \bar{X}} > \frac{1 + p^3 - pX^2}{1 + 3 \bar{p}^2} - \frac{1 - p^3}{1 + \bar{X} + \bar{X}}.
\]

The RHS of this expression is concave in \( \bar{\pi} \) because its second derivative with respect to \( \bar{\pi} \) is negative

\[
\frac{2 \bar{p}^3}{1 + 3 \bar{p}^2} - \frac{6 (1 - p^3) \bar{X}(1 + \bar{X})}{(1 + \bar{X} + \bar{X})^2 < 0}.
\]

Hence, if the RHS of (A17) is positive with the highest and the lowest possible values of \( \bar{X} \), then it is positive for all possible \( \bar{X} \) values. Setting \( \bar{X} = 1 \) in the RHS of (A17) gives

\[
\frac{2 - 3 p - 3 p^2 + 4 p^3 + 3 p^4}{3 (1 + 3 p^2)}
\]

which is always positive for all \( p \in [0, 1/2] \). Setting \( \bar{X} = p \) in (A17) gives

\[
p \frac{1 - 3 p + 3 p^2}{1 + 3 p^2} > 0.
\]

Thus, \( \bar{p} > p \).

**Proof of Proposition 3.** In symmetric equilibrium, the payoff to the merged entity is equal to

\[
\hat{\pi}(\bar{p}^*, \hat{p}^*) = \hat{\pi}^* \left[ (1 - \bar{X}^*)(\bar{X} - \bar{p}^* + \bar{X}^*)^k + k \int_{0}^{\bar{X}^* - \bar{p}^*} (\bar{X} - \bar{p}^* + \bar{X}^*)^{k-1} d\bar{X} \right].
\]

while the payoff to a nonmerging firm is equal to

\[
\hat{\pi}(\bar{p}^*, \hat{p}^*) = \hat{\pi}^* \left[ 1 - \bar{X} - \bar{p}^* + \bar{p}X^* \right] + \int_{0}^{\bar{X}^* - \bar{p}^*} (\bar{X} - \bar{p}^* + \bar{X}^*)^{k-1} d\bar{X}.
\]

(i) To prove this statement, we set \( k = 2 \) in the profits difference \( \hat{\pi}^* / k - \pi^* \) and study its sign when \( \bar{X} \to p^n_1 (= 1/\sqrt{3}) \). For the profit of a merging firm, we have

\[
\lim_{\bar{X} \to 1/\sqrt{3}} \frac{\hat{\pi}^*}{2} = \frac{(\bar{p}_1)^{\bar{p}^*} - 2}{3 \sqrt{3}}.
\]
where, as in the proof of Proposition 2, \( \tilde{p}_i \equiv \lim_{\tau \to \tau^*} \tilde{p}^\tau \). We have shown above that \( \tilde{p}_i < p^\tau = 1/2 \). Therefore,

\[
\lim_{\tau \to \tau^*} \frac{\tilde{p}^\tau}{2} < \frac{(p^\tau)^{y-2}}{3\sqrt{3}}.
\]

which implies that

\[
\lim_{\tau \to \tau^*} \left[ \frac{\tilde{p}^\tau}{2} - \pi^* \right] < \frac{(1/2)^{y-2}}{3\sqrt{3}} - \frac{(p_i)^2 \left[ 1 - (p_i)^y \right]}{n(1 - p_i^2)}.
\]  \((A19)\)

where, again, as in the proof of Proposition 2, \( p_i \equiv \lim_{\tau \to \tau^*} p^\tau \). If we demonstrate that \((A19)\) is negative, then the result follows. For this, we need that

\[
p_i > \sqrt{\frac{n(1 - 3^{-1/2})(1/2)^{y-2}}{3\sqrt{3}(1 - 3^{-3/2})}}.
\]  \((A20)\)

To show that \((A20)\) indeed holds, we now invoke the FOC in the premerger market; when \( x \to 1/\sqrt{3} \) the FOC writes

\[
1 - (p_i)^y - p_i \frac{1 - 3^{-1/2}}{1 - 3^{-3/2}} = 0.
\]  \((A21)\)

Now, using \((A21)\), if we replace \( p_i \) by \( \left[ \frac{n(1 - 3^{-1/2})}{3\sqrt{3}(1 - 3^{-3/2})} \right]^{1/2} \) in this expression, we get

\[
1 - \left[ \frac{n(1 - 3^{-1/2})}{2^{2-3}\sqrt{3}(1 - 3^{-3/2})} \right]^{1/2} \frac{(1/2)^{y-2}}{3\sqrt{3}} - \frac{(1 - 3^{-1/2})}{1 - 3^{-3/2}}.
\]  \((A22)\)

This last expression is always positive for \( n \geq 3 \). Because \((A21)\) is decreasing in \( p_i \), then \((A20)\) must hold.

(ii) Using the definition of \( p_i^\tau \), we have that \( 1 - (p_i^\tau)^y = k(p_i^\tau)^4 \). Therefore, we can write

\[
\lim_{\tau \to \tau^*} \frac{\tilde{p}^\tau}{k} - \pi^* = \lim_{n \to \infty} \left[ \frac{p_i^\tau (\tilde{p})^{y-1} \left( 1 - (p_i^\tau)^y \right) - (p_i)^2 (1 - (p_i)^y)}{n(1 - p_i^2)} \right] < \lim_{n \to \infty} \left[ \frac{p_i^\tau (1 - p_i^\tau)}{2^{2-1}} - \frac{p_i^{y} (1 - (p_i)^y)}{n(1 - p_i^2)} \right],
\]

where the inequality follows from the fact that \( \tilde{p}_i < p^\tau = 1/2 \). Note that \( 1 - (p_i^\tau)^y > 1 - (p_i^\tau)^y = k(p_i^\tau)^4 \). Thus,

\[
\lim_{n \to \infty} \left[ \frac{(p_i^\tau)^{y+1}}{2^{2-1}} - \frac{(p_i)^{y} (1 - (p_i)^y)}{n(1 - p_i^2)} \right] < \frac{(p_i^\tau)^4}{1 - p_i^\tau} \lim_{n \to \infty} \left[ \frac{p_i^\tau (1 - p_i^\tau)}{2^{2-1}} - \frac{(p_i)^2 (1 - (p_i)^y)}{n(1 - p_i^2)} \right] = 0,
\]

which shows that for any \( k \), merging is not profitable whenever search costs and the number of competitors is sufficiently high.

(c) To prove this, we show that \( \lim_{\tau \to \tau^*} [\tilde{p}^\tau - k \pi^*] > 0 \). Notice that

\[
\lim_{\tau \to \tau^*} \tilde{p}^\tau = \tilde{p}_i k \int_0^{1/\pi^*} (\varepsilon + \tilde{p}_i)^{y-1} (\varepsilon + \tilde{p}_i)^{k-1} d\varepsilon,
\]  \((A23)\)

where \( \tilde{p}_i \equiv \lim_{\tau \to \tau^*} \tilde{p}^\tau \) and \( \tilde{p}_i \equiv \lim_{\tau \to \tau^*} \tilde{p}^\tau \). Because \( \tilde{p}_i \) solves the FOC \((10)\), we can replace \( \tilde{p} \) in \((A23)\) and write

\[
\lim_{\tau \to \tau^*} \tilde{p}^\tau = \tilde{p}_i k \int_0^{1/\pi^*} (\varepsilon + \tilde{p}_i)^{y-1} (\varepsilon + \tilde{p}_i)^{k-1} d\varepsilon = \frac{k \tilde{p}_i^2}{n} (1 - \pi^\tau).
\]

We note that the polynomial \( y(1 - y) \) is increasing in \( y \) for all \( y \leq (n + 1)^{-1/2} \). Therefore, because \( \tilde{p}^\tau > p^\tau \), we can write that \( \lim_{\tau \to \tau^*} \tilde{p}^\tau > k \lim_{\tau \to \tau^*} \pi^* \), where \( p^\tau \equiv \lim_{\tau \to \tau^*} p^\tau \).

Proof of Proposition 4. The proof is similar to the proof of Proposition 1. We first claim that there is a price pair \((\hat{\pi}^*, \hat{p}^*)\) that satisfies the FOCs \((18)\) and \((19)\), and then argue that such a pair of prices is unique. Because the proofs of these two claims are similar to the corresponding ones in Proposition 1, we omit them here and refer the reader to our working paper. There we also prove that “large” deviations are not profitable, either.

It remains to be shown that \( \bar{\pi} - \hat{p}^* > \bar{\pi} - \tilde{p}^* \) and \( \hat{p}^* > \bar{p}^* \). Consider the case in which \( s \) is sufficiently large. Because we assume that \( k \leq 10 \), \( s \to 1/8 \) \((\bar{\pi} \to 1/2)\) suffices. It takes a few steps to check that the solution to the FOCs \((18)\) and \((19)\) is \( \hat{p}^* = p^\tau = 1/2 = \bar{\pi} \) and \( \hat{p}^* = p^\tau = (1 + k)^{-1/2} \) when \( s \to 1/8 \); therefore, \( \hat{p}^* > \bar{p}^* \). Given that \( \hat{p}^* = p^\tau = 0 \) when \( s \to 1/8 \), proving that \( \bar{\pi} - \hat{p}^* > \bar{\pi} - \tilde{p}^* \) boils down to showing that \( \bar{\pi} - \tilde{p}^* > 0 \). We know \( \bar{\pi} \) satisfies \( \int_s^1 k(\varepsilon - x)k^{k-1} d\varepsilon = s = 0 \), or

\[
\frac{k(1 - \bar{\pi}) - \bar{\pi}(1 - \bar{\pi})}{k + 1} - s = 0.
\]  \((A24)\)
Equation (A24) can be rewritten as

\[ \bar{\pi} = \frac{k + \bar{x}^{\frac{1}{k+1}}}{k+1} - s. \]

Deducting \( \hat{p}^* \) on both sides of this equality gives

\[ \bar{\pi} - \hat{p}^* = \frac{k + \bar{x}^{\frac{1}{k+1}}}{k+1} - \hat{p}^* - s. \]  \hspace{1cm} (A25)

When \( s \to 1/8 \), \( \hat{p}^* = (1 + k)^{-1/k} \). As a result, when \( s \to 1/8 \), equation (A25) writes

\[ \bar{\pi} - \hat{p}_m^* = \frac{k + \bar{x}^{\frac{1}{k+1}}}{k+1} - \frac{1}{(1+k)^2} - \frac{1}{8} \]  \hspace{1cm} (A26)

Note now that the RHS of (A26) increases in \( \bar{\pi} \). Therefore, using the lowest admissible value for \( \bar{\pi} \), we can write

\[ \bar{\pi} - \hat{p}_m^* > \frac{k + \left( \frac{1}{2} \right)^{\frac{1}{k+1}}}{k+1} - \frac{1}{(1+k)^2} - \frac{1}{8} > 0 \]

for all \( 2 \leq k < 10 \).

Proof of Proposition 5. (i) We first show that the merging stores increase their profits after the merger. The difference between the profit per product of the merged entity, \( \hat{\pi} / k \), and the typical premerger profit of a firm, \( \pi^* \), equals

\[ \frac{\hat{\pi}}{k} - \pi^* = \hat{p}^* \left[ \frac{1}{n} \int_0^{\pi^*} (1-x) + \frac{1}{n} (x^\pi - \hat{p}^* n) \right] - \hat{p}^* (1 - p^m). \]

Because \( \hat{p}^* \) is an equilibrium price, then, given the nonmerging firm’s price, \( \hat{\pi} (\hat{p}^*) \) is greater than \( \hat{\pi} (\hat{p}) \) for any \( \hat{p} \neq \hat{p}^* \). Therefore, replacing \( \hat{p}^* \) by \( \hat{p}^* \) gives

\[ \frac{\hat{\pi}}{k} - \pi^* > \hat{p}^* \left[ \frac{1}{n} \int_0^{\pi^*} (1-x) + \frac{1}{n} (x^\pi - \hat{p}^* n) \right] - \hat{p}^* (1 - p^m). \]  \hspace{1cm} (A27)

We now note that \( \frac{1}{\bar{\pi}} = \int_x^{\pi^*} \varepsilon^{d-1} \, d\varepsilon \) and is decreasing in \( k \). Therefore, the RHS of (A27) is greater than when we set \( k = n-1 \), which gives

\[ \hat{p}^* \left[ \frac{1}{n} \int_0^{\pi^*} (1-x) + \frac{1}{n} (x^\pi - \hat{p}^* n) \right] = \hat{p}^* \left( n(1-\pi^{n-1}) + (n-1)(\pi^\pi - \hat{p}^* n) \right) - \hat{p}^* (1 - p^m). \]  \hspace{1cm} (A28)

Observe next that this expression is decreasing in \( \bar{\pi} \) because its derivative with respect to \( \bar{\pi} \) is proportional to \( -\bar{\pi}^{-2} n(n-1)(\bar{\pi} - 1) < 0 \). Therefore, (A28) is larger than when we set \( \bar{\pi} = 1 \), which gives

\[ \frac{\hat{p}^*}{n(n-1)} \left( n(n-1) - (n-1)(\pi^\pi) \right) - \frac{\hat{p}^*}{n (1 - p^m)} (1 - p^m) = \frac{\hat{p}^*}{n (1 - p^m)} (1 - p^m) - \frac{\hat{p}^*}{n (1 - p^m)} (1 - p^m). \]

Finally, note that the expression \( \hat{p}^*(1 - \pi^\pi) \) is increasing in \( \hat{p}^* \) because its derivative with respect to \( \hat{p}^* \) is equal to \( 1 - (n+1) \pi^\pi > 0 \). We then conclude that \( \hat{\pi}/k - \pi^* > 0 \) if \( \hat{p}^* > p^* \). However, as argued in the proof of Proposition 2, the last inequality is true because \( \hat{p}^* \) is the price of the firms that are visited last.

(ii) It has been shown in the proof of Proposition 4 that when search cost is large, \( \hat{p}^* \to p^*_1 \) and \( \hat{p}^* \to \frac{1}{2} \). Therefore, we have

\[ \lim_{\tau \to 1/2} \left( \frac{\hat{\pi}}{k} - \pi^* \right) = \lim_{\tau \to 1/2} \left[ \hat{p}^* \left( 1 - (\bar{x} - \hat{p}^* + \hat{p}^*) ^{\frac{1}{k}} + \frac{1}{n-k} \int_0^{\tau-\hat{p}^*} (e + \hat{p}^*)^{-\frac{1}{k}} (e + \hat{p}^*)^{\frac{1}{k}} d\varepsilon \right) \right] - \hat{p}^* \left( \frac{\bar{x} - \hat{p}^* + \hat{p}^*}{n-k} (1 - \bar{x}^{\frac{1}{k}}) + \int_0^{\tau-\hat{p}^*} (e + \hat{p}^*)^{-\frac{1}{k}} (e + \hat{p}^*)^{\frac{1}{k}} d\varepsilon \right) \right] \]

\[ = \frac{1}{k} p^*_1 (1 - p^*_1) - \frac{1}{2} \left( p^*_1 \right)^{\frac{1}{k}} \int_0^{1} e^{\varepsilon^{-1}} d\varepsilon\]

\[ = \frac{1}{k} p^*_1 (1 - p^*_1) - \frac{(p^*_1)^{\frac{1}{k}}}{2} \int_{1/2}^{1} \varepsilon^{d-1} \ln \varepsilon \, d\varepsilon > 0. \]
Then,
\[
\lim_{x \to 1/2} \left[ \hat{\pi}^* - \tilde{\pi}^* \right] = \frac{p_n^*}{k} \left( 1 - (p_n^*)^k \right) - \frac{1}{2} \frac{(p_n^*)^k}{k + 1} (1 - 2^{k-1})
\]
\[
= \frac{p_n^*}{k + 1} \left( 1 - \frac{1}{2} \right) + \frac{1}{4(k + 1)} \left[ 4p_n^* - 1 \right] > 0,
\]
where the first inequality follows form replacing \( n \) by \( k + 1 \).

**Proof of Proposition 6.** (i) We first note that the equilibrium of Proposition 4 has \( \hat{p}^* > \tilde{p}^* \) when the search cost is sufficiently high. The difference between post- and premerger total industry profits is \( \Delta \Pi \equiv \hat{\pi}^* + (n - k)\tilde{\pi}^* - n\pi^* \).

Using the expressions for profits above, we have
\[
\Delta \Pi = \hat{p}^* \left( 1 - (x - \tilde{p}^* + \hat{p}^*)^k \int_0^{\pi - \hat{p}^*} (\varepsilon + \hat{p}^*)^{k-1} d\varepsilon \right)
\]
\[+ \tilde{p}^* \left( (x - \tilde{p}^* + \hat{p}^*)^k (1 - \tilde{\pi}^{n-k}) + (n - k) \int_0^{\pi - \tilde{p}^*} (\varepsilon + \tilde{p}^*)^{k-1} d\varepsilon \right) - p^* (1 - p^m).
\]
Note now that this expression is clearly increasing in \( \hat{p}^* \) (the derivative of the first line, by the FOC, is zero and that of the second line is positive). Hence,

\[\Delta \Pi > \Delta \Pi |_{\hat{p} = \tilde{p}} = \hat{p}^* (1 - \tilde{p}^m) - p^* (1 - p^m) > 0, \tag{A29}\]
as shown in the proof of Proposition 5.

(ii) In the premerger market, consumer surplus is given by
\[
CS^* = \frac{1 - x}{1 - x} \int_{x}^{1} (\varepsilon - p^*) d\varepsilon + n \int_{p^*}^{\pi} (\varepsilon - p^*) d\varepsilon - \frac{1 - \pi^*}{1 - x}.
\]
In the postmerger market, consumer surplus is given by \( CS = \tilde{CS} + \tilde{CS} - Sc \) where \( \tilde{CS}, \tilde{CS}, \) and \( Sc \) are given by (20), (21), and (22), respectively.

When \( s \to 1/8, x \to 1/2, p^* \to 1/2 \) and \( \hat{p}^* \to p_n^* \). Then, we can establish the comparison
\[
\lim_{x \to 1/8} \left[ \tilde{CS} + \tilde{CS} - Sc - CS^* \right] = \int_{p_n^*}^{1} k (\varepsilon - p_n^*)^k d\varepsilon - \frac{1}{8} > 0.
\]

The proof is now complete.

**References**


