The Expected Returns and Valuations of Private and Public Firms

(Previously titled: The Cross-Section of Industry Investment Returns)

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Abstract

Industry characteristics explain the cross section of investment returns among industries consisting primarily of private firms as well as among industries composed mostly of public firms. For both types of industries, common asset pricing models explain the cross-sectional variation of characteristic-based investment returns. Tobin’s $q$ and its cross sectional variation are very similar across private and public firms. An industry’s characteristics, not the fraction of private firms in it, determines the industry’s cost of capital. Assuming that managers of private firms are less affected by investor misvaluation our results are consistent with a rational interpretation of the role of characteristics.

JEL Classification: G0, G12, G31.

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1 Introduction

The role of firm characteristics in describing the cross section of average stock returns has led to the claim that mispricing is prevalent in the economy. Daniel and Titman (1997) show that characteristics dominate covariances in explaining the cross section of average stock returns.\(^1\) These findings are part of the backbone of the evidence suggesting investors exhibit behavioral biases (see the discussion in Barberis and Thaler (2003)). However, Lin and Zhang (2012) show that in general equilibrium, just like covariances, firm characteristics are sufficient statistics for expected stock returns, and expected stock returns are determined endogenously jointly with covariances (as in the consumption approach of Lucas, 1978) and firm characteristics (as in the investment approach of Cochrane, 1991). Therefore, the search for mispricing through running horse races of covariances against characteristics is pointless. Moreover, characteristics will dominate covariances in return regressions since, as Lin and Zhang (2012) show, the former are measured more precisely. However, this says nothing about mispricing; finding evidence that characteristics dominate covariances provides evidence that is consistent with both rational and irrational pricing.

In this paper, we examine the determinants of the cross section of industry investment returns, derived from the \(q\)-theory of investment (Cochrane, 1991, Liu, Whited and Zhang, 2009) within two groups of industries. The first is a group of industries which consist primarily of privately held firms, which we intermittently refer to as private industries, whereas most firms in the second group of industries are public firm and hence we intermittently refer to that group as public industries. We also study the investment returns of portfolios consisting of both types of industries. We use the NBER industry productivity database that aggregates both public and private firms, and identify private industries as those industries with a low ratio of sales (employees) of listed firms in the industries to total industry sales (employees). Examining investment returns of all firms, and focusing particularly on private firms, allows us to address four important issues.

First, it has been established that investment returns are equal to stock returns.\(^2\) Therefore, if the role of characteristics in investment returns in a sample that includes primarily private firms is the same as their role in investment returns of a sample of mostly public firms, this lends some support to ruling out mispricing as an explanation for the role of these characteristics. The reasons for this is that private firms have no stock prices and that their managers are less susceptible to misvaluation as we argue below. Instead, the role of characteristics is likely to stem from their presence in the first order investment conditions of firms’ optimal investment decisions.

Second, this is the first paper to address the risk-return relation of all firms. If a factor is a true aggregate risk factor it should price all equity, whether it belongs to public or private firms, assuming equity holders of both public and private firms require a premium for bearing the factor’s risk. To date the literature has only examined the risk-return relation of public firms and therefore it has not been possible to establish whether common risk factors are actually sources of aggregate uncertainty or are relevant only for firms that are publicly listed on the stock exchange.

Third, the estimates of the cost of capital for private firms are notoriously difficult to obtain because of the lack of stock prices. However, by using investment returns, we can obtain the first estimates of the cost of capital of private firms from asset pricing models. Because most firms in the economy are private, being able to obtain a risk based measure of the cost of capital is crucial to optimal decision making. Our paper assesses the only means, to the best of our knowledge, of achieving this.

Fourth, following Belo, Xue and Zhang (2013), we obtain valuation ratios (that is, Tobin’s average \(Q\)) implied by firms’ first-order conditions with respect to investment. Subsequently, we compare the valuation ratios as well as the cross-section of valuation ratios of private and public industries. To the best of our knowledge, ours is the first paper to examine the valuation of private firms and to compare them to those of public firms. On the one hand, private firms might have higher valuation ratios than public firms.

\(^2\)Cochrane (1991) demonstrates this theoretically and provides evidence at the aggregate level. Liu, Whited, and Zhang (2009) show that investment returns are equal to stock returns for portfolios sorted on characteristics that give a large spread in stock returns.
due to less dispersed ownership structure and hence fewer manager-shareholder agency problems. On the other hand, public firms might be less financially constrained and hence have higher valuation ratios as they might be better able to exercise their growth options. The comparison between the two is an important empirical question which we examine.

While all previous assessments of risk, return, the cost of equity capital and valuation ratios have focused on public firms, the importance of private firms in the economy should not be underestimated and is an economically important topic. For instance, Asker, Farre-Mensa and Ljungkvist (2012) estimate that in 2007 private U.S. firms accounted for 54.5% of aggregate non-residential fixed investment, 67.1% of private sector employment, 57.6% of sales, and 20.6% of aggregate pre-tax profits. Thus, private firms are an important, but often neglected, part of the US economy.\(^3\)

The empirical results we present are a novel contribution because the extant literature has not focused on the role of characteristics and risk-adjusted returns for all firms including private firms due to the lack of stock return data for private firms. Our approach of using investment returns, derived from the \(q\)-theory of investment, circumvents the need for stock return data and is therefore particularly useful for examining private firms. Since Cochrane (1991), Restoy and Rockinger (1994), and Liu, Whited and Zhang (2009) show empirically that investment returns are equal to stock returns, the results we present are consistent with those that would have been obtained had stock return data been available.

Our main findings can be summarized as follows. First, we show that characteristics that have been shown to explain the cross section of stock returns, namely the investment to capital ratio \((I/K)\), the return on assets \((ROA)\) (see Hou, Xue and Zhang, 2012) and lagged investment returns are determinants of the cross section of investment returns of both industry portfolios with a relatively large fraction of private firms as well as of industry portfolios with a relatively small fraction of private firms. Therefore, because characteristics share the same role in the determination of average investment returns for both private and public firms, their role in determining investment returns is unlikely to

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\(^3\) The vast majority of firms in the U.S. are closely-held corporations. The latest Census indicates seven million corporate tax filers, of which only about 8,000 are public firms.
stem from stock mispricing simply because private firms have no stock price. Rather the role of characteristics stems from their fundamental part in the first order conditions for investment decisions (Lin and Zhang (2012)). We also find that idiosyncratic volatility is an important determinant of the cross section of investment returns of both portfolios with a large fraction of private firms as well as of portfolios with a large fraction of public firms. Idiosyncratic volatility could have a role for one of two reasons. First, under-diversification which may be present in private firms where firms are more likely to be closely held. Second, Bartram, Brown, and Stulz (2011) argue that the high idiosyncratic volatility of U.S. stock returns is related to high levels of investor protection and research intensity at the country level and high levels of research and development at the firm level. They relate these characteristics to a greater level of growth options and the opportunities to exercise them in the U.S.. Given that idiosyncratic volatility is important cross-sectionally in both public and private firms, it is unlikely to arise from under-diversification, but rather from growth options.

Second, a three factor model derived from the $q$-theory of investment, similar to that in Hou, Xue and Zhang (2012), composed of the "market" investment return, an $I/K$ factor and an ROA factor performs well in explaining the cross-section of investment returns of twenty characteristic-based industry portfolios formed using mainly private firms. The portfolios are composed of five $I/K$ portfolios, five ROA portfolios, five portfolios sorted by lagged investment returns and five portfolios sorted by idiosyncratic volatility. The model also performs well in terms of small pricing errors and a large cross-sectional $R^2$. This finding is the case irrespective of the amount of private firms in each portfolio. Therefore, because the risk factor model affects both public and private firms they are likely to be true aggregate risk factors in that they are aggregate sources of uncertainty in the economy. Overall, whether looking at the role of characteristics or risk factors, we find that they are crucial in explaining the cross section of all industries, as well as the cross-section of private industries and public industries separately.

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4The factor model of Hou, Xue and Zhang (2012) includes a size factor as well. We find that size, as measured by the size of the real capital stock of industries, cannot explain the cross section of industries' investment returns. This result could be the consequence of us using the size of assets whereas Hou, Xue and Zhang using market capitalization for size.
Third, based on the estimates from the three factor model, we calculate the cost of capital (expected asset return) for all industries and industries with varying degrees of private firms in them. The differences in these estimates across private and public firms are generally small suggesting that private and public firms have similar costs of equity. There is certainly no systematic difference in the cost of capital in the sense that private firms always have a higher (lower) cost of capital than public firms. In fact, we find that the differences in the cost of equity capital between private and public firms are largest in the portfolios with high or low characteristics. This indicates that the characteristics, and not the fact that the portfolio contains more of less private firms, are driving any differences in the cost of capital. Therefore, using investment returns and a three factor model based on Hou, Xue and Zhang (2012) to calculate the expected returns for private firms is a useful risk adjusted method that firms can employ to calculate their weighted average cost of capital. In addition, given that the cost of capital from the investment return approach is similar for public and private firms, given a portfolio formation characteristic, private firms can use similar characteristics public firms investment returns to proxy their own cost of capital, particularly when the private firms do not have an extreme value of one of the characteristics we examine. To our knowledge, we provide the first empirical support for this method of calculating the cost of capital for private firms.

The results that private and public industries have similar costs of capital, and in particular that there is no systematic difference across the two types of firms, might seem surprising given the lack of liquidity of private firms and the potential under-diversification of their owners. However, these findings are consistent with Moskowitz and Vissing-Jørgensen (2002) who use estimates of private firm value and profits at the aggregate level and study the returns to aggregate entrepreneurial investment. They find that in spite of the poor diversification of the owners of private firms, the returns to aggregate private equity are not higher than the returns to public equity. We differ from Moskowitz and Vissing-Jørgensen along several dimensions. First, we examine the determinants of the cross section of a large number of industries rather than the time-series of the

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5 Due to lack of data on industries’ capital structure in our database we can provide evidence on the weighted average cost of capital (WACC) but not on the cost of equity and the cost of debt separately.
aggregate market. Second, we estimate the returns on investment using production and capital stock data and not on equity value estimates. Moreover, as we do not have data on capital structure we cannot directly compare the cost of equity for private and public industries.

Our fourth finding regards the valuation ratios of private industries and of public industries. We find that private industries have valuation ratios and a cross sectional variation of valuation ratios that are very similar to those of public industries. Thus, the possibly better corporate governance of private firms and the easier access of public firms to capital seem to offset each other. Moreover, under the assumption that managers of private firms are less influenced by investors’ misvaluation, the fact that the cross-section of valuation ratios of private and public firms are very similar supports the conjecture that managers of public firms are also largely unaffected by investor misvaluation. This finding provides further support to Belo, Xue and Zhang (2013) that firms’ valuation ratios derived from the neoclassical model of investment represent the intrinsic value of the firms.

At first blush, it might be thought that the findings we present regarding the role of characteristics and mispricing should be considered cautiously. The reason for this is that the lack of stock prices does not necessarily imply that investment returns are not affected by overvaluation or undervaluation of the firm. For example, if a certain characteristic indicates that a public firm’s stock is overpriced and subsequent stock returns are abnormally negative, then the same characteristic could be associated with abnormally high real investment due to managers’ overvaluation of investment projects followed by negative abnormal investment returns for private firms. However, to the extent that managers of firms, and especially of private firms, are less affected by investors’ misvaluation concerning the firm than investors in the stock market, our results are consistent with a rational-based explanation for the role of characteristics in explaining expected stock returns.

Our claim that the results are most consistent with a rational based explanation are based on a number of factors that lead us to believe that the investment returns
of private firms are less likely to be affected by investors’ misvaluations. First, when managers possess private information on which they base their expectations and rational decisions they are likely to ignore investors’ misvaluations. Given that private firms are likely to be characterized by more asymmetric information, the influence of investor sentiment is further diminished for these firms. This is collaborated in Hribar and Quinn (2010) who examine the trading patterns of managers and find evidence that they can see through market sentiment. Second, as noted by Polk and Sapienza (2009), if the market misprices firms according to their level of investment, managers may try to boost short-run share prices by catering to current sentiment. Managers with shorter shareholder horizons should cater more. This mechanism is unlikely to exist within private firms. Stein (1996) argues that managers with short horizons should be aggressively investing when investors are overly optimistic. Asker, Farre-Mensa and Ljungkvist (2011) present evidence consistent with managers of public firms being short-termist and managers of private firms not being short-termist. Third, while managers of private firms could still raise capital through private placements when their firms are overvalued, being non short-termist implies they will use the proceeds for investment in T-bills rather than undertake negative NPV projects (Stein, 1996). Fourth, Cooper and Priestley (2011) find that the investment-future stock return relation can be explained without recourse to arguments based on overinvestment or investor overreaction. In particular, they find that differences in systematic risk between high and low investment firms can explain the differences in average stock returns between high and low investment firms.

Overall, while we can not fully rule out that investment returns of private firms are unaffected by sentiment or other behavioral biases, it is certainly the case that they are less likely to be. Therefore, our findings that the same characteristics and risk factors are relevant for both private and public firms points to the conclusion that the role of characteristics in both private and public firms’ investment returns and the previous reported role of them in stock returns, is unlikely to be related solely to mispricing.

The rest of the paper is organized as follows. In Section 2, we illustrate the equivalent role of characteristics and covariances in returns. Section 3 describes the data and variable
construction. Section 4 presents the empirical findings. The paper concludes in Section 5.

2 The Equivalent Role of Characteristics and Covariances

In this section of the paper, we follow Lin and Zhang (2012) and show the equivalence between the role of characteristics and covariances. In the typical consumption economy with no production the agent’s first order consumption problem results in the following well known expression for expected returns:

\[ E_t[M_{t+1} r_{i,t+1}^s] = 1, \]  

(1)

where \( M_{t+1} \) is the stochastic discount factor and \( r_{i,t+1}^s \) is the gross return on stock \( i \). Cochrane (2005) shows how to use the definition of covariance to write expression (1) in terms of a beta pricing model:

\[ E_t[r_{i,t+1}^s] - r_f = \beta_i^M \lambda_M, \]  

(2)

where \( r_f = \frac{1}{E_t[M_{t+1}]} \) is the risk free rate, \( \beta_i^M = -\text{cov}(r_{i,t+1}^s, M_{t+1})/\text{var}(M_{t+1}) \) is the loading of \( r_{i,t+1}^s \) on \( M_{t+1} \), and \( \lambda_M \) is the price of risk defined as \( \text{var}(M_{t+1})/E_t[M_{t+1}] \).

Now turning to a production economy with adjustment costs, Cochrane (1991) shows that stock returns can be written in terms of characteristics:

\[ r_{i,t+1}^s = \frac{\pi_{i,t+1}}{1 + a \left( \frac{I_{i,t}}{K_{i,t}} \right)}, \]  

(3)

where \( \pi_{i,t+1} \) is firm \( i \)'s productivity given a set of random aggregate shocks, \( I_{i,t} \) is firm investment, \( K_{i,t} \) is firm capital stock, and \( a \) is an adjustment cost parameter. Lin and Zhang (2012) focus on the equivalence between these two approaches:
where the first term presents the expression for expected returns in terms of covariances and the final term in terms of characteristics. Rearranging makes the relationship between covariances and characteristics clearer:

\[
\beta_i^M = \frac{E_t [\pi_{i,t+1}] - r_f}{\lambda_M (1 + a \left( \frac{I_{i,t}}{K_{i,t}} \right))},
\]

(5)

In a general equilibrium framework with positive adjustment costs, expected stock returns, covariances and characteristics all become endogenous. There is no causal relation among these variables. Specifically, no causality runs from covariances to expected returns, from characteristics to expected returns, or vice versa. Therefore, showing that risk factors (covariances) or characteristics are important in stock return regressions does not mean that they explain expected returns. We can say nothing about the rationality of prices from these approaches. However, we can say nothing about irrationality either. The point is that characteristics can show up in the cross section of returns because of their role in the firm’s first order investment decision or because of mispricing.

Now consider Cochrane (1991) who shows that

\[
r_{i,t+1}^s = \frac{\pi_{i,t+1}}{1 + a \left( \frac{I_{i,t}}{K_{i,t}} \right)} = r_{i,t+1}^I,
\]

(6)

where \( r_{i,t+1}^I \) is the firm’s investment return. The equivalence between stock returns and investment returns allows us to use investment returns for private firms. This enables us to address two central and important issues. First, if characteristics and loadings on risk factors are important in the determination of expected returns, and are similar for private and public firms’ investment returns, then the role of characteristics in general is likely to be due to the first order production decisions of firms and not due to mispricing. That is, to the extent that managers of private firms are less affected by investor sentiment or valuation mistakes regarding their firms than investors in the stock market and than
managers of public companies, finding that characteristics drive the cross section of returns would lend some support to the idea that it is the fundamental first order investment decision that explains the role of characteristics in the cross-section of returns. Second, what is the cost of capital for private firms and does it differ from that of public firms? This issue has not been addressed before in a risk-return framework.

There is a further advantage with asset pricing tests that use private firms as part of the sample. If a factor that is related to returns is a "true" risk factor then it is a necessary condition that it is a source of aggregate uncertainty which affects all firms in the economy, assuming some level of diversification for owners of privately held firms. To our knowledge, the extant literature has focussed asset pricing tests entirely on returns of public firms because of the availability of stock returns. Consequently, there is no possibility to assess whether these factors are an aggregate source of uncertainty. By including private firms, we are able to assess whether risk factors are an aggregate source of uncertainty.

3 Data and Variable Construction

We use the Bartelsman, Becker and Gray 2009 NBER-CES Manufacturing Industry Productivity Database (which we refer to as the NBER database), available on the NBER website, as well as the Compustat database. The NBER database contains annual 4-digit SIC industry-level data on output, investment, capital stock and other industry-related variables for all 4-digit manufacturing industries in the US for the period 1958-2005. The data covers 459 manufacturing industries and are collected from various government sources, with many of the variables taken directly from the Census Bureau’s Annual Survey of Manufacturers (ASM) and Census of Manufacturers. The ASM is a survey of around 60,000 establishments, carried out by the Census Bureau. Bartelsman and Gray (1996) provide a detailed description of the database.

Our primary variable of interest is the rate of return on investment. Liu, Whited and Zhang (2009) assume a Cobb-Douglas production function and a quadratic adjustment cost function and derive the investment return as follows:
\[ r'_{i,t+1} = \frac{(1 - \tau_{t+1}) \left[ \alpha \frac{Y_{i,t+1}}{K_{i,t+1}} + \frac{a}{2} \left( \frac{I_{i,t+1}}{K_{i,t+1}} \right)^2 \right] + \tau_{t+1} \delta_{i,t+1} + (1 - \delta_{i,t+1}) \left[ 1 + (1 - \tau_{t+1}) \alpha \left( \frac{I_{i,t+1}}{K_{i,t+1}} \right) \right]}{1 + (1 - \tau_{t+1}) a \left( \frac{I_{i,t}}{K_{i,t}} \right)} \]  

(7)

where \( \alpha \) is the share of capital in production, \( Y \) is sales, \( K \) is the stock of capital, \( I \) is investment, \( \delta \) is capital depreciation and \( a \) is an adjustment cost parameter. A larger value of \( a \) implies that the industry is facing higher adjustment costs of investment.

As Liu, Whited and Zhang (2009) note, the investment return given in equation (7) is the ratio of the marginal benefit of an additional unit of installed capital (marginal \( q \)) to the marginal cost of installing an extra unit of capital. The term \((1 - \tau_{t+1}) \left[ \frac{Y_{i,t+1}}{K_{i,t+1}} \right]\) is the marginal after-tax profit produced by an extra installed unit of capital. The term \((1 - \tau_{t+1}) \left[ \frac{a}{2} \left( \frac{I_{i,t+1}}{K_{i,t+1}} \right)^2 \right]\) is the marginal after-tax reduction in adjustment costs caused by having an extra unit of installed capital. The term \( \tau_{t+1} \delta_{i,t+1} \) is the marginal depreciation tax shield, and the last term is the marginal continuation value of an extra unit of capital net of depreciation.\(^6\)

To calculate industry investment returns at the aggregate industry level we need several data items and estimates. We use the value of shipment data item from the NBER database, deflated by a value of shipment deflator in order to obtain data on real industry output, \( Y \). We use the real capital stock series from the NBER database for the capital stock \( K \). Investment, \( I \), is given by total capital expenditures, deflated by a deflator for that series in order to obtain investment in real terms, where both capital expenditure per industry and the investment deflator are from the NBER database. We follow Liu, Whited and Zhang and measure \( \tau_{i} \), the corporate tax rate, as the statutory corporate income tax rate. The source for the tax data is the Commerce Clearing House annual publications. We use Compustat data on depreciation and amortization (item DP) to compute industry-level rates of depreciation as follows. For each industry in each year, we sum the depreciation of all firms in that industry and divide by the sum of capital

\(^{6}\) Note that the price of an installed unit of capital is equal to its marginal value (marginal \( q \)), which under optimality equals the marginal cost of investment given by \( a \left( \frac{I_{i,t+1}}{K_{i,t+1}} \right) \). Thus, the last term in (7) reflects the value of the undepreciated extra unit of capital.
stocks of all firms on Compustat in the industry.

For the industry-specific capital share parameter $\alpha$ and the adjustment cost parameter $a$ we use the estimates in Belo, Xue and Zhang (2010). Belo Xue and Zhang use GMM on the investment Euler equation and on a valuation equation to estimate these parameters for the Fama French 48 industries (as well as characteristic-based portfolios). For each of the four-digit SIC code industries in our sample we assign the $\alpha$ and $a$ parameters of the two-digit industry from the 48 Fama and French industries that industry belongs to.$^7$

A potential problem when using the NBER database to calculate industry investment returns is the fact that the data are only for US-based variables. That is, there is no information in this database on the stock of capital of US industries held abroad, as opposed to the Compustat data which includes data on total firm capital held domestically and abroad. Note, however, that the required return on investment in the stock of capital held in the US should not be affected by the exclusion of capital held in other countries for the following reason. If a firm undertakes an investment project in the US it will require a rate of return on that investment that either corresponds to the risk of the project, or is related to some behavioral biases the firms’ managers have. Thus, it is possible to study the risk-return relation for such projects independently of capital held in foreign countries. This is similar to examining the cross section of average stock returns in a sub-sample of the CRSP database, for example in a sub-sample that contains NYSE stocks only. Any asset pricing model would contend that average returns of firms in that sub-sample of firms are related to their riskiness or to some characteristics.

Common measures of the return on real investment such as the return on investment ($ROI$), the return on assets ($ROA$) or the return on equity ($ROE$) might be imprecise for several reasons. First, these measures assume that the marginal return on investment equals the average return on investment. However, the return on investment is likely diminishing. Second, the denominators of $ROA$ and $ROE$ do not account for adjustment costs of investment. Third, the numerators account only for the cash flow part of investment but disregard the part that is due to the undepreciated capital and the reduction in

$^7$We remove from the sample industries which do not have data for all years from 1958 to 2005, which reduces our sample from 459 industries to 449 industries.
future adjustment costs caused by installing capital in the present. The investment-based model derives an investment return expression that is more precise as it captures the missing elements that the popular measures fail to include.

The important aspect of the NBER database is that we are able to separate out, to a large extent, private and public firms. In particular, we separate industries into two groups, based on two measures that aim to separate private from public firms. The first is the fraction of the sales of public firms to total industry sales and the second is the fraction of the number of employees of the public firms in the industry to the total number of employees in the industry. Our rationale is that industries for which the fraction of sales (employees) of public firms to total industry sales (employees) is low include predominately private companies. Thus examining the cross section of returns for such industries enables a closer inspection of whether characteristics play a role in determining the cross section of investment returns among private firms.

When separating out private firms, we examine data 308 industries for which data is available both in Compustat as well as in the NBER database. For each year, we split industries into four groups as follows. The first group consists of industries which are below the median fraction of sales of public firms to total industry sales, the second group includes the industries in the lowest 25% fraction of public firms’ sales to total industry sales. The third group is the group of industries above the median fraction of sales and the fourth group is the top 25% industries. We repeat the split into four groups by the fraction of employees. We conjecture that when the fraction of sales of public firms to total industry sales is low, a large fraction of firms in that industry are private firms. Thus, finding that the cross sectional results hold for the group of industries with the lower fraction of public firm sales to total industry sales would constitute evidence that characteristics are important determinants of the cross section of investment returns among private firms, and would lend support to the rational explanation for the role of characteristics in determining the cross section of returns.

We use sales data from Compustat, aggregated over all firms in each industry for the sales of public firms in each industry and we use the non-deflated value of shipment series
from the NBER database for total industry sales. We note that the data on sales from
Compustat includes data for the sales of all public firms within an industry, including the
sales from operations abroad of these firms. In contrast, the data on the value of shipment
at the NBER includes sales (including sales abroad) of only US-based establishments.
Hence the ratio of the sales of public firms to the sales of the aggregate industry might
be biased upward in general and quite likely the bias varies across industries. However,
this upward bias is not a big concern for the following reason. Consider an industry for
which the ratio of sales of public firms to total industry sales is low (in the lowest 25%
group) and is hence is categorized as an industry with mostly private firms. The maximum
ratio of public firm sales to total industry sales for that group in our sample is 12% if
we restrict the share of public firms to be strictly positive in each year and 7% without
that restriction.\footnote{Throughout the paper we present the results without the restriction that the fraction of sales of public firms is strictly positive in each year. Imposing this restriction yields results which are very similar.} Therefore, the fraction of US-based public firm sales in that industry
is at most 12% and due to the bias it is likely lower. Hence, while some industries with
mostly private firms might incidentally, due to a very high bias, be in the group that we
term public industries, our private industries group indeed contains industries with a high
fraction of private firms and our tests capture the investment returns of industries with a
high fraction of privately held firms.

4 Empirical Results

This section of the paper presents results on the determinants of the cross section of
investment returns at the four-digit manufacturing industry level. The industry portfolios
consist of both public and private firms. In the first part of the analysis, we examine the
cross sectional determinants for the set of industries that are made up of predominately
private firms. We subsequently repeat the analysis using the set of industries that are
made of predominately public firms. This enables a closer inspection of the differences in
the determinants of average investment returns between public and private firms. Finally,
we examine all industries in order to examine the aggregate economy.
In the cross sectional regressions, we focus on the following characteristics. The investment to capital ratio and the return on assets (ROA), both of which explain the cross section of average stock returns (Hou, Xue and Zhang, 2012). We also examine whether lagged investment returns and a measure for idiosyncratic volatility explain the cross section of average returns.

Following the factor model in Hou, Xue and Zhang (2012), we perform asset pricing tests by examining the cross sectional patterns of investment returns when using three investment return based risk factors. These factors are a "market" investment return factor, an \( I/K \) factor and an ROA factor. Next, we investigate whether the cost of capital calculated from the asset pricing model varies between public and private firms within the manufacturing sector. Finally, we study valuation ratios and the cross-section of valuation ratios for both public and private industries, based on the first-order conditions for optimal investment derived in Belo, Xue and Zhang (2013).

### 4.1 Characteristics and the cross section of private firms’ investment returns

In Table 1, we run year-by-year cross sectional Fama MacBeth regressions of investment returns for industries in excess of the risk free rate on industry characteristics. The industries that we focus on in Table 1 are those that are dominated by private firms. We examine industries that have below the median sales of public firms, below 25% of sales by public firms as well as below the median and the lowest 25% in terms of employment in public firms. Panel A of Table 1 reports the results for univariate cross sectional regressions of investment returns on the one year lagged investment to capital ratio. That is, we regress investment returns in year \( t \) on the ratio of investment in year \( t - 1 \) to capital in year \( t - 2 \). The first two rows report the results for factions of sales that are below the median and the 25%, and the remaining rows report results splitting by number of employees. Consistent with the result for stock returns (see Xing, 2008), the coefficients on the investment to capital ratio are negative for private firms. The estimates are very similar irrespective of whether we split firms by sales or number of employees.
However, in absolute terms, the coefficients are larger when looking at the low 25%, that is, when we focus on the sample that includes more private firms. This indicates that the investment to capital ratio effect is stronger in private firms. Note that the estimates are statistically significant with a \( t \)-statistics ranging from 7.69 to 11.98. The size of the coefficient in Xing (2008, Table 3) is considerably larger (-4.75) relative to our estimates (-1.21 to -1.47). This can be explained by the fact that we use investment returns and industry portfolios whereas Xing uses individual stock returns. The \( R^2 \)s range from 5.6% to 7.3%, versus 1% in Xing, 2008. This first result in Panel A of Table 1 indicates that the investment effect in investment returns is prevalent among private firms.

Panel B of Table 1 reports the results for ROA. Hou, Xue and Zhang (2012) show that the \( q \)-theory of investment implies a positive relation between ROE and future stock returns. Given a certain level of investment, a firm’s riskiness must increase with ROE to justify the level of investment. The intuition is as follows. Consider two firms with a given investment to capital ratio. As investment is determined by expected future cash flows and by risk, the firm with higher ROE, that is higher expected cash flows, must also have higher risk to explain that its investment to capital ratio is not higher. The same intuition applies to ROA which we use in our tests because we lack data on industries capital structure. Indeed Hou, Xue and Zhang (2012) show that the risk premium on a stock return factor defined as the excess return of high ROE stocks over low ROE stocks is 0.60% per month and is statistically significant. Looking at Panel B, the coefficient on ROA is 0.09 in three cases and 0.08 in the fourth case and they are statistically significant, with a \( t \)-statistics between 7.28 and 9.53 and the \( R^2 \)s around 6%. Reassuringly, there is little difference in the estimates given how we split the industries into private firms, that is, by sales or employees. The finding that the coefficient estimates are the same across the sample that varies between below the median and low 25% indicates that the cross sectional relation between investment returns and ROA is independent of the amount of private firms in the sample.

In Panel C of Table 1, we present the results where we regress the current year’s investment returns on one-year lagged investment returns. The coefficients on last year’s
investment returns are in all but one case 0.09 (the remaining case 0.08) and they are all statistically significant with $t$-statistics around 3. The $R^2$s range from 1.9 to 2.7. Interestingly, just like the case of stock returns, we find a momentum effect in investment returns of private firms: high investment returns in the previous year are followed by high investment returns in the subsequent year. This result is consistent with Liu and Zhang (2013) who find a momentum effect in investment returns of public firms. The momentum effect in stock returns is often interpreted as some type of misvaluation effect. However, its presence in the investment returns of private firms suggests an alternative explanation is warranted.

The results for idiosyncratic volatility appear in Panel D Table 1. To measure idiosyncratic volatility we form a "market" portfolio by equal-weighting the investment returns of all 449 industries using the NBER database. We use this "market" portfolio to estimate idiosyncratic volatilities using the full sample period. We first regress the excess investment returns of each industry (using the risk free rate from Kenneth French's website) on the "market" portfolio's excess returns using the whole sample. The standard deviation of the residuals from this regression is our proxy for the industry idiosyncratic volatility. Idiosyncratic volatility is an important determinant of the investment returns for private firms. The estimated coefficients range from 0.31 to 0.34, they are statistically significant the $R^2$s are around 5%. This result is qualitatively similar to Mueller (2011).9 Fu (2009) finds that idiosyncratic volatility is positively related to expected stock returns, while Ang, Hodrick, Xing, and Zhang's (2006) find a negative relation between idiosyncratic volatility and expected stock returns.

Panel E of Table 1 presents multiple regression results, where the regressors are the variables used in the univariate regressions in the previous Panels. The signs of the coefficients remain unchanged but there are some changes in their magnitudes. For example, the coefficients on the investment to capital ratio fall to become more negative and the

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9 Mueller (2011) uses proxies for the value of equity of unlisted firms and finds similar results. Our approach is different than Mueller's as we use production data and examine investment returns rather than equity returns. Moreover, Mueller measures idiosyncratic risk as the degree of lack of diversification of a firm's owners whereas we derive idiosyncratic risk from market model regressions in which the market return is the investment return of a broad index of all of the US manufacturing industries investment returns.
coefficients on idiosyncratic volatility fall by around 50%. However, there is certainly joint predictive power of the four variables as can be seen from the increases in the $R^2$ s which now range from 21 to 25 per cent.

Overall the results in Table 1 show that characteristics that are important for explaining the cross section of stock returns are also important for explaining the cross section of investment returns of private firms. The only coefficient that changes substantially as we vary the amount of private firms from below the median sales (employees) to low 25% is the investment to capital ratio. In the next part of the paper, we look at firms that are predominately public in order to assess if the role of characteristics is any different for these firms.

Given the large size of the private company sector in the economy, our results are important and are unlikely to be driven only by public firms and lend support to the risk-based explanations for the role of characteristics in explaining average stock returns based on the investment first-order condition.

### 4.1.1 Public Firms

Table 1 illustrates that characteristics explain the cross section of investment returns for industry portfolios composed of mainly private companies. We now turn to looking at industry portfolios that are composed of mainly public firms. This enables us to establish whether characteristics have a different role in private and public firms.

Panel A of Table 2 presents the results for the investment to capital ratio when the industry portfolios are sorted by above the median sales (employees) by public firms and those industries that have sales (employees) in the high 25%. The coefficients on $I/K$ do vary somewhat across the above median and high 25% groups, but not across sales and number of employees. The coefficients become less negative for the industry portfolios as the proportion of public firms increases. This can be seen from comparing the results with those in Panel A of Table 1 where they are nearly twice as large (in absolute value) and by comparing the estimates in Panel A of Table 2 where the coefficient is less negative for the high 25% group than for the above the median group.
Clearly, the role of the investment to capital ratio in explaining the cross section of investment returns is stronger for private firms than for public firms. This can also be seen from comparing the $R^2$s which are around twice as large in the regressions that use predominately private firms as compared to the regressions that use predominately public firms. This finding is interesting since one of the behavioral explanations for the investment effect in stock return is a slow reaction of the market to overinvestment by empire building managers (Titman, Xie and Wei, 2004). This explanation is less likely to hold for private firms, for which agency conflicts between managers and shareholders are less likely to be prevalent. The other behavioral explanation for the investment effect in stock returns is market overreaction to firm growth (Cooper, Gulen and Schill, 2008). As there is no market price for private firms this explanation might also be less likely to hold for the investment effect within private firms. Our results, and in particular those that show the size of the coefficient is greater for the sample with a higher fraction of private firms, where mispricing might be thought to be less prevalent, lend support to the rational-based explanation of the investment effect. This is consistent with recent findings by Cooper and Priestley (2011) that the spread in stock returns between low investment firms and high investment firms can be largely explained by loadings on macroeconomic risk factors.

The estimated coefficients of 0.08 on $ROA$ are the same across the four groups as seen in Panel B of Table 2. This is also very similar to the estimates for the sample that includes predominately private firms (in Panel B of Table 1 the estimates are all 0.09 except for one case where it is 0.08). In addition, the $R^2$s are roughly similar for the sample that includes predominately private firms and predominately public firms, around 5 to 6 per cent. Therefore, unlike the investment to capital ratio, the explanatory power of $ROA$ is not effected by whether the firms are private or public, but there is clearly a consistent role for this characteristics for both private firms and public firms.

The results in Panel C of Table 2 show that momentum in investment returns is an important determinant of the cross section of investment returns for public firms. The estimated coefficients range from around 0.3 for the above median sales (employees)
sample to 0.4 for the high 25 sales (employee) samples. The $R^2$s are also relatively high between 15 and 20 percent indicating that momentum is the single most important determinant of the investment returns of public firms and the importance increases with the fraction of public firms in the industry. This finding is reinforced when we contrast these results to those in Panel C of Table 1 where momentum is shown to be the least important determinant of the cross section of investment returns of private firms.

The results from regressing investment returns of public firms on lagged idiosyncratic volatility are reported in Panel D of Table 2. Idiosyncratic volatility has a substantial impact on the cross section of investment returns. The estimated coefficients are somewhat larger than those for private firms and the $R^2$s are also larger. This might appear to be somewhat surprising given the low diversification of the owners of private firms (see Moskowitz and Vissning–Jørgensen (2002)). However, the findings are consistent with the real growth options explanation in Bartram, Brown, and Stulz (2011) where high U.S. stock return volatility is explained by high levels of investor protection, aggregate research intensity, and high levels of firm research and development, all of which lead to the more valuable and easily exercisable growth options. This indicates that public firms have more growth options than private firms.

The multivariate results for public firms are presented in Panel E. The various coefficient estimates are consistent with the univariate results, except that the coefficients on $I/K$ are, in an absolute sense, considerably larger now. The effect of idiosyncratic volatility is smaller for all of the groups in the multivariate regression. In fact, if we now compare the multivariate regression results in Panel E for both Table 1 and Table 2, we find very similar sizes of coefficients across all four characteristics. This indicates that the characteristics role in explaining the cross section of investment returns is remarkably similar across private and public firms. The only major difference across private and public firms is that the $R^2$s are between 21 and 25 per cent for the private firms and between 32 and 36 per cent for the public firms.

The results in Tables 1 and 2 show that characteristics that are important determinants of stock returns play a central role in explaining the investment returns of both public
firms as well as of private firms. In Table 3, we consider both private and public firms together in a sample that simply looks at all industries and therefore provides insight to the role of characteristics across all firms. The second column of Table 1 reports the results for univariate cross sectional regressions of investment returns on the one year lagged investment to capital ratio, the coefficient on the investment to capital ratio is negative and it is statistically significant, with a $t$-statistic of 10.99. The third column of Table 1 reports the results for $ROA$, the coefficient on $ROA$ is 0.09 and it is highly statistically significant, with a $t$-statistic of 11.66 and the $R^2$ is 4.76%. The fourth column of Table 1 presents the results for one-year lagged investment returns. The coefficient on last year’s investment returns is 0.23 and is statistically significant and the $R^2$ is 10.57%. The result for idiosyncratic volatility appears in the fifth row of Table 1. Idiosyncratic volatility is an important determinant of investment returns with a coefficient of 0.33 and a $t$-statistic of 4.25 and the $R^2$ is 7.37%. The last column of Table 1 shows multiple regression results, where the regressors are the variables used in the univariate regressions in the previous columns. The signs of the coefficients remain unchanged and the $R^2$ increases to 27%. There are two notable quantitative changes relative to the univariate regressions. First, the coefficient on the lagged investment to capital ratio doubles (in absolute value) from -1.02 to -2.09. Second, the coefficient on idiosyncratic volatility nearly halves, as it drops from 0.33 to 0.17. All the estimates remain statistically significant indicating the four characteristics have different roles.

The findings in Table 3 confirm that characteristics are important in describing the cross sectional of all industries in the economy. The findings in Tables 1, 2, and 3 lend substantial support to the conjecture that the role of characteristics in explaining cross sectional patterns in returns is due to rational behavior.

### 4.2 Asset Pricing Tests

In this section of the paper, we assess whether three of the four factors from the model of Hou, Xue, and Zhang (2012) can explain the cross-section of average investment returns of the twenty portfolios formed according to $I/K$, $ROA$, momentum and idiosyncratic
volatility using the cross-sectional regression approach of Fama and MacBeth (1973).

The three factors of the of Hou, Xue and Zhang (2012) model that we are using are the market portfolio, formed by equal-weighting the investment returns of all industries. The market portfolio in our sample earns on average 11.29% with a $t$-ratio of 11.80. The $I/K$ factor return in year $t$ is defined as the excess investment return in year $t$ of the low 33% investment-to-capital industries in year $t-1$ over the return on the top 33% investment-to-capital industries in year $t-1$. The $I/K$ factor earns a substantial premium of 10.14% and is highly statistically significant with a $t$-ratio of 11.77. The return on the $ROA$ factor in year $t$ is defined as the year $t$ excess return of the top 33% $ROA$ industries in year $t-1$ over the bottom 33% $ROA$ industries in year $t-1$. The average investment return on the $ROA$ factor is 11.80% with a $t$-ratio of 14.48.

The asset pricing tests are undertaken using the Fama and MacBeth (1973) procedure which involves a first step in which a time series regression is employed to estimate the factor loadings (betas) of the portfolio returns. The second step runs cross-sectional regressions of investment returns on the estimated betas in order to estimate the prices of risk. The use of annual data rules out using the typical rolling regression approach to estimate betas for each period. Instead, we use full sample estimates to obtain factor loadings (betas) and in the second step we estimate a cross-sectional regression of average investment returns in each year on the factor loadings estimated over the full sample. This is the method recommended and employed by Lettau and Ludvigson (2001) for quarterly data over a relatively short time series sample such as ours, and discussed in Cochrane (2005).

Table 4 reports the results from estimating of the prices of risk from the three factor model when using investment returns from industry portfolios that are weighted such that they hold mainly private firms:

\[
r_i = \lambda^0 + \lambda^m \widehat{\beta}_{i,m} + \lambda^{I/K} \widehat{\beta}_{i,I/K} + \lambda^{ROA} \widehat{\beta}_{i,ROA} + \epsilon_i.
\]  

(8)

where $r_i$ is the average return in the $i$th portfolio, $\lambda^0$ is a constant, $\lambda^{I/K}$ is the price of risk associated with the $I/K$ factor, $\widehat{\beta}_{i,I/K}$ is the beta with respect to the $I/K$ factor, $\lambda^{ROA}$
is the price of risk associated with the \( ROA \) factor, \( \hat{\beta}_{i,ROA} \) is the beta associated with the \( ROA \) factor, and \( e_i \) is the residual. We also report the cross-sectional \( \bar{R}^2 \) which, following Jagannathan and Wang (1996) and Lettau and Ludvigson (2001), is calculated as
\[
\bar{R}^2 = \frac{\text{Var}_c(\bar{r}_i) - \text{Var}_c(e_i)}{\text{Var}_c(\bar{r}_i)}
\]
where \( \text{Var}_c \) is the cross-sectional variance, \( \bar{r}_i \) is the average investment return and \( \bar{e}_i \) is the average residual. We also assess the performance of the model by calculating the square root of the squared pricing error across all twenty portfolios and across each group of five portfolios separately. Finally, we report a statistic that tests whether the pricing errors are jointly zero. This is a Chi-square test given as
\[
\hat{\alpha}'\text{cov}(\hat{\alpha})^{-1}\hat{\alpha}
\]
where \( \hat{\alpha} \) is the vector of average pricing errors across the twenty portfolios and \( \text{cov} \) is the covariance matrix of the pricing errors.

The first row of Table 4 reports the estimated prices of risk when using the sample that focuses on mainly private firms. First, we use industries that have below the median fraction of public firms sales and then industries that their ratio of public firms’ sales to total industry sales is in the bottom 25% percentile. We subsequently repeat the analysis when industries are sorted between private the public using the number of employees. The results in the first panel show that for firms with less than the median fraction of sales from public all three factors are important in describing the cross-section of average investment returns. The market price of risk 12.2% per annum and is statistically significant with corresponding \( t \)-statistic of 8.27. The price of risk associated with the \( I/K \) factor is 7.3% per annum with a \( t \)-statistic of 5.59 and the price of risk associated with the \( ROA \) factor is 14.5% per annum with a \( t \)-statistic of 10.17. The cross-sectional \( \bar{R}^2 \) is 0.85 indicating an excellent fit. In addition, the pricing errors are economically small, ranging from 2.1% per annum to 0.5% per annum with an average over all portfolios of 1.3% per annum. However, it should be noted that it is possible to reject the null hypothesis of jointly zero pricing errors.

The next row of Table 4 presents the results for the set of industries that contain more private firms, defined as those with less than 25% of sales from listed firms. The estimated prices of risk are very similar to those in the first row. The only major difference is that the price of risk associated with \( ROA \) falls from 14.5 to 10.2 per cent per annum. There
is a slight reduction in the $R^2$ to 0.79 and the pricing errors rise to 1.9% per annum on average across all the portfolios, however, they remain economically small.

The next two rows of the Table report the results when using the number of employees as the variable for separating out private and public industries. The results are remarkably similar to those that use sales. Whether we consider the separation of industries below the median or less than 25th percentile of the fraction of employees from listed firms, the estimated prices of risk are quite consistent with those based on sales as are the $R^2$s and the size of the pricing errors.

Table 4 indicates that a three factor asset pricing model can successfully explain the cross section of a sample of industries that contain mainly private firms. To our knowledge this is the first time that private firms’ expected returns are related to systematic risk factors that have been shown to be important in the determination of the cross section of listed firms’ expected returns.

In Table 5, we report the estimated prices of risk from the three factor model when we use mainly public firms in the sample. Focusing on the first two rows that use sales to split the sample into public and private, the public firms who have above the median fraction of public firms’ sales record positive and statistically significant prices of risk for the $I/K$ and $ROA$ factors, but a negative estimated price of risk for the market factor. This latter result is interesting since it mirrors the findings in the stock return literature that the market price of risk is negative. That we do not find the same effect in the private sector is also interesting. In terms of the model specification metrics, both the $R^2$ and the pricing errors are similar to those presented in Table 4 which used mainly private firms. If we compare the two extreme differences, the private firms in the low 25% of Table 4 and the public firms in the high 25% of Table 5 the prices of risk for the $I/K$ and $ROA$ factors are somewhat larger for the private industries, and the estimates on the market price of risk are opposite in sign. The model performs slightly better for the public firms in terms of having a higher $R^2$ (0.88 versus 0.78) and slightly smaller pricing errors (1.6% per annum versus 1.9% per annum).

The next two rows of the Table report the results when using the number of employees
as the mechanism for separating out private and public firms. The results are similar to those when using sales. The only major difference is that the price of risk associated with the \( I/K \) factor is not statistically significant, although the model performs just as well.

The findings from the three factor model confirm that these factors are a source of aggregate uncertainty in the sense that they are important for all firms, not just public firms. It is evident that the three factor model motivated from the \( q \)-theory of investment is able to successfully explain the cross-sectional differences in the twenty portfolios formed on four characteristics that include a substantial number of private firms. This is an important finding since it rules out, at least to some extent, the possibility that characteristics are driven by mispricing of stock prices. A large part of the sample has no stock price and, therefore, investors cannot under or over value many of these assets based on their characteristics. Coupled with the likely scenario that managers of private firms are less likely to be affected by investor sentiment, the results point to the conclusion that, first, the three fundamentals factors are related to the risk and return characteristics of firms and second, the risk and return characteristics of non-public firms are similar to those of public firms.

We can confirm the aggregate nature of the risk factors by assessing the performance of the model using all industries and thus ignoring the split between private and public. Table 6 reports the results where we start by considering the performance of the single factor CAPM in order to contrast it with the three factor model. To this end, we estimate

\[
    r_i = \lambda^0 + \lambda^m \hat{\beta}_{i,m} + \epsilon_i. \tag{9}
\]

where \( r_i \) is the actual investment return on the \( i \)th portfolio, \( \hat{\beta}_{i,m} \) is the estimate of portfolio \( i \)'s market beta, \( \lambda^0 \) is the intercept which should equal the risk free rate of return, \( \lambda^m \) is the estimate of the market price of risk, and \( \epsilon_i \) is a residual.

The second row of Table 6 reports the estimates and shows that the market price of risk is estimated to be 19.9% per annum and is statistically significant with a \( t \)-statistic of 14.55. The estimated intercept of -8.7% per annum, however, is a long way from the average risk free rate over the sample of 6%. Notwithstanding this, the CAPM does a
much better job at explaining the cross section of investment returns than the cross section of stock returns as reflected in the positive estimate of the market price of risk and the cross-sectional $R^2$ which is 0.45. The remainder of the second row presents information on the size of the pricing errors. The average pricing error is 3.4% per annum for the $I/K$ portfolios and 1.7% per annum for the $ROA$ portfolios. The average pricing errors are largest for the momentum portfolios at 3.5%. The pricing errors for the idiosyncratic volatility portfolios are 2.3 percent per annum. Across all portfolios the average pricing error is 2.7% per annum and the Chi-square test rejects the null that the twenty pricing errors are jointly zero. To summarize, the CAPM provides a positive estimate of the market price of risk, something which in itself is different from recent results that use stock market returns, and a provides a cross sectional $R^2$ and pricing errors that are reasonable.

The third row of Table 6 reports the estimates of the prices of risk from the three factor model:

$$r_i = \lambda^0 + \lambda^m \hat{\beta}_{i,m} + \lambda^{I/K} \hat{\beta}_{i,I/K} + \lambda^{ROA} \hat{\beta}_{i,ROA} + e_i.$$  \hfill (10)

where $\lambda^{I/K}$ is the price of risk associated with the $I/K$ factor, $\hat{\beta}_{i,I/K}$ is the beta with respect to the $I/K$ factor, $\lambda^{ROA}$ is the price of risk associated with the $ROA$ factor, $\hat{\beta}_{i,ROA}$ is the beta associated with the $ROA$ factor, and $e_i$ is the residual. The results show that all three factors are important in describing the cross-section of average investment returns. The market price of risk drops substantially to a more realistic value of 6.8% per annum and is statistically significant. The price of risk associated with the $I/K$ factor is 11% per annum with a $t$-statistic of 12.11 and the price of risk associated with the $ROA$ factor is 12.5% per annum with a $t$-statistic of 13.69. The cross-sectional $R^2$ is 0.69, a substantial improvement on the CAPM.

The pricing errors for the three factor model are substantially lower than for those reported from the CAPM. For example, across all twenty portfolios the average pricing error is 1.9% as opposed to 2.7% for the CAPM and this extent in the fall of the pricing errors is observed across all four sets of portfolios except the idiosyncratic volatility portfolios.
However, the Chi-square test rejects the null hypothesis that the twenty pricing errors are jointly zero. Finally, the intercept, while not equal to the risk free rate of return, is much closer at 4.4%.

The remainder of Table 6 reports estimates from cross-sectional regressions that drop one of the factors one at a time. The aim here is to try and establish if all of the factors are economically important. First, we drop the ROA factor and estimate the regression with the market factor and the I/K factor. Both of these factors are statistically significant and have estimates of 16.1% and 8.8% per annum, respectively. However, the $R^2$ is lower than that of the three factor model at 0.53 and the intercept is large and negative at -4.7%. The pricing errors, while substantially smaller for the I/K portfolios, are on average smaller than when estimating the CAPM at 2.3% per annum and the Chi-square test rejects the null of jointly zero pricing errors.

The next row reports the results when dropping the I/K factor. The price of risk associated with the market factor is estimated at 14.5% per annum with a $t$-statistic of 9.23 and the ROA price of risk is 13.5% per annum with a $t$-statistic of 14.83. This version of the model does better than the CAPM but worse than the model that includes the market factor and the I/K factor. The cross-sectional $R^2$ is 0.50 and the average pricing error is 2.7% per annum, the same as the the CAPM, but larger than the three factor model and the two factor model that includes the market factor and the I/K factor. The Chi-square test of jointly zero pricing errors rejects the null hypothesis.

In the final row, we report the results from the two factor model that drops the market factor. Both the I/K and ROA factors have positive and statistically significant prices of risk of 10.3 and 12.6 per cent respectively, and the $R^2$ is 0.70. The average pricing errors across all portfolios are small at 2.0% per annum, slightly larger than the three factor model. However, the intercept is 1.5%, a long way from the risk free rate of return.

Considering the performance of the different versions of the model, the three factor model produces estimates of the prices of risk that are all statistically significant. Moreover, they are economically important as well since dropping any one of the factors individually leads to either a lower $R^2$, higher average pricing errors, or estimates of the
intercept that is further away from the risk free rate. These findings from the three factor model confirm that these factors are a source of aggregate uncertainty in the sense that they are important for all firms, not just listed firms.

It is evident that the three factor model motivated from the q-theory of investment is able to successfully explain the cross-sectional differences in the twenty portfolios formed on four characteristics that include a substantial number of unlisted firms. This is an important finding since it rules out, at least to some extent, the possibility that characteristics are driven by mispricing of stock prices. A large part of the sample has no stock price and, therefore, investors cannot under or over value many of these assets based on their characteristics. Coupled with the likely scenario that managers of unlisted firms are less likely to be affected by investor sentiment, the results point to the conclusion that, first, the three fundamentals factors are related to the risk and return characteristics of firms and second, the risk and return characteristics of non-listed firms are similar to those of listed firms.

4.3 The Cost of Capital for Listed and Unlisted Firms

We now examine whether the cost of capital, namely expected investment returns that are calculated from the three factor model, vary between industries with a high ratio of sales of public firms to total sales and industries with a low ratio of sales of public firms to total sales. As seen in the previous tables, average investment returns vary considerable with industry characteristics. This part of the paper aims to answers the question of whether expected investment returns vary between public and private firms. The results are presented in Table 7.

The second and third columns of Table 7 report average and expected investment returns when we use all industries and the rows report different portfolios. There are some clear patterns in both actual and expected investment returns. For all the four characteristics, the portfolios have average investment returns and expected investment returns that match up well, consistent with the small pricing errors reported in the cross sectional tests. What is interesting is when we compare the average and, particularly, expected re-
turns between samples that have different proportions of public firms. Comparing the below median and above median sorting columns, we find similar expected investment returns for all but a few of the extreme portfolios. Similar findings are observed when we compare the 25% lowest and 25% highest sorting. This indicates that for most of the portfolios the expected investment returns between portfolios that include more public firms are similar to those that include more private firms. Interestingly, there is certainly no systematic differences in the expected returns across the portfolios with a different fraction of public firms that would indicate a private firm effect in the cost of capital. Any differences that are observed are likely to be a results of a difference in the value of a particular characteristic, for example a higher (lower) $I/K$ ratio rather than being due to the firms being public or private.\textsuperscript{10}

The pattern in expected investment returns is seen more clearly in Table 8 which reports the differences in the expected investment returns of portfolios sorted according to the fraction of public firms sales using both sales and the number of employees. We report the difference between below and above the median and lowest 25% and highest 25%. At the extreme portfolios, for losers and winners the differences are noticeable. However, other differences are generally small. On average the absolute differences are 2 and 2.8 percent per annum for the below the median minus above the median and the low 25% minus high 25%, respectively. What is particularly interesting is that there are no systematic differences in the costs of equity capital between the portfolios that include more or less public and private firms. This is an important finding and provides new evidence that the private equity premium is similar to the public equity premium. There are two interesting implications from this results. First, risk adjusted estimates of the cost of capital for private firms, notoriously difficult to obtain, can be estimated from the investment returns of these firms. Second, since the cost of capital from the investment return approach is similar for public and private firms, given a characteristic, then private firms can use similar characteristics public firms stock returns to proxy their cost of capital, especially if they do not have an extreme value of a particular characteristic.

\textsuperscript{10}Similar results are obtained when splitting industries into private the public using the number of employees and are available on request.
The results that private and public firms have the same cost of capital might seem surprising given the lack of liquidity of private firms and the potential under-diversification of their owners. However, the findings are consistent with Moskowitz and Vissing-Jørgensen (2002) who use estimates of private firms value and profits and study the returns to entrepreneurial investment. They find that in spite of poor diversification the returns to private equity are not systematically higher than the return to public equity.

4.4 Valuation and Private Firms

We now examine two aspects related to the valuation ratio, namely Tobin’s $Q$, of private and public industries, derived from the neoclassical investment model. First, we compare the valuation ratios of the two groups. This comparison is interesting for several reasons. First, as Belo, Xue and Zhang (2013) note, valuation ratios derived from the neoclassical investment model are close to firms’ intrinsic value. Because private firms are less likely to be affected by investor misvaluations, finding that the valuation ratios of private industries are close to the valuation ratios of public industries would lend further support to the conjecture that indeed public firms’ valuation ratios derived from the investment model represent intrinsic values. Second, a-priori it is not clear which type of industry should have higher valuation ratios. On one hand private firms are more financially constrained than public firms and so the valuation ratios of private firms could be lower. On the other hand public firms have a more dispersed ownership structure than private firms and therefore might have more manager-shareholder agency problems, entailing lower valuation ratios for public firms relative to private firms. Thus, it is an empirical question which of the valuation ratios is higher. The second aspect of valuation ratios of public and private firms that we would like to examine is the cross section of Tobin’s $Q$ of private firms and of public firms within the same industry. However we lack data at the individual level firm. Therefore we examine the cross section of 4-digit SIC code private industries and the cross section of four-digit SIC code public industries, where both the private and public four-digit industries belong to the same two-digit SIC code industry group. That is, within each two-digit level industry we form a group of four-digit private
industries and another group of four-digit public industries. Subsequently, for each of the
two-digit industries we examine the cross section of Tobin’s $Q$ for public industries and
for private industries.

Belo, Xue and Zhang (2013) specify a neoclassical model of investment to derive a
valuation equation. Tobin’s $Q$ is a non-linear function of the investment to capital ratio
and is given by

\[ Q_{it} = 1 + (1 - \tau_t) \eta^v \left( \frac{I_{it}}{K_{it}} \right)^{v-1}, \tag{11} \]

where $\eta > 0$ is the slope adjustment cost parameter and $v > 1$ is the curvature
adjustment cost parameter.$^{11}$ Using the estimates of Belo, Xue and Zhang (2010) we
calculate a time series of Tobin’s $Q$ for each of the four-digit industries in our sample.

Table 9 presents the results. Panel A examines private industries. The average level of
$Q$ for the bottom quintile $Q$ industries is 1.30, whereas the average level of $Q$ for the top
quintile group is 3.41. The standard deviation of $Q$ rises considerably with $Q$. The third
row shows that average returns are negatively related to $Q$, as they are for stock returns.
As seen in Panel B, the results are very similar for public firms. Both the ratios themselves
as well as their cross section seem very similar in Panels A and B. Panel C presents the
valuation ratios at the aggregate industry level. The results resemble those of Panels A
and B. Therefore the factors that increase the valuation of private firms relative to that
of public firms seem to be offset by other factors so that the valuations are highly similar
in the data.

Table 10 reports the cross section of valuation ratios within two-digit SIC code indus-
tries.$^{12}$ Consistent with the results in Table 10, both Tobin’s average $Q$ as well as the
cross section of $Q$ are very similar for private industries and for public industries that
belong to the same two-digit SIC code industry group. For example, for SIC code 20, the
average of $Q$ for private industries ranges from 1.63 to 3.12, whereas the average of $Q$ for
public industries varies from 1.74 to 3.20.

\[ \text{We obtain equation (11) in our paper by dividing both sides of equation (5) in Belo, Xue and Zhang (2013) by } K_{it+1}. \]

\[ \text{Three of the twenty two-digit SIC manufacturing industries are omitted due to insufficient number of four-digit SIC industries within those two-digit SIC industries.} \]
Overall Tables 9 and 10 show that the valuation of firms and industries relative to their assets are very similar regardless of whether they are private and public. This finding is quite consistent with the other findings in the paper and with the results in Moskowitz and Vissing-Jørgensen (2002).

5 Conclusion

This paper examines the determinants of the cross sectional variation in average investment returns for industry portfolios composed mainly of privately held firms and for industries consisting of mostly publicly listed firms. Investment returns are derived from the $q$-theory of investment (see Liu, Whited and Zhang, 2009). We use the NBER Productivity database to calculate investment returns at the aggregate industry level, which includes both public and private firms. The NBER Productivity database contains detailed data on real capital stock, real investment and sales for 459 manufacturing industries from 1958 to 2005.

We find that characteristics that are important determinants of the cross section of stock returns, namely the investment to capital ratio, the return on assets, lagged returns and idiosyncratic volatility, also explain the cross sectional variation of both public and private firms’ investment returns. Given that private firms have no stock price and if the managers of private firms are less susceptible to investor sentiment and misvaluations, our results lend some support for a rational based interpretation of the role of characteristics in the cross section of returns.

We also test the performance of a three factor version of the four factor model of Hou, Xue and Zhang (2012). We test the models using twenty characteristic-based single sorted portfolios as test assets. The multifactor models perform well in describing the cross section of investment returns. This is a noteworthy finding since this is the first test of an asset pricing model over all assets, including private firms. For a candidate risk factor to be a "true" risk factor, it must be an aggregate factor that affects all firms. We show that these three factors affect all firms and not only public firms. This finding is reinforced by our results that show the factors work well when we vary the fraction of
public and private firms in the test assets.

The asset pricing tests have economically important implications for cost of capital calculations for private firms. The cost of capital for private firms is difficult to measure using risk based measures. This is because of the lack of stock prices for these firms. We show that it is possible to use investment returns to calculate the cost of capital. Moreover, an alternative way to calculate the cost of capital is to use proxy firms from the public market and use their investment returns. While this method has been used in the past, we show that it is a reliable benchmark to use. Finally we show that the valuation rations, namely Tobin’s $Q$, and their cross sectional variation are very similar for public and private firms.
References


Table 1  
Cross Sectional Regressions of Investment Returns on Characteristics for Private Industries

This table reports coefficients from Fama MacBeth cross sectional regressions for four groups. In each year the first group includes the industries in the NBER database for which the fraction of the sales of public firms to total industry sales is below the median for that year. The second group contains industries for which this ratio is below the 25% percentile. In each year the third group consists of industries for which the ratio of employees of public firms to total industry employees is below the median for that year, whereas the fourth group is composed of industries for which this ratio is below the 25% percentile. The frequency of the data is annual and the sample period is from 1960 to 2005. The table reports average intercepts and slopes from the cross sectional regressions. \( t \)-statistics are in parentheses. \( \overline{R^2} \) is the average \( R^2 \) of the cross sectional regressions.

### Panel A - Investment to Capital

<table>
<thead>
<tr>
<th></th>
<th>( ^\hat{\gamma}_0 )</th>
<th>( ^\hat{\gamma}_{I/K} )</th>
<th>( \overline{R^2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Below the Median Sales</td>
<td>0.15 ( (8.11) )</td>
<td>-1.21 ( (-11.98) )</td>
<td>5.55</td>
</tr>
<tr>
<td>Low 25% Sales</td>
<td>0.17 ( (8.03) )</td>
<td>-1.47 ( (-7.72) )</td>
<td>7.16</td>
</tr>
<tr>
<td>Below the Median Employees</td>
<td>0.14 ( (8.25) )</td>
<td>-1.26 ( (-10.10) )</td>
<td>5.74</td>
</tr>
<tr>
<td>Low 25% Employees</td>
<td>0.18 ( (8.24) )</td>
<td>-1.50 ( (-7.69) )</td>
<td>7.32</td>
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</table>

### Panel B - Return On Assets

<table>
<thead>
<tr>
<th></th>
<th>( ^\hat{\gamma}_0 )</th>
<th>( ^\hat{\gamma}_{ROA} )</th>
<th>( \overline{R^2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Below the Median Sales</td>
<td>-0.02 ( (-1.72) )</td>
<td>0.09 ( (8.82) )</td>
<td>5.68</td>
</tr>
<tr>
<td>Low 25% Sales</td>
<td>-0.02 ( (-1.79) )</td>
<td>0.09 ( (7.28) )</td>
<td>6.67</td>
</tr>
<tr>
<td>Below the Median Employees</td>
<td>-0.03 ( (-2.61) )</td>
<td>0.08 ( (9.53) )</td>
<td>5.34</td>
</tr>
<tr>
<td>Low 25% Employees</td>
<td>-0.02 ( (-1.57) )</td>
<td>0.09 ( (7.50) )</td>
<td>6.71</td>
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### Panel C - Momentum

<table>
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<th>( ^\hat{\gamma}_{MOM} )</th>
<th>( \overline{R^2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Below the Median Sales</td>
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<td>0.09 ( (3.35) )</td>
<td>1.86</td>
</tr>
<tr>
<td>Low 25% Sales</td>
<td>0.05 ( (3.76) )</td>
<td>0.09 ( (3.01) )</td>
<td>2.72</td>
</tr>
<tr>
<td>Below the Median Employees</td>
<td>0.03 ( (2.42) )</td>
<td>0.09 ( (3.14) )</td>
<td>1.90</td>
</tr>
<tr>
<td>Low 25% Employees</td>
<td>0.05 ( (4.00) )</td>
<td>0.08 ( (2.95) )</td>
<td>2.56</td>
</tr>
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### Panel D - Idiosyncratic Volatility

38
<table>
<thead>
<tr>
<th></th>
<th>$\hat{\gamma}_0$</th>
<th>$\hat{\gamma}_{idvol}$</th>
<th>$R^2$ (%)</th>
</tr>
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<td>(-0.68)</td>
<td>(3.63)</td>
<td></td>
</tr>
<tr>
<td>Low 25% Sales</td>
<td>-0.01</td>
<td>0.33</td>
<td>5.53</td>
</tr>
<tr>
<td></td>
<td>(-0.48)</td>
<td>(2.92)</td>
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</tr>
<tr>
<td>Below the Median Employees</td>
<td>-0.02</td>
<td>0.31</td>
<td>4.50</td>
</tr>
<tr>
<td></td>
<td>(-1.22)</td>
<td>(3.47)</td>
<td></td>
</tr>
<tr>
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<td>5.51</td>
</tr>
<tr>
<td></td>
<td>(-0.39)</td>
<td>(2.94)</td>
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### Panel E - Multiple Regressions

<table>
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<th>$\hat{\gamma}_0$</th>
<th>$\hat{\gamma}_{I/K}$</th>
<th>$\hat{\gamma}_{ROA}$</th>
<th>$\hat{\gamma}_{MOM}$</th>
<th>$\hat{\gamma}_{idvol}$</th>
<th>$R^2$ (%)</th>
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<td>0.20</td>
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<tr>
<td></td>
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<td>(-19.85)</td>
<td>(9.95)</td>
<td>(5.15)</td>
<td>(2.10)</td>
<td></td>
</tr>
<tr>
<td>Low 25% Sales</td>
<td>0.10</td>
<td>-2.35</td>
<td>0.11</td>
<td>0.13</td>
<td>0.17</td>
<td>25.05</td>
</tr>
<tr>
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<td>(-11.77)</td>
<td>(7.59)</td>
<td>(4.34)</td>
<td>(1.55)</td>
<td></td>
</tr>
<tr>
<td>Below the Median Employees</td>
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<td>-2.13</td>
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<td>0.14</td>
<td>0.18</td>
<td>21.10</td>
</tr>
<tr>
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<td>(4.74)</td>
<td>(-14.26)</td>
<td>(9.60)</td>
<td>(5.07)</td>
<td>(1.92)</td>
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<td>0.18</td>
<td>25.00</td>
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<td>(-12.07)</td>
<td>(7.76)</td>
<td>(4.29)</td>
<td>(1.61)</td>
<td></td>
</tr>
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</table>
Table 2
Cross Sectional Regressions of Investment Returns on Characteristics for Public Industries

This table reports coefficients from Fama MacBeth cross sectional regressions for four groups. In each year the first group includes the industries in the NBER database for which the fraction of the sales of public firms to total industry sales is above the median for that year. The second group contains industries for which this ratio is in the top 25% percentile. In each year the third group consists of industries for which the ratio of employees of public firms to total industry employees is above the median for that year whereas the fourth group is composed of industries for which this ratio is in the top 25% percentile. The frequency of the data is annual and the sample period is from 1960 to 2005. The table reports average intercepts and slopes from the cross sectional regressions. t-statistics are in parentheses. $R^2$ is the average $R^2$ of the cross sectional regressions.

<table>
<thead>
<tr>
<th>Panel A - Investment to Capital</th>
<th>$\hat{\gamma}_0$</th>
<th>$\hat{\gamma}_{I/K}$</th>
<th>$R^2$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Above the Median Sales</td>
<td>0.10</td>
<td>-0.72</td>
<td>3.31</td>
</tr>
<tr>
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<tr>
<td>High 25% Sales</td>
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<tr>
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<tr>
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<td>0.10</td>
<td>-0.71</td>
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<td>(-5.35)</td>
<td></td>
</tr>
<tr>
<td>High 25% Employees</td>
<td>0.11</td>
<td>-0.56</td>
<td>3.70</td>
</tr>
<tr>
<td></td>
<td>(6.56)</td>
<td>(2.35)</td>
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<table>
<thead>
<tr>
<th>Panel B - Return On Assets</th>
<th>$\hat{\gamma}_0$</th>
<th>$\hat{\gamma}_{ROA}$</th>
<th>$R^2$ (%)</th>
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</thead>
<tbody>
<tr>
<td>Above the Median Sales</td>
<td>-0.03</td>
<td>0.08</td>
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<tr>
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<td>(12.04)</td>
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<tr>
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<td>4.89</td>
</tr>
<tr>
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<td>(-3.79)</td>
<td>(11.97)</td>
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</tr>
<tr>
<td></td>
<td>(-1.06)</td>
<td>(10.98)</td>
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<table>
<thead>
<tr>
<th>Panel C - Momentum</th>
<th>$\hat{\gamma}_0$</th>
<th>$\hat{\gamma}_{MOM}$</th>
<th>$R^2$ (%)</th>
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<tbody>
<tr>
<td>Above the Median Sales</td>
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<td>0.32</td>
<td>16.02</td>
</tr>
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<td>(4.67)</td>
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<tr>
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<td>0.38</td>
<td>19.30</td>
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<td>(5.11)</td>
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<td>0.29</td>
<td>14.79</td>
</tr>
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<td></td>
<td>(1.28)</td>
<td>(5.06)</td>
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</tr>
<tr>
<td>High 25% Employees</td>
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<td></td>
<td>(1.49)</td>
<td>(5.43)</td>
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</table>
### Panel D - Idiosyncratic Volatility

<table>
<thead>
<tr>
<th></th>
<th>$\hat{\gamma}_0$</th>
<th>$\hat{\gamma}_{idvol}$</th>
<th>$R^2$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.45</td>
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<tr>
<td>High 25% Sales</td>
<td>-0.06</td>
<td>0.47</td>
<td>17.59</td>
</tr>
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<td></td>
<td>(-1.73)</td>
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<tr>
<td>Above the Median Employees</td>
<td>-0.03</td>
<td>0.36</td>
<td>13.88</td>
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<td></td>
<td>(-1.32)</td>
<td>(3.22)</td>
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<tr>
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<td>-0.04</td>
<td>0.47</td>
<td>17.48</td>
</tr>
<tr>
<td></td>
<td>(-1.41)</td>
<td>(3.10)</td>
<td></td>
</tr>
</tbody>
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### Panel E - Multiple Regressions

<table>
<thead>
<tr>
<th></th>
<th>$\hat{\gamma}_0$</th>
<th>$\hat{\gamma}_{I/K}$</th>
<th>$\hat{\gamma}_{ROA}$</th>
<th>$\hat{\gamma}_{MOM}$</th>
<th>$\hat{\gamma}_{idvol}$</th>
<th>$R^2$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Above the Median Sales</td>
<td>0.05</td>
<td>-1.96</td>
<td>0.09</td>
<td>0.28</td>
<td>0.21</td>
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<tr>
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<td>(-17.00)</td>
<td>(8.79)</td>
<td>(5.81)</td>
<td>(3.17)</td>
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<td>0.08</td>
<td>0.30</td>
<td>0.22</td>
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<td>(3.55)</td>
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<td>(6.04)</td>
<td>(5.70)</td>
<td>(3.04)</td>
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<tr>
<td>Above the Median Employees</td>
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<td>-1.83</td>
<td>0.08</td>
<td>0.28</td>
<td>0.20</td>
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<td>(7.87)</td>
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<td>-2.05</td>
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<td>0.21</td>
<td>36.59</td>
</tr>
<tr>
<td></td>
<td>(4.61)</td>
<td>(-12.64)</td>
<td>(6.51)</td>
<td>(6.04)</td>
<td>(3.04)</td>
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</table>
Table 3
Aggregate Cross Sectional Regressions with Characteristics

This table reports coefficients from Fama MacBeth cross sectional regressions of industry investment returns on industry characteristics. Data is from the NBER Productivity database which contains data on all 459 US manufacturing industries with data on the of capital stock, investment, output and other variables aggregated over public and private companies. The frequency of the data is annual and the sample period is from 1960 to 2005. The table reports average intercepts and slopes from the cross sectional regressions. \( t \)-statistics are in parentheses. \( \overline{R^2} \) is the average \( R^2 \) of the cross sectional regressions.

<table>
<thead>
<tr>
<th>( \hat{\gamma} )</th>
<th>( ^\wedge \gamma_0 )</th>
<th>( ^\wedge \gamma_{I/K} )</th>
<th>( ^\wedge \gamma_{ROA} )</th>
<th>( ^\wedge \gamma_{MOM} )</th>
<th>( ^\wedge \gamma_{idvol} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\gamma}_0 )</td>
<td>0.14</td>
<td>(9.63)</td>
<td>0.04</td>
<td>(3.93)</td>
<td>0.04</td>
</tr>
<tr>
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</tr>
<tr>
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<td>(11.66)</td>
<td>0.09</td>
<td>(11.66)</td>
<td>0.09</td>
</tr>
<tr>
<td>( \hat{\gamma}_{MOM} )</td>
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<td>(4.06)</td>
<td>0.25</td>
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<td>0.25</td>
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<tr>
<td>( \hat{\gamma}_{idvol} )</td>
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<td>(4.25)</td>
<td>0.17</td>
<td>(3.22)</td>
<td>0.17</td>
</tr>
<tr>
<td>( R^2 )</td>
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<td>4.76</td>
<td>10.57</td>
<td>7.37</td>
<td>27.15</td>
</tr>
</tbody>
</table>

\( \overline{R^2} \):
Table 4
Cross Sectional Regressions with Risk Factors for Private Industries

We perform a set of cross sectional regressions of investment returns on factor loadings. The three factor model is

\[ r_i = \gamma_0 + \gamma_{MKT} \hat{\beta}_{i,MKT} + \gamma_{I/K} \hat{\beta}_{i,I/K} + \gamma_{ROA} \hat{\beta}_{i,ROA} + \epsilon_i, \]

where \( r_i \) is the investment return, \( \hat{\beta}_{i,MKT} \) is the factor loading on the market investment return portfolio, \( \hat{\beta}_{i,I/K} \) is the factor loading on the \( I/K \) investment return portfolio, \( \hat{\beta}_{i,ROA} \) is the factor loading on the \( ROA \) investment return portfolio, and \( \epsilon_i \) is the residual. The factor loadings are estimated over the full sample period. The table reports the constant and the estimated prices of risk (\( t \)-values in parenthesis). Below median refers to industries for which the ratio of sales of public firms in the industry to total industry sales is below the median of all industries. Low 25% refers to industries for which this ratio is below the 25% percentile. \( \bar{R}^2 = \frac{Var_c(\bar{r}_i) - Var_c(\bar{e}_i)}{Var_c(\bar{r}_i)} \), where \( Var_c \) is the cross-sectional variance, \( \bar{r}_i \) is the average investment return and \( \bar{e}_i \) is the average residual. \( \bar{R}^2 \) is the adjusted \( R^2 \). We define the pricing error for a given portfolio \( i \) as the difference between the actual investment return and the expected investment return according to the cross-sectional test; \( p.e. \) represents the square root of the aggregate squared pricing errors across all portfolios in each division (\( p \)-value in brackets). The sample period is 1960 to 2005.

<table>
<thead>
<tr>
<th>( \gamma_0 )</th>
<th>( \gamma_{MKT} )</th>
<th>( \gamma_{I/K} )</th>
<th>( \gamma_{ROA} )</th>
<th>( \bar{R}^2 )</th>
<th>( p.e. \bar{\pi} )</th>
<th>( p.e.ROA )</th>
<th>( p.e.MOM )</th>
<th>( p.e.VOL )</th>
<th>( p.e.ALL )</th>
<th>( X^2_{ALL} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Below median Sales</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(-0.031)</td>
<td>(0.122)</td>
<td>(0.073)</td>
<td>(0.145)</td>
<td>(0.850)</td>
<td>(0.013)</td>
<td>(0.012)</td>
<td>(0.021)</td>
<td>(0.005)</td>
<td>(0.013)</td>
<td>(37.482)</td>
</tr>
<tr>
<td>( (2.78) )</td>
<td>( (8.27) )</td>
<td>( (5.59) )</td>
<td>( (10.17) )</td>
<td>( (0.01) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low 25% Sales</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(-0.025)</td>
<td>(0.121)</td>
<td>(0.079)</td>
<td>(0.102)</td>
<td>(0.785)</td>
<td>(0.018)</td>
<td>(0.024)</td>
<td>(0.015)</td>
<td>(0.015)</td>
<td>(0.019)</td>
<td>(36.875)</td>
</tr>
<tr>
<td>( (2.67) )</td>
<td>( (8.70) )</td>
<td>( (5.35) )</td>
<td>( (6.71) )</td>
<td>( (0.01) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Below median Employees</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.002)</td>
<td>(0.093)</td>
<td>(0.075)</td>
<td>(0.137)</td>
<td>(0.823)</td>
<td>(0.013)</td>
<td>(0.011)</td>
<td>(0.017)</td>
<td>(0.011)</td>
<td>(0.013)</td>
<td>(41.461)</td>
</tr>
<tr>
<td>( (0.19) )</td>
<td>( (7.26) )</td>
<td>( (5.94) )</td>
<td>( (9.56) )</td>
<td>( (0.00) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low 25% Employees</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(-0.033)</td>
<td>(0.127)</td>
<td>(0.086)</td>
<td>(0.111)</td>
<td>(0.821)</td>
<td>(0.013)</td>
<td>(0.027)</td>
<td>(0.015)</td>
<td>(0.011)</td>
<td>(0.016)</td>
<td>(38.092)</td>
</tr>
<tr>
<td>( (3.35) )</td>
<td>( (8.94) )</td>
<td>( (6.19) )</td>
<td>( (7.34) )</td>
<td>( (0.01) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Cross Sectional Regressions with Risk Factors for Public Industries

We perform a set of cross sectional regressions of investment returns on factor loadings. The three factor model is

\[ r_i = \gamma_0 + \gamma_{MKT}\beta_{i,MKT} + \gamma_{I/K}\beta_{i,I/K} + \gamma_{ROA}\beta_{i,ROA} + \epsilon_i, \]

where \( r_i \) is the investment return, \( \beta_{i,MKT} \) is the factor loading on the market investment return portfolio, \( \beta_{i,I/K} \) is the factor loading on the \( I/K \) investment return portfolio, \( \beta_{i,ROA} \) is the factor loading on the \( ROA \) investment return portfolio, and \( \epsilon_i \) is the residual. The factor loadings are estimated over the full sample period. The table reports the constant and the estimated prices of risk (\( t \)-values in parenthesis). Above median refers to firms that have a fraction of sales from the public firms that is above the median, whereas High refers to the industries in the top 25\% percentile of that fraction. \( R^2 = \frac{Var_c(\bar{r}_i) - Var_c(\bar{r}_i)}{Var_c(\bar{r}_i)} \), where \( Var_c \) is the cross-sectional variance, \( \bar{r}_i \) is the average investment return and \( \bar{r}_i \) is the average residual. \( \bar{R}^2 \) is the adjusted \( R^2 \). We define the pricing error for a given portfolio \( i \) as the difference between the actual investment return and the expected investment return according to the cross-sectional test; \( p.e. \) represents the square root of the aggregate squared pricing errors across all portfolios in each division (\( p \)-value in brackets). The sample period is 1960 to 2005.

<table>
<thead>
<tr>
<th>( \gamma_0 )</th>
<th>( \gamma_{MKT} )</th>
<th>( \gamma_{I/K} )</th>
<th>( \gamma_{ROA} )</th>
<th>( \bar{R}^2 )</th>
<th>( p.e. )</th>
<th>( p.e. )</th>
<th>( p.e. )</th>
<th>( p.e. )</th>
<th>( \chi^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| \begin{tabular}{c}
Above median Sales
\end{tabular} | \begin{tabular}{c}
\begin{tabular}{c}
0.158  \\
(10.83)
\end{tabular}  \\
\begin{tabular}{c}
-0.076  \\
(3.93)
\end{tabular}  \\
\begin{tabular}{c}
0.049  \\
(3.85)
\end{tabular}  \\
\begin{tabular}{c}
0.128  \\
(9.04)
\end{tabular}  \\
\begin{tabular}{c}
0.849  \\
(9.04)
\end{tabular}  \\
\begin{tabular}{c}
0.020  \\
(9.04)
\end{tabular}  \\
\begin{tabular}{c}
0.011  \\
(9.04)
\end{tabular}  \\
\begin{tabular}{c}
0.014  \\
(9.04)
\end{tabular}  \\
\begin{tabular}{c}
0.014  \\
(9.04)
\end{tabular}  \\
\begin{tabular}{c}
0.015  \\
(9.04)
\end{tabular}  \\
\begin{tabular}{c}
127.376  \\
[0.00]
\end{tabular}
\end{tabular} | \begin{tabular}{c}
\begin{tabular}{c}
High 25\% Sales
\end{tabular}  \\
\begin{tabular}{c}
\begin{tabular}{c}
0.177  \\
(9.50)
\end{tabular}  \\
\begin{tabular}{c}
-0.094  \\
(4.05)
\end{tabular}  \\
\begin{tabular}{c}
0.048  \\
(3.14)
\end{tabular}  \\
\begin{tabular}{c}
0.095  \\
(4.99)
\end{tabular}  \\
\begin{tabular}{c}
0.885  \\
(9.04)
\end{tabular}  \\
\begin{tabular}{c}
0.019  \\
(9.04)
\end{tabular}  \\
\begin{tabular}{c}
0.006  \\
(9.04)
\end{tabular}  \\
\begin{tabular}{c}
0.017  \\
(9.04)
\end{tabular}  \\
\begin{tabular}{c}
0.022  \\
(9.04)
\end{tabular}  \\
\begin{tabular}{c}
0.016  \\
(9.04)
\end{tabular}  \\
\begin{tabular}{c}
21.124  \\
[0.34]
\end{tabular}
\end{tabular} |
| \begin{tabular}{c}
\begin{tabular}{c}
Above median Employees
\end{tabular}  \\
\begin{tabular}{c}
\begin{tabular}{c}
0.155  \\
(1214)
\end{tabular}  \\
\begin{tabular}{c}
-0.073  \\
(4.19)
\end{tabular}  \\
\begin{tabular}{c}
0.050  \\
(3.79)
\end{tabular}  \\
\begin{tabular}{c}
0.136  \\
(9.69)
\end{tabular}  \\
\begin{tabular}{c}
0.832  \\
(9.69)
\end{tabular}  \\
\begin{tabular}{c}
0.026  \\
(9.69)
\end{tabular}  \\
\begin{tabular}{c}
0.011  \\
(9.69)
\end{tabular}  \\
\begin{tabular}{c}
0.016  \\
(9.69)
\end{tabular}  \\
\begin{tabular}{c}
0.009  \\
(9.69)
\end{tabular}  \\
\begin{tabular}{c}
0.015  \\
(9.69)
\end{tabular}  \\
\begin{tabular}{c}
38.831  \\
[0.01]
\end{tabular}
\end{tabular} | \begin{tabular}{c}
\begin{tabular}{c}
High 25\% Employees
\end{tabular}  \\
\begin{tabular}{c}
\begin{tabular}{c}
0.161  \\
(10.34)
\end{tabular}  \\
\begin{tabular}{c}
-0.074  \\
(3.56)
\end{tabular}  \\
\begin{tabular}{c}
0.001  \\
(0.07)
\end{tabular}  \\
\begin{tabular}{c}
0.111  \\
(5.59)
\end{tabular}  \\
\begin{tabular}{c}
0.921  \\
(5.59)
\end{tabular}  \\
\begin{tabular}{c}
0.022  \\
(5.59)
\end{tabular}  \\
\begin{tabular}{c}
0.013  \\
(5.59)
\end{tabular}  \\
\begin{tabular}{c}
0.012  \\
(5.59)
\end{tabular}  \\
\begin{tabular}{c}
0.011  \\
(5.59)
\end{tabular}  \\
\begin{tabular}{c}
0.015  \\
(5.59)
\end{tabular}  \\
\begin{tabular}{c}
44.110  \\
[0.00]
\end{tabular}
\end{tabular} |
| \begin{tabular}{c}
\begin{tabular}{c}
44
\end{tabular}
\end{tabular} |
Table 6
Cross Sectional Regressions with Risk Factors for all Industries

We perform a set of cross sectional regressions of investment returns on factor loadings. The three factor model is

\[ r_i = \gamma_0 + \gamma_{MKT} \beta_{i,MKT} + \gamma_{I/K} \beta_{i,I/K} + \gamma_{ROA} \beta_{i,ROA} + \epsilon_i, \]

where \( r_i \) is the investment return, \( \beta_{i,MKT} \) is the factor loading on the market investment return portfolio, \( \beta_{i,I/K} \) is the factor loading on the \( I/K \) investment return portfolio, \( \beta_{i,ROA} \) is the factor loading on the \( ROA \) investment return portfolio, and \( \epsilon_i \) is the residual. The factor loadings are estimated over the full sample period. The table reports the constant and the estimated prices of risk from various cross-section regressions that include different combinations of the factors (t-values in parenthesis). \( R^2 = [Var_c(\bar{r}_i) - Var_c(\bar{\epsilon}_i)] / Var_c(\bar{r}_i) \), where \( Var_c \) is the cross-sectional variance, \( \bar{r}_i \) is the average investment return and \( \bar{\epsilon}_i \) is the average residual. \( R^2 \) is the adjusted \( R^2 \). We define the pricing error for a given portfolio \( i \) as the difference between the actual investment return and the expected investment return according to the cross-sectional test; \( p.e. \) represents the square root of the aggregate squared pricing errors across all portfolios in each division (p-value in brackets). The sample period is 1960 to 2005. The test assets are twenty portfolios, five each according to the \( I/K \) ratio, \( ROA \), lagged investment, and idiosyncratic volatility.

### Panel A: All Firms

<table>
<thead>
<tr>
<th>( \gamma_0 )</th>
<th>( \gamma_{MKT} )</th>
<th>( \gamma_{I/K} )</th>
<th>( \gamma_{ROA} )</th>
<th>( \bar{R}^2 )</th>
<th>( p.e._{\bar{r}} )</th>
<th>( p.e._{ROA} )</th>
<th>( p.e._{MOM} )</th>
<th>( p.e._{VOL} )</th>
<th>( p.e._{ALL} )</th>
<th>( \chi^2_{ALL} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.087</td>
<td>0.199</td>
<td></td>
<td></td>
<td>0.445</td>
<td>0.034</td>
<td>0.017</td>
<td>0.035</td>
<td>0.023</td>
<td>0.027</td>
<td>41.923</td>
</tr>
<tr>
<td>(9.01)</td>
<td>(14.55)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[0.00]</td>
</tr>
<tr>
<td>0.044</td>
<td>0.068</td>
<td>0.109</td>
<td>0.125</td>
<td>0.687</td>
<td>0.009</td>
<td>0.015</td>
<td>0.029</td>
<td>0.024</td>
<td>0.019</td>
<td>55.790</td>
</tr>
<tr>
<td>(4.00)</td>
<td>(4.63)</td>
<td>(12.11)</td>
<td>(13.69)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[0.00]</td>
</tr>
<tr>
<td>-0.047</td>
<td>0.160</td>
<td>0.088</td>
<td></td>
<td>0.534</td>
<td>0.016</td>
<td>0.016</td>
<td>0.036</td>
<td>0.022</td>
<td>0.023</td>
<td>11.937</td>
</tr>
<tr>
<td>(5.07)</td>
<td>(11.84)</td>
<td>(9.53)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[0.88]</td>
</tr>
<tr>
<td>-0.033</td>
<td>0.145</td>
<td></td>
<td>0.135</td>
<td>0.500</td>
<td>0.035</td>
<td>0.015</td>
<td>0.032</td>
<td>0.025</td>
<td>0.027</td>
<td>131.749</td>
</tr>
<tr>
<td>(2.65)</td>
<td>(9.23)</td>
<td>(14.83)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[0.00]</td>
</tr>
<tr>
<td>0.015</td>
<td></td>
<td>0.126</td>
<td>0.697</td>
<td>0.011</td>
<td>0.014</td>
<td>0.030</td>
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<td>0.020</td>
<td></td>
<td>14.258</td>
</tr>
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<td>(1.84)</td>
<td></td>
<td>(13.72)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[0.77]</td>
</tr>
</tbody>
</table>
Table 7
Expected and Actual Investment Returns

This Table reports the average investment returns (AR) and the expected investment returns (ER) from the three factor model based on Hou, Xue and Zhang (2012). The columns report the average and expected investment returns for All industries, industries that have below the median sales from public firms, industries that have above the median sales from public firms, industries that their fraction of sales coming from public firms is less than the 25th percentile and industries for which the fraction of sales of public firms is in the top 25th percentile.

<table>
<thead>
<tr>
<th>Port</th>
<th>All AR</th>
<th>ER</th>
<th>Below Median AR</th>
<th>ER</th>
<th>Above Median AR</th>
<th>ER</th>
<th>Low 25% AR</th>
<th>ER</th>
<th>High 25% AR</th>
<th>ER</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low I/K</td>
<td>0.162</td>
<td>0.154</td>
<td>0.186</td>
<td>0.187</td>
<td>0.131</td>
<td>0.101</td>
<td>0.214</td>
<td>0.194</td>
<td>0.137</td>
<td>0.118</td>
</tr>
<tr>
<td>2</td>
<td>0.143</td>
<td>0.138</td>
<td>0.139</td>
<td>0.105</td>
<td>0.147</td>
<td>0.165</td>
<td>0.153</td>
<td>0.126</td>
<td>0.175</td>
<td>0.179</td>
</tr>
<tr>
<td>3</td>
<td>0.117</td>
<td>0.120</td>
<td>0.112</td>
<td>0.125</td>
<td>0.117</td>
<td>0.110</td>
<td>0.120</td>
<td>0.130</td>
<td>0.121</td>
<td>0.090</td>
</tr>
<tr>
<td>4</td>
<td>0.093</td>
<td>0.065</td>
<td>0.087</td>
<td>0.100</td>
<td>0.107</td>
<td>0.086</td>
<td>0.099</td>
<td>0.119</td>
<td>0.108</td>
<td>0.109</td>
</tr>
<tr>
<td>High I/K</td>
<td>0.039</td>
<td>0.074</td>
<td>0.020</td>
<td>0.027</td>
<td>0.057</td>
<td>0.096</td>
<td>0.010</td>
<td>0.029</td>
<td>0.056</td>
<td>0.102</td>
</tr>
<tr>
<td>Low ROA</td>
<td>0.041</td>
<td>0.011</td>
<td>0.041</td>
<td>0.040</td>
<td>0.037</td>
<td>0.054</td>
<td>0.044</td>
<td>0.039</td>
<td>0.049</td>
<td>0.054</td>
</tr>
<tr>
<td>2</td>
<td>0.079</td>
<td>0.069</td>
<td>0.080</td>
<td>0.100</td>
<td>0.081</td>
<td>0.055</td>
<td>0.090</td>
<td>0.098</td>
<td>0.076</td>
<td>0.068</td>
</tr>
<tr>
<td>3</td>
<td>0.094</td>
<td>0.133</td>
<td>0.093</td>
<td>0.114</td>
<td>0.094</td>
<td>0.119</td>
<td>0.088</td>
<td>0.135</td>
<td>0.097</td>
<td>0.085</td>
</tr>
<tr>
<td>4</td>
<td>0.136</td>
<td>0.144</td>
<td>0.122</td>
<td>0.121</td>
<td>0.152</td>
<td>0.135</td>
<td>0.134</td>
<td>0.112</td>
<td>0.148</td>
<td>0.169</td>
</tr>
<tr>
<td>High ROA</td>
<td>0.202</td>
<td>0.195</td>
<td>0.208</td>
<td>0.170</td>
<td>0.194</td>
<td>0.196</td>
<td>0.243</td>
<td>0.215</td>
<td>0.229</td>
<td>0.223</td>
</tr>
<tr>
<td>Losers</td>
<td>0.078</td>
<td>0.068</td>
<td>0.095</td>
<td>0.125</td>
<td>0.061</td>
<td>0.039</td>
<td>0.113</td>
<td>0.125</td>
<td>0.051</td>
<td>0.019</td>
</tr>
<tr>
<td>2</td>
<td>0.080</td>
<td>0.096</td>
<td>0.087</td>
<td>0.088</td>
<td>0.072</td>
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<tr>
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<td>0.142</td>
<td>0.131</td>
<td>0.224</td>
<td>0.223</td>
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<td>0.086</td>
<td>0.081</td>
<td>0.080</td>
<td>0.073</td>
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<td>0.102</td>
<td>0.104</td>
<td>0.078</td>
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<td>0.196</td>
<td>0.209</td>
<td>0.225</td>
<td>0.196</td>
<td>0.212</td>
<td>0.239</td>
<td>0.288</td>
<td>0.258</td>
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</table>
Table 8
Expected Investment Return Differences

This Table reports the difference in expected investment returns (ER) from the three factor model based on Hou, Xue and Zhang (2012). The columns report the differences in expected investment returns between below and above the median fraction of sales by public firms (low-high) and between the industries for which the ratio of public firm sales is below the 25th percentile and industries for which that ratio is in the top 25th percentile (low25-high25).

<table>
<thead>
<tr>
<th>Port</th>
<th>Sales</th>
<th>Employees</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>low-high</td>
<td>low25-high25</td>
</tr>
<tr>
<td>Low I/K</td>
<td>0.037</td>
<td>0.045</td>
</tr>
<tr>
<td>2</td>
<td>-0.017</td>
<td>-0.022</td>
</tr>
<tr>
<td>3</td>
<td>0.03</td>
<td>-0.013</td>
</tr>
<tr>
<td>4</td>
<td>-0.01</td>
<td>0.006</td>
</tr>
<tr>
<td>High I/K</td>
<td>-0.038</td>
<td>-0.031</td>
</tr>
<tr>
<td>Low ROA</td>
<td>-0.021</td>
<td>-0.039</td>
</tr>
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<td>0.011</td>
<td>0.017</td>
</tr>
<tr>
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<td>0.037</td>
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<td>0.015</td>
<td>-0.014</td>
</tr>
<tr>
<td>High ROA</td>
<td>-0.008</td>
<td>-0.018</td>
</tr>
<tr>
<td>Losers</td>
<td>0.046</td>
<td>0.065</td>
</tr>
<tr>
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<td>0.015</td>
</tr>
<tr>
<td>3</td>
<td>0.004</td>
<td>-0.012</td>
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<tr>
<td>4</td>
<td>0.005</td>
<td>0.036</td>
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<tr>
<td>Winners</td>
<td>-0.064</td>
<td>-0.117</td>
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<tr>
<td>Low IVOL</td>
<td>-0.023</td>
<td>-0.006</td>
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<td>-0.02</td>
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<td>0.018</td>
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<td>High IVOL</td>
<td>-0.009</td>
<td>-0.02</td>
</tr>
<tr>
<td>Average</td>
<td>0.021</td>
<td>0.028</td>
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</tbody>
</table>
Table 9
The Cross-Section of Q for Private and Public Industries

We sort industries into quintiles by Tobin’s Q and report the mean and standard deviation of Q within each quintile. Tobin’s Q is calculated from firms’ first order conditions for optimal investment. The table reports mean and standard deviation of Q for each quintile for both private industries and public industries, as well as the mean and standard deviation of investment returns for these industries. In each year private industries are the industries for which the ratio of public firms’ sales to total industry sales is equal or below the median of the ratio for that year, whereas public industries are those with a ratio above the median. Panel C reports the results at the aggregate industry level. The data is from 1960 to 2005.

<table>
<thead>
<tr>
<th>Private Firm Industries</th>
<th>Q</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>High Q</th>
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<tbody>
<tr>
<td>Q</td>
<td>1.30</td>
<td>1.59</td>
<td>1.86</td>
<td>2.24</td>
<td>3.45</td>
</tr>
<tr>
<td>std(Q)</td>
<td>0.14</td>
<td>0.13</td>
<td>0.17</td>
<td>0.25</td>
<td>1.51</td>
</tr>
<tr>
<td>R(%)</td>
<td>13.68</td>
<td>13.86</td>
<td>11.72</td>
<td>11.11</td>
<td>3.82</td>
</tr>
<tr>
<td>std(R)(%)</td>
<td>31.60</td>
<td>33.65</td>
<td>23.38</td>
<td>25.99</td>
<td>28.07</td>
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</table>

<table>
<thead>
<tr>
<th>Public Firm Industries</th>
<th>Q</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>High Q</th>
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</thead>
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<tr>
<td>Q</td>
<td>1.32</td>
<td>1.63</td>
<td>1.91</td>
<td>2.31</td>
<td>3.58</td>
</tr>
<tr>
<td>std(Q)</td>
<td>0.14</td>
<td>0.14</td>
<td>0.18</td>
<td>0.25</td>
<td>1.20</td>
</tr>
<tr>
<td>R(%)</td>
<td>10.74</td>
<td>11.52</td>
<td>12.42</td>
<td>12.47</td>
<td>5.94</td>
</tr>
<tr>
<td>std(R)(%)</td>
<td>14.60</td>
<td>21.27</td>
<td>45.02</td>
<td>53.86</td>
<td>35.78</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>All Industries</th>
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<th>3</th>
<th>4</th>
<th>High Q</th>
</tr>
</thead>
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<td>1.88</td>
<td>2.27</td>
<td>3.52</td>
</tr>
<tr>
<td>std(Q)</td>
<td>0.14</td>
<td>0.13</td>
<td>0.17</td>
<td>0.24</td>
<td>1.37</td>
</tr>
<tr>
<td>R(%)</td>
<td>12.47</td>
<td>12.31</td>
<td>12.46</td>
<td>11.40</td>
<td>5.01</td>
</tr>
<tr>
<td>std(R)(%)</td>
<td>25.00</td>
<td>28.22</td>
<td>35.71</td>
<td>41.81</td>
<td>32.42</td>
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</table>
### Table 10
The Cross-Section of $Q$ for private and public industries that belong to the same 2-digit SIC code group

We sort four-digit industry SIC industries into groups according to their two-digit industry SIC code. Within each of the two-digit group we sort industries into private and public industries, where in each year private industries are industries for which the ratio of public firms’ sales to total industry sales is equal or below the median of the ratio for the two-digit industry for that year, whereas public industries are those with a ratio above the median. We report the cross sectional mean values of $Q$ within each quintile for both private and public four-digit SIC industries belonging to the same two-digit SIC group. Tobin’s $Q$ is calculated based on firms’ first order conditions for optimal investment. The sample is from 1960 to 2005.

<table>
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