Bundling and Quality Assurance

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Abstract
We consider a repeated moral hazard model of product quality choice with multiproduct firms and imperfect private monitoring by consumers. When consumers are small, receive imperfect private signals of product quality, and have heterogeneous preferences, consuming multiple products from the same firm improves monitoring. But monitoring is a public good, and consumers free ride on each others’ monitoring. Product bundling improves product quality by constraining consumers to use better monitoring and punishment strategies. The social and private value of bundling is even larger if one of the two goods is a durable and the other is a complementary non-durable, or if consumers can only attribute a negative signal to a pair of complementary products and not an individual product.

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1 Introduction

The Latin expression falsus in uno, falsus in omnibus, meaning “false in one, false in everything,” aptly depicts consumers’ expectations about product quality. Consumers often expect a firm’s entire product line to be of poor quality when they learn that one of a firm’s products is of poor quality. Similarly, positive experience with one of a firm’s products may be a stimulus for consumers to purchase other products from the firm. Multiproduct firms, aware of this important dimension of consumer behavior, can use product bundling to intensify consumer monitoring and to thereby increase product quality.

Although tying and bundling have been viewed with suspicion by US courts – and are deemed per se illegal in some circumstances – it is common for quality assurance to be used as an explicit efficiency defense in antitrust cases.\(^1\) In a landmark case from the 1930s, IBM unsuccessfully argued that tying the sale of paper tabulating cards to the lease of tabulating machines was a necessary and legitimate business practice. According to IBM, even small deviations in the size or thickness of the cards, or the presence of slime or carbon spots, “could cause inaccuracies in the function of the machine, serious in their consequences and difficult to trace to their source, with consequent injury to the reputation of the machines and the good will of the lessors.”\(^2\) A similar argument was successfully used by General Motors while defending their business practice of requiring their dealers to use only General Motors parts in the aftermarket service and repairs of their cars. “Defective parts, preventing efficient operations of cars, bring dissatisfaction with automobiles themselves. The material result is blame of the manufacturer and consequent loss of sales.”\(^3\)

This paper analyzes the economic incentives of a firm to bundle experience goods. Formally, we extend Klein and Leffler’s (1981) repeated moral hazard model of product quality to consider product bundling by multiproduct firms. In our model, a single branded firm competes with a fringe of competitive manufacturers who produce unbranded, low-quality products. There are many long-lived consumers who have heterogeneous preferences for the branded firm’s products. In each period, the branded firm decides whether to invest in product quality. Although consumers do not observe product quality directly, they do observe im-

\(^1\)See Jefferson Parish Hop. Dist. No. 2 v. Hyde (466 U.S. 2 [1984]) and Mozart C. v. Mercedes-Benz of North America Inc. (833 F. 2d 1342, 1348 [9th Cir. 1987]).

\(^2\)IBM v. United States (298 U.S. 131 [1936]). While accepting the need for quality assurance the Supreme Court affirmed the lower court’s decision, observing that IBM could include contractual restrictions in its leases requiring lessees to only use cards that met the necessary specifications for accurate functioning.

\(^3\)Pick Mfg. Co. v. General Motors Corp. et al. (80 F. 2d 641 [7th Cir. 1935]).
perfect private signals of quality which they can use to update their beliefs about product quality over time.

The main contribution of our paper is to show that bundling increases consumer and producer surplus by eliminating free-riding by consumers. With imperfect private monitoring, consumers who purchase multiple products from the branded firm are better able to monitor the branded firm’s choice of product quality and can respond more quickly when the firm shirks on quality. Absent bundling, consumers would pursue their individual interests without taking into account the impact that their purchase decisions have on the branded firm’s incentives. Product bundling implements more effective monitoring and punishment strategies by excluding consumers who would otherwise mix and match their purchases.\footnote{The same intuition holds in a model with single good when consumers demand different amounts of the good. In a companion paper, Dana and Spier (2014), we examine a simple model of heterogeneous consumer demand and show that quantity forcing can be used to exclude consumers who purchase too little and free ride on consumers who purchase more units.}

Note that the assumption that there are many consumers, each of which is small, is important because it implies consumers ignore the impact of their purchase decisions on the firm’s current and future effort decisions, which creates the free-riding problem. The assumption that monitoring is private is important because otherwise the quality of information generated by monitoring is independent of whether consumers buy multiple products from the firm. And finally the assumption that monitoring is imperfect is important for the same reason.

We extend our model to consider imperfect attribution, where consumers cannot attribute a negative signal to the specific individual product that generated it. For example, either hardware or software errors may prevent a consumer from print from their personal computer. Consumers who purchase only one of the branded firm’s products are likely to place some or all of the blame for a bad signal on the low quality fringe firms. Consumers’ propensity to blame the fringe exacerbates the branded firm’s incentive problem as it implies that consumers who mix and match are even less effective at monitoring the branded firm’s behavior. However, we find that product bundling is so useful in this environment that the incentives for high quality are unchanged when the attribution problem is introduced.

We also extend our model to consider the tying of consumable or aftermarket service to the sale of a durable good, and show that bundling is is more valuable in this environment. When the time between purchases is long, monitoring is typically less effective. However, if the firm bundles, or ties, its durable product to a complementary non-durable product, then it can both increase monitoring use bundling to convert its profits on the durable into a discretionary stream of
profits on the non-durable product that makes high quality easier to sustain. We find that product bundling is so useful that the incentives for high quality are unchanged when one non-durable is transformed into an infinitely-lived durable.

The next section of the paper discusses some of the related literature. Section 3 describes the model and characterizes the optimal pricing and product quality for the benchmark case in which product quality is observable, and for a multiproduct firm with and without product bundling when product quality is unobservable at the time of purchase. Section 3 also characterizes when bundling is used and demonstrates that it increases consumer and producer surplus whenever it is used. Section 4 considers a second extension in which consumers have trouble attributing signals to specific products. Section 5 considers an extension of the model in which one of the two goods is an infinitely-lived durable goods. Section 6 discusses other extensions. In each of these cases we show, perhaps counterintuitively, that bundling is even more likely to be privately profitable and socially desirable. Section 7 concludes.

2 Literature

Although there is a large literature on bundling and tying in the legal and economic literatures, the relationship between bundling and quality assurance has received relatively little attention. Bork (1978) and Posner and Easterbrook (1981) argued informally that when consumers use low-quality products along with high-quality products, then low overall performance may be erroneously attributed to the producer of the high-quality product. By tying or bundling the products together, the seller can protect its reputation. Iacobucci (2003) refined this argument further, highlighting the importance of the attribution problem (see also Bar-Gill, 2006). Our work provides additional rigor and clarity to these arguments by providing an explicit treatment of the dynamic mechanisms by which consumers learn about product quality. In contrast to Iacobucci, we find

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5 Adams and Yellen (1976) and McAfee, McMillan and Whinston (1989) have argued that bundling can be an effective form of price discrimination, allowing a monopolist to extract greater rents from consumers. See also Fang and Norman (2006). Other papers, particularly Elhauge and Nalebuff (2014), and argue that bundling or tying facilitates price discrimination because firms can use consumption of tied non-durable goods as a proxy for consumers’ valuation of a durable good. Other papers, including Whinston (1990) and Nalebuff (2004), have argued that bundling can be used as a weapon by an incumbent to foreclose competition in the market (see the survey by Nalebuff, 2008). Salinger (1995) emphasizes that bundling may also reduce the costs of production and distribution of products – it is more efficient for automobile manufacturers, for example, to bundle tires with cars than for consumers to do it themselves.

6 For additional references and discussion of the legal literature, see Iacobucci (2003, at page 437).
that the consumers’ attribution problem strengthens the benefits of bundling but
is not necessary for bundling to be privately or socially desirable.

Our work is also related to the literature on umbrella branding, some of which
also considers multiproduct versions of Klein and Leffler (1981). In a perfect
public monitoring model related to Bernheim and Whinston’s (1990) model of
multimarket contact, Andersson (2002) shows that as long as the products are
asymmetric and at least one of the incentive constraints for producing high quality
bends, joint production is profitable for a monopolist. Choi (1998) models the firm
as introducing a sequence of new products and shows that the firm can leverage
its future product introduction rents to credibly reduce the cost of signaling high
quality in the present. Cabral (2009) and Cai and Obara (2006) show that joint
production can reduce equilibrium path punishments when product markets are
symmetric and there is imperfect public monitoring. Note that none of the pa-
pers in this literature emphasize the value of bundling or purchase restraints, only
additional advantages that multiproduct firms have over single-product firms.

Some papers are more closely related to our durable good extension. Like
us, Schwartz and Werden (1996) suggest that tying a durable to a non-durable
can shift the rents from the durable to the non-durable and help overcome the
information problem. They consider a model of private information in which tying
can signal quality. In contrast, our model is a repeated moral hazard model, and
is the only paper to demonstrate that the advantage of bundling is greatest when
there is asymmetric information about both goods.

3 Model

A unit mass of infinitely-lived consumers use a single unit of each of two experience
goods, product 1 and product 2, in each period of their lives. These experience
goods are available from two sources, a branded firm (sometimes referred to as the
firm or the monopolist) and a competitive fringe. The competitive fringe produces
only low-quality products. The marginal cost, price, profits and consumer surplus
associated with low-quality products from the competitive fringe are normalized

\textsuperscript{7} Wernerfelt (1998) and Cabral (2000) also offer theories of umbrella branding as a signal
of product quality. Montgomery and Wernerfelt (1992) consider a free-entry repeated moral
hazard model in which some consumers observe quality and find that umbrella branding reduces
the variation in quality. Pepall and Richards (2002) consider demand-side and supply-side
economies of scope to explain umbrella branding.

\textsuperscript{8} Hakenes and Peitz (2008) also consider imperfect public monitoring, but they assume (as
we do) that there is a positive probability of accurately observing that the firm produced a
low-quality product.
to zero.\footnote{We could alternatively assume that the consumer could self-supply a low quality product. An industrial consumer who purchases a large industrial machine and forgoes the service contract on that machine might service the machine with its own (nonspecialized) employees.}

The branded firm can produce either high or low-quality products. Consumers view low-quality products from the branded firm as identical to the low-quality products from the competitive fringe. The firm’s cost of producing low-quality products is zero. A consumer’s willingness to pay for high-quality products, denoted by $v_1$ and $v_2$, are distributed with symmetric probability density function $f(v_1, v_2)$ on $[0, \bar{v}] \times [0, \bar{v}]$. We assume that the density $f(v_1, v_2)$ is strictly positive on its support, which implies that $v_1$ and $v_2$ are not perfectly correlated. We also assume that $\bar{v} > c$ where $c > 0$ is the common marginal cost of producing a high-quality product.

We will now define some useful notation. Suppose that consumers believe that the quality of the branded products is high. If the products are sold separately at prices $p_1$ and $p_2$, the demand for each product can be written as $d(p_i) = \int_0^\bar{v} f(v_i, v_j) dv_i dv_j$. The mass of consumers who buy both products can be written as $D^m(p_1, p_2) = \int_0^\bar{v} \int_0^\bar{v} f(v_1, v_2) dv_1 dv_2$, and the mass of consumers who buy product $i$ alone can be written as $d^m_i(p_1, p_2) = d(p_i) - D^m(p_1, p_2)$. If the two products are bundled together and are sold at price $2p_b$, the demand for the bundle is

$$D^b(p_b) = \int_{v_1 + v_2 \geq 2p_b} f(v_1, v_2) dv_1 dv_2. \quad (1)$$

Expressing the bundled price as $2p_b$ facilitates comparison of the per unit bundled price, $p_b$, to the single-product prices, $p_1$ and $p_2$.

In our model, consumers do not directly observe product quality at the time of purchase. Instead, they receive imperfect private signals of quality immediately after purchase. If the quality of product $i$ is low, then $1 - \pi \in (0, 1)$ is the probability that the consumer will receive a private negative signal alerting him or her to that fact, and $\pi$ is the corresponding probability that no negative signal is received.\footnote{A low-quality good from the monopolist and a low-quality good from the fringe are both valued at zero, and both generate a negative signal with probability $1 - \pi$. The simplest interpretation of our assumptions is that the signal is purely information, as opposed to a product failure which would directly impact a consumer’s ex post willingness to pay. Alternatively, the signal can be interpreted as a product failure with an associated negative cost that is offset by a positive benefit (identical to the benefit generated by the high-quality good) received when a failure does not occur.} If a consumer purchases two products from the firm and both products are low quality, then the consumer may receive one or two negative signals. It is natural to assume that the signals are independently distributed so
the probability of receiving at least one negative signal is $1 - \pi_b = 1 - \pi^2$, but we do not impose this and assume only that $\pi_b < \pi$ and $\pi_b > 2\pi - 1$. The former holds if the signals are not perfectly correlated and the latter holds because the probability of a negative signal from two products can’t exceed the sum of the probability of a negative signal with each product independently. If the quality of product $i$ is high, then no negative signal is generated.

The timing of the game is as follows. At time $t = 0$ consumers learn their valuations, which remain fixed over time, and the monopolist chooses whether to sell the two products separately or as a bundle. In every subsequent period, $t = 1, 2, 3, ...$, the monopolist chooses its product quality and chooses its price (or prices), and then consumers decide whether to purchase from the monopolist or from the fringe. Finally, the consumer privately observes imperfect signals of product quality.\textsuperscript{11} Finally, we assume the firms, and consumers, have a common discount factor $\delta$.

### 3.1 Observable Quality

As a benchmark, suppose that the consumers can perfectly observe product quality at the time of purchase. The firm will, of course, only produce high-quality products in this case. If it produced low-quality products, consumers would view these products as undifferentiated from the fringe products and would not be willing to pay a premium for them.

Suppose that the firm sells two high-quality products separately, allowing consumers to purchase one, both, or neither. Then the prices that maximize the firm’s period profits are $p_1 = p_2 = p^* = \arg \max (p_i - c) d(p_i)$. If the firm bundles the two products, then the bundled price that maximizes the firm’s total period profits is $p^*_b = \arg \max 2(p_b - c) D_b(p_b)$. We assume that the profit maximizing prices are uniquely defined by the first-order conditions. Finally, we assume that monopolist earns strictly higher profits selling the two products separately than by selling them as a bundle:

$$2(p^* - c)d(p^*) > 2(p^*_b - c)D_b(p^*_b).$$

That is, when product quality is observed by consumers at the time of purchase, the monopolist would choose strictly prefer to sell the products separately. Hence we are assuring that in the absence of asymmetric information, there is no advantage of bundling.

\textsuperscript{11}We restrict our attention to pure bundling, not mixed bundling, and do not allow for resale.
Example: Suppose that \( c = .25 \) and that \( f(v_1, v_2) \) is uniformly distributed on \([0,1] \times [0,1]\) so the valuations are independently distributed. If the firm does not bundle, then the optimal prices and quantities are \( p^* = 5/8 \) and \( d(p^*) = 3/8 \). Half of the total demand is from consumers who buy both products and half is from consumers who buy just one product. The firm’s profit is \( 2(3/8)^2 = 9/32 \).

When the firm bundles, then the optimal price is \( p^*_b = 1/2 \), so the total price of the bundle is \( 2p^*_b = 1 \), and \( D^b(p_b) = 1/2 \). The firm’s profit is \( 1/4 \), which is lower than without bundling.\(^{12}\)

3.2 Unobservable Quality

In this section, we focus on the most profitable symmetric, stationary perfect Bayesian equilibria. In these equilibria, high-quality products are produced whenever feasible. Consumers believe that the monopolist will continue to produce high-quality products as long as they have not experienced any negative signals in the past. If a consumer ever receives a negative signal – hard evidence that the firm deviated and produced a low-quality product – the consumer believes that the quality of both of the firm’s products will be low in all future periods. The firm loses these consumers, as they are no longer willing to pay a premium for what they believe to be low-quality products. These off-the-equilibrium-path beliefs generate the harshest punishment for a deviation and hence select the equilibrium in which the firm has the strongest incentives to produce high-quality products.

Our analysis will proceed in several steps. First, we consider a benchmark case in which the two products are produced by two separate firms. Second, we consider the case in which the firm bundles its two products so the customers must buy both together. Third, we consider the case in which the firm sells the two products separately and unbundled, so consumers are free to purchase only one product from the firm and the other from the competitive fringe.

Single-Product Firms

Suppose the products are produced by two separate firms, firm 1 and firm 2, and that the managers of the two firms make their quality and pricing decisions independently of one another. If a consumer sees a negative signal from product

\(^{12}\)Of course for other parameter values, bundling can make the firm strictly better off, but this example is useful because it illustrates the value of bundling as a mechanism assure product quality when bundling isn’t otherwise profitable. The consumer surplus is \(.5(3/8)^2 + .5(3/8)^2 = (3/8)^2\) without bundling and \(1/6\) with bundling. Thus, the consumers are better off with bundling even if high product quality is sustainable in either case.
the consumer believes that the quality of product $i$ will be low in all future periods.

Incentive compatibility for the single-product firms requires that

$$d(p_i) \frac{p_i - c}{1 - \delta} \geq d(p_i) \frac{p_i}{1 - \delta \pi}. \quad (2)$$

The left-hand side is the present discounted value of a producing high-quality product $i$. The right-hand side is the profit from a deviation to low quality for product $i$. The probability that a consumer who purchased a low-quality product in period $t-1$ receives a negative signal is $1 - \pi$, and so the probability that a consumer will purchase again in period $t$ is $\pi$. From the firm $i$’s perspective, the benefit of deviating to low quality is that it can lower its production cost from $c$ to zero. The firm’s cost of deviating to low quality is that the consumers will gradually learn over time and the number of consumers remaining in the market will shrink.

When $\delta$ is sufficiently large, the incentive compatibility constraint is slack and firm $i$ will charge the monopoly price, $p_i = p^\ast$. For $\delta < \bar{\delta}_s$, where

$$\bar{\delta}_s = \frac{c}{\pi c + (1 - \pi)p^\ast}, \quad (3)$$

the incentive constraint cannot be satisfied at the monopoly price, but can be satisfied at a higher price. From equation (2), the incentive compatibility constraint is satisfied when

$$p_i \geq \frac{c(1 - \delta \pi)}{\delta(1 - \pi)}. \quad (4)$$

However, for $\delta \leq \bar{\delta}_s$, where

$$\bar{\delta}_s = \frac{c}{\pi c + (1 - \pi)\bar{v}}, \quad (5)$$

the incentive constraint cannot be satisfied without setting $p_i \geq \bar{v}$ which implies that sales and profits are zero. The following proposition follows immediately.

**Proposition 1.** When product quality is unobservable and the two branded products are sold by single-product firms, high quality is sustainable with positive sales if and only if $\delta > \bar{\delta}_s$. The prices charged in the highest profit subgame perfect Nash equilibrium is

$$p_s(\delta) = \begin{cases} 
\frac{c(1 - \delta \pi)}{\delta(1 - \pi)} & \text{if } \delta \in [\bar{\delta}_s, \bar{\delta}_s) \\
 p^\ast & \text{if } \delta \in [\bar{\delta}_s, 1].
\end{cases}$$

where $\lim_{\delta \to \bar{\delta}_s} p_s(\delta) = \bar{v}$ (and profits go to zero) as $\delta$ approaches $\bar{\delta}_s$ from above.
Comparing the single-product firm’s incentive compatibility constraint, equation (2), to the standard incentive constraint with perfect public monitoring, which is \((p_i - c)/(1 - \delta) \geq p_i\), it is clear that when a consumer learns only from his or her own private experience, and not from other consumers’ experiences, the incentive constraint is more difficult to satisfy. So a policy change which makes consumer learning public, or more generally just improves consumer learning (lowers \(\pi\)), such as the law requiring restaurants to post cleanliness ratings in their windows, which was analyzed by Jin and Leslie (2003), can increase product quality and social welfare.

**Bundled-Product Firm**

Next, suppose that one firm produces both products and bundles them together. The consumer beliefs that support the high-quality equilibrium are analogous to those in the previous section. A consumer believes that quality of both products will be high unless the consumer witnesses a product failure. Specifically, if one or both products fail in a given period, the consumer believes that the quality of both products will be low in the future and is no longer willing to pay a price premium in excess of the outside option from the fringe.

Recall that \(2p_b\) is the total price of the bundle, \(p_b\) is the average price per unit in the bundle. Incentive compatibility requires that the firm prefers to maintain high quality for both products rather than deviate and lower the quality of both. This condition may be written as

\[
2D^b(p_b) \frac{p_b - c}{1 - \delta} \geq 2D^b(p_b) \frac{p_b}{1 - \delta \pi_b}. \tag{6}
\]

The left-hand side is the present discounted value of selling a bundle of high-quality products. The right-hand side is the present discounted value of deviating and selling a bundle of two low-quality products when consumers expect high-quality products. As consumers observe negative signals, the consumer base shrinks.

The firm could also deviate by producing just one low-quality product, however in the Appendix we show that if that if any other deviation is profitable, then it must be profitable for the firm to deviate in the quality of both products, so it is sufficient for us to verify that the incentive constraint (6) is satisfied.

If the incentive compatibility constraint is slack then the firm will charge the observable-quality monopoly price, \(p^*_b\). At the monopoly price, the incentive constraint is not satisfied for \(\delta < \delta_b\), where

\[
\delta_b = \frac{c}{\pi_b c + (1 - \pi_b)p^*_b},
\]
but the incentive compatibility constraint may be satisfied at a higher price. Using equation (6), this price must satisfy
\[ p_b \geq \frac{c(1 - \delta \pi_b)}{\delta(1 - \pi_b)}. \] (7)

However, for \( \delta \leq \delta_b \), where
\[ \delta_b = \frac{c}{\pi_b c + (1 - \pi_b)\bar{v}}, \]
the incentive constraint cannot be satisfied without setting \( p_b \geq \bar{v} \) which implies sales and profit of the monopolist would be zero. Also, note that clearly \( \pi_b < \pi \) implies \( \delta_b < \delta_s \), so the lower bound for which high quality is sustainable for a bundling firm is smaller than the lower bound for which high quality is sustainable for single product firms.

The following proposition follows immediately.

**Proposition 2.** When product quality is unobservable and the two branded products are bundled, high quality is sustainable if and only if \( \delta \geq \delta_b \). The price charged for the bundle in the highest profit subgame perfect Nash equilibrium is \( 2p_b(\delta) \) where
\[
p_b(\delta) = \begin{cases} 
\frac{c(1 - \delta \pi_b)}{\delta(1 - \pi_b)} & \text{if } \delta \in [\delta_b, \bar{\delta}_b) \\
p^*_b & \text{if } \delta \in [\bar{\delta}_b, 1],
\end{cases}
\]
where \( \lim_{\delta \downarrow \delta_b} p_b(\delta) = \bar{v} \) (and profits go to zero) as \( \delta \) approaches \( \delta_b \) from above.

**Multiproduct Firm**

Now suppose instead that the firm produces both products, but does not bundle them. We will see that this case has some of the advantages of bundling, but without the main disadvantage, which is the foregone profits due to excluding single-product purchases.

When the discount factor \( \delta \) is sufficiently close to 1, then there exists a perfect Bayesian equilibrium where the firm produces high-quality products. Formally, the incentive compatibility constraint for the multiproduct firm can be written as
\[
(d^m_{p1}(p_1, p_2) + D^m(p_1, p_2)) \left( \frac{p_1 - c}{1 - \delta} \right) + (d^m_{p2}(p_1, p_2) + D^m(p_1, p_2)) \left( \frac{p_2 - c}{1 - \delta} \right) \\
\geq d^m_{p1}(p_1, p_2) \left( \frac{p_1}{1 - \delta \pi} \right) + d^m_{p2}(p_1, p_2) \left( \frac{p_2}{1 - \delta \pi} \right) \\
+ D^m(p_1, p_2) \left( \frac{p_1 + p_2}{1 - \delta \pi_b} \right). \] (8)

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Another possible deviation for the firm is to produce one low-quality product and one high-quality product, but in the Appendix, using an argument very similar to the one used in the bundling case, we show that if any other deviation is profitable, than so is deviating forever in both product qualities, so it is sufficient to check that incentive condition (8) is satisfied.

The multiproduct firm’s incentive compatibility constraint is a composite of the single-product and bundled-product firm incentive constraints. Specifically, the single-product and bundled-product firm incentive constraints, equations (4) and (7), can be written as

\[ S_i(p, \delta) = (p_i - c) - p_i \left( \frac{1 - \delta}{1 - \delta \pi} \right) \geq 0, \]

and

\[ B(p, \delta) = (p_b - c) - p_b \left( \frac{1 - \delta}{1 - \delta \pi_b} \right) \geq 0. \]

Using this notation, the multiproduct firm’s incentive compatibility constraint can be rewritten as a weighted linear combination of \( S_i(\cdot) \) and \( B(\cdot) \),

\[ M(p_1, p_2, \delta) = d_1^m(p_1, p_2) S_1(p_1, \delta) + d_2^m(p_1, p_2) S_2(p_2, \delta) + D^m(p_1, p_2) B \left( \frac{p_1 + p_2}{2}, \delta \right) \geq 0, \]

It is straightforward to show that \( \partial S_i(\cdot) / \partial \delta > 0 \) and \( \partial B(\cdot) / \partial \delta > 0 \), so \( M(p_1, p_2, \delta) \) is also increasing in \( \delta \). Raising the discount factor relaxes the multiproduct firm’s incentive compatibility constraint.

**Proposition 3.** When product quality is unobservable and the two branded products are sold by a multiproduct firm, there exists a \( \delta_m > 0 \) such that high quality is sustainable with positive sales if and only if \( \delta > \delta_m \); there exists a \( \delta_m < 1 \) such that high quality is sustainable at the observable-quality monopoly prices if and only if \( \delta \geq \tilde{\delta}_m \); and finally \( \delta_m < \tilde{\delta}_m \).

**Proof.** Clearly \( M(p_1, p_2, \delta) \) is continuous and increasing in \( \delta \) for all \( p_1, p_2 \). Since \( M(p_1, p_2, 0) < 0 \) for all \( p_1, p_2 \), and since \( M(p_1, p_2, 1) > 0 \) for all \( p_i > c \), it follows that there exists a unique \( \delta(p_1, p_2) > 0 \) such that \( M(p_1, p_2, \delta(p_1, p_2)) = 0 \). Clearly \( \delta_m \) is equal to \( \min_{p_1, p_2} \delta(p_1, p_2) \). So there exists \( \delta_m \) and associated prices, \( \hat{p}_1, \hat{p}_2 \), such that \( M(\hat{p}_1, \hat{p}_2, \delta_m) = 0 \) and \( M(p_1, p_2, \delta) < 0 \) for all \( p_1, p_2 \), and all \( \delta < \delta_m \). Moreover, since \( M(p_1, p_2, \delta) \) is increasing in \( \delta \), the incentive constraint can clearly be satisfied for any \( \delta > \delta_m \).

Since \( M(p_1, p_2, \delta) \) is continuous and increasing in \( \delta \); since \( M(p^*, p^*, 1) > 0 \), because \( S_i(p^*, 1) > 0 \) and \( B(p_b^*, 1) > 0 \); and since \( M(p^*, p^*, 0) < 0 \), there clearly
exists a unique $\delta_m \in (0, 1)$ such that $M(p^*, p^*, \delta_m) = 0$ and $M(p^*, p^*, \delta) \geq 0$ if only if $\delta \geq \delta_m$.

Finally, since $p^*$ is the unconstrained monopoly price, $M(p^*, p^*, \delta_m)$ must be locally increasing in these prices when $p_i = p^*, i = 1, 2$. A small price increase has no effect on the left hand side of (8), because the first derivative with respect to price is equal to zero at an unconstrained maximum, but the right hand side of (8) is clearly decreasing in price. So there exists prices such that $M(p_1, p_2, \delta_m) > 0$, which implies $\delta_m < \delta_m$.

$$\square$$

### 3.3 Comparison - The Value of Bundling

A comparison of bundling to the multiproduct firm without bundling demonstrates that bundling can increase both producer and consumer surplus.

The following is the main result of the paper.

**Proposition 4.** High quality can be supported in equilibrium for the greatest range of discount factors under bundling: $0 < \delta_b < \delta_m \leq \delta_s < 1$. For $\delta \in (\delta_b, \delta_m)$, a high-quality equilibrium exists only if the firm bundles its products, and bundling increases both firm profits and consumer surplus.

**Proof.** By definition $B(\bar{v}, \delta_b) = 0$, and clearly $S_i(\bar{v}, \delta_b) < 0$, $\forall i$ since $\delta_b < \delta_s$, so clearly $M(p_1, p_2, \delta_b) < 0$ in a neighborhood of $p_1 = \bar{v}$ and $p_2 = \bar{v}$. More precisely, there exists $\epsilon$ such that $M(p_1, p_2, \delta_b) < 0$ for all $p_1 \in (\bar{v} - \epsilon, \bar{v})$ and $p_2 \in (\bar{v} - \epsilon, \bar{v})$.

Moreover, $M(p_1, p_2, \delta_b) < 0$ for all $p_1 < \bar{v}$ and $p_2 < \bar{v}$ because $S_i(p_i, \delta)$, $\forall i$, and $B((p_1 + p_2)/2, \delta)$ are strictly increasing in $p_1$ and $p_2$.

Now suppose $\hat{p}_1$ and $\hat{p}_2$ are prices at which high quality can be sustained when $\delta = \delta_m$, the lowest discount factor at which high quality is sustainable. We have just shown that $M(\hat{p}_1, \hat{p}_2, \delta_b) < 0$, so since $M$ is increasing in $\delta$, it must be that $\delta_m > \delta_b$. This implies that bundling increases profits and consumer surplus for $\delta \in (\delta_b, \delta_m)$ since otherwise product quality is low, and profits and consumer surplus are zero.

Profits and consumer surplus are clearly higher with bundling for $\delta \in (\delta_b, \delta_m)$ since they are both equal to zero when high quality cannot be sustained.

The proof that $\delta_m \leq \delta_s$ is simple. Since the firm can set $\hat{p}_1 = \bar{v}$, the multiproduct firm can always act like a single-product firm selling only good 2, and similarly by setting $\hat{p}_2 = \bar{v}$, act like a single-product firm selling only good 1. So the multiproduct firm can support high quality for at least one product if $\delta_m \leq \delta_s$. High quality can be supported for both goods for all $\delta \leq \delta_s$. $\square$

We can also compare the multiproduct firm without bundling to the single-product firms. Multiproduct firms benefit from the superior monitoring of customers who buy both products.
Proposition 5. In the highest profit equilibrium, for \( \delta \in (\hat{\delta}_m, \hat{\delta}_s) \), producer and consumer surplus are strictly higher with a multiproduct firm than with two single-product firms.

Proof. For all \( \delta \in (\hat{\delta}_m, \hat{\delta}_s) \) the incentive constraint is binding for at least one single product firm, that is, \( S_i(p^*, \delta) < 0 \) for some \( i \). Let \( \hat{p}_i \) denote the prices charged by the single profit firms in the most profitable equilibrium. Clearly \( M(\hat{p}_1, \hat{p}_2, \delta) > 0 \), since \( S_i(\hat{p}_i, \delta) \geq 0 \) and this implies \( B(\frac{\hat{p}_1 + \hat{p}_2}{2}, \delta) > 0 \) and \( M \) is a weighted average of \( S_i \) and \( B \). So for any \( \delta \) the distortion in the prices is smaller (and prices are lower) for the multiproduct firm, which implies both profits and consumer surplus is higher.

Our theoretical analysis of imperfect private monitoring and multiproduct firms is consistent with the empirical evidence in Jin and Leslie (2009), which finds that restaurants that belong to chains have higher cleanliness ratings than restaurants that are not members of chains. The relevant implication of our theory is that consumers who dine in multiple locations learn faster about the cleanliness of the chain, so high quality is easier to support in equilibrium.

Finally, note that our analysis assumed symmetry, but only minor aspects of the problem depend on symmetry. First, symmetry was used to prove that it was sufficient to consider a single incentive constraint. If we relax symmetry sufficiently, the firm might prefer to deviate only in the quality of one product. And the firm might strictly prefer to sell only one high quality product, so sustaining high quality for both products is no longer synonymous with maximizing profitability. Second, symmetry was used to argue that \( \hat{\delta}_b < \hat{\delta}_s \). When it is easier to maintain high quality for one product than a second product, then bundling will help maintain quality for both, but in no longer follows the bundling sustains high quality easier for both products than for just the easiest of the two products.

4 Bundling and the Attribution Problem

This section of the paper analyzes a variant of the model in which the two products sold are used together and consumers are unable to accurately attribute the negative signal to one of their two product purchases. An important implication of this assumption is that consumers will blame the fringe firm whenever they believe a priori that the branded firm’s product is high quality and the fringe firms’ products are low quality, which weakens the branded firm’s incentives to provide high quality.
Suppose that consumers each purchase two goods, but may purchase zero, one, or two products from the fringe. Fringe products are all low quality and produce negative signals with probability $1 - \pi$. High-quality products from the branded firm never produce negative signals. Low-quality products from the branded firm, like fringe products, produce negative signals with probability $1 - \pi$. Suppose that when either or both products produce negative signals, the consumer cannot distinguish which product or products produced the signal or how many signals were received. So when a consumer cannot distinguish whether one or both products are of low quality. So if the consumer believes he or she purchased one high-quality product and one low-quality fringe product, then he or she will attribute all negative signals to the fringe, and will therefore not change his or her beliefs about quality following a deviation by the firm.

**Single-Product Firms**

With single-product firms it is now much more difficult to sustain high quality. Consumers who purchased one product from the fringe and the other from a branded firm will always blame the fringe and never punish the branded firm. Only consumers that purchase from both single-product branded firms will ever punish the branded firms. In the most profitable equilibrium these consumers will punish both firms when they observe a negative signal, even though they think that only one firm deviated.

The incentive constraint is

$$d(p_i) \frac{p_i - c}{1 - \delta} \geq d_i^m(p_1, p_2) \frac{p_i}{1 - \delta} + D^m(p_1, p_2) \frac{p_i}{1 - \delta \pi}$$

(10)

The deviation profit for a single-product firm on the right-hand side depends on $d_i^m(\cdot)$ and $D^m(\cdot)$ because the consumers’ future purchase decisions depend on whether they purchase one or two goods from the branded firms. The first term is the stream of profits from consumers who never punish the firm even though it deviated. The second term is the stream of profits from consumers who buy both branded products and as a consequence exit with probability $1 - \pi$ each period. Since $d(p_i) = d_i^m(p_1, p_2) + D^m(p_1, p_2)$ this simplifies to

$$D^m(p_1, p_2) \frac{p_i - c}{1 - \delta} - d_i^m(p_1, p_2) \frac{c}{1 - \delta} \geq D^m(p_1, p_2) \frac{p_i}{1 - \delta \pi}$$

(11)

or

$$\frac{p_i - c}{1 - \delta} - \frac{d_i^m(p_1, p_2) c}{D^m(p_1, p_2) 1 - \delta} \geq \frac{p_i}{1 - \delta \pi}$$

(12)

which is clearly more difficult to satisfy than the single product incentive constraint in the absence of the attribution problem, equation (2). That is, the
values of $\delta$ at which (12) holds with equality when $p = p^*$ and $p = \bar{v}$ are strictly larger the $\delta_s$ and $\delta_\bar{v}$ respectively, so the range of $\delta$ for which high quality can be sustained is strictly smaller.

**Bundled-Product Firm**

Incentive compatibility for a bundled-product firm is different because the firm sells only to consumers who buy two products. This incentive compatibility constraint is

$$2D^b(p_b)\frac{p_b - c}{1 - \delta} \geq 2D^b(p_b)\frac{p_b}{1 - \delta \pi_b},$$

(13)

since a deviating firm loses a fraction $(1 - \pi_b)$ of its consumers (all two-product consumers) each period. This is identical to the incentive constraint for bundling without the attribution problem, equation (6). Hence, as long as the firm is bundling its products, introducing the attribution problem does not make it any more difficult for the firm to maintain high quality.

**Multiproduct Firm**

The incentive compatibility constraint for a multiproduct firm is different without bundling because some consumers purchase one product and never punish the firm (they always blame the fringe), while other consumers purchase two products from the firm and will punish punish the firm by stopping their purchases of both products following any negative signal. The incentive compatibility constraint is

$$\left(\begin{array}{c} d^m_1(p_1, p_2) + D^m(p_1, p_2) \\ d^m_2(p_1, p_2) + D^m(p_1, p_2) \end{array}\right) \frac{p_1 - c}{1 - \delta} + \left(\begin{array}{c} d^m_1(p_1, p_2) + D^m(p_1, p_2) \\ d^m_2(p_1, p_2) + D^m(p_1, p_2) \end{array}\right) \frac{p_2 - c}{1 - \delta \pi_b} \geq p_1 + p_2 - 2c \frac{1 - \delta}{1 - \delta \pi_b},$$

(14)

A deviating firm loses none of its single-product consumers each period – these consumers attribute the bad signal to the fringe – but loses a fraction $(1 - \pi_b)$ of its two-product consumers each period (so a fraction $\pi_b$ remain after each period).

Note that equation (14) is very similar to our earlier incentive constraint for the multiproduct firm (8), with one important difference: The consumers who purchase just one product are not able to monitor the firm at all.

The incentive compatibility constraint can be simplified to

$$\frac{p_1 + p_2 - 2c}{1 - \delta} - \left(\begin{array}{c} d^m_1(p_1, p_2) + d^m_2(p_1, p_2) + 2D^m(p_1, p_2) \\ D^m(p_1, p_2) \end{array}\right) \frac{c}{1 - \delta} \geq \frac{p_1 + p_2}{1 - \delta \pi_b},$$

(15)
which is clearly more difficult to satisfy at any price then the constraint for the
bundled firm. That is, the values of $\delta$ at which (14) holds with equality when
$p = p^*$ and $p = \bar{v}$ are strictly larger $\delta_m$ and $\bar{\delta}_m$ respectively (which were defined
without the attribution problem), so the range of $\delta$ for which high quality can be
sustained by a multiproduct firm is strictly smaller.

More importantly, the range of $\delta$ for which high quality can be sustained by a
firm that bundles is unchanged, so the range of $\delta$ for which bundling is necessary
to sustain high quality is even greater with the attribution problem than it was
in our initial model.

So the attribution problem makes high quality harder to sustain for both
single product and multiproduct firms, but multiproduct firms that bundle their
products can sustain high quality just as easily when the attribution problem is
present as when it is absent.

5 Bundling Durable and Non-durable Goods

Many examples of bundling, particularly some of the most legally contested ex-
amples, involve a durable good tied to a non-durable complementary good. IBM
computers and punch cards, printers and ink, and automobiles and spare parts
are obvious well-known examples. Because of the empirical relevance of these
examples, we now describe an extension in which one good is durable which lasts
forever, and the second good is nondurable which is purchased every period.

We use $n$ and $d$ (short for nondurable and durable) to denote the two prod-
ucts. The cost of supplying the high quality nondurable good is $c_n$ per unit, and
the cost of supplying one unit of the durable good is $C_d$. To maintain notational
comparability with the earlier sections, we define $c_d$ to be the equivalent perpe-
tuity cost of the durable good, so $\frac{c_d}{1 - \delta} = C_d$. Similarly, $p_n$ is the unit price of
the nondurable good, and $P_d$ is the price charged for the durable good at $t = 1$.
We define $p_d$ as the equivalent perpetuity payment, so $\frac{p_d}{1 - \delta} = P_d$. (From the con-
sumer’s perspective, the decision to purchase the durable for price $P_d$ is equivalent
to a commitment to rent the durable for price $p_d$ in each period in perpetuity.)

The consumer receives a negative signal with probability $1 - \pi_i$, $i = n, d$ in
each period if the consumer purchases only one low-quality product, and with
probability $1 - \pi_b$ if the consumer purchases two low-quality products. If we
assume that $f$ is symmetric, $c_d = c_n$, and $\pi_d = \pi_n$, then Proposition 4 and 5
still hold and the only difference between our current assumptions and our initial
assumptions is that one of the two products is a durable good, is purchased at
time $t = 1$ only, and lasts forever.
**Single-Product Firm**

In the benchmark case where the durable and nondurable products are sold by different firms, it is clear that there cannot exist an equilibrium in which the firm selling the durable good chooses high quality. The reason for this is simple: since the consumers only purchase the durable product once, a single-product firm would have absolutely no incentive to create a better product. This is the standard problem with experience goods. The single-product firm selling the nondurable good does have some incentive to provide high quality in order to encourage repeat purchases. In particular, the incentive constraint for a single-product firm selling the nondurable good is the same as in equation (2).

**Bundled-Product Firm**

If the firm bundles its products the incentive constraint is very different. Consistent with the literature, we assume that bundling (or tying) prevents the consumer from deriving any positive value from consuming one of the firm’s products and one of the fringe products. If a consumer with valuation $v_d$ purchases the durable from the branded firm at time 1, he only gets the valuation $v_d$ at time $t$ if he also purchases the consumable product from the branded firm in at time $t$. The consumer is free to stop purchasing the nondurable product from the firm at any time, but if he does so he will forego the valuation of the durable good, $v_d$. So, once the consumer has purchased a high-quality durable good from the firm, the consumer’s valuation for one unit of a high-quality nondurable good is equal to his or her valuation for the two products together, $v_n + v_d$.

The bundled contract offered by the firm at time $t = 1$ includes a per-period price for the nondurable good, $p_n$, and a price $P_d = \frac{p_d}{1-\delta}$ for the durable good. Thus, a consumer pays $P_d + p_n$ for the durable good plus one unit of the nondurable good at time $t = 1$, and $p_n$ in each period for the nondurable thereafter. Importantly, this contract does not require that the consumer make future purchases of the nondurable good from the firm. If the consumer is dissatisfied with the products, the consumer can simply stop purchasing the nondurable good. For expositional ease, we will imagine for the moment that the firm can commit to charge $p_n$ in all future periods, although we will see that this is inessential for the results since, in equilibrium, the optimal nondurable price $p_n$ is incentive compatible for the firm over time.

Using the notation adopted earlier, $2p_b = p_d + p_n$, so $p_b$ is the average per unit perpetuity price of the bundle and $D^b(p_b)$ is the per period demand for the
bundle. Incentive compatibility at time $t = 1$ can be written

$$D^b(p_b) \left( \frac{p_d - c_d}{1 - \delta} \right) + D^b(p_b) \left( \frac{p_n - c_n}{1 - \delta} \right) \geq D^b(p_b) \left( \frac{p_d}{1 - \delta} \right) + D^b(p_b) \left( \frac{p_n}{1 - \delta \pi_b} \right).$$

Letting $2c_b = c_d + c_n$ and rearranging terms we can rewrite this as

$$D^b(p_b) \left( \frac{p_n - 2c_b}{1 - \delta} \right) \geq D^b(p_b) \left( \frac{p_n}{1 - \delta \pi_b} \right).$$

Note the strong similarity between (17) and equation (6), the incentive constraint for a firm that produces two nondurable goods. Indeed, if $p_n = 2p_b$ and the corresponding durable price is $p_d = 0$, then these two incentive constraints are identical! Note also that the incentive constraint at $t = 2$ and beyond is

$$D^b(p_b) \left( \frac{p_n - c_n}{1 - \delta} \right) \geq D^b(p_b) \left( \frac{p_n}{1 - \delta \pi_b} \right),$$

which is clearly satisfied if constraint (17) is satisfied. So if the firm has the incentive to produce high-quality products at $t = 1$, it will also have the incentive to continue to produce a high-quality nondurable products in every period thereafter.

We have just demonstrated an important result. By contractually tying the sale of the non-durable to the durable good, and by selling the durable good below cost and marking up the price of the nondurable good, the firm can achieve the same level of profits as in the prior setting where both products were nondurable. The branded firm can essentially leverage the incentives from the nondurable experience good to create strong incentives to produce a high-quality durable good as well. The optimal prices for the firm, and the range of parameter values for which high quality is sustainable, is exactly the same as in Proposition 2. Specifically, when $\delta \geq \delta_b$ then high quality is sustainable, and the firm will charge $P_d = 0$ for the durable good and $p_n = 2p_b(\delta)$ for the nondurable good.\(^{13}\)

\(^{13}\)One might think that the firm would be tempted to lower the price for the consumable below $2p_b(\delta)$ at time $t \geq 2$ since the firm’s cost of supplying the nondurable product is $c_n < c_b$ and the cost of producing the durable good is sunk. However, lowering the price would not generate any additional sales, since the size of the market is effectively capped at $D^b(p_b)$, the mass of consumers who purchased the durable at $t = 1$.\(^{19}\)
Finally, consider a multiproduct firm that sells the durable for \( P_d = \frac{p_d}{1 - \delta} \) at time \( t = 1 \) and the nondurable at a price \( p_n \) at \( t = 1 \) and in every period thereafter. We can again represent the firm’s per-period demand in three parts: the consumers who buy both products; \( D^m(p_d, p_n) \); and the consumers who buy only the durable good, \( d^m_d(p_d, p_n) \) and the consumers who buy only the nondurable good, \( d^m_n(p_d, p_n) \). As before, consumers who purchase one product have only one opportunity each period to observe a negative signal while consumers who purchase both products have two opportunities each period.

If the multiproduct firm sells both products, their incentive constraint at time \( t = 1 \) is

\[
(d^m_d(p_d, p_n) + D^m(p_d, p_n)) \left( \frac{p_d - c_d}{1 - \delta} \right) + (d^m_n(p_d, p_n) + D^m(p_d, p_n)) \left( \frac{p_n - c_n}{1 - \delta} \right) \\
\geq (d^m_d(p_d, p_n) + D^m(p_d, p_n)) \left( \frac{p_d}{1 - \delta} \right) \\
+ d^m_n(p_d, p_n) \left( \frac{p_n}{1 - \delta \pi_n} \right) + D^m(p_d, p_n) \left( \frac{p_n}{1 - \delta \pi_b} \right) \tag{19}
\]

The left-hand side is the present value of the profits from the sales of the durable, expressed as a stream, plus the profit from the sales of the nondurable. The right-hand side is the present value of the profits from the sales of the durable, expressed as a stream, which does not change if the firm deviates, plus the present value of the profits from the sales of the nondurable to customers who buy only the nondurable, which declines each period as consumers receive a negative signal about the quality of the nondurable good (probability \( 1 - \pi_n \)), plus the present value of the profits from the sales of the nondurable to customers who buy both products, which declines even faster because these consumers stop purchasing the nondurable after receiving a negative signal about either product (probability \( 1 - \pi_b \)).

Comparing the incentive constraint to the analogous incentive constraint in our initial model of multiproduct firms (8) establishes that incentive compatibility is significantly harder to achieve when one good is a durable good. The left-hand side of our new incentive compatibility constraint is the same as before. However, the right-hand side of our new incentive compatibility constraint is markedly different. Here, if the multiproduct firm shirks, then the consumers

\[\text{If incentive constraint was satisfied at } t = 1, \text{ then the multiproduct firm’s incentive constraint will also be satisfied for all } t \geq 2. \text{ It is also easy that if to see that the firm will charge the same price for the nondurable good as the single-product firm if the constraint doesn’t bind, and the firm will charge a price given by the incentive constraint if the constraint does bind.}\]
cannot punish the firm by ceasing to purchase the durable – the price $P_d = \frac{P_d}{1-\delta}$ paid at time $t = 1$ is sunk. So high quality is harder to sustain at $t = 1$ when one product is a durable good.\footnote{The thresholds $\delta_m$ and $\hat{\delta}_m$ would both be larger.}

The incentive problem associated with a durable experience good can be mitigated, but not eliminated, if the firm could lease the durable good to consumers. Suppose that instead of selling the product to the consumer for price $P_d$ at time $t = 1$, the monopolist could rent the product to the consumer at price $p_d$ in each period. Then, after seeing a negative signal, the consumer would rationally infer that the quality was low and would cease to rent the durable product. This would allow even the single-product firm to achieve high quality for a sufficiently high discount factor, and would allow the multiproduct firm to do just as well as it did in the earlier analysis. However, leasing the durable does not increase quality as effectively as tying the two goods.

Finally, note that our analysis in Section 3 of the paper was for symmetric distributions and costs, so formally our generalization to durable goods is valid only when $f$ is symmetric, $c_d = c_n$, and $\pi_d = \pi_n$, which may seem less natural in this context. But as we discussed in Section 3, there is no reason to think asymmetries would alter the fundamental importance of bundling as a quality assurance mechanism.

Finally note that it is easy to combine the attribution problem with the assumption that one good is an infinitely-lived durable, and show that the firm’s ability to sustain high quality remains unchanged as long as bundling is feasible. However, in the absence of bundling combining the attribution problem and allowing one good to be a durable good makes sustaining high quality even more difficult.

6 Other Extensions

In addition to analyzing durable goods and the attribution problem, there are several other variations of our model worth discussing.

One extension we could analyze, but don’t, is to introduce behavioral commitment types that always produce high quality. With such commitment types it follows that for $\delta$ close to one the unique equilibrium is the high-quality equilibrium. We have avoided analyzing this straightforward extension because we are explicitly interested in identifying the range of discount factors, $\delta$, for which bundling is necessary to sustain higher quality, and, as we have already seen, this range is bounded away from one.
Also note that while our model is a repeated moral hazard model, all of the results hold if there is just a single effort choice in the first period. This is relevant because some applications fit this assumption better. For example products may benefit from investment in design but then changes in design might be observable. So for example, the gas mileage and safety of a car may be determined mainly by ex ante investments.

We look at two issues in more detail. First, we explore a variant of our model with homogeneous consumers. While think consumer heterogeneity is an important dimension of the problem we study, the issues we study arise even if consumers are homogeneous. That is, free riding is a potential problem even when consumers are homogeneous.

Second, we look at ways to relax the assumption that the negative signal is perfectly informative. This is clearly the strongest assumption we make and it is important to understand how one might relax it.

6.1 Homogeneous Consumers

Our model emphasizes the free-riding problem that arises when consumers have heterogeneous preferences, but even if consumers are homogeneous they may still underinvest in monitoring. As long as the consumer is small, he or she does not internalize the impact of their monitoring on product quality. As a consequence bundling can still benefit the firm by enforcing a consumption level that consumers would collectively like to commit to ex ante. While intuitive, this isn’t immediately obvious in our model because we assumed that the products were perfect complements. If instead the products were substitutes (or even imperfect complements), then consumers could be better off committing to by both products ex ante, but not be willing to do so ex post. So high quality might be feasible only with a bundled (or volume) discount.

To illustrate this, suppose consumers are homogeneous with valuation $v$ for one unit of the high-quality product and valuation $(1 + \gamma)v$ for two units of the high-quality product where $\gamma < 1$. That is, consuming both goods yields two signals, but if one good is high quality the value of the other being high quality is diminished. Without bundling the highest price the firm can charge and sell both goods is $p = \gamma v$, and at this price it can sustain high quality if

$$\left(\frac{\gamma v - c}{1 - \delta}\right) + \left(\frac{\gamma v - c}{1 - \delta}\right) \geq \left(\frac{2\gamma v}{1 - \delta\pi_b}\right).$$

(20)

However with bundling the highest price firm can charge for the bundle is $p =$
(1 + γ)v, and at this price it can sustain high quality if
\[
\left(\frac{(1 + γ)v/2 - c}{1 - δ}\right) + \left(\frac{(1 + γ)v/2 - c}{1 - δ}\right) \geq \left(\frac{(1 + γ)v}{1 - δπ_b}\right).
\]
which is clearly satisfied for a wide range of δ.

We explore this volume discount interpretation of product bundling more extensively in our companion paper, Dana and Spier (2014).

6.2 A Model with Both Public and Private Signals

One characteristic of our model that is restrictive is the assumption that a bad signal is perfectly informative, that is, that the probability of a negative signal conditional on high quality is zero. While this simplifies our analysis, and it isn’t trivial to relax this assumption, we believe our insights are robust.

First, note that models like Cai and Obara (2006) and Green and Porter (1984) relax this assumption by assuming public signals and having punishment occur on the equilibrium path. However our main insight is that private signals introduce free riding, and with private signals we do not have the coordination device that these models use to start a punishment period, and without such coordination the firm’s quality decision does not change following just one negative signal so consumers won’t punish following a bad signal.

One way to resolve this is to extend the model by introducing a public signal. Suppose that the probability of receiving a negative private signal is 1 − πh and 1 − πl when quality is high and when quality is low respectively, and similarly 1 − πb and 1 − πl are the private negative signal rates when two goods are purchased from the firm. On the equilibrium path, if quality is high, consumers ignore their negative private signals. However, suppose that a negative public signal occurs each period as well. Specifically, suppose one consumer is chosen at random and their private signal is revealed to everyone.

Following Cai and Obara (2006), we now consider a public trigger strategy equilibrium in which the firm produces high quality products until a negative signal is observed, and then following the public signal consumers stop buying from the firm and the firm only produces low quality. We will focus on an equilibrium in which consumers stop purchasing from the firm if it receives any negative signal, though it is worth noting that profits would be even higher in an equilibrium with finite length punishment periods (like Green and Porter, 1984).

Regardless of the market structure demand is a function only of the public signal so we can write the firm’s (or firms’) incentive compatibility constraints as
\[
\frac{p_i - c}{1 - δμ_h} \geq \frac{p_i}{1 - δμ_l}.
\]

(22)
where $1 - \mu^h$ is the chance of a negative public signal given high effort and $1 - \mu^l$ is the probability of a negative public signal given low effort. Because negative public signals can occur on the equilibrium path it no longer follows that high quality is feasible for sufficiently high $\delta$. In particular, not that if high quality is feasible, it must be feasible when $p = \bar{v}$ and $\delta = 1$, so take those as given we can write a sufficient condition for high quality to be feasible as

$$\frac{1 - \mu^h}{1 - \mu^l} \geq \frac{\bar{v} - c}{\bar{v}}. \quad (23)$$

That is, the probability of a negative public signal given high effort must be sufficiently small relative to the probability of a negative public signal given low effort. If this condition holds, then high quality is sustainable for all $\delta \geq \delta$ where

$$\delta = \frac{c}{\mu^l c + (\mu^h - \mu^l)\bar{v}}. \quad (24)$$

Now suppose the public signals are a function of the private signals. In particular, the probability of a negative public signal is increasing in the average number of negative private signals and most importantly that the ratio $\frac{1 - \mu^h}{1 - \mu^l}$ is decreasing in the number of consumers buying both products from the firm.

To see that bundling can still increase profits and consumer surplus, consider a simple example. Imagine that the public signal is simply equal to the private signal of one randomly chosen consumer. Then in the single product environment the public signal is just the private signal of one customer who purchases from the firm only if his or her valuation exceeds the price which is some probability $\rho_1$ where 1 denotes 1 product. So $1 - \mu^h = \rho_1 (1 - \pi^h)$ and $1 - \mu^l = \rho_1 (1 - \pi^l)$.

Now suppose that

$$\frac{1 - \pi^h}{1 - \pi^l} \geq \frac{\bar{v} - c}{\bar{v}}. \quad (25)$$

where the $\rho$'s cancel, so high quality is not sustainable with single product firms.

This also implies that high quality is not sustainable with multiproduct firms if the public signal is a random consumer’s private signal about just one product. But let’s instead define the public signal as a random consumer’s experience with two products. So a random consumer is chosen and if the consumers buys 0 or 1 products, or gets a positive signal about at least one product, not public signal is generated, but if the consumer buys two products and gets a negative private signal about both products, then the negative public signal is generated. So $1 - \mu^h = \rho_2 (1 - \pi^h)^2$ and $1 - \mu^l = \rho_2 (1 - \pi^l)^2$. Since clearly

$$\frac{1 - \pi^h}{1 - \pi^l} > \frac{(1 - \pi^h)^2}{(1 - \pi^l)^2} \quad (26)$$
it follows that if
\[ \frac{\bar{v} - c}{\bar{v}} \in \left( \frac{1 - \pi^h}{1 - \pi^l}, \left(\frac{1 - \pi^h}{1 - \pi^l}\right)^2 \right) \] (27)

Then high quality is sustainable under bundling, but not in the single product environment. More importantly, the chance of randomly choosing a consumer who buys two products is much higher under bundling, \( \rho^m_2 < \rho^b_2 \), so from equation (24) it is clear that \( \delta^b < \delta^m \), so bundling supports high quality for a broader range of discount factors.

Another approach to relaxing the assumption that negative signals perfectly reveal low effort is to introduce a bad commitment type. In this case, negative signals would be attributed mainly to the bad commitment type and so the firm would be punished on the equilibrium path.

Yet another approach would be to suppose effort is chosen at the start of the game as opposed to each period mixed strategy equilibrium were the firm mixed between high and low effort. In this case, it is rational for consumers to punish low outcomes because the rationally associated it with low effort.

7 Conclusion

We extend Klein and Leffler’s (1981) repeated moral hazard model of product quality to the case of multiproduct firms and imperfect monitoring. In our model, consumers are small, heterogeneous, and receive imperfect private signals about the quality of their past purchases. Through bundling, the multiproduct firm constrains consumers to gather better information about the firm’s product quality decisions. Using this information, consumers are collectively better able to punish the firm when it sells low-quality products. This in turn gives the firm a stronger incentive to only produce high-quality products. Intuitively, the multiproduct firm will rationally adopt a bundling strategy only when bundling enables the firm to offer high quality and the resulting loss of market share is offset by the higher margins that high-quality products command.

The private and social benefits of bundling are even higher when consumers are unable to attribute a negative result to the particular product that caused it. In the absence of product bundling, many consumers purchase only one product from the multiproduct firm and don’t hold that firm to blame when they have a bad experience. As a consequence of these free riders, and the associated attribution problem, shirking becomes even more attractive to the multiproduct firm. With bundling, although consumers cannot directly verify which product produced a negative signal, they can nevertheless easily deduce that the branded firm is to blame.
The model and the results rely on several key assumptions. First, we assumed that consumers were small and had heterogeneous preferences. If there were a single consumer, that consumer would internalize the impact of his or her behavior on the firm’s quality choice and bundling would be unnecessary. Second, we assumed that consumers received imperfect private signals of product quality. If consumers received perfect private signals instead, then the quality of monitoring would not depend on whether or not the consumer purchased both products.\(^{16}\) Finally, if the signals were publicly observed instead of privately observed, then the consumers’ information would be independent of their private purchase decisions and bundling would be ineffective.

While our model focused on the private and social desirability of pure bundling, the framework could be extended to consider the role of mixed bundling. Recall that quality-enhancing benefit of pure bundling in our model came at a cost. With bundling, consumers who placed high value on one branded product were inefficiently priced out of the market. The branded firm can potentially mitigate this cost through a mixed-bundling strategy, offering a limited number of individual units in addition to the bundled units. By rationing individual unit sales, the branded firm can keep the proportion of consumers buying multiple units just high enough to satisfy its incentive constraint. More generally, with a continuum of consumer types, the firm might offer a pure bundle discount in order to increase the proportion of consumers who purchase multiple products.\(^{17}\)

Our model had that feature that, absent quality assurance concerns, the branded firm would never find it profitable to bundle its products together. Bundling is only privately and socially attractive when the branded firm cannot otherwise maintain a reputation for high quality. In reality, there are other benefits that that the branded firm can derive by bundling, such as price discrimination and market foreclosure. In a more general model, bundling might increase product quality but also deter entry by potential competitors.\(^{18}\) The social costs of these anticompetitive effects would need to be weighed against the social benefits of quality improvements. Theoretical and empirical research combining these effects may be fruitful avenues for future research.\(^{19}\)

\(^{16}\)The conditions for sustaining high quality in equilibrium would be the same for a single-product firm, a multiproduct firm, and a multiproduct firm that bundles.

\(^{17}\)However in a model with such heterogeneity, the firm would also be able to use bundling to price discriminate, making the analysis much more complicated.

\(^{18}\)More generally, one could explore how competition among branded firms changes the analysis. Intuitively, the mix-and-match behavior we analyze here should occur more easily when consumers are choosing among multiple high quality firms, however the analysis of the incentive constraints, particularly with the attribution problem, would be considerably more complicated.

\(^{19}\)One implication is that empirically demonstrating bundling improves quality would be suggestive, but not conclusive, evidence the bundling is efficient.
References


8 Appendix

In this appendix we show that it is sufficient to check that the firm does not want to deviate in the quality of both of its products simultaneously and in perpetuity. Specifically, we show that if any deviation is profitable, then deviating in the quality of both products forever is also profitable. So if deviating in both is not profitable, deviating in just one product is not profitable.

We begin by analyzing the incentive constraints for the bundling firm (equation 6). Consider a high-quality equilibrium. The firm prefers to deviate and produce one low-quality product when

\[
\frac{2p_b - 2c}{1 - \delta} < \frac{2p_b - c}{1 - \delta \pi}
\]

Multiplying both sides by \(1 - \delta \pi\), subtracting and adding \(c\) from the right hand side, and rearranging terms gives

\[
c > \left[\frac{1 - \delta \pi}{1 - \delta} - 1\right] (2p_b - 2c) = \delta (1 - \pi) \frac{2p_b - 2c}{1 - \delta} \tag{28}
\]

The left-hand side is the cost savings from deviating and producing one low-quality product. The right-hand side is the punishment from the one-time deviation. With probability \(1 - \pi\) the consumer detects the deviation and stop buying from the firm, in which case the firm loses the perpetuity of profit margins \(2p_b - c\).

The firm prefers to deviate and produce two low quality products in perpetuity if

\[
\frac{2p_b - 2c}{1 - \delta} < \frac{2p_b}{1 - \delta \pi_b}
\]

Multiplying both sides by \((1 - \delta \pi_b)/2\), subtracting and adding \(c\) from the right-hand side, and rearranging terms gives

\[
c > \delta (1 - \pi_b) \frac{p_b - c}{1 - \delta} \tag{29}
\]

The left-hand side is the one-time, per unit benefit from deviating (the unit cost savings). The right-hand side is the punishment from a one-time deviation.

The right-hand side of (29) is smaller than the right-hand side of (28) if and only if \(1 - \pi_b < 2(1 - \pi)\). This is clearly true when the signals are uncorrelated \((\pi_b = \pi^2)\) and when they are uncorrelated \((\pi_b = \pi)\). More generally, it holds for \(\pi_b > 2\pi - 1\) which is true by our earlier assumption. So if the firm finds it profitable to deviate and produce one low-quality product, it is also profitable to deviate by producing two low-quality products.
Now we turn to the multiproduct firm. Because \( f(v_1, v_2) \) is symmetric, the optimal prices for the multiproduct firm satisfy \( p_1 = p_2 \). To simplify notation, let \( p = p_1 = p_2 \) be the prices and let \( d^m = d^m_1(p, p) = d^m_2(p, p) \) and let \( D^m = D^m_1(p, p) \).

Consider a high-quality equilibrium where the firm sells volume \( d^m + D^m \) of each of the two products. First, consider a deviation where the firm lowers the quality of product 2 alone in perpetuity. This deviation is profitable when

\[
(2d^m + 2D^m)\frac{p - c}{1 - \delta} < d^m\frac{p - c}{1 - \delta} + d^m\frac{p}{1 - \delta\pi} + D^m\frac{2p - c}{1 - \delta\pi} \tag{30}
\]

The first term on the right-hand side is the profit stream from the consumers who purchase just the high-quality product (product 1) in each period. These consumers never exit the market. The second term is the profit stream from consumers who purchase just the low-quality product (product 2) in each period. These consumers exit at a rate \( 1 - \pi \) each period.

The third term is the profit stream from consumers who purchase both products, one high-quality and one low-quality product. Subtracting \( d^m \left( \frac{p - c}{1 - \delta} \right) \) from both sides and subtracting and adding \( \frac{d^m + D^m}{1 - \delta\pi} c \) to the right hand side, and combining terms this becomes

\[
(d^m + 2D^m)\frac{p - c}{1 - \delta} < (d^m + 2D^m)\frac{p - c}{1 - \delta\pi} + (d^m + D^m)\frac{c}{1 - \delta\pi}. \tag{31}
\]

Rearranging terms and isolating \( c \) on one side gives

\[
c > \delta(1 - \pi)\left(\frac{d^m + 2D^m}{d^m + D^m}\right)\frac{p - c}{1 - \delta}. \tag{32}
\]

We will now show that if (32) holds, then the firm would want to deviate and produce two low-quality products in a one-time deviation. A one-time deviation is profitable for the multiproduct firm when

\[
(2d^m + 2D^m)\frac{p - c}{1 - \delta} < (2d^m + 2D^m)p + 2d^m\pi\delta\frac{p - c}{1 - \delta} + 2D^m\pi\delta\frac{p - c}{1 - \delta}. \tag{33}
\]

This may be rewritten as

\[
(d^m + D^m)\frac{p - c}{1 - \delta} < (d^m + D^m)(1 - \delta)\frac{p - c}{1 - \delta} + \delta(\pi d^m + \pi_b D^m)\frac{p - c}{1 - \delta} + (d^m + D^m)c. \tag{34}
\]

Collecting the terms that include \( p - c \) on the left hand side gives us

\[
[\delta(d^m + D^m) - \delta(\pi d^m + \pi_b D^m)]\frac{p - c}{1 - \delta} < (d^m + D^m)c. \tag{35}
\]
Rearranging terms, this becomes

\[ c > \delta \left( 1 - \frac{\pi d^m + \pi_b D^m}{d^m + D^m} \right) \frac{p - c}{1 - \delta}. \]  

(36)

The right-hand side of (36) is larger than the right-hand side of (32). So if the firm benefits from deviating with low quality for one product, it will also be profitable to deviate with low quality for two products.