Crowded Trades: An Overlooked Systemic Risk for Central Clearing Counterparties

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Abstract

Counterparty risk might hamper trade and worsen a financial crisis. A central clearing counterparty (CCP) insures traders against counterparty default and thus benefits trade. Default of the CCP itself however introduces a new systemic risk. Standard CCP risk management currently overlooks the risk associated with crowded trades. This paper identifies such risk, measures it, and proposes a margin methodology that accounts for it. An application to CCP data illustrates that for the two peak margin days this hidden risk is about a third of total risk.
1 Introduction

The 2007-2008 financial crisis and the more recent European debt crisis renewed the debate on how to best measure and regulate systemic risk (see, e.g., Bisias et al., 2012; Brunnermeier and Oehmke, 2013). Regulators advocate a central clearing counterparty (CCP) to mitigate the risk of market failure (see, e.g., Dodd-Frank and EMIR). A market might fail when agents are reluctant to trade as they fear counterparty default. The introduction of a CCP could solve such trade deadlock as the CCP effectively becomes the counterparty to each trade.

A CCP might reduce counterparty risk but it does not eliminate it. In fact, counterparty exposures concentrate in the CCP. Counterparty default becomes systemic in nature as CCP default affects all clearing members at the same time. Regulators are aware as, for example, Bernanke (2011) emphasized that financial stability strongly depends on resiliency of a CCP. The ESRB (2012, p. 16) writes: “Mandatory clearing will turn CCPs into systemic nodes in the financial system.”

Risk management at the level of CCPs becomes a first-order concern. The Bank for International Settlements (BIS) and the International Organization of Securities Commissions (IOSCO) cooperated to develop recommendations for CCPs as a standalone entity (BIS-IOSCO, 2004) and as a “financial market infrastructure (BIS-IOSCO, 2012).” They recommend that a CCP measures its credit exposures to participants and limits them through margin requirements or other control mechanisms. The standard approach is to set initial margins on a member by member basis. This margin scales with the “product” of the net positions in a member’s trade portfolio and the volatility of returns on these net positions, appropriately accounting for correlations between them (see, e.g., Hedegaard, 2012). This approach captures the tail risk in a member’s portfolio return. “Return” here is expressed in absolute terms, i.e., in currency units (e.g., $ or €) as opposed to percentages.

1 Examples of CCP failures are Caisse de Liquidation, Paris (1974), the Kuala Lumpur Commodity Clearing House (1983), and the Hong Kong Futures Guarantee Corporation (1987) (see Hills et al., 1999). Boissel et al. (2014) find that during the 2008-2011 European sovereign debt crisis the repo market behaved as if CCP default was imminent.
Margins therefore scale with how quickly losses can develop in a member’s portfolio until the next time the CCP visits to collect margin, typically a day later. An example of such approach is SPAN, the margin methodology developed by the Chicago Mercantile Exchange (CME) and implemented by 54 exchanges and clearing houses around the world.

This paper identifies and studies crowded trades as a risk to CCPs that is overlooked in current member-by-member margin methodologies. It explores the intuition that losses in members’ portfolios become more correlated when their trades crowd on a single security or risk factor. If this factor experiences a large shock in value then multiple members experience large variation margin calls simultaneously. If members are unable to post new collateral to fulfill these calls, then a CCP might have to cover losses not on a single portfolio, but on many portfolios at the same time.

The paper develops four sets of results based on CCP “aggregate exposure,” the CCP risk measure proposed by Duffie and Zhu (2011). Aggregate exposure is defined as the sum of losses in members’ portfolios, profits excluded (as the sum would otherwise be zero as for every buyer there is a seller). Losses are what the CCP is effectively exposed to when it inherits trade portfolios from members in default. For (conditionally) normal return distributions, analytical expressions are derived for the mean and the standard deviation of aggregate exposure. The derivation is based on absolute moments for the normal distribution (Nabeya, 1951) and moments for the truncated normal (Rosenbaum, 1961). The expression for standard deviation is a contribution; the expression for mean is taken from of Duffie and Zhu (2011).

The first result is that CCP aggregate exposure becomes more volatile when its members’ trades crowd, all else equal. The right tail of its distribution becomes “fatter,” i.e., extreme losses become

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2I focus on initial margins throughout and will refer to them as “margin” for brevity. It is not to be confused with “variation margin,” which is the collateral collected or returned based on mark-to-market changes in member portfolio values. For example, suppose member 1 is long $1 worth of index futures and member 2 is short. Consider a CCP that collects margin at the end of the day. Let daily index returns have mean zero and a standard deviation of 5%. The CCP collects say 7 times standard deviation as initial margin. Then both member 1 and 2 have to post $0.35 worth of collateral with the CCP. Suppose the return realization was +8% (and the distribution remained unchanged). The new initial margin for both members then becomes 7*0.05*1.08=$0.38. The variation margin call is $0.08 for member 1 and $-0.08 for member 2.
more likely. The all-else-equal clause implies that risk remains unchanged in each member’s individual portfolio. The margins required based on standard margin methodologies therefore remain unchanged. Crowded trade risk—shortened to crowded risk—turns out to be hidden risk for CCPs.

Second, the extent to which member trades crowd can be computed as an index, CrowdIx. This index is constructed as the ratio of two volatilities. The numerator is the volatility of actual aggregate exposure. The denominator is the “maximized” volatility obtained after re-allocating member trades to a single risk factor while keeping their individual portfolio risk fixed. Each member becomes a buyer or a seller of a single risk factor (to the maximum extent possible). A key strength of the CrowdIx measure is that it does not require specifying any risk factor a priori. CrowdIx computation only requires the covariance matrix of member portfolio returns (“P&Ls”).

Third, an alternative margin methodology is proposed to account for crowded risk: Margin(A). It extends standard practice whereby tail risk is computed as the “delta-normal” Value-at-Risk (VaR). This VaR is simply the mean plus alpha times the standard deviation (Jorion, 2007). For example, the CCP who provided the data for this study calculates margins this way with alpha equal to seven. The proposed new margin methodology follows this practice by computing Margin(A) simply as the VaR of aggregate exposure (i.e., aggregate loss). It has several appealing features.

1. Homogeneity of degree one in member portfolio risk implies that Margin(A) decomposes naturally across members. Each member is margined according to the “shadow cost” of its portfolio in terms of (aggregate) CCP risk. The polluter pays, i.e., those who joined the crowded trade pay for the elevated CCP risk caused by it.

2. Computing margins is trivial as all expressions are analytical. No heavy-duty simulations are required. This is particularly attractive for CCPs that intend to collect margins intradaily, have many members, and clear many securities.

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3 The single risk factor could be a security’s return, a particular portfolio of securities, or the market portfolio. All yield the same value of CrowdIx. Note that the textbook case of all investors trading the market portfolio corresponds to maximum crowded risk for a CCP.
3. The risk factor that is the subject of the crowded trade can be tracked down methodically based on another set of analytic results.

4. The VaR approach extends standard practice and is therefore more likely to be widely accepted. Many clearing houses currently base their margin calculation on the “delta-normal VaR” of a member’s trade portfolio (Lam, Sin, and Leung, 2004, Table 1).

Fourth, real-world data are used to illustrate that crowded risk matters. Proprietary CCP data was obtained for equity trades in Denmark, Finland, and Sweden. The sample runs from October 19, 2009 through September 9, 2010. The actual margins that the CCP imposed are compared to the margin implied by Margin(A), both through time and in the cross-section (across clearing members). The actual daily aggregate margin is mostly one, two, or at most three hundred million euro, except for two peaks of half a billion euro. CrowdIx reaches its highest levels exactly on these two days. Appropriately accounting for crowded risk therefore would require margins that are at least two hundred million euro higher. In the cross-section, the allocation of aggregate margin across members also differs substantially between the two methods. For example, on one of these peak days (April 23, 2010) member 12 was charged more than €60 million whereas the alternative measure would have charged less than €10 million. Member 41 on the other hand was charged about €40 million but should have been charged more than double that amount under Margin(A). Inspection of their portfolios reveals that the latter member joined a crowded trade whereas the former member did not.

The paper contributes to several literatures. One literature studies ways to gauge how systemically risky a financial institution is (reviewed in, for example, Bisias et al., 2012). One approach is to analyze how its market value co-varies with total market value in the left tail (e.g., CoVaR or Systemic Expected Shortfall proposed by Adrian and Brunnermeier (2011) and Acharya et al. 4

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4Counterparty risk exists for equity trades as the actual transfer of securities and money takes place at the settlement date, typically a couple of days after the trade was concluded. Equity trades are therefore like short-term forward contracts.
An alternative approach is to collect financial institutions’ exposures to a set of risk factors and identify risk through scenario analysis (e.g., Duffie, 2012; Brunnermeier, Gorton, and Krishnamurty, 2013). This would be a natural task for the newly established Office of Financial Research. In fact, such exposure information could be used to calculate CrowdIx and Margin(A) to gauge the risk of an extreme “aggregate loss” in the financial system, and which institutions contribute most to it.

The paper further contributes to a literature on how to allocate total systemic risk appropriately across all that contribute to it. Brunnermeier and Oehmke (2013) review various allocation rules. The proportional rule allocates risk according to an institution’s individual risk divided by the sum of all institutions’ individual risk (Urban et al., 2003), ignoring correlations across institutions. Standard margin methodologies allocate risk this way. The with-and-without rule allocates it based on the difference in systemic risk when an institution is included or excluded (Merton and Perold, 1993). The Euler-or-gradient rule is the marginal version of this rule (Patrik, Bernegger, and Rüegg, 1999; Tasche, 2000). Finally, the Shapley rule allocates according to Shapley value, a concept from cooperative game theory (Tsanakas, 2009; Tarashev, Borio, and Tsatsaronis, 2009).

Brunnermeier and Oehmke (2013, p. 62-63) characterize a good allocation rule as follows:

“Ideally, the allocation should be such that (i) the sum of all risk contributions equals the total systemic risk and (ii) each risk contribution incentivizes financial institutions to (marginally) take on the appropriate amount of systemic risk. However, capturing both total and marginal risk contributions in one measure is a challenging task, because the relationship between the two may be non-linear. In fact, the marginal contribution of one institution may depend on the risks taken by other institutions.”

Margin(A) is an Euler-or-gradient rule that meets this characterization. In particular, an institution’s margin does depend on the risks that others take as it accounts for crowded trades. Admittedly, Margin(A) only applies to plain vanilla linear products such as equity, forwards, or futures.
Non-linear derivatives require linearization before Margin(A) can be applied. An alternative non-linear model-based approach is suggested by Brunnermeier and Cheridito (2013).

A related contemporary paper is Cruz Lopez et al. (2014). The authors propose “CoMargin” as an alternative margin methodology that accounts for correlations across members’ portfolios. It sets margin for a particular member at a level high enough to withstand losses conditional on one or two other members being in financial distress. It is based on CoVaR that was introduced by Adrian and Brunnermeier (2011). The relative merit of Margin(A) over CoMargin is that it avoids conditioning and therefore any discussion on who the appropriate clearing members are to condition on. No clearing member would volunteer to be selected as, by construction, his portfolio would correlate highly (perfectly in the one-member CoMargin case) with the benchmark portfolio. He would therefore be charged the most. And, one cannot take the aggregate portfolio across all clearing members as benchmark portfolio as this portfolio is zero by definition (for every clearing member buying there is one selling).

Finally, a couple of issues merit discussion. First off, the analytic expressions for the mean and standard deviation of aggregate exposure critically depend on returns being normally distributed. That said, the approach does account for time-varying volatility and correlations as tomorrow’s return distribution depends on the commonly used RiskMetrics covariance matrix, i.e., an exponentially weighted moving average of the outer product of historical daily returns. Technically, RiskMetrics is a restricted case of GARCH(1,1).5

Second, aggregate exposure itself is not normally distributed in spite of member portfolio values being normally distributed. The actual VaR confidence level implied by the mean plus alpha times standard deviation approach can only be computed through simulations. One can however still rely on Chebyshev’s inequality to obtain a lower bound for the implied VaR level (see also footnote 9).

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5It is well-known that conditionally normal returns in GARCH can cause fat tails in the unconditional distribution. Several empirical studies find that this explains some of the excess kurtosis in returns, but not all of it (Bollerslev, Chou, and Kroner, 1992, p. 19-20).
Third, I presented this paper’s findings at several CCPs and learned about natural applications of CrowdIx and Margin(A). CrowdIx, for example, is a natural statistic to include in the periodic risk management reports that a CCP sends to its clearing members. Margin(A) is viewed as a useful measure of how much each member contributes to aggregate risk. But, replacing the existing margin methodology by Margin(A) is likely to affect their members’ trading behavior in non-trivial ways. In particular, it introduces additional uncertainty in the trading process as it is hard for traders to predict what the crowded trade will be and future margin requirements on current trades therefore become more uncertain (in addition to the volatility uncertainty that is already there in current margin methodologies). Endogenizing trade decisions is beyond the scope of this paper and left for future work (e.g., Menkveld, 2014). One CCP considered Margin(A) useful to inform low-frequency decisions on how much each member should contribute to the default fund.

The remainder of the manuscript is organized as follows. Section 2 presents the model, identifies “crowded risk,” develops the crowded-trade risk index, CrowdIx, and the alternative margin methodology, Margin(A). Section 3 uses CCP data to illustrate the use of CrowdIx and Margin(A). Section 4 concludes.

2 Simple model to capture crowded risk

2.1 Model description

Central clearing counterparty (CCP) risk is analyzed in the framework proposed by Duffie and Zhu (2011). Let a period be defined by the frequency at which a CCP collects margin. For example, a period will be one day for our data sample. Consider a set of $I$ securities with one-period returns that are normally distributed:

$$R \sim N(0, \Omega).$$

(1)

Let the column vector $n_j$ be the start-of-period yet-to-settle trade portfolio of member $j$, where
\( j \in \{1, \ldots, J\} \). This is the portfolio that the CCP effectively insures against counterparty default. Once settlement has taken place for a particular trade, it will be removed from the trade portfolio as it no longer requires insurance. \( n_j \) will be referred to as trade portfolio or portfolio in the remainder of the manuscript. Mathematically, the aggregate exposure \( A \) associated with the trade portfolios of all member firms is

\[
A = \sum_{j=1}^{J} E_j, \quad (2)
\]

where \( E_j \) captures loss (as a positive number) in member \( j \)'s portfolio

\[
E_j = -\min(X_j, 0), \quad (3)
\]

where \( X_j \) is the one-period return (or profit and loss, “P&L”) on member \( j \)'s yet-to-settle trade portfolio

\[
X_j = n_j'R. \quad (4)
\]

Now collect all members’ P&Ls in column vector \( X \) and one obtains

\[
X \sim N(0, \Sigma), \quad \Sigma = N'\Omega N, \quad (5)
\]

where \( N \) is a \( I \times J \) matrix with member \( j \)'s trade portfolio \( n_j \) as column vector \( j \). The covariance matrix \( \Sigma \) of all members’ portfolio returns is the key object of all subsequent crowded risk analysis.

The above structure was proposed by Duffie and Zhu (2011) in their analysis of CCP risk. They argue that aggregate exposure \( A \) captures CCP risk (p. 78):

“For given collateralization standards, the risk of loss caused by a counterparty default is typically increasing in average expected exposure.”

We extend the Duffie and Zhu (2011) analysis by focusing on CCP tail risk that, in addition to the mean of aggregate exposure, also requires computing its standard deviation. The following results will turn out to be useful.
Aggregate-exposure mean. Duffie and Zhu (2011) find that aggregate-exposure mean is

\[ E(A) = \sum_{j=1}^{J} \sqrt{\frac{1}{2\pi} \sigma_j}, \]  

(6)

where \( \sigma_j \) is the standard deviation of member \( j \)'s portfolio return \( (n'jR) \), i.e., the square root of \( \sigma_{jj} \) in \( \Sigma \) of eqn. (5).

Aggregate-exposure standard deviation. The aggregate exposure standard deviation is calculated based on two sets of results: absolute moments of a bivariate normal distribution (Nabeya, 1951) and moments of a truncated bivariate normal distribution (Rosenbaum, 1961). All derivations are in Appendix A.

The aggregate-exposure standard deviation is

\[ \text{std}(A) = \sqrt{\sum_{k,l} \left( \frac{\pi - 1}{2\pi} \right) \sigma_k \sigma_l M(\rho_{kl})}, \]  

(7)

where \( \rho_{kl} \) is the correlation between the portfolio return of members \( k \) and \( l \) (also retrieved from \( \Sigma \) in eqn. (5)) and

\[ M(\rho) = \left[ \frac{\frac{1}{2} \pi + \arcsin (\rho)}{\pi - 1} \right] \rho + \sqrt{1 - \rho^2 - 1}. \]  

(8)

Note that \( M(-1) = -1/(\pi - 1) \), \( M(0) = 0 \), and \( M(1) = 1 \).

The \( M(.) \) function is the engine of all crowded-risk analysis. It maps correlations in portfolio returns into correlations in portfolio exposures. Figure 2 plots the function and illustrates that it leaves positive correlations almost untouched (as it is close to a 45 degree line), but reduces the size of negative correlations. The intuition is that, unlike aggregate return, aggregate exposure cannot benefit from a profit in one account to compensate for the loss in another. The clearing house cannot make up for the portfolio loss of members in default by collecting from its non-defaulted
members who (by the zero-sum nature of trading) enjoy a portfolio profit. In some sense, negative correlations become less useful. Positive correlations do not die as a loss in one account becomes more likely when there is a loss in another account. In essence, the CCP insures “left-tail draws” from the portfolio profit distribution\(^6\) but does not benefit in the cross-section from offsetting right-tail draws. It does suffer from other left-tail draws.

Finally, note that aggregate exposure is defined based on the start-of-period trade portfolio. It therefore does not account for likely changes in trade portfolios throughout the trading period. This intra-period portfolio change risk is potentially important but out of scope here (and left for future work). The focus is on how to improve on existing margin methodologies (that also ignore intra-period trades) by recognizing the systemic risk caused by crowded trades.

### 2.2 CrowdIx: A measure of crowded-trade risk

**Definition 1** CrowdIx for \(\Sigma\) (the covariance matrix of one-period returns on the \(J\) members’ portfolios) is defined as

\[
\text{CrowdIx} = \frac{\text{std}(A)}{\text{std}(\tilde{A})},
\]

\[(9)\]

where \(\text{std}(\tilde{A})\) is the standard deviation of aggregate exposure when all members’ trades are re-allocated to a single risk factor, leaving individual members’ portfolio risk unchanged. The first-fit-descending binning algorithm is used to implement the re-allocation.

CrowdIx compares the standard deviation of aggregate exposure based on actual trade portfolios, to the standard deviation for a benchmark of re-allocating member trades to a single risk factor, any factor will do (see also footnote 3). First off, note that CrowdIx is based on standard deviation only, as the aggregate-exposure mean does not change when only re-allocating trades. More specifically, the benchmark portfolio is constructed such that each member’s portfolio risk remains unchanged.

\(^6\)Note that these correspond to right-tail draws from the portfolio loss distribution.
Consider for example the case of 4 clearing members, $J = 4$. All trades are equal in size and risk factors are independent identically distributed random variables. Suppose member 1 bought risk factor 1 from member 2, and member 3 bought risk factor 2 from member 4. The benchmark portfolio is easily created by re-allocating the latter trade to risk factor 1. Such perfect re-allocation is not possible in general. Suppose that member 1 bought risk factor 1 from member 2, member 2 bought risk factor 2 from member 3, and member 3 bought risk factor 3 from member 1. In this case it is impossible to re-allocate all trades to a single risk factor under the condition that all members’ individual portfolio risk is unchanged.

The general approach is to re-allocate all trades to a single risk factor to the maximum extent possible. Actual maximization is a non-trivial problem as the set of possible re-allocations is simply too big. It consists of $3^J$ possibilities as each member becomes a buyer or seller of the single risk factor or lands in a “remainder bin.” In our data, for example, the size of this set would be $3^{55} \approx 1.7 \times 10^{26}$. The approach taken here is to revert to the first-fit-descending (FFD) binning algorithm (Coffman, Garey, and Johnson, 1996). The optimization becomes $O(J \log J)$ instead of $O(3^J)$, and therefore manageable. Members are sorted from largest to smallest portfolio risk. They are then sequentially assigned to be either buyer, seller, or land in a residual category. The objective is to maximize the total amount transacted in this risk factor. The residual category is ignored. It is expected to be small given a highly skewed distribution in member portfolio risk due to lots of small-risk portfolios. A formal treatment of the optimization associated with construction of the benchmark portfolio and a detailed description of the FFD algorithm to solve it is in Appendix B.

**Lemma 1** CrowdIx has a lower bound:

$$CrowdIx \geq \sqrt[3]{\frac{1}{\left\lfloor \frac{J}{2} \right\rfloor}},$$

(10)

where the function $\lfloor x \rfloor$ finds the largest integer smaller than or equal to $x$.

A detailed proof is in Appendix B.
The minimum is attained when all members’ portfolios are equally risky and portfolio risk is spread perfectly across orthogonal risk factors. In this case, none of the off-diagonal elements in the portfolio risk covariance matrix $\Sigma$ is positive. There simply is no positive correlation across clearing members’ P&Ls in such situation.

If one is able to perfectly re-allocate all trades to a single risk factor then the sum across off-diagonals is unaffected (as aggregate profit remains zero). It effectively adds a mean-preserving spread to the off-diagonals. This raises the aggregate-exposure standard deviation as the $M(\cdot)$ function is convex (see eqn. (A7) in Appendix A) and Jensen’s inequality applies. In other words, the additional positive correlations in member portfolio returns are left untouched when converting them to exposure correlations as $M(1) = 1$, whereas the additional negative correlations shrink as $M(-1) = -1/(\pi - 1)$.$^7$

Note that the lower bound of CrowdIx decreases in the number of clearing members, $J$. This is not surprising given that adding clearing members creates more opportunities for spreading aggregate risk across members, and for creating diversity of trade.

Finally, strictly speaking, CrowdIx cannot be bounded from above as it might exceed one in pathological cases. The reason is that perfect re-allocutation might not be feasible. The earlier case of three trades among clearing members serves as an example. In practice, the FFD procedure seems to always yield more crowded benchmark portfolios. In our data sample CrowdIx stayed far below one. Its maximum was 0.83 (see Section 3).

**CrowdIx for a subset of agents?** CrowdIx could be adapted to measure the extent to which a subset of agents crowd *among themselves*. This is particularly relevant for a CCP who is worried about liquidation risk. Consider the subset of all intermediaries. If, for example, they all default at the same time, a CCP is likely to pay a fire sale premium when offloading the inherited portfolios to any remaining “slow moving capital” (Menkveld, 2013). The natural benchmark portfolio for

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$^7$In this case the function $M(\cdot)$ is only evaluated at +1 or -1 as re-allocating to a single risk factor implies that portfolio return correlations become either +1 or -1 (members are either buyers or sellers of the same risk factor).
an adapted CrowdIx measure is one where all intermediaries’ bets are on one side of a single risk factor. The numerator and denominator only sum over this subset of agents, and the denominator re-allocates trades to one side of a single risk factor. Note that the denominator is calculated more easily as one is not constrained by the CrowdIx constraint that there needs to be an equal amount of buying and selling for the single risk factor across all members.\(^8\)

2.3 Example to illustrate crowded risk

A simple example illustrates the concept of crowded risk. Consider two securities that are available for trade. Their one-period returns are distributed as two independent standard normal variables. Let four members implement two trades.

Crowded trades. Suppose member 1 buys a unit of security 1 from member 2, and member 3 buys a unit of the same security from member 4. In this case all members’ trades crowd on the same security. The trade portfolio matrix therefore is

\[
N = \begin{pmatrix}
1 & -1 & 1 & -1 \\
0 & 0 & 0 & 0
\end{pmatrix}
\quad \text{and} \quad
\Sigma = N' \Omega N = \begin{pmatrix}
1 & -1 & 1 & -1 \\
-1 & 1 & -1 & 1 \\
1 & -1 & 1 & -1 \\
-1 & 1 & -1 & 1
\end{pmatrix},
\]

which implies the following mean and covariance for exposures \(E\), where \(E = (E_1, E_2, \ldots, E_J)'\) and \(E_j\) is as in eqn. (3):

\[
E(E) = \sqrt{\frac{1}{2\pi}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \text{var}(E) = \frac{1}{2\pi} \begin{pmatrix}
\pi - 1 & -1 & \pi - 1 & -1 \\
-1 & \pi - 1 & -1 & \pi - 1 \\
\pi - 1 & -1 & \pi - 1 & -1 \\
-1 & \pi - 1 & -1 & \pi - 1
\end{pmatrix}.
\]

The mean and standard deviation of aggregate exposure are therefore

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\(^8\)This however can only be implemented if the volume transacted by the subset of agents is less than half of total volume as otherwise there is insufficient money on the other side of the single risk factor trade.
E(\textit{A}) = 4 \sqrt{\frac{1}{2\pi}} \approx 1.60 \quad \text{and} \quad \text{std}(\textit{A}) = 2 \sqrt{\frac{\pi - 2}{\pi}} \approx 1.21, \quad \text{respectively.} \quad (13)

CrowdIx is one in this case as std(\tilde{\textit{A}}) = \text{std}(\textit{A}) since the benchmark portfolio equals the actual portfolio.

**Noncrowded trades.** Suppose that members 3 and 4 trade security 2 instead of security 1, i.e., both trades now exhibit perfect diversity. The trade portfolio matrix and the covariance matrix of portfolio returns are

\[
N = \begin{pmatrix}
1 & -1 & 0 & 0 \\
0 & 0 & 1 & -1 \\
-1 & 1 & 0 & 0 \\
0 & 0 & 1 & -1 \\
0 & 0 & -1 & 1 \\
\end{pmatrix}
\]

and the covariance matrix of portfolio profits is

\[
\Sigma = \begin{pmatrix}
1 & -1 & 0 & 0 \\
-1 & 1 & 0 & 0 \\
0 & 0 & 1 & -1 \\
0 & 0 & -1 & 1 \\
\end{pmatrix}
\]

(14) \hspace{1cm} (15)

The mean and covariance matrix of exposures \textit{E} are:

\[
\text{E}(\textit{E}) = \sqrt{\frac{1}{2\pi}} \begin{pmatrix}
1 \\
1 \\
1 \\
1 \\
\end{pmatrix}, \quad \text{var}(\textit{E}) = \frac{1}{2\pi} \begin{pmatrix}
\pi - 1 & -1 & 0 & 0 \\
-1 & \pi - 1 & 0 & 0 \\
0 & 0 & \pi - 1 & -1 \\
0 & 0 & -1 & \pi - 1 \\
\end{pmatrix}
\]

(16)

The mean and standard deviation of aggregate exposure (\textit{A} = \sum_{j=1}^{J} \textit{E}_{j}) are therefore

\[
\text{E}(\textit{A}) = 4 \sqrt{\frac{1}{2\pi}} \approx 1.60 \quad \text{and} \quad \text{std}(\textit{A}) = 2 \sqrt{\frac{\pi - 2}{2\pi}} \approx 0.85, \quad \text{respectively.} \quad (17)
\]

CrowdIx is \sqrt{\frac{1}{2}}=0.71 in this case because std(\tilde{\textit{A}}) = 2 \sqrt{\frac{\pi - 2}{\pi}}. The index reaches its lower bound (see Lemma 1).
In sum, the aggregate-exposure mean is the same for the two polar cases, but the standard deviation is 41% higher in the crowded risk case. The example only involves four clearing members, yet, crowded trades make a large aggregate loss a lot more likely. One measure of tail risk, the 97.5% Value-at-Risk as approximated by the delta-normal method (Jorion, 2007, p. 260), is $1.60 + 1.96 \times 0.85 = 3.27$ in the noncrowded case and $1.60 + 1.96 \times 1.21 = 3.97$ in the crowded case. It is 21.4% higher in the crowded case. Figure 3 graphs the distribution for both cases based on simulations. The heavy right tail for the crowded case illustrates the increased likelihood of an extremely large loss. We will return to this Value-at-Risk measure and the associated aggregate loss simulations when we implement the basic ideas on real-world data in Section 3.

2.4 Margin(A): A margin methodology that accounts for crowded-trade risk

This section develops an alternative margin methodology that accounts for crowded-trade risk. Rather than thinking about collateral on a member by member basis, it starts thinking about total collateral needed at the level of the CCP. In other words, how much collateral is needed if it is CCP tail risk that one is worried about rather than member by member tail risk? First, we propose a Value-at-Risk (VaR) approach to capture CCP tail risk based on its aggregate exposure ($A$). It turns out that this aggregate risk naturally decomposes across clearing members based on Euler’s homogeneous function theorem. The polluter pays in the sense that those who are part of the crowded trade internalize the risk they impose at the aggregate level, i.e., CCP tail risk. Finally, the crowded trade can be traced down by computing CCP tail risk sensitivity to candidate risk factors. All results are analytical and are developed in the next three subsections.
2.4.1 Margin(A) as a CCP (tail) risk measure

Standard margin methodologies charge a margin based on tail risk in member portfolio returns. Examples are SPAN of the Chicago Mercantile Exchange (CME) and CoH of the CCP who provided us with the data sample. CoH bases margin on member-portfolio VaR according to the “delta-normal method,” which is one of the three standard VaR approaches according to Jorion (2007). The delta-normal VaR computes VaR as the return mean minus alpha times its standard deviation.

Let CCP tail risk be defined as the delta-normal VaR of aggregate loss, i.e.,

\[ \text{Margin}(A) := \text{mean}(A) + \alpha \text{std}(A), \]

where “Margin(A)” captures the collateral needed as a function of the aggregate-exposure distribution.\(^9\)

Margin(A) has several attractive features. First, it accounts for crowded risk. Second, an analytical decomposition result developed in the next subsection (2.4.2) allocates this risk appropriately across members, i.e., the more a member joins crowded trades the more he has to contribute. Third, it is easily computed as the mean and standard-deviation results are analytical (particularly relevant in an environment of many securities, many clearing members, and the high-frequency monitoring that might be required in the presence of sub-millisecond trading). Fourth, an analytical result developed in subsection 2.4.3 helps identifying the crowded-trade securities/risk factors. Fifth, it follows standard practice.

2.4.2 Decomposing Margin(A) across members – the polluter pays

The mean and standard deviation of aggregate exposure are homogeneous of degree one in member portfolio risk (see eqn. (6) and (7), respectively).

\(^9\) The delta-normal VaR is useful even if returns are non-normal. Chebyshev inequality \( (P[|X - \mu| \geq k\sigma] \leq 1/k^2) \) yields an upper bound for the VaR probability. In CoH \( \alpha \) is set to seven and its VaR probability therefore is at most \( 1/7^2 \approx 0.02 \) (normality implies \( 1.3 \times 10^{-12} \)).
Euler’s homogeneous function theorem applies and yields:

\[
\text{Margin}(A) = \sum_{k=1}^{J} \sigma_k \left( \frac{\partial}{\partial \sigma_k} \text{Margin}(A) \right) = \sum_{k=1}^{J} \sigma_k \left( \frac{\partial}{\partial \sigma_k} \text{mean}(A) + \alpha \frac{\partial}{\partial \sigma_k} \text{std}(A) \right). \tag{19}
\]

The decomposition naturally allocates aggregate risk to what each member contributes to it, i.e., the number of risk units in his portfolio, \(\sigma_k\), times how sensitive CCP risk is to his risk, \(\frac{\partial}{\partial \sigma_k} \text{Margin}(A)\).

A member contributes in proportion to its shadow cost expressed in terms of aggregate risk, i.e., CCP risk.

Straightforward calculations using

\[
\frac{\partial \text{std}(A)}{\partial \sigma_k} = \sum_{l=1}^{J} \frac{1}{\text{std}(A)} \left( \frac{\pi - 1}{2\pi} \right) \sigma_l M(\rho_{kl}) \tag{20}
\]

yield

\[
\text{Margin}(A) = \sum_{k=1}^{J} \sigma_k \left\{ \sqrt{\frac{1}{2\pi}} + \frac{\alpha}{\text{std}(A)} \left( \frac{\pi - 1}{2\pi} \right) \sigma_k \right\} + \sum_{l \in \{1, \ldots, J\} \backslash \{k\}} \frac{\alpha}{\text{std}(A)} \left( \frac{\pi - 1}{2\pi} \right) \sigma_l M(\rho_{kl}) \right\} \right. \tag{21}
\]

The decomposition of \(\text{Margin}(A)\) identifies two parts for each member’s contribution to CCP risk. The first part is a member-specific “old” part that depends on \(\sigma_k\) only. The second part is the “new” part that considers the correlation of a member’s portfolio return with other members’ portfolio returns. The \(M(.)\) function captures the “punishment” for crowded trades. Positive correlations with other members’ portfolio returns carry the full weight, whereas negative correlations get attenuated (see also discussion in Section 2.1).
### 2.4.3 Sensitivity of Margin(A) to candidate risk factors – identifying the crowded trade

To discover on what security or risk factor member portfolios crowd, it is useful to measure how Margin(A) changes if one more unit of risk is added to the risk factor of interest, say $f$ (e.g., a particular security’s return, the market return, or a Fama-French factor). The resulting Margin(A) sensitivity result is

$$\frac{\partial}{\partial \sigma^f} \text{Margin}(A) = \frac{\partial}{\partial \sigma^f} \text{mean}(A) + \alpha \frac{\partial}{\partial \sigma^f} \text{std}(A). \tag{22}$$

Straightforward calculations yield

$$\frac{\partial}{\partial \sigma^f} \text{Margin}(A) = \sum_{j=1}^{J} \sqrt{\frac{1}{2\pi} \frac{\sigma^f}{\sigma_j}} B_{jj} + \alpha \left( \frac{\pi - 1}{4\pi} \right) \frac{\sigma^f}{\sigma^A} \sum_{k,l \in \{1, \ldots, J\}} \left[ 2M'(\rho_{kl}) B_{kl} + \left( \frac{\sqrt{1 - \rho_{kl}^2} - 1}{\pi - 1} \right) \left( \frac{\sigma_l}{\sigma_k} B_{kk} + \frac{\sigma_k}{\sigma_l} B_{ll} \right) \right] \tag{23}$$

with

$$M'(\rho) = \frac{\frac{1}{2} \pi + \arcsin(\rho)}{\pi - 1}, \quad B_{kl} := n_k' \beta \beta' n_l, \quad \text{and} \quad \beta = \text{cov}(R, r^f)/\text{var}(r^f). \tag{24}$$

The partial derivatives of $\text{mean}(A)$ and $\text{std}(A)$ to $\sigma^f$ are derived in Appendix A.

Inspecting the partial derivative of Margin(A) to $\sigma^f$ leads to several observations. First, the expression is linear in $\sigma^f$, the volatility of factor $f$. Second, a key factor is $B_{kl}$ that captures not only how trade portfolio profits load on the factor (e.g., $B_{kk}$ for member $k$’s portfolio), but also the extent to which they crowd on that factor ($B_{kl}, k \neq l$). Third, the effect that $B_{kl}$ has on aggregate exposure is governed by the derivative of the $M(.)$ function. The slope of this function is higher in the positive domain than in the negative domain (see Figure 1), i.e., a risk change in the factor kicks in stronger in areas where portfolio risk crowds relative to areas where it does not.
3 Real-world application of CrowdIx and Margin(A)

3.1 Data and summary statistics

This final section explores CrowdIx and Margin(A) further by applying them to actual data. The sample available for analysis consists of about a year’s worth of “trade” reports filed by clearing members to their central clearing counterparty, EMCF. It also contains the margin EMCF required each member to post each day. The sample period is October 19, 2009 through September 10, 2010. The reports cover transactions in stocks that are listed in Denmark, Finland, or Sweden.

EMCF cleared trades for almost all exchanges that traded these stocks at the time: NASDAQ OMX, Chi-X, Bats, Burgundy, and Quote MTF. The only exchange whose Nordic trades it did not clear was Turquoise. This London-based exchange had a very small market share at the time.

Fifty-seven clearing members traded these stocks (see Appendix C). They span a variety of financial firms, e.g., international brokerage firms (e.g., Goldman Sachs and JPMorgan), local banks (e.g., Nordea and Swedbank), and high frequency traders (e.g., GETCO and Knight). Fifty-five of them were active in the sample period. Members are anonymized and referred to by a two digit code (random numbers between 0 and 100).

[insert Table 1 here]

Table 1 provides some summary statistics. The sample consists of 1.4 million reports in 242 securities across 228 trading days. This implies that on average a stock traded about 25 times per day. Signed volume (buy volume gets a positive sign, sell volume gets a negative sign) sums to zero for each stock each day. This result serves as a straightforward data integrity check.

Overall statistics reported in Panel A lead to a couple of observations. First, clearing members collectively filed about 6000 trade reports to the CCP each day. Trade size is heavily skewed to the right as the average number of shares in a single trade report is 25,600 (€287,600) whereas its

10EMCF later merged with the U.S. based DTCC to become EuroCCP.
median is only 2,600 (€36,100). The largest trade was for roughly 19 million shares worth €142 million.

Cross-sectional statistics show substantial difference in activity across clearing members and across stocks. Panel B reports how average activity is distributed across clearing members. The cross-sectional mean of the average daily number of reports filed by a clearing member is 114.5; the median is 64.9. The least active member filed a trade report less than once a month whereas the most active member filed 736.4 reports daily. Daily volume statistics show similar skewness. The cross-sectional statistics for stock activity also exhibit skewness to the right although to a lesser degree. Panel C shows that the mean and median for the average daily number of reports per stock are 26.0 and 20.6, respectively. The least active stock traded less than once a month on average whereas the most active stock traded 84.2 times per day.

3.2 Aggregate-exposure distribution and the implied margin

Margin(A), the margin based on aggregate exposure (see Section 2.4), is calculated for each day in the sample. A time-varying covariance matrix for returns is calculated as the exponentially weighted moving average (EWMA) of the outer product of historical daily returns. Such covariance matrix has become a standard in industry and is used by, for example, MSCI’s RiskMetrics. The parameter used for the daily frequency is 0.94, following RiskMetrics. It implies that the most recent week carries (1-0.95^2)=27% of total weight; the most recent month carries 71% of total weight. EMCF’s proprietary methodology CoH also uses EWMA estimates for the covariance matrix. The CCP inherited CoH from Fortis Bank Global Clearing who wrote in the annual accounts (p. 21): “E.g. equity prices are stressed by seven standard deviations (EWMA or implied), giving a confidence level of 99.98%.” I follow this lead and set \( \alpha=7 \) for Margin(A).

[insert Figure 4 here]
Figure 4 illustrates the time variation in the actual margin collected by the CCP, and what should have been collected according to Margin(A). For the remainder of the analysis, Margin(A) was scaled up by 6.13 so as to make both margins have the same (unconditional) average. The focus is on comparing both margin methodologies in terms of the time variation in margin and the cross-sectional variation in margin, not on its level per se.

Panel A illustrates that through time both aggregate margin series are highly correlated. Their empirical distributions are both heavily skewed with two extreme peaks occurring on April 23, 2010 and May 10, 2010. Margins typically hover around €150 million, but shoot up to about half a billion euro or more for these two days. The highest peak corresponds to a “macro shock” as it followed an announcement of a Eurozone bailout program a couple of days after Greece was bailed out. The announcement was on a Sunday, and the extreme margin calls came on Monday after a day of heavy trading and lots of volatility. The second-highest peak was more of a surprise as it was triggered by an idiosyncratic event. On April 22, at noon local time, Nokia reported first-quarter earnings that were far below analyst expectations. It was perceived as the company’s failure to compete with the growing smart phone market. The stock price declined by about 15% from noon until the end of the day, and volume increased by almost 400% in this period relative to the morning period.

The more striking result revealed in Panel A is that when margins shoot up, Margin(A), the one that accounts for crowded risk, shoots up substantially more. For example, the total “bailout-program” margin collected by the CCP was €511 million, whereas Margin(A) would have required €747 million for that day, i.e., €236 million more. The total “Nokia day” margin collected by the CCP was €495 million whereas Margin(A) would have collected €644 million, €149 million more.\footnote{As mentioned in the introduction, the countercyclicality concern applies to Margin(A) as well as to the actual margin collected. One might not actually want to collect collateral when the market is “under stress.” It remains relevant though to calculate what is needed appropriately (and perhaps collect it outside of stress periods).}
Panel B illustrates that it is crowding that drives the wedge between the two margin methodologies. It zooms in on the month surrounding the two peak margin events and then overlays the crowding index, CrowdIx. CrowdIx is 72% for the Nokia peak and 62% for the bailout-program peak. The CrowdIx sample mean is 47% with a standard deviation of 8%. A level of 62% or higher therefore indicates extreme crowding.

The disproportionately large jump in Margin(A) relative to CCP-imposed margin deserves further study. The new margin methodology accounts for crowding and CrowdIx does indeed reach extreme levels in this period. As Margin(A) is based on an easily computed analytical mean and standard deviation, it is worth studying the actual distribution to verify the existence of a “fat” right tail due to crowding. The approach taken here is to simulate returns from the standard normal, calculate the aggregate exposure realization for each simulation, and plot its distribution (similar to what was done for the example in Section 2.3, presented in Figure 3). To provide benchmarks, the same procedure was followed for the median- and minimum-CrowdIx day. The distribution for these benchmark days are rescaled so as to match the mean of the day of interest.

[insert Figure 5 here]

Figure 5 reveals that the distribution for the “Nokia day” does indeed exhibit a fat right tail. The rescaled distributions for the median- and minimum-CrowdIx benchmark days show considerable less mass in the right tail. The 90% quantile for the Nokia day is €244 million compared to €182 and €163 million for the median- and minimum-CrowdIx benchmark days, respectively. The 99.9% quantile (corresponding to an extreme event once every four years) is €472 million for the Nokia day relative to €335 million and €257 million for the two benchmark days. Tail risk defined this way appears to be between 34% and 84% higher due to crowding.
3.3 Aggregate margin decomposition across members

The Margin(A) decomposition result of eqn. (21) is used to analyze which members contribute most to CCP risk. In particular, is there substantial variation across members in the extent to which they join crowded trades? In the latter case, Margin(A) will allocate aggregate margin differently across members and is therefore a meaningful innovation relative to existing margin methodologies.

Figure 6 illustrates the issue with three scatterplots of actual margin posted by a member relative to what they should have posted based on Margin(A). Panel A depicts the scatterplot for July 29, 2010. It is the median-CrowdIx day in the sample (CrowdIx=0.46) and could therefore be considered “representative.” If both margin methodologies agree on how to allocate aggregate margin across members, then all points in the scatterplot should be on a single line. The plot illustrates that this is clearly not the case. Member 6 is perhaps the best example. The CCP required him to post more than €15 million whereas Margin(A) would have made him post only €5 million, less than a third of what was posted.

The bailout-program scatterplot in Panel B identifies a few members who contribute strongly to the high level of crowding on this day (CrowdIx=0.62). Member 41 for example posted less than €150 million but should have posted €250 million. His trade portfolio characterized in the exhibit below the plot shows that its ten largest positions are short positions. Given the macro nature of the event, market return is the likely candidate for the crowded trade (Section 3.4 will show that it is). Other members appear to be closer to a 45 degree line. The margin required for member 12 for example is about the same for the two margin methodologies. His portfolio is also shown and unlike member 41 its ten largest positions are neither fully long nor fully short in the market portfolio.
One interpretation of the bailout-program observations is that member 41 is arbitraging across the index futures market and the cash market. One could say that it is therefore not systemically risky. In this particular case the cash CCP is different from the index futures CCP (in case there is one). Any potential for offsetting gains for member 41 are therefore out of reach for the cash CCP. Economically, any further margin calls due to quick recovery of the market (requiring member 41 to post “variation margin” to mark his position to market) should in this case not be a problem for member 41 as such recovery would free up margin on the long position in the index futures market. The key point is that the cash CCP is simply not aware of any offsetting positions and therefore the large Margin(A) contribution that it makes member 41 post reflects the enormous shadow cost of his position to the cash CCP. It is part of prudential risk management.

The Nokia day depicted in Panel C suggests that many members were contributing to the high crowded risk (CrowdIx=0.72), yet some members with large portfolio positions stayed away from the crowded bet. Member 12 for example posted €60 million but should have posted less than €10 million based on Margin(A). It is no surprise that the Nokia stock is not in the top ten of his trade portfolio. Member 41 on the other hand contributed substantially to CCP risk as he would have been charged more than €80 million based on Margin(A), but posted less than half of it. His portfolio reveals a significant Nokia exposure, more than a fifth of his trade portfolio is Nokia.

3.4 Aggregate margin sensitivity to risk factors

The Margin(A) sensitivity result of eqn. (22) is useful in identifying the crowded risk factors. This partial derivative result shows how aggregate margin changes when a particular risk factor becomes one unit more risky (measured in terms of standard deviation). Perhaps more informative are elasticities which are easily calculated by multiplying the partial derivative with the ratio of the standard deviation of the candidate risk factor and Margin(A), i.e., \( e^{Margin(A)}_{\sigma_f} = \frac{\partial}{\partial \sigma_f} Margin(A) \frac{\sigma_f}{Margin(A)} \).

To illustrate the sensitivity analysis, Margin(A) is analyzed for the same three days as used in the decomposition analysis: the median-CrowdIx day, the bailout-program day, and the Nokia day.
The three factors considered here are: the market index return (STOXXNordic30), the European telecom sector return (STOXXTelecom), and the Nokia stock return. The telecom sector was chosen as the Nordic countries seem to be telecom-heavy through firms like Tele2, Ericsson, and Nokia.

[insert Table 2 here]

The sensitivity results presented in Table 2 lead to a couple of observations. First, the strongest findings are elasticities larger than one. Perhaps the most interesting one is an elasticity of 1.05 for the Nokia-day margin with respect to the Nokia stock return. If one increases the daily Nokia stock volatility by one percentage point, the aggregate margin would increase by €147 million. Elasticities for the market and for telecom are an order of magnitude smaller for this day, i.e., 0.19 and 0.00 respectively. These elasticities are extraordinary as on the “representative” median-CrowdIx day the elasticity for the market is 0.91, for the telecom sector it is 0.46, and for Nokia it is 0.15. It illustrates how an idiosyncratic event can have “systemic” consequences. Adding one percentage point to a single stock’s volatility generates a larger additional collateral call (€147 million) than adding one percentage point to market volatility (€116 million).

Second, for the bailout-program day Margin(A) is particularly sensitive to the market return and to the telecom sector return. Elasticities are 0.98 and 0.83, respectively. This result is not surprising given that it was a macro event.

4 Conclusion

Crowded trades constitute a risk to a CCP that is overlooked in standard margin methodologies. Standard practice is to base margin on the tail risk in member portfolio returns, computed member by member. Correlations across member portfolios due to crowded-trades are ignored. The paper’s main message is that they matter as they drive a wedge between CCP tail risk and the “sum” of tail risk in all of its members’ portfolios.
The paper develops two products that are useful to manage such crowded-trade risk. First, it proposes a crowding index, CrowdIx, as diagnostic tool to measure the extent to which there are crowded trades. Its strength is that it only needs the covariance matrix of clearing members’ portfolio returns. That is, one does not need to specify the crowded trade security or risk factor \(a priori\).

Second, an alternative margin methodology, Margin(A), is proposed that accounts for the additional risk due to crowded trades. It takes a fundamentally different approach relative to standard margin methodologies, as it first computes the aggregate collateral needed at the level of the CCP and then disaggregates it across clearing members. One attractive feature is that each member’s contribution depends on how much his portfolio contributes to risk at the aggregate level. It therefore makes a member internalize the externality it imposes on others through crowded trades. The polluter pays. It is easily computed as all results are analytical. Computationally burdensome simulations are avoided. This is particularly relevant in an electronic world that increasingly features sub-millisecond trading, lots of clearing members, and many securities. Finally, Margin(A) comes with a set of analytical results that help identifying the crowded-trade security or risk factor.

Finally, the paper applies these two products, CrowdIx and Margin(A), to CCP data to illustrate their relevance and strength. CrowdIx and Margin(A) were calculated for a 2009-2010 sample of trades in Nordic stocks by members of a large European central clearing counterparty.

Appendix

A Aggregate-exposure mean and standard deviation

The mean and standard deviation of aggregate exposure is derived in this appendix. It is derived for the case where the distribution of member portfolio profit is standard normal, i.e., all diagonal elements of \(\Sigma\) are equal to one. The more general case is easily derived by scaling these results.
appropriately.

The mean and standard deviation of individual exposures follow immediately from results on
the absolute moments of a bivariate normal (Nabeya, 1951):

\[ E(E_j) = \frac{1}{2} E(|Z|) = \sqrt{\frac{1}{2\pi}}, \quad \text{and} \quad \text{std}(E_j) = \sqrt{\frac{1}{2} E(Z^2) - (E(E_j))^2} = \sqrt{\frac{\pi - 1}{2\pi}}, \quad \text{(A1)} \]

where \( Z \) is a standard normal random variable.

The mean and standard deviation of aggregate exposure \( A = \sum_j E_j \) further requires calculation
of how correlation among portfolio returns \( X_j \) translates into correlation among exposures \( E_j = -\min(X_j, 0) \). This result is derived as follows:

1. Rosenbaum (1961, eqn. (5)) implies the following result for a truncated bivariate normal
distribution constructed from two correlated standard normals, \( Z_1 \) and \( Z_2 \), and truncation
points set equal to zero:

\[ L(0, 0; \rho) E(\tilde{Z}_1 \tilde{Z}_2) = \rho L(0, 0; \rho) + \frac{\sqrt{1 - \rho^2}}{2\pi} \quad \text{(A2)} \]

where \( \rho \) is the correlation between \( Z_1 \) and \( Z_2 \), \( L(h, k; \rho) \) is the probability mass in the top-right
quadrant defined by truncation points \( h \) and \( k \), and \( \tilde{Z}_1 \) and \( \tilde{Z}_2 \) represent the truncated random
variables.

2. \( L(h, k; \rho) \) cannot be obtained as an analytical expression in general, but for the special case
\( h = k = 0 \) it can be calculated explicitly (see, e.g., Balakrishnan and Lai, 2009, p. 495):

\[ L(0, 0; \rho) = \frac{1}{4} + \frac{1}{2\pi} \arcsin(\rho). \quad \text{(A3)} \]

3. The exposure correlation now follows from the following steps:
(a) The factor \( L(0,0;\rho) \) on the LHS of eqn. (A2) can be deleted as the corresponding moment for exposures does not involve truncation. The product of exposures is zero in the other three quadrants.

(b) The expression for \( L(0,0;\rho) \) in eqn. (A3) is inserted on the RHS of eqn. (A2).

(c) The covariance between exposures is then derived by subtracting the squared exposure mean (i.e., \( 1/2\pi \)) from the exposure moment.

(d) The correlation is derived by dividing this covariance by the exposure variance which equals \( (\pi - 1)/2\pi \) (see eqn. A1).

The result of these steps is the \( M(.) \) function that maps portfolio return correlations into exposure correlations, see eqn. (8). I conducted simulations and compared the simulations based \( M(.) \) function to the closed-form solution of \( M(.) \) derived here to convince myself that the solution is correct.

**Sensitivity to a particular security/risk factor.** The partial derivative of the mean of aggregate exposure to risk factor \( f \) is:

\[
\frac{\partial}{\partial \sigma^f} E(A) = \sum_{j=1}^{J} \sqrt{\frac{1}{2\pi}} \frac{\partial}{\partial \sigma^f} \sigma_j = \sum_{j=1}^{J} \sqrt{\frac{1}{2\pi}} \frac{\partial}{\partial \sigma^f} \sqrt{n_j' \beta \beta' n_j + n_j' \tilde{\Sigma} n_j} \tag{A4}
\]

where \( \tilde{\Sigma} \) is the covariance matrix of the residual after regressing a member’s portfolio return on the factor \( f \).

The partial derivative of \( \text{std}(A) \) with respect to \( \sigma^f \) is:

\[
\frac{\partial}{\partial \sigma^f} \text{std}(A) = \left( \frac{\pi - 1}{4\pi} \right) \sigma^f \sum_{k,i} \left[ 2M'(\rho_{ki})B_{ki} + (M(\rho_{ki}) - \rho_{ki}M'(\rho_{ki})) \left( \frac{\sigma_i}{\sigma_k} B_{ik} + \frac{\sigma_k}{\sigma_i} B_{ki} \right) \right] \tag{A5}
\]

with
\[ M'(\rho) = \frac{\frac{1}{2} \pi + \arcsin(\rho)}{\pi - 1}, \quad B_{kl} := n_k' \beta' n_l, \quad \text{and} \quad \beta = \frac{\text{cov}(R, r^f)}{\text{var}(r^f)}. \quad (A6) \]

Straightforward algebra yields eqn. (23).

\[ M(\rho) \text{ is convex.} \quad M(\rho) \text{ is a convex function as the second derivative is positive:} \]

\[ M''(\rho) = \frac{1}{(\pi - 1) \sqrt{1 - \rho^2}} > 0. \quad (A7) \]

**B The construction of CrowdIx and its properties**

Let the aggregate member portfolio risk be defined as:

\[ T = \text{tr}(\Sigma) = \sum_{j=1}^{J} \sigma_j^2, \quad (A8) \]

where \( \sigma_j^2 := \sigma_{jj} \), i.e., the element in row \( j \) and column \( j \) of \( \Sigma \) where the latter is defined in eqn. (5).

The crowding index, CrowdIx, is defined as the ratio of two standard deviations. The numerator is the standard deviation of aggregate exposure based on true member portfolios. The denominator is its standard deviation for a benchmark case where i) total risk, \( T \), is unchanged, ii) individual member portfolio risk, \( \sigma_j^2 \), is unchanged, but iii) trades are re-allocated to the maximum extent possible to a single risk factor. This section first formalizes the optimization associated with creating the benchmark case, then proposes an algorithm to solve it, and finally obtains its lower bound.
B.1 Identifying the maximum-crowding case

If all members would trade the same risk factor, then \( \exists n \in \mathbb{R}^I \) such that \( \forall j: \)

\[
X_j = v_j \times (n'R), \quad \text{with } v_j \in \mathbb{R}.
\]

Then

\[
\Sigma = n'\Omega n \times \left( v_j v'_j \right)_{j \times J}.
\]

Without loss of generality, let \( n'\Omega n = 1 \). For member by member portfolio risks to remain unchanged, one needs \( \forall j: \)

\[
v_j = \sigma_j. \quad \text{(A9)}
\]

In addition, the aggregate (signed) trade has to be zero (for every buyer there should be a seller):

\[
\sum_{B} v_j = \sum_{S} v_j, \quad \text{(A10)}
\]

where \( B \) denotes the set of buyers and \( S \) denotes the set of sellers. The trade re-allocation problem becomes:

\[
\max_{B,S,R} \left( \min \left( \sum_{j \in B} v_j, \sum_{j \in S} v_j \right) \right) \quad \text{(A11)}
\]

where \( B \cup S \cup R \) is a partition of the set \( \{1, \ldots, J\} \), and \( R \) is a remainder set. In the example of Section 2.3, one partition that obtains the maximum value is \( B = \{1, 3\}, S = \{2, 4\} \), and \( R = \emptyset \). If perfect re-allocation is not possible, then a member is taken from the remainder bin and his risk is split into one part that gets assigned to the least full bin, \( B \) or \( S \), and the other part remains in the remainder bin. This is done to ensure eqn. (A10) holds, for every unit transacted there is a buyer and a seller so that the trade re-allocation becomes feasible.

The trade re-allocation problem is a combinatorial problem that is NP hard. It closely resembles

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12This subsection greatly benefited from Jean-Paul Renne’s excellent discussion of an earlier version of the paper.
a one-dimensional bin packing problem (Coffman, Garey, and Johnson, 1996). Can all items be packed into two bins of size \( \left( \sum \sigma^2_j \right) / 2 \)? If not, what is the largest amount that can be packed into two such bins? Brute force requires \( 3^J \) steps to solve the problem. The \( 3^J \) approach simply tries all combinations as for every member one can try to put him in either the buyer bin, the seller bin, or in the residual bin.

B.2 The FFD algorithm

The first-fit-descending (FFD) algorithm reduces finding a trade re-allocation solution from \( O(3^J) \) to \( O(J \log J) \). It is known to do reasonably well relative to other standard approaches for the offline\(^{13} \) bin-packing problem. In particular, it does well for the two standard algorithm quality analyses:

- **Average-case analysis:** If item size is drawn from \( U[0, 1/2] \) for one-unit bins then Coffman, Garey, and Johnson (1996, p. 39) claim “FFD is typically optimal.”

- **Worst-case analysis:** If all items are smaller than 1/2 then FFD does as well its closest contender MFFD (modified-first-fit-descending) (Coffman, Garey, and Johnson, 1996, p. 16-19).

B.3 CrowdIx construction

CrowdIx uses FFD and is computed as follows:

1. Set the bin size equal to \( T/2 \).

2. Sort clearing members according to their portfolio return risk in descending order.

\(^{13}\)For online algorithms the items arrive in some given order and must be assigned to bins as they arrive, *without* knowledge of the items yet to arrive (Coffman, Garey, and Johnson, 1996, p. 2). Offline algorithms have full information on the size of all items when assigning a particular item to a bin.
3. Assign members to bins sequentially. Start with the first bin and add the member to the bin if it fits. That is, only add him if adding his portfolio risk to the risk already in this bin does not exceed the bin size. Say we are considering assigning member 4 and members 1 and 3 are already in this bin, then we will only add member 4 to this bin if \( \sigma_1^2 + \sigma_3^2 + \sigma_4^2 \leq T/2 \). If he does not fit, then consider assigning him to the next bin. If all existing bins were tried and still unsuccessful, open a new bin and assign the member to that bin (note that this is always possible since a member’s risk cannot exceed half total risk).

4. Make clearing members in first bin buyers and members in the second bin sellers of the single risk factor.

5. Visit the remainder bin and re-assign its member’s portfolio risk partially to the first bin or the second bin depending on which one is least full. Note that there cannot be more than one member in this remainder bin as otherwise the sum across members’ portfolio risk would exceed \( T \). Re-assign the member’s portfolio risk only so much as to make the bins equally full. This ensures that trade re-allocation is internally consistent, for every buyer there is a seller.

In the best case this procedure can reassign member portfolio risk perfectly so that the first two bins are filled perfectly (as is the case in the noncrowded example discussed in Section 2.3). In most real-world cases the bins will not be filled entirely. The space that remains is likely to be small as i) there is considerable variation in member portfolio risk and ii) largest risk members are assigned first.

---

14 Suppose not. Consider the case where there are \( N \geq 2 \) members who landed in the third (remainder) bin. Let \( X \) be the maximum across the space still available in the first and second bin. Then the risk of each member in the third bin exceeds \( X \). The total risk in the system in this case is at least \( 2(T/2 - X) + NX > T \) as \( N \geq 2 \). This cannot be true.
B.4 Lower bound CrowdIx (Lemma 1)

Assume that the number of clearing members, $J$, is even; the case of odd $J$ will be dealt with later. Total risk $T$ achieves lowest aggregate exposure when distributed evenly across all clearing members and members trade bilaterally in risk factors that are orthogonal. Say member 1 buys risk factor 1 from member 2, member 3 buys risk factor 2 from member 4, etc. This creates a portfolio return covariance matrix $\Sigma$ with the following structure:

$$
\Sigma = \begin{bmatrix}
Q & -Q & 0 & 0 & \cdots \\
-Q & Q & 0 & 0 & \\
0 & 0 & Q & -Q & \\
0 & 0 & -Q & Q & \\
\vdots & \ddots & & & \\
\end{bmatrix}
$$

(A12)

The standard deviation of aggregate exposure associated with $\Sigma$ is calculated based on eqn. (7).

Each block

$$
\begin{bmatrix}
Q & -Q \\
-Q & Q \\
\end{bmatrix}
$$

(A13)

of the block-diagonal matrix $\Sigma$ contributes

$$
2 \left(1 - \frac{1}{\pi - 1}\right) Q,
$$

(A14)

because $M(-1) = 1/(\pi - 1)$. Therefore

$$
\text{std}(A) = \sqrt{\left(\frac{J}{2}\right) \left(\frac{\pi - 1}{2\pi}\right) 2 \left(\frac{\pi - 2}{\pi - 1}\right) \frac{\text{tr}(\Sigma)}{J}},
$$

(A15)

where $A$ is the aggregate exposure associated with $\Sigma$. This expression simplifies to eqn. (9).

The portfolio profit covariance matrix for the benchmark portfolio of maximum crowded risk becomes:
The standard deviation of the aggregate exposure associated with $\tilde{\Sigma}$ is calculated in the same way as the standard deviation for the actual portfolio. Each block again contributes
\[ 2\left(1 - \frac{1}{\pi - 1}\right) Q. \tag{A17} \]
As there are $(J/2)^2$ such blocks relative to the $J/2$ blocks in the case of eqn. (A12), the result follows immediately, i.e.,
\[ \text{std}(A)/\text{std}(\tilde{A}) = \sqrt{\frac{2}{J}}. \tag{A18} \]

The case of an odd number of clearing members is treated the same way as the case of $J - 1$ clearing members after merging the last two members into a single member.

Finally, note that CrowdIx can exceed its lower bound in essentially two ways: through crowded trades (as emphasized in this paper) or through unequal dispersion of aggregate risk across clearing members. The latter channel is perhaps best understood by taking this to the extreme. Suppose that the aggregate risk is allocated to only clearing members 1 and 2. One becomes the buyer of a risk factor and the other becomes a seller. In this case, there is perfect crowding by construction and CrowdIx equals one. CrowdIx therefore captures concentration of risk by either some members’ portfolios becoming disproportionately risky or by all members trading the same risk factor. The former case is covered by standard member-by-member margin methodologies, the latter case is not.\(^{15}\)

\(^{15}\)Conceptually, one could “decompose” CrowdIx along these two components. Consider the alternative scenario of keeping the risk in member portfolios fixed and creating maximum diversity of trade by making members trade...
C Clearing members EMCF (December 2010)

<table>
<thead>
<tr>
<th>Bank Name</th>
<th>Other Name</th>
</tr>
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<tbody>
<tr>
<td>ABN AMRO Clearing Bank N.V.</td>
<td>Numis Securities Ltd</td>
</tr>
<tr>
<td>BNP Paribas Securities Services S.A.</td>
<td>UBS Ltd</td>
</tr>
<tr>
<td>Bank of America Merrill Lynch</td>
<td>Barclays Capital Securities Ltd.</td>
</tr>
<tr>
<td>Citibank Global Markets and Citibank International</td>
<td>Alandsbanken Abp</td>
</tr>
<tr>
<td>JPMorgan Securities Ltd.</td>
<td>Alandsbanken Sverige AB</td>
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<tr>
<td>Goldman Sachs International</td>
<td>Amagarbanken A/S</td>
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<td>Skandinaviska Enskilda Banken</td>
<td>Arbejdernes Landsbank A/S</td>
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<tr>
<td>KAS BANK N.V.</td>
<td>Avanza Bank AB</td>
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<td>Parel S.A.</td>
<td>Carnegie Bank A/S</td>
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<tr>
<td>Deutsche Bank AG</td>
<td>Dexia Securities France</td>
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<tr>
<td>Citigroup</td>
<td>E-Trade Bank</td>
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<td>Eik Bank A/S</td>
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<td>DnB NOR Bank</td>
<td>GETCO Ltd.</td>
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<td>Deutsche Bank (London Branch)</td>
<td>Handelsbanken</td>
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<td>HSBC Trinkaus &amp; Burkhardt</td>
<td>Jefferies International Ltd.</td>
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<td>Istituto Centrale delle Banche Popolari Italiane SpA</td>
<td>Knight Capital Markets</td>
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<td>Interactive Brokers</td>
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<td>Nykredit A/S</td>
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<tr>
<td>Credit Agricole Cheuvreux</td>
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<td>Credit Suisse Securities (Europe) Ltd</td>
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<td>Morgan Stanley Securities Ltd.</td>
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</table>

Source: Zhu (2011)

References


orthogonal securities to the maximum extent possible. CrowdIx computed for this scenario, say CrowdIx2, captures concentration risk solely due to inequality in risk across member portfolios. One could decompose the distance of CrowdIx to its lower bound (Lemma 1) into a part due to this concentration risk, CrowdIx2 - lower_bound_CrowdIx, and one due to members trading the same portfolio, CrowdIx-CrowdIx2. The practical difficulty is in creating the scenario of “maximum diversity of trade.”


Hedegaard, Esben. 2012. “How Margins are Set and Affect Prices.” Manuscript, NYU.


This table presents summary statistics based on 1,434,946 Nordic “trade” reports sent to the clearing house by 55 clearing members. The sample covers 242 stocks listed in Denmark, Finland, or Sweden. Each report contains a time stamp to the second, an anonymized clearing member ID, the symbol of a stock, a price, a buy or sell indicator, and the size of the transaction in terms of shares. The sample period covers 228 trading days. It starts on October 19, 2009 and ends on September 9, 2010. The sum of signed volume is zero for each stock.

<table>
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<tr>
<th></th>
<th>Mean</th>
<th>Std</th>
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<th>Median</th>
<th>Max</th>
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<td><strong>Panel A: Overall summary statistics</strong></td>
<td></td>
<td></td>
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<tr>
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<td>699.0</td>
<td>1,135.0</td>
<td>6,426.5</td>
<td>7,663.0</td>
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<td>Daily volume (in mln shares)</td>
<td>160.9</td>
<td>42.1</td>
<td>8.1</td>
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<td>0.0</td>
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<td><strong>Panel B: Cross-sectional summary statistics, based on clearing-member averages</strong></td>
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<td>Daily number of reports</td>
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<td>46.9</td>
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<td>222.4</td>
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<td><strong>Panel C: Cross-sectional summary statistics, based on stock averages</strong></td>
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<td>Daily number of reports</td>
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<td>Daily volume (in mln euro)</td>
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<td>124.0</td>
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Table 2: CCP risk sensitivity to security/risk factor

This table shows how sensitive CCP risk is to a particular security or risk factor. CCP risk is measured by the aggregate margin that it would collect when accounting for crowded risk, i.e., Margin(A). Sensitivity could be used to identify securities or risk factors that members crowded on. Sensitivity is reported in two ways. The table shows what the change in Margin(A) is when one percentage point is added to the daily volatility of a particular risk factor. It further reports the elasticity of Margin(A) to the volatility of the risk factor. Three days were picked from the sample: the median-CrowdIx day and the two days for which the CCP charged highest aggregate margin. The risk factors considered are the market return (STOXXNordic30), the Nokia stock return, and the telecom sector return (STOXXTelecom).

<table>
<thead>
<tr>
<th>Date</th>
<th>CrowdIx</th>
<th>Risk factor</th>
<th>Margin(A) (million euro)</th>
<th>ΔMargin(A) on Δσ_f=0.01 (million euro)</th>
<th>Elasticity</th>
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<tbody>
<tr>
<td>Median CrowdIx day</td>
<td>Jul 29, 2010</td>
<td>0.46</td>
<td>Market</td>
<td>128</td>
<td>81</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>Nokia</td>
<td>128</td>
<td>11</td>
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<td></td>
<td></td>
<td></td>
<td>Telecom</td>
<td>128</td>
<td>46</td>
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<tr>
<td>Bailout program</td>
<td>May 10, 2010</td>
<td>0.62</td>
<td>Market</td>
<td>747</td>
<td>307</td>
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<td></td>
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<td>Nokia</td>
<td>747</td>
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<td>Telecom</td>
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<td>298</td>
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<td>Nokia reports Q1</td>
<td>Apr 23, 2010</td>
<td>0.72</td>
<td>Market</td>
<td>644</td>
<td>116</td>
</tr>
<tr>
<td></td>
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<td></td>
<td>Nokia</td>
<td>644</td>
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<td></td>
<td></td>
<td></td>
<td>Telecom</td>
<td>644</td>
<td>-2</td>
</tr>
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</table>
Figure 1: Crowded vs. noncrowded risk

This figure illustrates two extreme cases: crowded trades in Panel A and noncrowded trades in Panel B. The $n_j$ vector represents the position of member $j$ after trading. The graphs depict the result after two trades: member 1 traded with member 2 and member 3 traded with member 4.

Panel A: Crowded trades

Panel B: Noncrowded trades
Figure 2: $M(.)$ function to map portfolio return correlation into exposure correlation

This figure illustrates $M(.)$ that maps correlations in member portfolio returns ($X_j$) to correlations in member portfolio exposures ($E_j = -\min(0, X_j)$).
This figure graphs the density estimate (based on 100,000 draws) of the aggregate exposure when trades crowd and when they do not. The estimate is based on a simple setting. Two securities are available for trade. Their one-period returns are distributed as two independent standard normal variables. Four members implement two trades. In the case of crowded trades member 1 buys security 1 from member 2 and member 3 buys the same security from member 4. The aggregate exposure is defined as $A = \sum_{j} -\min(X_j, 0)$, where $X_j$ is the profit on member $j$’s trade portfolio. The noncrowded setting is the same except that members 3 and 4 trade security 2 instead of security 1.
Figure 4: Aggregate daily margin: actual margin vs. Margin(A)

This figure plots both the actual total margin that the CCP collected and Margin(A), the alternative margin methodology that accounts for risk due to crowded trades. Panel A graphs these series for the full sample period. Panel B zooms into the period with highest realized aggregate margin and overlays CrowdIx, a measure of crowded risk.

Panel A: Full sample period
Panel B: Mid-April through mid-May 2010

Apr 22: Nokia publishes Q1 results
May 2: Eurozone and IMF agree to bailout Greece
May 9: EU announces bailout program

Apr 21 2010
Apr 24 2010
Apr 27 2010
Apr 30 2010
May 03 2010
May 06 2010
May 09 2010
May 12 2010
Figure 5: CCP aggregate exposure distribution

This figure plots the simulated probability density function of CCP aggregate exposure for April 23, 2010. This was one of two peak days in terms of the total margin that the CCP collected. It was the period right after Nokia announced its disappointing first quarter results. The crowding index was particularly high on that day: CrowdIx=0.72. The aggregate exposure distribution is based on 100,000 simulations of security returns that are each assumed to be normally distributed. To illustrate the enhanced right-tail risk due to crowding, the plot also contains the distributions for the median- and minimum-CrowdIx days as a benchmarks. The aggregate exposure for the benchmark days was multiplied by the ratio of CCP margin on the “Nokia” day and the CCP margin on the benchmark day in order to make them comparable. The exhibit below the graph reports the 90%, 99%, and the 99.9% quantile for each distribution. The figure mirrors Figure 3 that was created to illustrate that crowded trades magnify CCP tail risk.

<table>
<thead>
<tr>
<th>Date</th>
<th>CrowdIx</th>
<th>Q(0.90) (million euro)</th>
<th>Q(0.99) (million euro)</th>
<th>Q(0.999) (million euro)</th>
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<td>Nokia reports Q1</td>
<td>Apr 23, 2010</td>
<td>0.72</td>
<td>244</td>
<td>374</td>
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<td>Median CrowdIx day benchmark</td>
<td>Jul 29, 2010</td>
<td>0.46</td>
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<td>269</td>
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<tr>
<td>Min CrowdIx day benchmark</td>
<td>Nov 12, 2009</td>
<td>0.31</td>
<td>163</td>
<td>216</td>
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Figure 6: Scatterplot of actual margin vs. Margin(A)

This figure contains three scatterplots of the margin that members actually posted versus what they would have posted based on Margin(A). The plots correspond to three days in the sample: the median-CrowdIx day and the two days for which the CCP charged highest aggregate margin. The exhibit below the scatterplots contains the ten largest positions in the trade portfolio of a member in the top-left corner and a member in the bottom-right corner.

Panel A: A representative day, i.e., CrowdIx at median level (July 29, 2010)
Panel B: Bailout program (May 10, 2010)

![Diagram showing the relationship between member margin actually posted and crowdIx]

<table>
<thead>
<tr>
<th>Clearing member 41</th>
<th>Clearing member 12</th>
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</thead>
<tbody>
<tr>
<td>Stock</td>
<td>NetPos (mln €)</td>
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<tr>
<td>ER</td>
<td>-99.9</td>
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<tr>
<td>NOKI</td>
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<tr>
<td>HMB</td>
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<td>TLS1V</td>
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<td>FSPAA</td>
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<td>SHBA</td>
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<td>ZEN</td>
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</table>
Panel C: Nokia reports Q1 (April 23, 2010)

![Graph showing the relationship between member margin actually posted and model-implied member margin.]

Crowdlx = 0.72

<table>
<thead>
<tr>
<th>Clearing member 41</th>
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<tbody>
<tr>
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<td>SEN</td>
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<table>
<thead>
<tr>
<th>Clearing member 12</th>
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<tbody>
<tr>
<td>Stock</td>
<td>NetPos (mln €)</td>
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