Financial Fragility and Central Bank Liquidity Operations *

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September 22, 2015

JOB MARKET PAPER
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Abstract

At the end of 2011 and the beginning of 2012, the ECB engaged in large scale liquidity provision to the European commercial banking system in exchange for eligible collateral. The interest rate against which banks could borrow from the ECB was substantially below prevailing interest rates on the unsecured interbank market. Theoretically, the provision of cheap liquidity by the central bank should have an expansionary effect on output. But when commercial banks are undercapitalized, cheap liquidity provision can actually be contractionary in the short run. Commercial banks shift out of private loans, since they are not eligible as collateral, to purchase assets that are eligible as collateral, leading to a contraction in private credit, investment and output. I construct a DSGE model that contains balance sheet constrained financial intermediaries that finance private loans and long term government bonds. The central bank provides cheap liquidity to commercial banks, for which commercial banks need to post collateral in the form of government bonds. I calibrate the model on data from the European periphery. Although bank balance sheets recover faster when cheap central bank funding is available, with positive long run effects on investment and output, the cumulative impact of the central bank policy is approximately zero. I therefore investigate a direct recapitalization by the fiscal authority, for which the crowding out of private credit does not occur, and find that it is much more effective in advancing the economic recovery.

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*I acknowledge the generous support of the Dutch Organization for Sciences, through the NWO Research Talent Grant No. 406-13-063. I am grateful to Sweder van Wijnbergen, Christian Stoltenberg, Bjoern Bruggeman and Wouter den Haan for helpful comments and suggestions.

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1 Introduction

At the end of 2011 and the beginning of 2012, the ECB engaged in large scale liquidity provision to the European commercial banking system in order to prevent a serious credit contraction, and allow commercial banks to repay €200 billion of (unsecured) bank bonds (European Central Bank, 2012). Commercial banks took out approximately €1000 billion under the LTRO program\(^1\) in exchange for eligible collateral. The interest rate against which banks could borrow from the ECB was substantially below prevailing interest rates rates on the unsecured interbank market.

Theoretically, the provision of cheap liquidity by the CB (central bank) should have an expansionary effect on output: lower funding costs for commercial banks\(^2\) increase bank profits. Bank balance sheets improve, and allow an expansion of credit to the real economy, fuelling an economic expansion, a subsidy effect. When commercial banks are balance sheet constrained after a financial crisis, however, cheap liquidity provision can actually be contractionary in the short run, due to a so-called ‘collateral effect’: commercial banks need to post collateral in order to obtain cheap liquidity from the CB. Private loans, however, are usually not eligible for collateral purposes. But since commercial banks are balance sheet constrained after a financial crisis, they have to shed private loans in order to be able to purchase additional government bonds. Private credit provision falls, with a contractionary impact on investment and output.

Although the most important reason for the ECB to undertake the LTRO was to prevent a serious credit contraction (European Central Bank, 2012), the focus of my paper will be on the macroeconomic effects when the CB provides cheap liquidity to an undercapitalized commercial banking system. Because the subsidy effect and the collateral effect work in opposite directions with respect to private credit provision (and through that channel on investment and output), it is not clear whether or not the LTRO was contractionary or expansionary. This trade-off will be the key focus of my paper.

There are several issues related to this question: what is the role of financial fragility? European banks, for example, have been undercapitalized since the start of the Great Recession in 2007 (International Monetary Fund, 2011; Hoshi and Kashyap, 2014). Under what conditions does the liquidity support have effects on the real economy? Even if the collateral effect possibly dominates the short run, will the CB intervention help the recovery in the long run? If we view cheap liquidity provision by the CB as an indirect recapitalization, how does it compare with a direct recapitalization by the fiscal authority? These are the issues I will look at in the current paper.

\(^{1}\)Long Term Refinancing Operations. In normal times, the ECB provides liquidity to commercial banks against eligible collateral under the MRO (Main Refinancing Operations) and the LTRO. MRO’s have a maturity of one week. LTRO’s have a maturity of 3 months in normal times. In December 2011, and February 2012, the ECB initiated two LTRO’s with a maturity of 36 months, at an interest rate equal to the MRO rate. Whenever I am talking about the LTRO, I will refer to the two extraordinary LTRO operations with an extended maturity of 36 months. Whenever I talk about regular LTRO operations, I will explicitly talk about “normal LTRO operations”

\(^{2}\)Throughout the paper, I will use ‘commercial banks’ and ‘financial intermediaries’ interchangeably to denote the same group of economic agents.
Many of these unconventional monetary policy measures have only been introduced since the Great Recession of 2007-2009, and the subsequent European sovereign debt crisis. Therefore, there exist relatively few papers that evaluate the macroeconomic consequences of liquidity provision by the CB. Gertler and Kiyotaki (2010) and Bocola (2015) model discount window lending by the CB to balance sheet constrained financial intermediaries within a DSGE framework, but do not require commercial banks to provide collateral in exchange for CB liquidity. Bocola (2015) finds that LTRO’s, subsidized long-term loans in his setup, have a (small) positive effect on lending and output. Schabert (2015) and Hoermann and Schabert (2015) incorporate a collateral constraint for refinancing operations, and find that CB balance sheet policies can neutralize an increase in firm’s borrowing costs, and stabilize inflation and output when conventional monetary policy hits the zero lower bound. Brunnermeier and Sannikov (2013) find that interest rate cuts lead to an appreciation of long-term nominal assets, thereby recapitalizing financial institutions and mitigating the economic downturn. Giannone et al. (2012) find a positive effect on commercial bank lending and real activity from ECB credit intermediation policies.

The innovation of my paper is that it introduces the possibility of cheap collateralized CB borrowing for balance sheet constrained commercial banks. My goal is to investigate the macroeconomic effects of such a policy, and whether it is expansionary, as standard monetary theory would suggest. I therefore develop a DSGE model which contains balance sheet constrained financial intermediaries, similar to Gertler and Karadi (2011). Commercial banks have a portfolio choice between private loans, which are used to purchase productive ‘physical’ capital and long-term government bonds, see Van der Kwaak and Van Wijnbergen (2014). Commercial banks are financed through net worth, deposits and CB liquidity. CB liquidity requires commercial banks to post collateral in the form of government bonds, whereas private loans are not eligible. Hence an increase in recourse to CB funding forces commercial banks to purchase more government bonds for collateral purposes. Credit conditions affect the real economy through their effect on private lending by commercial banks, which determines the amount of aggregate investment in the real economy. I explicitly model the CB balance sheet, which consists of liquidity support to commercial banks on the asset side, and household deposits on the liabilities side. CB profits and losses are transferred to the fiscal authority period by period. The CB has the possibility to decrease the nominal interest rate on the liquidity support with respect to regular deposit funding.

My paper connects to several strands of the literature. First of all, it relates to a large literature on the LOLR (Lender of Last Resort). One of the first to describe LOLR financing in times of financial crisis by the central bank is Bagehot (1873). Similarly, Friedman and Schwartz (1963) argue that the series of bank failures could have been prevented if the Fed had acted as a LOLR. LOLR lending by the Federal Reserve during the 2008 financial crisis prevented a credit crunch according to Bernanke (2013). Drechsler et al. (2014) and Giannone et al. (2012) empirically investigate the LOLR policies of the ECB. Drechsler et al. (2014) find that weakly capitalized banks borrowed more from the LOLR.
Since European commercial banks have been undercapitalized since the start of the 2008 financial crisis (International Monetary Fund 2011; Hoshi and Kashyap 2014), including balance sheet constrained commercial banks is a key element in my analysis. My paper therefore connects to the literature on financial frictions. Early papers in this literature are Gertler and Karadi (2011), Gertler and Kiyotaki (2010) and Gertler and Karadi (2013). Commercial banks face an endogenous leverage constraint that limits the size of the balance sheet for a given amount of net worth. Kirchner and van Wijnbergen (2012), Gertler and Karadi (2013) and Van der Kwaak and Van Wijnbergen (2014) introduce a portfolio choice whereby commercial banks finance both private loans and government debt. Gertler and Karadi (2013) and Van der Kwaak and Van Wijnbergen (2014) contain the possibility of capital losses on government bonds by introducing long term government debt, in a way similar to Woodford (1998, 2001).

My paper also connects to the literature on collateral constraints. The key paper here is Kiyotaki and Moore (1997), where borrowing is limited by the requirement to pledge collateral. Kiyotaki and Moore (1997) show how such a collateral constraint can amplify shocks, since the credit limit depends on the price of collateral. A higher collateral price leads to more borrowing, and more demand for capital that can subsequently be pledged as collateral, which leads to a higher price that increases the credit limit in turn. Other papers with collateral constraints are Schabert (2015) and Hoermann and Schabert (2015), which also fall in the literature on LOLR. They introduce a cash-in-advance constraint for the purchase of goods. Since new cash can only be obtained from the CB by pledging (scarce) collateral, CB balance sheet operations are not neutral anymore. But the collateral constraint applies to households, which are not balance sheet constrained, unlike the commercial banks in my model.

The last literature to which my paper connects is the literature on sovereign debt accumulation, which arises in my model through the collateral effect. Drechsler et al. (2014) find that weakly capitalized banks pledged riskier collateral, and actively invested in distressed sovereign debt after the start of the European sovereign debt crisis in 2010. Crosignani (2014) develops a general equilibrium model in which banks shift into domestic sovereign debt when they are undercapitalized: domestic sovereign debt has the highest payoff in the good state of the world, the only state which banks care about, since they will be bankrupt in the bad state of the world. Becker and Ivashina (2014) find empirical evidence for crowding out of private loans by increased holdings of government bonds, and find evidence for financial repression by governments.

2 Model

2.1 Households

Households are infinitely lived, and exhibit identical preferences and asset endowments. Each household consists of bankers and workers. There is perfect consumption insurance within the household. Households obtain utility from consumption $c_t$, while labor $h_t$ provides disutility. Households receive income from labor at wage rate $w_t$. Households can invest in 1 period debt
$W_t$, which consists of deposits $d_t$ and loans to the CB (Central Bank) $d_{cb}^t$, i.e. $W_t = d_t + d_{cb}^t$. Investment of $W_{t-1}$ in period $t-1$ yields repayment of principal $W_{t-1}$ and interest $r^d_t$ in period $t$. Households can also invest in government bonds with return $r^b_t$ on their holdings $q_{t-1}^b s_{t-1}^{b,h}$, where $q_{t-1}^b$ is the bond price in period $t-1$, and $s_{t-1}^{b,h}$ the number of bonds purchased in period $t-1$. Besides that, they receive income from profits $\Pi_t$ from the production sector and the financial sector. Income is used for consumption $c_t$, investment in 1 period debt $W_{t-1}$ and investment in government bonds $s_{t-1}^{b,h}$ at price $q^b_t$. Households pay a cost for the intermediation of government bonds though, which is quadratic in the deviation of the number of bonds from the level $\hat{s}_{b,h}$. The government levies lump sum taxes $\tau_t$. Households maximize expected life-time utility subject to the budget constraint:

$$\max_{\{c_t, h_t, W_t, s_{t}^{b,h}\}} E_t \left[ \sum_{i=0}^{\infty} \beta^i \left\{ \log (c_{t+i} + v) - \chi \frac{h_{t+i}^{1+\varphi}}{1+\varphi} \right\} \right]$$

subject to

$$c_t + \tau_t + W_t + q_t^b s_{t}^{b,h} + \frac{k_{s,h}}{2} (s_t^{b,h} - \hat{s}_{b,h})^2 = w_t h_t + (1 + r^d_t) W_{t-1} + (1 + r^b_t) q_{t-1}^b s_{t-1}^{b,h} + \Pi_t$$

This will give rise to the following first order conditions:

$$c_t : \lambda_t = (c_t - \beta c_{t-1})^{-1} - \beta v E_t \left[ (c_{t+1} - \beta c_t)^{-1} \right], \quad (1)$$

$$h_t : \chi h_t^\varphi = \lambda_t w_t, \quad (2)$$

$$W_t : E_t \left[ \frac{\beta \lambda_{t+1}}{\lambda_t} (1 + r^d_{t+1}) \right] = 1, \quad (3)$$

$$s_{t}^{b,h} : E_t \left[ \frac{\beta \lambda_{t+1}}{\lambda_t} \left( \frac{(1 + r^d_{t+1}) q_t^b}{q^b_t + k_{s,h}} \frac{(s_t^{b,h} - \hat{s}_{b,h})}{s_t^{b,h} - \hat{s}_{b,h}} \right) \right] = 1, \quad (4)$$

where $\lambda_t$ is the marginal utility of consumption. The household’s stochastic discount factor is $\beta \Delta_{t,t+1} = \beta \lambda_{t+1}/\lambda_t$.

### 2.2 Financial intermediaries

Financial intermediaries channel funds from savers to borrowers. They invest in two asset classes: private loans to intermediate goods producers $s_{t}^{k,p}$ and government bonds $s_{t}^{b,p}$. Total assets $p_{j,t}$ are given by:

$$p_{j,t} = q_j^k s_{j,t}^{k,p} + q_j^b s_{j,t}^{b,p},$$

where $q_j^k$ is the price of private loans, and $q_j^b$ the price of government bonds. Intermediaries fund their assets through net worth $n_{j,t}$, risk-free household deposits $d_{j,t}$ and CB (Central Bank) liquidity $d_{cb,j,t}$.

$$p_{j,t} = n_{j,t} + d_{j,t} + d_{cb,j,t},$$
New net worth is the difference between the return on assets and the return on debt funding, augmented by financial sector support of the government \((n^g_{j,t} = \tau_t^g n_{j,t-1})\) and repayment of earlier provided support \((\tilde{n}^g_{j,t} = \tilde{\tau}^n_{t} n_{j,t-1})\), where \(\tau^g_t\) and \(\tilde{\tau}^n_t\) will be defined in section 2.4. New net worth is given by:

\[
n_{j,t+1} = (1 + r^b_{t+1}) q_t^b s^b_{j,t} + (1 + r^b_{t+1}) q_t^b s^b_{j,t} - (1 + r^d_{t+1}) d_{j,t} - (1 + r^d_{t+1}) d_{j,t} + n^g_{j,t+1} - \tilde{n}^g_{j,t+1} = (1 + r^b_{t+1} + \tau^g_{t+1} - \tilde{\tau}^n_{t+1} + \tilde{n}^g_{j,t+1}) q_t^b s^b_{j,t} - (1 + r^d_{t+1} + \tau^g_{t+1} - \tilde{\tau}^n_{t+1}) d_{j,t},
\]

where \(r^b_t\) is the net real return on private loans in period \(t\), \(r^d_t\) the net real return on government bonds, \(r^{cb}_t\) the net real return on deposits and \(r^{cb}_t\) the net real return on CB funding. The financial intermediary is allowed to continue operating with probability \(\theta\), which is i.i.d. and exogenous, both in time and the cross-section, see [Gertler and Karadi (2011)]. When the intermediary is forced to stop operating, all net worth is paid out to the household, the ultimate owners of the financial intermediary. The financial intermediary is interested in maximizing the expected future discounted profits:

\[
V \left( s^b_{j,t-1}, s^b_{j,t-1}, d_{j,t-1}, d^{cb}_{j,t-1} \right) = \max \mathbb{E}_t \left[ \beta \Lambda_{t+1} \left\{ (1 - \theta) n_{j,t+1} + \theta V \left( s^b_{j,t}, s^b_{j,t}, d_{j,t}, d^{cb}_{j,t} \right) \right\} \right]
\]

Households face an agency problem when deciding on the amount of funds to save through deposits, as in [Gertler and Karadi (2011) 2013]: financial intermediaries have the capability to divert assets when moving from the current to the next period. Depositors can force the intermediary into bankruptcy in that case, but will only recoup a fraction \(1 - \lambda_a\) of asset class \(a\). The remaining fraction \(\lambda_a\) of each asset class is paid out as a dividend to the household owning the intermediary. Depositors, however, anticipate this possibility, and will in equilibrium only provide deposits up to the point where the continuation value of the intermediary is larger or equal than the opportunity cost of diverting the assets.

\[
V \left( s^b_{j,t-1}, s^b_{j,t-1}, d_{j,t-1}, d^{cb}_{j,t-1} \right) \geq \lambda q_t^b s^b_{j,t} + \lambda \delta q_{t}^b s^b_{j,t}.
\]

Financial intermediaries have access to CB liquidity \(d^{cb}_{j,t}\), but are required to post collateral in the form of government bonds. The commercial bank remains the legal owner of the bond, and will receive the interest payments from the bond after repayment of \(d^{cb}_{j,t}\) to the CB in period \(t+1\), unless the financial intermediary becomes insolvent, a case I abstain from through a proper calibration. The collateral constraint has the following functional form:

\[
d^{cb}_{j,t} \leq \theta q_t^b s^b_{j,t},
\]
where \( \theta_t \) is the haircut parameter that regulates the collateral policy of the CB, see section 2.4. A low \( \theta_t \), or a high haircut, indicates that a commercial bank will obtain little CB financing for a given amount of government bonds. The optimization problem of the financial intermediary can now be formulated:

\[
V_{j,t} = \max_{\{s_{j,t-1}^k, s_{j,t-1}^b, d_{j,t-1}, d_{j,t}^b\}} E_t \left[ \beta \Lambda_{t,t+1} \left\{ (1 - \theta) n_{j,t+1} + \theta V_{j,t+1} \right\} \right], \\
\text{s.t.} \ \\
V_{j,t} \geq \lambda_b q_{t} s_{j,t}^k + \lambda_b q_{t} s_{j,t}^b, \\
n_{j,t} + d_{j,t} + d_{j,t}^b \geq \lambda_b q_{t} s_{j,t}^k + \lambda_b q_{t} s_{j,t}^b, \\
\eta_t \geq (1 + r_{t+1}^b + \tau_{t+1}^b - \bar{r}_t^b) d_{j,t-1} + (1 + r_{t+1}^b + \tau_{t+1}^b - \bar{r}_t^b) q_{t-1} s_{j,t-1}^b, \\
\eta_t \geq (1 + r_{t+1}^b + \tau_{t+1}^b - \bar{r}_t^b) d_{j,t-1} - (1 + r_{t+1}^b + \tau_{t+1}^b - \bar{r}_t^b) d_{j,t-1}, \\
\theta_t q_{t} s_{j,t}^b \geq \phi_{j,t}^b.
\]

where we have abbreviated the value function of the financial intermediary by \( V_{j,t} = V \left( s_{j,t-1}^k, s_{j,t-1}^b, d_{j,t-1}, d_{j,t}^b \right) \).

After going through the optimization in appendix A.1, we find the following first order conditions:

\[
\frac{\lambda_b}{\lambda_k} E_t \left[ \Omega_{t,t+1} \left( r_{t+1}^b - r_{t+1}^d \right) \right] = E_t \left[ \Omega_{t,t+1} \left( r_{t+1}^b - r_{t+1}^d \right) \right] + \theta_t \left( \frac{\omega_t}{1 + \mu_t} \right),
\]

\[
\lambda_b \left( \frac{\mu_t}{1 + \mu_t} \right) = E_t \left[ \Omega_{t,t+1} \left( r_{t+1}^b - r_{t+1}^d \right) \right],
\]

\[
\frac{\omega_t}{1 + \mu_t} = E_t \left[ \Omega_{t,t+1} \left( r_{t+1}^b - r_{t+1}^d \right) \right],
\]

\[
\eta_t = E_t \left[ \Omega_{t,t+1} \left( 1 + r_{t+1}^b + \tau_{t+1}^b - \bar{r}_{t+1} \right) \right],
\]

\[
(1 + \mu_t) \eta_t n_{j,t} \geq \lambda_b q_{t} s_{j,t}^k + \lambda_b q_{t} s_{j,t}^b.
\]

where \( \mu_t \) is the Lagrangian multiplier on the bank’s balance sheet constraint [5], or equivalently [11], and \( \omega_t \) the Lagrangian multiplier on the collateral constraint [6]. \( \eta_t \) denotes the shadow value of an additional unit of net worth. \( \Omega_{t,t+1} = \beta \Lambda_{t,t+1} \{ (1 - \theta) + \theta (1 + \mu_{t+1}) \eta_{t+1} \} \) is the intermediaries’ stochastic discount factor, and can be interpreted as the household’s stochastic discount factor \( \beta \Lambda_{t,t+1} \), augmented by an additional term to incorporate the effect of the financial frictions.

The first order condition [7] is the condition that pins down the bank’s portfolio choice on the asset side of the balance sheet. The left hand side denotes the marginal benefit to the financial intermediary from investing an additional unit of private loans, valued by the intermediaries’ stochastic discount factor, and corrected by the term \( \lambda_b / \lambda_k \) to reflect the fact that the financial friction is more severe for private loans than for government bonds. The right hand side denotes the marginal cost of giving up an additional unit of government bonds, measured by the credit spread between government bonds and the deposit rate. But government bonds also derive value from the fact that they serve as collateral with which intermediaries can obtain CB funding.
which is reflected by the second term on the right hand side of equation (7).

Equation (8) is the first order condition for private loans. We clearly see that the presence of a binding bank balance sheet constraint ($\mu_t > 0$ in equation (11)) limits the ability of commercial banks to arbitrage away the difference between the expected rate of return on private loans and deposits, since they cannot expand the balance sheet.

Equation (9) denotes a portfolio choice on the liabilities side of the balance sheet: an increase in the credit spread between deposits and CB liquidity $r^d_{t+1} - r^cb_{t+1}$ increases the collateral value of government bonds $\omega_t$, everything else equal, and leads to a shift into government bonds, see (7), which allows the commercial bank to increase the amount of CB funding.

Equation (10) shows the shadow value of an additional unit of net worth $\eta_t$, which is equal to the gross return on deposits, augmented by the financial sector support per unit of net worth $\tau^u_{t+1}$ and the repayment per unit of net worth $\bar{\tau}^u_{t+1}$. Equation (11) denotes the balance sheet constraint, and limits the amount to which the bank’s balance sheet can be expanded in terms of current net worth $n_{j,t}$. Using these first order conditions, I can derive several lemma’s:

**Lemma 1:** there is no first order effect from CB liquidity provision on private credit if the bank balance sheet constraint (11) is not binding.

**Proof:** When equation (11) is not binding, $\mu_t$ is equal to zero. From equation (8), we see that the expected return on private loans must equal the expected rate on deposits. The term on the left hand side of equation (7) is zero, and is not affected by $\omega_t$, which captures the effects from CB liquidity provision.

**Lemma 2:** when the interest rate on deposits $r^d_{t+1}$ and on CB liquidity facilities $r^cb_{t+1}$ are the same, CB liquidity provision and the haircut parameter $\theta_t$ do not matter for the allocation.

**Proof:** when $r^d_{t+1} = r^cb_{t+1}$, we find from equation (9) that $\omega_t = 0$. This implies that the second term on the right hand side of equation (7) drops out. This is the only first order condition where $\omega_t$, which measures the collateral value of government bonds, and the haircut parameter $\theta_t$ show up. Hence, CB liquidity provision does not affect the portfolio decision between private loans and government bonds, and neither the allocation.

**Lemma 3:** when the CB liquidity policy does not affect the allocation ($\omega_t = 0$), it does not matter for the allocation whether I take the collateral constraint (6) to be binding or not.

**Proof:** When $\omega_t = 0$, the collateral value of government bonds is zero, and the allocation is not affected. The interest rate on deposits and CB liquidity provision is the same, hence commercial banks are indifferent between the two funding sources. Hence I can take the collateral constraint to be binding in my simulations.
Lemma 4: CB liquidity provision and the collateral policy directly affect the allocation only when the bank balance sheet constraint (11) is binding and the interest rate on deposits $r_{d,t+1}$ is not equal to the interest rate on CB liquidity facilities $r_{cb,t+1}$.

Proof: Lemma 4 automatically follows from combining lemma’s 1-3.

Because of lemma 4, and the fact that European commercial banks have been undercapitalized since 2007, I will take both the bank’s balance sheet constraint (11) and the collateral constraint (6) to be binding in my simulations.

Aggregate law of motion net worth

The law of motion for aggregate net worth consists of the net worth of the bankers that are allowed to continue operating, together with the aggregate net worth given to new bankers, which is equal to a fraction $\chi$ of previous period assets $p_{t-1}$. Together with net government support $n_{g,t} - \tilde{n}_{g,t}$, I obtain the following law of motion:

$$n_t = \theta \left[ (r_k^b - r_{d,t}^b) q_{k,t-1}^k s_{k,t-1}^k + (r_{b,t}^b - r_{d,t}^b) q_{b,t-1}^b s_{b,t-1}^b + (r_{d,t}^b - r_{cb,t}^b) d_{cb,t-1}^b + (1 + r_{d,t}^b) n_{t-1} + \chi p_{t-1} + n_t^q - \tilde{n}_t^q \right].$$

(12)

I introduce the variable $\omega_k^t$ to denote the fraction of assets invested in private loans:

$$\omega_k^t = q_k^t s_k^t / p_t.$$

(13)

In my simulations, I will refer to this variable as “Portfolio weight claims”.

2.3 Production sector

The production factor is modeled in standard New-Keynesian fashion. I will shortly outline the setup below, with a more detailed exposition in appendix A.2.

2.3.1 Intermediate Goods Producers

A continuum of intermediate goods producers, that face perfect competition, acquire capital $k_{i,t-1}$ from capital producers at the end of period $t - 1$ for a price $q_k^{t-1}$ through a state-contingent loan $s_{k,i,t}^{k-1} = k_{i,t-1}$ from the financial intermediaries. Next period’s profits can credibly be pledged to the intermediaries, as in Gertler and Kiyotaki (2010). After realization of the shocks, the producers hire labour $h_{i,t}$ at a wage $w_t$, and start producing intermediate goods with previous period capital $k_{i,t-1}$ and labor $h_{i,t}$. After production, the intermediate goods producers pay a state-contingent net real return $r_k^t$ over claims issued in period $t$, with the following production technology:

$$y_{i,t} = a_t (\xi k_{i,t-1})^\alpha h_{i,t}^{1-\alpha}.$$
Quality of capital $\xi_t$ and total factor productivity $a_t$ are driven by exogenous AR(1) processes.

Output $y_{i,t}$ is sold to retail firms for a price $m_t$. The effective capital stock (after depreciation) is sold to the capital producers for a price $q^k_t$ and the proceeds are used to pay back the loans and a net return to the financial intermediaries.

2.3.2 Capital Producers

Capital producers purchase the effective capital stock that is left after production (including depreciation), $\xi_t k_{t-1}$, from the intermediate goods producers. They also purchase an amount $i_t$ of final goods, and convert the old capital stock and newly purchased final goods into new capital. The newly produced capital stock $k_t$ is subsequently sold to the intermediate goods producers at the same price $q^k_t$ that was paid for the capital after production. The capital producers face convex adjustment costs, so that for every unit $i_t$ only $1 - \Psi(i_t)$ units of capital are produced, with $i_t = i_t / i_{t-1}$ representing the change in the investment level. The expression for the capital stock after the capital producers have produced (or output of capital producers) is then:

\[ k_t = (1 - \delta)\xi_t k_{t-1} + (1 - \Psi(i_t)) i_t, \quad \text{with} \quad \Psi(i_t) = \frac{\gamma}{2} (i_t - 1)^2 \]

2.3.3 Retail Firms

A continuum of differentiated retail firms indexed by $i \in [0, 1]$ transform intermediate goods $y_{i,t}$ into differentiated retail goods $y_{f,t} = y_{i,t}$ under perfect monopolistic competition. Each period, only a random portion $(1 - \psi)$ of retail firms is allowed to reset their prices $P_{f,t}$, while the other firms must keep their prices fixed, see Calvo (1983) and Yun (1996). Retail firms face the demand function $y_{f,t} = \left( P_{f,t} / P_t \right)^{-\epsilon} y_t$, with $\epsilon > 1$ and price index $P_t^{1-\epsilon} = \int_0^1 P_{f,t}^{1-\epsilon} df$.

2.3.4 Final Goods Producers

Final goods producers purchase the differentiated retail goods $y_{f,t}$ to produce final goods. They face the following technology constraint:

\[ y_t^{(\epsilon-1)/\epsilon} = \int_0^1 y_{f,t}^{(\epsilon-1)/\epsilon} df, \]

where $\epsilon$ represents the elasticity of substitution between goods bought from the retail firms. Final good producers operate in a perfectly competitive market. Hence they take prices as given, and sell their goods for the same price $P_t$. Final goods are sold to households and government for consumption, and to capital producers as input for investment.
2.4 Government

2.4.1 Fiscal authority

The government issues $b_t$ long term bonds in period $t$, and raises $q^b_t b_t$ in revenue with $q^b_t$ the market price of bonds. I parametrize the maturity structure of government debt like Woodford (1998, 2001). A bond issued in period $t - 1$ pays a cash flow $r_c$ in period $t$, $\rho r_c$ in period $t + 1$, $\rho^2 r_c$ in period $t + 2$, etc. The rate of return $r^b_t$ on a bond purchased in period $t - 1$ is given by:

$$1 + r^b_t = \frac{r_c + \rho q^b_t}{q^b_{t-1}},$$

(14)

where $\rho$ pins down the maturity of government debt, see for more details Van der Kwaak and Van Wijnbergen (2014). The government also raises revenue by levying lump sum taxes on the households. Government purchases are constant in real terms: $g_t = G$. Furthermore the government may provide assistance to the financial intermediaries by injecting capital $n^g_t$, and it receives repayment of previously administered support ($\tilde{n}^g_t$) and CB profits $\Pi^b_{cb}$. So the budget constraint becomes:

$$q^b_t b_t + \tau_t + \tilde{n}^g_t + \Pi^b_{cb} = g_t + n^g_t + (1 + r^b_t) q^{b}_{t-1} b_{t-1}.$$  

(15)

The tax rule of the government is given by a rule which makes sure the intertemporal government budget constraint is satisfied (Bohn, 1998):

$$\tau_t = \bar{\tau} + \kappa_b (b_{t-1} - \bar{b}) + \kappa_n n^g_t, \quad \kappa_b \in (0, 1], \quad \kappa_n \in [0, 1].$$  

(16)

$\bar{b}$ is the steady state level of debt. $\kappa_n$ controls the way government transfers to the financial sector are financed. If $\kappa_n = 0$, support is financed by new debt. $\kappa_n = 1$ implies that the additional spending is completely financed by increasing lump sum taxes. I parametrize government support as follows:

$$n^g_t = \tau^n_t n_{t-1}, \quad \zeta \leq 0, \quad l \geq 0,$$

(17)

$$\tau^n_t = \zeta \xi_{t-1}.$$  

Thus the government provides funds to the financial sector if $\zeta < 0$ (a negative shock $\xi_{t-1}$ to the quality of capital). Depending on the value of $l$, the government can provide support instantaneously ($l = 0$), or with a lag ($l > 0$). Furthermore, $\vartheta$ indicates the extent to which the

\[ \text{The duration of the bond is equal to } \sum_{j=1}^{\infty} \beta^j (\rho^{-1} r_c) \sum_{j=1}^{\infty} \beta^j (\rho^{-1} r_c) \]
government needs to be repaid:

\[ \tilde{n}_t^g = \vartheta n_{t-c}^g, \quad \vartheta \geq 0, \quad c \geq 1. \] (18)

\( \vartheta = 0 \) means the support is a gift from the government. In case \( \vartheta = 1 \), the government aid is a zero interest loan, while a \( \vartheta > 1 \) implies that the financial intermediaries have to pay interest over the support received earlier. \(^4\) The parameter \( c \) denotes the amount of time after which the government aid has to be paid back.

### 2.4.2 Central Bank Conventional Interest Rate Policy

The Central Bank sets the nominal interest rate on deposits \( r_t^n \) according to a standard Taylor rule, in order to minimize output and inflation deviations:

\[ r_t^n = (1 - \rho_r) \left( r^n + \kappa_\pi (\pi_t - \bar{\pi}) + \kappa_y \log(y_t / y_{t-1}) \right) + \rho_r r_{t-1}^n + \varepsilon_{r,t}, \] (19)

where \( \varepsilon_{r,t} \sim N(0, \sigma_r^2) \), and \( \kappa_\pi > 1 \) and \( \kappa_y > 0 \) (active monetary policy). The parameter \( \bar{\pi} \) is the target inflation rate. The real interest rate on deposits then equals:

\[ 1 + r_t^d = \frac{(1 + r_t^n)}{\pi_t}. \] (20)

### 2.4.3 Central Bank Balance Sheet Policy

Besides conventional interest rate policy, the CB also provides liquidity \( d_t^{cb} \) to the commercial banking system. In order to obtain access to this facility, commercial banks are required to provide collateral in the form of government bonds, see equation (6). Collateral is needed in order for the creditor to recoup the principal in case of a debtor’s bankruptcy. A haircut is applied to protect the creditor from capital losses on the collateral.

The CB has two instruments for its balance sheet policy. It controls the nominal interest rate \( r_t^{n,cb} \) on its liquidity facility, and the haircut parameter \( \theta_t \) applied to the collateral. For a given interest rate \( r_t^{n,cb} \) and haircut parameter \( \theta_t \), the CB provides as much liquidity as demanded by the commercial banks, in line with the Fixed Rate Full Alotment policy of the ECB after October 2008.\(^5\) The CB can lower the nominal interest rate \( r_t^{n,cb} \) in times of crisis with respect to the interest rate on regular deposit funding \( r_t^n \) by increasing the credit spread \( \Gamma_t^{cb} \):

\[ r_t^{n,cb} = r_t^n - \Gamma_t^{cb}. \] (21)

\(^4\)The case where \( \vartheta > 1 \) happened in the Netherlands, where financial intermediaries received government aid with a penalty rate of 50 percent. EU state support rules usually require financial intermediaries to repay previously received state support with a penalty rate.

\(^5\)Before October 2008, the ECB used to auction a given amount of liquidity against eligible collateral, with the interest rate being determined in the auctioning process. In October 2008, the ECB switched to a Fixed Rate Full Alotment policy, under which the ECB sets the collateral haircuts and the interest rate, and provides commercial banks with the liquidity demanded [European Central Bank 2015].
The credit spread \( \Gamma^b_t \) between the nominal rate on deposits and the nominal rate on the CB liquidity is given by:

\[
\Gamma^b_t = \bar{\Gamma}^b + \kappa^b \left( \xi_t - \bar{\xi} \right). 
\]  
(22)

The second policy instrument is the haircut parameter \( \theta_t \) that is applied to the collateral. From equation (6) we see that a commercial bank delivering \( q^b s^b j,t \) units of CB liquidity, hence the haircut is \( 1 - \theta_t \). A higher value of \( \theta_t \) allows the commercial bank to obtain more CB funding for the same number of government bonds, which increases the collateral value of government bonds. The haircut parameter \( \theta_t \) is possibly time-varying, and is given by the following process:

\[
\theta_t = \bar{\theta} + \kappa^d,cb \left( \xi_t - \bar{\xi} \right). 
\]  
(23)

The asset side of the CB balance sheet consists of CB lending \( d^b_{t-1} \) to commercial banks on which it receives a nominal interest rate \( r^{n,cb}_{t-1} \). The liabilities consist of household deposits \( p^d_{t-1} \) on which it pays the nominal deposit rate \( r^{n}_{t-1} \). CB profits (or losses) are passed on to the fiscal authority period by period. Hence CB net worth is zero, and liabilities consist solely of household deposits. Real CB profits (or actually losses, since \( r^{n,cb}_{t-1} \leq r^{n}_{t-1} \)), are given by:

\[
\Pi^b_t = \left( 1 + \frac{r^{n,cb}_{t-1}}{\pi_t} \right) d^b_{t-1} - \left( 1 + \frac{r^{n}_{t-1}}{\pi_t} \right) d^b_{t-1} = \left( r^{cb}_{t} - r^{d}_{t} \right) d^b_{t-1}, 
\]  
(24)

where \( \pi_t \) is the gross inflation rate, and the net real return on CB liquidity \( r^{cb}_{t} \) is given by:

\[
1 + r^{cb}_{t} = \frac{1 + r^{n,cb}_{t-1}}{\pi_t}. 
\]

Note that without the CB intervention, commercial banks would have to pay the interest rate \( r^{d}_{t} \). Hence the CB losses \( \Pi^b_t \) can be interpreted as the subsidy given by the CB to the commercial banks.

### 2.5 Equilibrium conditions

In equilibrium, the total number of private loans \( k_t \) must equal the total number of loans provided by the financial intermediaries.\(^6\) Similarly, the total bond supply must equal the bonds purchased by the financial intermediaries.\(^7\)
by the household sector and financial intermediaries:

\[ k_t = s_{t}^{k,p}, \]
\[ b_t = s_{t}^{b,h} + s_{t}^{b,p}. \]  
(26)  
(27)

The aggregate resource constraint is given by:

\[ y_t = c_t + i_t + g_t + \frac{1}{2} \kappa_{s_{b,h}} \left( s_{t}^{b,h} - \tilde{s}_{b,h} \right)^2 \]  
(28)

3 Calibration

I calibrate the model on a quarterly frequency. The targets can be found in Table 1, while the parameter values can be found in Table 2 see appendix B. Most of the parameters are common in the literature on DSGE models, or frequently used in models containing financial frictions. I follow Gertler and Karadi (2011) and Van der Kwaak and Van Wijnbergen (2014) for these parameters. The financial sector and sovereign debt levels are calibrated to target data from the European periphery. Standard parameter values are the subjective discount factor \( \beta \), the habit formation parameter \( \psi \), the inverse Frisch elasticity \( \varphi \), the elasticity of substitution for final goods producers \( \epsilon \), the Calvo probability of keeping prices fixed \( \psi \), the capital share in output \( \alpha \), the investment adjustment parameter \( \gamma \), and the smoothing parameters for production \( \rho_{z} \) and the quality of capital \( \rho_{\xi} \). These values are the same as in Gertler and Karadi (2011) and Van der Kwaak and Van Wijnbergen (2014). Several government policy parameters are also taken from Van der Kwaak and Van Wijnbergen (2014): the cash flow payment to the bondholder \( r_{c} \), the tax feedback parameter \( \kappa_{b} \), the inflation feedback parameter of the Taylor rule \( \kappa_{\pi} \), the output feedback parameter of the Taylor rule \( \kappa_{y} \), and the interest rate smoothing parameter \( \rho_{r} \).

Other coefficients are adjusted in order to hit specific targets: the relative utility weight of labor \( \Psi \) is adjusted to have the steady state labor supply equal 1/3. The portfolio adjustment cost for households on bond holdings \( \kappa_{s_{b,h}} \) is set to 0.0025. Households intermediation costs for government bonds are introduced in order to have the return on bonds (including intermediation costs) and deposits equal in the non-stochastic steady state. But government bonds are a relatively straightforward financial product, and there is no reason to assume why capital markets would be less efficient in intermediation of government bonds than financial intermediaries, because contrary to private credit, financial intermediaries do not have an informational advantage regarding government bonds. I therefore set \( \kappa_{s_{b,h}} \) very small, and only slightly bigger than zero.

I target the steady state investment-GDP ratio, and the steady state government spending-GDP ratio, and set both to 0.2, in line with long term average values in the periphery.8 The depreciation rate \( \delta \) is adjusted, in order to target the steady state investment-GDP ratio and

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8The eurozone periphery consists of Cyprus, Greece, Ireland, Italy, Portugal, Slovenia and Spain in my calibration.
a steady state credit spread $\Gamma_k$ between private loans and deposits of 50 basispoints, which is the average difference over the period July 2010 to June 2011 between the interest rate on total loans to non-financial corporations and total overnight deposits to households and non-profit institutions in the periphery of the eurozone.

The steady state leverage ratio is targeted by taking monthly country level data on MFIs excluding the ESCB from the European Central Bank (2015). I aggregate total assets from the periphery countries, as well capital and reserves. I divide aggregate assets over aggregate capital and reserves to find a monthly time-series for the average leverage ratio, which was equal to 12 in the beginning of 2011. Due to the fact that the cash flows from private loans are the residual after wages have been paid out, private loans are more like equity than debt. Taking a leverage ratio of 12 would therefore overstate fluctuations in net worth. Hence I reduce the steady state leverage ratio to 6, a procedure also applied by Gertler and Karadi (2013). The parameter $\theta$ is set in such a way that the average survival period for bankers is 20 quarters. I calibrate the diversion parameter for government bonds $\lambda_b$ to be equal to 0.5 times the diversion rate for private loans $\lambda_k$, as in Gertler and Karadi (2013). The steady state value of the haircut parameter $\bar{\theta}$ is set to 0.95, implying a steady state haircut of 5% for loans taken from the liquidity facilities of the CB. The steady state spread $\Gamma_{cb}$ between the interest rate on deposits and CB liquidity facilities is equal to zero, reflecting the fact that these facilities only have value to financial intermediaries in times of financial crisis.

I calibrate the steady state government liabilities $\bar{q_b}\bar{b}$ to be equal to 100% of annual steady state output, in line with the average debt level in the periphery at the start of the crisis in 2011. The average duration of government bonds is 5 years and can be calculated from the stress tests performed by the European Banking Authority (2011). We set the steady state fraction of government bonds placed at commercial banks to be equal to 25% of total government bonds, approximately equal to the average fraction of periphery bonds placed at periphery banks. A financial crisis is initiated as in Gertler and Karadi (2011) by having a one standard deviation capital quality shock of $\sigma_\xi = 0.050$, and an autoregressive component of $\rho_\xi = 0.66$.

Several robustness checks regarding the parametrization of the model can be found in appendix F and show that the results are not qualitatively much different for different parameter values.

4 Results

In this section I will discuss the results from my model simulations. I will first simulate a financial crisis with no additional support measures to explain the general mechanism of the model. I then proceed to look at the effect of cheap liquidity provision by the CB. I next investigate the impact of varying the haircut parameter, and I conclude by comparing the effect of cheap CB liquidity provision with the effect of a debt-financed recapitalization of the financial sector by the fiscal authority.
4.1 Financial crisis impact, no additional policy

I start by inspecting the response of the economy to a financial crisis in figure [1] where the financial crisis is initiated through a capital quality shock of 5% compared to the steady state, as in [Gertler and Karadi (2011)]. For now, I abstain from a policy intervention by either the CB or the fiscal authority. The model response is very similar to [Van der Kwaak and Van Wijnbergen (2014)]: the capital quality shock deteriorates the quality of the private loans, and hence financial intermediaries suffer losses. Net worth deteriorates, and (commercial) bank balance sheet constraints tighten. Credit spreads and (expected) interest rates increase, reducing the demand for private loans and capital, which leads to a drop in the price of capital. Remember that the capital is sold by intermediate goods producers after production, and the proceeds are used to repay the loan from the commercial bank. Hence a lower capital price leads to additional losses on the outstanding loans to the intermediate goods producers. Commercial banks’ net worth deteriorates further, leading to a second round of interest rate increases.

But balance sheet tightening of commercial banks does not only affect credit spreads on private loans, but also induces commercial banks to sell government bonds. Bond prices go down, and impose capital losses on existing bondholders, which results in an additional fall in commercial banks’ net worth on top of the losses on private loans, further tightening bank balance sheet constraints. Since the interest rate on deposits and CB liquidity is the same, the collateral value of government bonds is zero, see equation [9]. Commercial banks shrink their balance sheet by selling government bonds to the household sector, thereby increasing the fraction of the balance sheet invested in private credit, see “Portfolio weight claims”, equation [13]. Remember that banks can only sell government bonds to households but not private loans. A lower bond price makes it more attractive for households (who are not balance sheet constrained) to purchase additional government bonds. When commercial banks offload government bonds to households, CB liquidity falls along by approximately 80% with respect to the steady state.

Lower private credit provision adversely affects the real economy: investment drops with 8% with respect to steady state, and a lower capital stock leads to lower wages and reduced household income. The wealth effect causes consumption to fall, and together with the fall in investment, a drop in output of almost 3% results.
Figure 1: Impulse response functions for the case with no additional policy. The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state.
4.2 No additional policy vs. CB intervention

Now consider the effect of cheap liquidity provision by the CB in figure 2. I compare the no intervention case from section 4.1 (blue, solid) with a policy that entails an increase in the credit spread $\Gamma_{cb}^t$ between the nominal interest rate on deposits and the CB liquidity facilities of 50 quarterly basis points on impact (red, slotted) in order to simulate the LTRO intervention\(^9\) by the ECB at the end of 2011, beginning of 2012.

The low interest rate induces commercial banks to shift from funding through regular deposits to cheap CB liquidity, which increases net worth everything else equal. The availability of cheap liquidity increases the (collateral) value of government bonds to commercial banks, see equation (9): not only does a bond produce a cash flow in the future, it also allows the intermediary to gain immediate access to cheap CB liquidity. But since intermediaries are balance sheet constrained, purchasing more government bonds also forces them to shift out of private loans (“Portfolio weight claims”, see equation (13)), which are not eligible as collateral. The credit spread between private loans and deposits increases by 75 basis points on impact compared with the no intervention case. Demand for private loans is reduced, and pushes down the price of capital further, leading to additional capital losses on private loans. Net worth deteriorates further, and tightens bank balance sheet constraints, leading to a second round of interest rate increases on private loans and additional capital losses. Net worth drops with respect to the no intervention case on impact. The shift out of private loans initially leads to a substantial drop in investment and pushes down the trough of output by almost 0.5%.

The lower funding costs on CB liquidity, however, quickly start to increase commercial bank profits, and lead to higher net worth compared with the no intervention case. But as long as the CB funding is cheaper than deposit funding, intermediaries will invest a larger fraction of their balance sheet in government bonds, thereby crowding out private credit. Commercial banks, however, are better capitalized after the cheap liquidity provision by the CB has ended, and have more space on their balance sheets to finance capital purchases by the intermediate goods producers. Investment increases with respect to the no intervention case, leading to a faster recovery along the entire time path once the intervention has ended.

In the short run, these results contradict the regular narrative concerning a monetary expansion, in which lower interest rates lead to higher output. Instead, the ‘collateral effect’ dominates the ‘subsidy effect’. The key reason is the fact that commercial banks are balance sheet constrained, and cannot increase their holdings of government bonds, necessary as collateral, without shedding private loans. In the long run, the cheap CB funding amounts to an indirect recapitalization of commercial banks and leads to a stronger economic recovery.

The results clearly introduce a trade-off for policymakers: if they are concerned about the

\(^9\)Under the LTRO program, commercial banks were allowed to borrow for 3 years at a nominal interest rate equal to the MRO rate, which is the short term policy rate of the ECB. The difference with the 1-year Euribor, a measure of unsecured interbank funding is at least 1%, see also figure 8 in appendix. I therefore take an annual spread between CB facilities and deposits of 2% since the LTRO has a maturity of 3 years, compared with a 1 year maturity of the 1-year Euribor. This gives a quarterly credit spread of 50 basis points.
Financial crisis impact, no additional policy vs. CB intervention

Figure 2: Impulse response functions for the case from section 4.1 with no additional policy (blue, solid) vs. a decrease in the nominal interest rate on CB liquidity facilities of 50 basis points on impact with respect to the nominal interest rate on regular deposit funding (red, slotted). The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state.
short run, providing cheap CB liquidity does not seem to be a good idea, due to the contractionary impact. But when they are concerned about the long run, the intervention helps the recovery by delivering a commercial banking system that is better capitalized.\textsuperscript{10} This raises two questions. First, is the cumulative effect of the CB intervention positive or negative? Second, can the haircut policy, indicated by $\theta_t$, affect the outcome, and if so how?

\textsuperscript{10}In appendix F I perform robustness checks, by varying several key parameters, and find that the results are robust under different parametrizations.
4.3 The cumulative intervention effect and the role of the haircut policy \( \theta_t \)

We saw in the previous section that the ‘collateral effect’ dominates in the short run, while the ‘subsidy effect’ dominates in the long run. This leads us to the question whether the cumulative effect of the cheap liquidity provision is positive or negative. I therefore introduce the cumulative discounted multiplier \( \mu_D \):

\[
\mu_D = \frac{\sum_s \beta^s \left( y_{t+s}^{cbp} - y_{t+s}^{np} \right)}{\sum_s \beta^s \left( d_{t+s}^{cb,cbp} - d_{t+s}^{cb,np} \right)},
\]

where \( x_{t+s}^{cbp} \) denotes the value of variable \( x \) in period \( t + s \) under the CB intervention, and \( x_{t+s}^{np} \) the value of variable \( x \) in period \( t + s \) under the no policy case. Figure 3 displays \( \mu_D \) versus the haircut parameter \( \theta_t \). We clearly see that \( \mu_D \) is approximately zero, and hardly changes as I vary \( \theta_t \). This results in two conclusions: first, cumulatively, the ‘subsidy effect’ and the ‘collateral effect’ offset each other, with a net effect that is hardly different from the case with no CB intervention. Second, \( \mu_D \) hardly changes as the haircut parameter \( \theta_t \) is varied. This suggests that the collateral policy of the CB has no macroeconomic effects. This seems counterintuitive, given the way the cheap liquidity provision by the CB affected the real economy in the previous section, see figure 2.

**CB intervention: discounted cumulative multiplier \( \mu_D \) vs. \( \theta_t \)**

![Discounted cumulative multiplier vs. \( \theta_t \)](image_url)

Figure 3: Discounted cumulative multiplier \( \mu_D \) vs. \( \theta_t \) for the CB intervention of section 4.2, where \( \theta_t \) is the haircut parameter of the CB.
We therefore turn to figure 4, which is the result from the same comparison between no additional policy and CB intervention as in figure 2 but for several steady state values of the haircut parameter $\theta_t$. Figure 4 shows the difference between the two policies, expressed as a percentage of steady state output. The upper panel displays the difference in CB lending, while the lower panel displays the difference in output.

**Variation in haircut parameter $\theta_t$**

![Graph showing difference in CB lending and output for different values of haircut parameter $\theta_t$.]

Figure 4: Both panels display the difference between the case where the nominal interest rate on CB liquidity facilities is lowered by 50 basis points on impact with respect to the nominal interest rate on regular deposit funding and the no intervention case for different steady state values of the haircut parameter $\theta_t$. The blue solid line refers to $\theta_t = 0.20$, the red slotted line to $\theta_t = 0.45$, the green dotted line to $\theta_t = 0.70$, and the black dashed line to $\theta_t = 0.95$. The upper panel displays the difference in CB lending, while the lower panel displays the difference in output.

Contrary to the results in figure 3, we see that the haircut parameter $\theta_t$ has a significant effect on output and CB lending. For a low value of the haircut parameter, $\theta_t = 0.20$, the CB policy has a relatively small effect. $\theta_t = 0.20$ implies a haircut of 80%, i.e. for each unit of CB liquidity, a commercial bank has to provide five times the value in government bonds. Clearly, the collateral value of government bonds is not very high. Even though the interest rate on CB funding is 50 basis points below regular deposit funding, commercial banks hardly take out additional CB liquidity, as can be seen from the blue solid line in figure 4. As the haircut parameter $\theta_t$ increases, the collateral value of government bonds increases. More cheap
funding becomes available for each bond purchased. Since commercial banks are balance sheet constrained, they shift out of private loans, and into government bonds. This can also be seen from equation (7): a higher $\theta_t$ will push up the next period return on private loans $r_{t+1}^k$ and decrease the next period return on government bonds $r_{t+1}^g$. This can only be achieved through a lower price of capital, and a higher bond price, which require the demand for private loans to fall, and for government bonds to rise. The larger $\theta_t$, the stronger the ‘collateral effect’, and the more contractionary the short run effect on output, which is almost 0.5% for $\theta_t = 0.95$. But at the same time, commercial banks take out more cheap debt funding from the CB (see figure 4), which accelerates the (indirect) recapitalization, and leads to a stronger long run recovery.

Figures 3 and 4 lead to a clear conclusion regarding the effectiveness of cheap liquidity provision by the CB: the cumulative effect from the intervention compared with the no intervention case is approximately zero. Hence the policy is not very effective in stimulating output, irrespective of the haircut parameter $\theta_t$. This rather ineffective policy, however, does not imply that the CB cannot affect the real economy. Quite the contrary, the haircut parameter $\theta_t$ allows the CB to influence the time path of the recovery, and shift output losses between periods. Setting a high $\theta_t$ induces commercial banks to shift into government bonds, with a short run contractionary effect, but indirectly recapitalizes commercial banks. Reducing $\theta_t$ mitigates the short run contractionary impact of the CB policy, but leaves commercial banks weaker capitalized once the CB intervention has ended. But no matter what haircut policy $\theta_t$ the CB pursues, the cumulative impact is approximately the same as when the CB does not intervene. This leaves the question whether other ways of recapitalizing the commercial banking system are more effective. I will address this question in the next section.
4.4 CB intervention vs. immediate recapitalization

We have seen in the previous section that the indirect recapitalization by the CB is rather ineffective when looking at the cumulative impact of the policy. I therefore investigate in figure 5 whether a direct recapitalization by the fiscal authority is more effective. The blue solid line refers to the CB intervention from section 4.2 while the red slotted line refers to an immediate debt-financed recapitalization of 1.25% of annual steady state output, see equation (17).

We clearly see that the debt-financed recap has a positive effect compared with the CB intervention, which is most striking in the initial quarters of the financial crisis: credit spreads, net worth, output and investment improve significantly. There are two key reasons why the recap seems to be more effective. First, commercial banks immediately receive new net worth, in contrast to the indirect recap by the CB, in which commercial banks are gradually recapitalized through lower interest rates on CB funding. A direct recap therefore immediately alleviates bank balance sheet constraints, allowing commercial banks to expand the balance sheet at once.

The second reason is the fact that, contrary to the CB intervention, a direct recap does not increase the collateral value of government bonds. The interest rate on household deposits and the CB liquidity facilities is the same under the direct recap policy, and hence the collateral value, indicated by $\omega_t$ in equation (9), is zero. Commercial banks do not have an incentive to load up on government bonds, as is the case under the CB intervention. The ‘collateral effect’ is therefore absent under the direct recap policy, see also equation (7), and only the ‘subsidy’ effect is present.

After approximately 10 quarters, the CB intervention has ended, and the commercial banks have been recapitalized under both the CB intervention and the direct recap policy. The collateral value of government bonds is now zero under both policies, and net worth and other credit market conditions are approximately the same, resulting in similar long run macroeconomic outcomes.
Financial crisis impact: CB intervention vs. immediate recap

Figure 5: Impulse response functions for the case from section 4.2 with a decrease in the nominal interest rate on CB liquidity facilities of 50 basis points on impact with respect to the nominal interest rate on regular deposit funding (blue, solid) vs. the case where the commercial banking system receives new net worth equal to 1.25% of annual GDP (red, slotted) through an immediate debt-financed recapitalization by the fiscal authority. The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state.
To confirm whether a direct recapitalization is more effective than an indirect recap by the CB, I calculate the discounted cumulative multiplier for the direct recapitalization policy in a similar way as in section 4.2:

\[
\mu_D = \frac{\sum_s \beta^s \left( y_{t+s}^{rp} - y_{t+s}^{np} \right)}{\sum_s \beta^s \left( n_{t+s}^{g,rp} - n_{t+s}^{g,np} \right)},
\]

where \( x_{t+s}^{rp} \) denotes the value of variable \( x \) in period \( t + s \) under the direct recap policy. In figure 6, I plot both the discounted cumulative multiplier from figure 3 (red, dotted), as well as the direct recap policy (blue, solid). Since the collateral value of government bonds \( \omega_t \) is zero under the direct recap policy, the haircut parameter \( \theta_t \) does not influence the portfolio decision of commercial banks, and hence the cumulative multiplier for the direct recap policy does not depend on \( \theta_t \). Figure 6 indeed confirms that the direct recap policy is much more effective in lifting output than the CB intervention. The discounted cumulative multiplier is close to one, compared with a cumulative multiplier that is close to zero for the CB intervention.

**Direct recap: discounted cumulative multiplier \( \mu_D \) vs. \( \theta_t \)**

![Figure 6: Discounted cumulative multiplier \( \mu_D \) vs. \( \theta_t \) for the CB intervention (red, dotted line), and immediate debt-financed recapitalization of 1.25% of annual steady state output (blue, solid). \( \theta_t \) is the haircut parameter of the CB.](image-url)
5 Conclusion

At the end of 2011 and the beginning of 2012, the ECB engaged in large scale liquidity provision to the European commercial banking system in order to prevent a serious credit contraction, and allow commercial banks to repay €200 billion of (unsecured) bank bonds [European Central Bank, 2012]. Commercial banks took out approximately €1000 billion under the LTRO program, in exchange for eligible collateral. The interest rate against which banks could borrow from the ECB was substantially below prevailing interest rates on the unsecured interbank market.

Theoretically, the provision of cheap liquidity by the CB should have an expansionary effect on output: lower funding costs for commercial banks increase bank profits. Bank balance sheets improve, and allow an expansion of credit to the real economy, fuelling the economic recovery, a subsidy effect. When commercial banks are balance sheet constrained, however, cheap liquidity provision can actually be contractionary in the short run, due to a so-called ‘collateral effect’: commercial banks need to post collateral in order to obtain cheap liquidity from the CB. Private loans, however, are usually not eligible for collateral purposes. But since commercial banks are balance sheet constrained after a financial crisis, they have to shed private loans in order to be able to purchase additional government bonds. Private credit provision falls, with a contractionary impact on investment and output.

I construct a DSGE model with balance sheet constrained financial intermediaries that finance private loans and long term government bonds. The CB provides liquidity to commercial banks, for which commercial banks need to post collateral in the form of government bonds. The CB lowers the interest rate on CB liquidity with respect to the interest rate on regular deposit funding in a financial crisis. I calibrate the model on data from the European periphery. I find that the collateral effect dominates the subsidy effect in the short run. In the long run, however, the subsidy effect dominates, because the cheap liquidity provision by the CB indirectly recapitalizes the commercial banks, benefiting the long run economic recovery.

But the cumulative impact of the policy, measured by a cumulative discounted multiplier, is close to zero or even slightly negative, and hardly changes upon variation of the haircut policy. This suggests that the haircut policy of the CB has no macroeconomic impact. I find that this is not the case: the haircut policy has a substantial effect on the time path of investment and output. A smaller collateral haircut increases the collateral value of government bonds. This leads to a larger contractionary impact of the collateral effect in the short run, but also increases the indirect recapitalization by the CB and improves the economic recovery in the long run. Changes in the haircut policy allow the CB to shift output losses between periods.

Because the cumulative impact of the CB intervention is close to zero or slightly negative, I conclude by investigating whether a direct debt-financed recapitalization by the fiscal authority is more effective. I find that this is indeed the case, the discounted cumulative multiplier of this intervention is close to one. There are two reasons. First, commercial banks are recapitalized immediately, in contrast to a gradual recapitalization through cheap liquidity provision by the CB. This leads to an immediate balance sheet expansion, with more lending to the real economy...
as a consequence. Second, the intervention does not affect the collateral value of government bonds, hence the ‘collateral effect’ is absent under this policy.

But a direct recapitalization might not always be feasible: when sovereign default risk is high, a case I do not explore in this paper, the room for additional debt issue to fund a bank recap might be absent and prevent a sovereign from implementing this policy option, see Van der Kwaak and Van Wijnbergen (2014) and the failed Spanish recapitalization attempt in May 2012. The second-best solution through cheap CB liquidity might then be an alternative way to recapitalize the banking system in such circumstances. Also note that a CB might implement such a policy in order to prevent a meltdown of the financial system (a case my model does not incorporate), of which the output costs are much higher than the short run contractionary effect that I find.
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A Mathematical derivations: Price stickiness & Monetary Policy

A.1 Financial intermediaries

In the main text, the collateral constraint is given by $d_{j,t}^b \leq \theta_t q_t^b s_{j,t}$. In this appendix I will apply a more general formulation, namely $d_{j,t}^b \leq \theta_t \kappa_t s_{j,t}^b$, where $\kappa_t$ can be equal to:

$$
\kappa_t = \left\{ \begin{array}{cl}
q_t^b & \text{“Regular collateral constraint”;}
1 & \text{“No risk-adjustment collateral constraint”.}
\end{array} \right.
$$
The law of motion for net worth, which includes recapitalizations by the government and financial sector repayments is given by:

\[
\begin{align*}
n_{j,t+1} &= (1 + r^k_{t+1}) q^k_{t} s^k_{j,t} + (1 + r^b_{t+1}) q^b_{t} s^b_{j,t} \\
&- (1 + r^d_{t+1}) d_{j,t} - (1 + r^c_{t+1}) d^c_{j,t} + n^g_{j,t+1} - \tilde{\eta}^g_{j,t+1} \\
&- (1 + r^k_{t+1}) q^k_{t} s^k_{j,t} + (1 + r^b_{t+1}) q^b_{t} s^b_{j,t} \\
&- (1 + r^d_{t+1}) d_{j,t} - (1 + r^c_{t+1}) d^c_{j,t} + \tau^g_{t+1} n_{j,t} - \tilde{\tau}^g_{t+1} n_{j,t} \\
&- (1 + r^k_{t+1}) q^k_{t} s^k_{j,t} + (1 + r^b_{t+1}) q^b_{t} s^b_{j,t} \\
&- (1 + r^d_{t+1}) d_{j,t} - (1 + r^c_{t+1}) d^c_{j,t} + (\tau^g_{t+1} - \tilde{\tau}^g_{t+1}) d_{j,t} - (1 + r^c_{t+1} + \tau^g_{t+1} - \tilde{\tau}^g_{t+1}) d^c_{j,t}.
\end{align*}
\]

Now we remember the optimization problem of the financial intermediary:

\[
V_{j,t} = \max_{\{s^k_{j,t}, s^b_{j,t}, d_{j,t}, d^c_{j,t}\}} E_t [\beta\Lambda_{t,t+1} \{ (1 - \theta) n_{j,t+1} + \theta V_{j,t+1} \}],
\]

subject to:

\[
\begin{align*}
V_{j,t} &\geq \lambda_k q^k_{t} s^k_{j,t} + \lambda_b q^b_{t} s^b_{j,t}, \\
n_{j,t} + d_{j,t} + d^c_{j,t} &\geq q^k_{t} s^k_{j,t} + q^b_{t} s^b_{j,t}, \\
n_{j,t} &= (1 + r^k_{t} + \tau^g_{t} - \tilde{\tau}^g_{t}) q^k_{t-1} s^k_{j,t-1} + (1 + r^b_{t} + \tau^g_{t} - \tilde{\tau}^g_{t}) q^b_{t-1} s^b_{j,t-1} \\
&- (1 + r^d_{t} + \tau^g_{t} - \tilde{\tau}^g_{t}) d_{j,t-1} - (1 + r^c_{t} + \tau^g_{t} - \tilde{\tau}^g_{t}) d^c_{j,t-1}, \\
\theta_{t}\kappa_1 s^b_{j,t} &\geq d^c_{j,t},
\end{align*}
\]

where we have abbreviated the value function of the financial intermediary by \(V_{j,t} = V \left( s^k_{j,t-1}, s^b_{j,t-1}, d_{j,t-1}, d^c_{j,t-1} \right)\).

We set up the accompanying Lagrangian of the problem:

\[
\mathcal{L} = (1 + \mu_1) E_t [\beta\Lambda_{t,t+1} \{ (1 - \theta) \left( (1 + r^k_{t} + \tau^g_{t} - \tilde{\tau}^g_{t}) q^k_{t} s^k_{j,t} + (1 + r^b_{t} + \tau^g_{t} - \tilde{\tau}^g_{t}) q^b_{t} s^b_{j,t} \\
- (1 + r^d_{t} + \tau^g_{t} - \tilde{\tau}^g_{t}) d_{j,t} - (1 + r^c_{t} + \tau^g_{t} - \tilde{\tau}^g_{t}) d^c_{j,t} \right) \} + \theta V \left( s^k_{j,t}, s^b_{j,t}, d_{j,t}, d^c_{j,t} \right)]
\]

\[
- \mu_k \kappa_1 q^k_{t} s^k_{j,t} - \mu_b \kappa_1 q^b_{t} s^b_{j,t} \\
+ \chi_t \left( (1 + r^k_{t} + \tau^g_{t} - \tilde{\tau}^g_{t}) q^k_{t-1} s^k_{j,t-1} + (1 + r^b_{t} + \tau^g_{t} - \tilde{\tau}^g_{t}) q^b_{t-1} s^b_{j,t-1} \\
- (1 + r^d_{t} + \tau^g_{t} - \tilde{\tau}^g_{t}) d_{j,t-1} - (1 + r^c_{t} + \tau^g_{t} - \tilde{\tau}^g_{t}) d^c_{j,t-1} \right)
\]

\[
\omega_t \left( \theta_{t}\kappa_1 s^b_{j,t} - d^c_{j,t} \right).
\]
This gives rise to the following first order conditions:

\begin{align}
\frac{\partial}{\partial s_{j,t}^{k,p}} \left[ V \left( s_{j,t}^{k,p}, s_{j,t-1}^{b,p}, d_{j,t-1}, d_{j,t}^{b} \right) \right] &= \chi_t \left( 1 + r_t^k + r_{t-1}^b + r_{t-1}^n - \tau_{t-1}^n \right) q_t^k, \\
\frac{\partial}{\partial s_{j,t-1}^{b,p}} \left[ V \left( s_{j,t}^{k,p}, s_{j,t-1}^{b,p}, d_{j,t-1}, d_{j,t}^{b} \right) \right] &= \chi_t \left( 1 + r_t^b + r_{t-1}^n - \tau_{t-1}^n \right) q_t^b, \\
\frac{\partial}{\partial d_{j,t-1}} \left[ V \left( s_{j,t}^{k,p}, s_{j,t-1}^{b,p}, d_{j,t-1}, d_{j,t}^{b} \right) \right] &= -\chi_t \left( 1 + r_t^d + r_{t-1}^n - \tau_{t-1}^n \right), \\
\frac{\partial}{\partial d_{j,t}^{b}} \left[ V \left( s_{j,t}^{k,p}, s_{j,t-1}^{b,p}, d_{j,t-1}, d_{j,t}^{b} \right) \right] &= -\chi_t \left( 1 + r_t^b + r_{t-1}^n - \tau_{t-1}^n \right) .
\end{align}

with complementary slackness conditions:

\begin{align}
\mu_t &= \left( V \left( s_{j,t-1}^{k,p}, s_{j,t-1}^{b,p}, d_{j,t-1}, d_{j,t-1}^{b} \right) - \lambda_k q_t^{k,j,t} = 0, \\
\chi_t &= \left( 1 + r_t^k + \tau_{t-1}^n - \tau_{t-1}^n \right) q_{t-1}^{k} s_{j,t-1}^{b,p} + \left( 1 + r_t^b + \tau_{t-1}^n - \tau_{t-1}^n \right) q_{t-1}^{b} s_{j,t-1}^{b,p} \\
\omega_t &= (\theta_t \kappa_t s_{j,t}^{b} - d_{j,t}^{b}) \omega_t = 0.
\end{align}

Now we apply the envelope theorem to find the derivatives:

\begin{align}
\frac{\partial}{\partial s_{j,t}^{k,p}} \left[ V \left( s_{j,t}^{k,p}, s_{j,t-1}^{b,p}, d_{j,t-1}, d_{j,t}^{b} \right) \right] &= \chi_t \left( 1 + r_t^k + \tau_{t-1}^n - \tau_{t-1}^n \right) q_{t-1}^{k}, \\
\frac{\partial}{\partial s_{j,t-1}^{b,p}} \left[ V \left( s_{j,t}^{k,p}, s_{j,t-1}^{b,p}, d_{j,t-1}, d_{j,t}^{b} \right) \right] &= \chi_t \left( 1 + r_t^b + \tau_{t-1}^n - \tau_{t-1}^n \right) q_{t-1}^{b}, \\
\frac{\partial}{\partial d_{j,t-1}} \left[ V \left( s_{j,t}^{k,p}, s_{j,t-1}^{b,p}, d_{j,t-1}, d_{j,t}^{b} \right) \right] &= -\chi_t \left( 1 + r_t^d + \tau_{t-1}^n - \tau_{t-1}^n \right), \\
\frac{\partial}{\partial d_{j,t}^{b}} \left[ V \left( s_{j,t}^{k,p}, s_{j,t-1}^{b,p}, d_{j,t-1}, d_{j,t}^{b} \right) \right] &= -\chi_t \left( 1 + r_t^b + \tau_{t-1}^n - \tau_{t-1}^n \right). \end{align}
Substitution of the envelope conditions (95) - (98) with (88) - (91), we find the following relation between the different assets:

\[ s_{j,t}^{b,p} : \lambda_k \left( \frac{\mu_t}{1 + \mu_t} \right) + \frac{\chi_t}{1 + \mu_t} = E_t \left[ \beta \Lambda_{t,t+1} \left\{ (1 - \theta) + \theta \chi_{t+1} \right\} \left( 1 + r_{t+1}^b + \tau_{t+1}^n - \bar{\tau}_{t+1}^n \right) \right], \]

\[ s_{j,t}^{b,p} : \lambda_b \left( \frac{\mu_t}{1 + \mu_t} \right) + \frac{\chi_t}{1 + \mu_t} = E_t \left[ \beta \Lambda_{t,t+1} \left\{ (1 - \theta) + \theta \chi_{t+1} \right\} \left( 1 + r_{t+1}^b + \tau_{t+1}^n - \bar{\tau}_{t+1}^n \right) \right], \]

\[ d_{j,t} : \frac{\chi_t}{1 + \mu_t} = E_t \left[ \beta \Lambda_{t,t+1} \left\{ (1 - \theta) + \theta \chi_{t+1} \right\} \left( 1 + r_{t+1}^b + \tau_{t+1}^n - \bar{\tau}_{t+1}^n \right) \right], \]

\[ d_{j,t}^{cb} : \frac{\chi_t}{1 + \mu_t} - \frac{\omega_t}{1 + \mu_t} = E_t \left[ \beta \Lambda_{t,t+1} \left\{ (1 - \theta) + \theta \chi_{t+1} \right\} \left( 1 + r_{t+1}^{cb} + \tau_{t+1}^n - \bar{\tau}_{t+1}^n \right) \right]. \]

Now we define the following variables:

\[ \eta_t = \frac{\chi_t}{1 + \mu_t}, \]

\[ \nu_t^k = \lambda_k \left( \frac{\mu_t}{1 + \mu_t} \right) + \frac{\chi_t}{1 + \mu_t} = \lambda_k \left( \frac{\mu_t}{1 + \mu_t} \right) + \eta_t \]

\[ \nu_t^b = \lambda_b \left( \frac{\mu_t}{1 + \mu_t} \right) + \frac{\chi_t}{1 + \mu_t} = \lambda_b \left( \frac{\mu_t}{1 + \mu_t} \right) + \nu_t \]

\[ \eta_t^{cb} = \frac{\chi_t}{1 + \mu_t} - \frac{\omega_t}{1 + \mu_t} = \eta_t - \frac{\omega_t}{1 + \mu_t} \]

We remember that we can rewrite (103) into \( \chi_t = (1 + \mu_t) \eta_t \). Substitution of (103) - (106). This gives rise to the following first order conditions:

\[ \nu_t^k = E_t \left[ \beta \Lambda_{t,t+1} \left\{ (1 - \theta) + \theta \left( 1 + \mu_{t+1} \right) \eta_{t+1} \right\} \left( 1 + r_{t+1}^b + \tau_{t+1}^n - \bar{\tau}_{t+1}^n \right) \right], \]

\[ \nu_t^b = E_t \left[ \beta \Lambda_{t,t+1} \left\{ (1 - \theta) + \theta \left( 1 + \mu_{t+1} \right) \eta_{t+1} \right\} \left( 1 + r_{t+1}^b + \tau_{t+1}^n - \bar{\tau}_{t+1}^n \right) \right], \]

\[ \eta_t = E_t \left[ \beta \Lambda_{t,t+1} \left\{ (1 - \theta) + \theta \left( 1 + \mu_{t+1} \right) \eta_{t+1} \right\} \left( 1 + r_{t+1}^b + \tau_{t+1}^n - \bar{\tau}_{t+1}^n \right) \right], \]

\[ \eta_t^{cb} = E_t \left[ \beta \Lambda_{t,t+1} \left\{ (1 - \theta) + \theta \left( 1 + \mu_{t+1} \right) \eta_{t+1} \right\} \left( 1 + r_{t+1}^{cb} + \tau_{t+1}^n - \bar{\tau}_{t+1}^n \right) \right]. \]
Now we assume a particular function for the value function, and will later check whether our first order conditions are consistent with it:

\[ V_{j,t} = \left( s_{j,t-1}^{k,p} \right)^{k,p} s_{j,t}^{b,p} + \nu_t q_{t}^{k,p} s_{j,t}^{b,p} - \eta_t d_{j,t} - d_{cb}^{b,p} \]

Substitution of the first order conditions (104) - (106) in the value function of the typical financial intermediary gives the following expression:

\[ V_{j,t} = \nu_t q_{t}^{k,p} s_{j,t}^{k,p} + \nu_t q_{t}^{b,p} s_{j,t}^{b,p} - \eta_t \left( \frac{\omega_t}{1 + \mu_t} \right) q_{t}^{b,p} \]

Now we substitute the expressions for the shadow values of the different asset classes in the expression for the expected discounted profits of the financial intermediary to obtain the following:

\[ V_{j,t} = \eta_t n_{j,t} + \frac{\mu_t}{1 + \mu_t} \left( \lambda_k q_{t}^{k,p} + \lambda_b q_{t}^{b,p} \right) \]

where the term with \( \omega_t \) drops out because of the slackness condition (94). We can now rewrite the leverage constraint:
expression:

\[ V_{j,t} = \max E_t \left[ \beta \Lambda_{t+1} \left\{ \left( 1 - \theta \right) n_{j,t+1} + \theta V_{j,t+1} \right\} \right] \]

\[ = E_t \left[ \beta \Lambda_{t+1} \left\{ \left( 1 - \theta \right) n_{j,t+1} + \theta \left( \eta_{t+1} n_{j,t+1} + \frac{\mu_{t+1}}{1 + \mu_{t+1}} \left( \lambda_k q_{t+1}^{k-p} s_{j,t+1}^{k-p} + \lambda_b q_{t+1}^{b-p} s_{j,t+1}^{b-p} \right) \right) \right\} \right] \]

\[ = E_t \left[ \beta \Lambda_{t+1} \left\{ \left( 1 - \theta \right) n_{j,t+1} + \theta \left( \eta_{t+1} n_{j,t+1} + \frac{\mu_{t+1}}{1 + \mu_{t+1}} (1 + \mu_{t+1}) \eta_{t+1} n_{j,t+1} \right) \right\} \right] \]

\[ = E_t \left[ \beta \Lambda_{t+1} \left\{ \left( 1 - \theta \right) n_{j,t+1} + \theta \left( 1 + \mu_{t+1} \right) \eta_{t+1} n_{j,t+1} \right\} \right] \]

Comparing with the initial guess for the solution, we obtain the following first order conditions:

\[ \nu_t^k = E_t \left[ \beta \Lambda_{t+1} \left\{ \left( 1 - \theta \right) + \theta \left( 1 + \mu_{t+1} \right) \eta_{t+1} \right\} \left( 1 + r_{t+1}^k + \tau_{t+1}^n - \tilde{\tau}_{t+1}^n \right) \right] , \tag{52} \]

\[ \nu_t^b = E_t \left[ \beta \Lambda_{t+1} \left\{ \left( 1 - \theta \right) + \theta \left( 1 + \mu_{t+1} \right) \eta_{t+1} \right\} \left( 1 + r_{t+1}^b + \tau_{t+1}^n - \tilde{\tau}_{t+1}^n \right) \right] , \tag{53} \]

\[ \eta_t = E_t \left[ \beta \Lambda_{t+1} \left\{ \left( 1 - \theta \right) + \theta \left( 1 + \mu_{t+1} \right) \eta_{t+1} \right\} \left( 1 + r_{t+1}^d + \tau_{t+1}^n - \tilde{\tau}_{t+1}^n \right) \right] , \tag{54} \]

\[ \eta_t^{cb} = E_t \left[ \beta \Lambda_{t+1} \left\{ \left( 1 - \theta \right) + \theta \left( 1 + \mu_{t+1} \right) \eta_{t+1} \right\} \left( 1 + r_{t+1}^{cb} + \tau_{t+1}^n - \tilde{\tau}_{t+1}^n \right) \right] . \tag{55} \]

We see that the solutions \[107\] - \[110\] and \[111\] - \[114\] coincide, and hence that our initial guess for the value function is correct. The law of motion for aggregate net worth is given by:

\[ n_t = \theta \left[ \left( r_t^k - r_t^d \right) q_{t-1}^{k-p} s_{t-1}^{k-p} + \left( r_t^b - r_t^{cb} \right) q_{t-1}^{b-p} s_{t-1}^{b-p} \right] + \left( r_t^d - r_t^{cb} \right) \omega p_{t-1} + n_t^g - \tilde{n}_t^g \tag{56} \]
A.1.1 Financial Sector First Order Conditions

The resulting first order conditions for the financial sector are now given by:

\[ \nu_t^k = \lambda_k \left( \frac{\mu_t}{1 + \mu_t} \right) + \chi_t \left( \frac{\lambda_t}{1 + \mu_t} \right) (57) \]

\[ \nu_t^b = \lambda_b \left( \frac{\mu_t}{1 + \mu_t} \right) + \chi_t \left( \frac{\lambda_t}{1 + \mu_t} - \left( \frac{\nu_t^k}{\nu_t^b} \right) \theta_t \left( \frac{\omega_t}{1 + \mu_t} \right) \right) (58) \]

\[ \eta_t = \frac{\chi_t}{1 + \mu_t} \theta_t \left( \frac{\omega_t}{1 + \mu_t} \right) (59) \]

\[ \eta_{t}^{cb} = \frac{\chi_t}{1 + \mu_t} - \frac{\omega_t}{1 + \mu_t} (60) \]

\[ \nu_t^k = E_t \left[ \delta \Lambda_{t+1} \left( 1 - \theta \right) + \theta \left( 1 + \mu_{t+1} \right) \eta_{t+1} \right] \left( 1 + r_{t+1}^b + \tilde{\tau}_{t+1}^n \right) \]

\[ \nu_t^b = E_t \left[ \delta \Lambda_{t+1} \left( 1 - \theta \right) + \theta \left( 1 + \mu_{t+1} \right) \eta_{t+1} \right] \left( 1 + r_{t+1}^b + \tilde{\tau}_{t+1}^n \right) \]

\[ \eta_t = E_t \left[ \delta \Lambda_{t+1} \left( 1 - \theta \right) + \theta \left( 1 + \mu_{t+1} \right) \eta_{t+1} \right] \left( 1 + r_{t+1}^b + \tilde{\tau}_{t+1}^n \right) \]

\[ \eta_{t}^{cb} = E_t \left[ \delta \Lambda_{t+1} \left( 1 - \theta \right) + \theta \left( 1 + \mu_{t+1} \right) \eta_{t+1} \right] \left( 1 + r_{t+1}^b + \tilde{\tau}_{t+1}^n \right) \]

\[ \left( 1 + \mu_t \right) \eta_{t}^{n,t} \geq \lambda_k q_t^k s_{t}^{k,p} + \lambda_b q_t^b s_{t}^{b,p} (61) \]

\[ n_t = \theta \left( \left( r_t^k - r_t^b \right) q_{t-1}^k s_{t-1}^{k,p} \right) + \left( r_t^d - r_t^{cb} \right) d_{t-1}^{b,p} + 1 + \omega_{p_{t-1} + n_t^g - \tilde{n}_t^g (62) \]

\[ d_t^{cb} = \theta_t \kappa_t s_{t}^{b,p} (63) \]

A.1.2 Further simplification of the F.O.C.’s for mathematical proofs

Now we combine some of the F.O.C.’s found in section A.1.1 to obtain a better economic understanding and more intuition. We start by combining (116) and (117), while substituting (118) for \( \chi_t / (1 + \mu_t) \) to obtain:

\[ \nu_t^b - \eta_t = \frac{\lambda_b}{\lambda_k} \left( \nu_t^k - \eta_t \right) - \left( \frac{\kappa_t}{\nu_t^b} \right) \theta_t \left( \frac{\omega_t}{1 + \mu_t} \right) \]

Substitution of the expressions (120), (121) and (122) results in the following equation:

\[ \frac{\lambda_b}{\lambda_k} E_t \left[ \Omega_{t+1} \left( r_t^k - r_t^d \right) \right] = E_t \left[ \Omega_{t+1} \left( r_t^b - r_t^{cb} \right) \right] + \left( \frac{\kappa_t}{\nu_t^b} \right) \theta_t \left( \frac{\omega_t}{1 + \mu_t} \right), (64) \]

where \( \Omega_{t+1} = \delta \Lambda_{t+1} \left( 1 - \theta \right) + \theta \left( 1 + \mu_{t+1} \right) \eta_{t+1} \) refers to the stochastic discount factor of the financial intermediaries, which is equal to the household’s stochastic discount factor, augmented
to incorporate the financial frictions.

Now we combine (118) and (119) to obtain the following relation between the shadow value on deposit funding and CB funding:

\[
\frac{\omega_t}{1 + \mu_t} = \eta_t - \eta_{\text{cb}}^t.
\]

Substitution of (122) and (123) gives rise to the following relation:

\[
\frac{\omega_t}{1 + \mu_t} = E_t \left[ \Omega_{t,t+1} \left( r_{t+1}^d - r_{t+1}^{\text{cb}} \right) \right].
\]

By these substitutions, we have removed the four shadow values \( \nu_k^t \), \( \nu_b^t \), \( \eta_{\text{cb}}^t \), and \( \chi_t \). Thus we get the following set of F.O.C.’s:

\[
\frac{\lambda_k}{\lambda_b} E_t \left[ \Omega_{t,t+1} \left( r_{t+1}^k - r_{t+1}^d \right) \right] = E_t \left[ \Omega_{t,t+1} \left( r_{t+1}^b - r_{t+1}^{\text{cb}} \right) \right] + \left( \frac{\kappa_t}{q_t^k} \right) \theta_t \left( \frac{\omega_t}{1 + \mu_t} \right),
\]

(70)

\[
\frac{\lambda_k}{1 + \mu_t} \eta_t = E_t \left[ \Omega_{t,t+1} \left( 1 + r_{t+1}^d + r_{t+1}^n - \tilde{r}_{t+1}^n \right) \right],
\]

(73)

\[
(1 + \mu_t) \eta_n j,t \geq \lambda_k q_t^k s_{j,t}^{k,p} + \lambda_b q_t^b s_{j,t}^{b,p},
\]

(74)

\[
\Omega_{t,t+1} = \beta \Lambda_{t,t+1} \left\{ (1 - \theta) + \theta (1 + \mu_{t+1}) \right\}. \tag{77}
\]

A.2 Production Process

A.2.1 Capital Producers

At the end of period \( t \), when the intermediate goods firms have produced, the capital producers buy the remaining stock of capital \((1 - \delta) \xi_t \kappa_{t-1}\) from the intermediate goods producers at a price \( q_t^k \). They combine this capital with goods bought from the final goods producers (investment \( i_t \)) to produce next period’s beginning of period capital stock \( k_t \). This capital is being sold to the intermediate goods producers at a price \( q_t^k \). We assume that the capital producers face convex adjustment costs when transforming the final goods bought into capital goods, set up such that changing the level of gross investment is costly. Hence we get:

\[
k_t = (1 - \delta) \xi_t \kappa_{t-1} + (1 - \Psi(t_t)) i_t, \quad \Psi(x) = \frac{\gamma}{2} (x - 1)^2, \quad t_t = i_t / i_{t-1}. \tag{78}
\]

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ξ_t represents a capital quality shock which will be discussed later. Profits are passed on to the households, who own the capital producers. The profit at the end of period t equals:

\[ \Pi_c^t = q^k_t k_t - q^k_t (1 - \delta) \xi_t k_{t-1} - i_t. \]

The capital producers maximize expected current and (discounted) future profits (where we substitute in (78)):

\[
\max_{\{i_{t+s}\}_{t=0}^\infty} E_t \left[ \sum_{i=0}^{\infty} \beta^s \Lambda_{t,t+s} \left( q^k_{t+s}(1 - \Psi(t_{t+s})) i_{t+s} - i_{t+s} \right) \right].
\]

Differentiation with respect to investment gives the first order condition for the capital producers:

\[
q^k_t (1 - \Psi(it)) - 1 - q^k_t \xi_t \Psi'(it) + \beta E_t \Lambda_{t,t+1} q^k_{t+1} i_{t+1} \gamma \psi'(t_{t+1}) = 0,
\]

which gives the following expression for the price of capital:

\[
\frac{1}{q^k_t} = 1 - \gamma \frac{(i_{t+1})^2 - \gamma i_{t+1} i_t - 1)}{i_{t+1} - 1} + \gamma \beta E_t \left[ \Lambda_{t,t+1} q^k_{t+1} i_{t+1} \gamma \psi'(t_{t+1}) \right]. \tag{79}
\]

### A.2.2 Intermediate Goods Producers

We remember that period t profits are given by:

\[ \Pi_{i,t} = \alpha m_t (\xi_t k_{i,t-1})^\alpha h_{i,t}^{-\alpha} + q^k_t (1 - \delta) \xi_t k_{i,t-1} - (1 + r^k_t) q^k_t (1 - \delta) \xi_t k_{i,t-1} - w_t h_{i,t}. \]

The intermediate goods producing firms maximize expected current and future profits using the household’s stochastic discount factor \( \beta^s \Lambda_{t,t+s} \) (since they are owned by the households), taking all prices as given:

\[
\max_{\{k_{i,t+s}, h_{i,t+s}\}_{t=0}^\infty} E_t \left[ \sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s} \Pi_{i,t+s} \right].
\]

The first order conditions belonging to this problem are given by:

\[
k_{i,t} : E_t \left[ \beta \Lambda_{t,t+1} q^k_t (1 + r^k_{t+1}) \right] = E_t \left[ \beta \Lambda_{t,t+1} (\alpha m_{t+1} y_{i,t+1}/k_{i,t} + q^k_{t+1} (1 - \delta) \xi_{t+1}) \right],
\]

\[
h_{i,t} : w_t = (1 - \alpha) m_t y_{i,t}/h_{i,t}.
\]

In equilibrium profits will be zero. By substituting the first order condition for the wage rate into the zero-profit condition \( \Pi_{i,t} = 0 \), we can find an expression for the ex-post return on capital:

\[
r^k_t = \left( q^k_{t-1} \right)^{-1} (\alpha m_t y_{i,t}/k_{i,t-1} + q^k_{t-1} (1 - \delta) \xi_t) - 1.
\]
Now we rewrite the first order condition for labor and the expression for the ex-post return on capital to find the factor demands:

\[ k_{i,t-1} = \alpha m_t y_{i,t} / q_{t-1}^k (1 + r_t^k) - q_t^k (1 - \delta) \xi_t, \]

\[ h_{i,t} = (1 - \alpha) m_t y_{i,t} / w_t. \]

By substituting the factor demands into the production technology function, we get for the relative intermediate output price \( m_t \):

\[ m_t = \alpha^{\alpha}(1 - \alpha)\alpha^{-1} a_t^{-1} \left( w_t^{1-\alpha} [q_{t-1}^k (1 + r_t^k) \xi_t^{-1} - q_t^k (1 - \delta)]^\alpha \right). \]  \( (80) \)

### A.2.3 Retail firms

Retail firms purchase goods \((y_{i,t})\) from the intermediate goods producing firms for a nominal price \(P_t^m\), and convert these into retail goods \((y_{f,t})\). These goods are sold for a nominal price \(P_{f,t}\) to the final goods producer. It takes one intermediate goods unit to produce one retail good \((y_{i,t} = y_{f,t})\). All the retail firms produce a differentiated retail good by assumption, therefore operate in a monopolistically competitive market, and charge a markup over the input price earning them profits \((P_{f,t} - P_t^m) y_{f,t}\).

Each period, only a fraction \(1 - \psi\) of retail firms is allowed to reset their price, while the \(\psi\) remaining firms are not allowed to do so, like in [Calvo (1983)] and [Yun (1996)]. The firms allowed to adjust prices are randomly selected each period. Once selected, they set prices so as to maximize expected current and future profits, using the stochastic discount factor \(\beta \Lambda_{t,t+s}(P_t/P_{t+s})\):

\[
\max_{P_{f,t}} \mathbb{E}_t \left[ \sum_{s=0}^{\infty} (\beta \psi)^s \Lambda_{t,t+s}(P_t/P_{t+s}) (P_{f,t} - P_t^m) y_{f,t+s} \right],
\]

where \(y_{f,t} = (P_{f,t}/P_t)^{-\epsilon} y_t\) is the demand function. \(y_t\) is the output of the final goods producing firms, and \(P_t\) the general price level. Symmetry implies that all firms allowed to reset their prices choose the same new price \((P_t^*)\). Differentiation with respect to \(P_{f,t}\) and using symmetry then yields:

\[
P_t^* / P_t = \frac{\epsilon \mathbb{E}_t \sum_{s=0}^{\infty} (\beta \psi)^s \Lambda_{t,s+1} P_t^s P_{t+s}^{-\epsilon} m_{t+s} y_{t+s}}{\epsilon - 1 \mathbb{E}_t \sum_{s=0}^{\infty} (\beta \psi)^s \Lambda_{t,s+1} P_t^s P_{t+s}^{-\epsilon} y_{t+s}}.
\]

Defining the relative price of the firms that are allowed to reset their prices as \(\pi_t = P_t^*/P_t\) and gross inflation as \(\pi_t = P_t/P_{t-1}\), we can rewrite this as:

\[
\pi_t = \frac{\epsilon}{\epsilon - 1} \frac{\Xi_{1,t}}{\Xi_{2,t}}, \tag{81}
\]

\[
\Xi_{1,t} = \lambda m_t y_t + \beta \psi E_t \pi_{t+1}^{\epsilon-1} \Xi_{1,t+1}, \tag{82}
\]

\[
\Xi_{2,t} = \lambda y_t + \beta \psi E_t \pi_{t+1}^{\epsilon-1} \Xi_{2,t+1}. \tag{83}
\]
The aggregate price level equals:

\[ P_t^{1-\epsilon} = (1 - \psi)(P_t^*)^{1-\epsilon} + \psi P_{t-1}^{1-\epsilon}. \]

Dividing by \( P_t^{1-\epsilon} \) yields the following law of motion:

\[ (1 - \psi)(\pi_t^*)^{1-\epsilon} + \psi \pi_t^{\epsilon-1} = 1. \] (84)

### A.2.4 Final Goods Producers

Final goods firms purchase intermediate goods which have been repackaged by the retail firms in order to produce the final good. The technology that is applied in producing the final good is given by

\[ y(\epsilon-1)/\epsilon = \int_0^1 y(\epsilon-1)/\epsilon df, \]

where \( y_{f,t} \) is the output of the retail firm indexed by \( f \). \( \epsilon \) is the elasticity of substitution between the intermediate goods purchased from the different retail firms. The final goods firms face perfect competition, and therefore take prices as given. Thus they maximize profits by choosing \( y_{f,t} \) such that

\[ P_t y_t - \int_0^1 P_{f,t} y_{f,t} df \]

is maximized. Taking the first order conditions with respect to \( y_{f,t} \), gives the demand function of the final goods producers for the retail goods. Substitution of the demand function into the technology constraint gives the relation between the price level of the final goods and the price level of the individual retail firms:

\[ y_{f,t} = \left( \frac{P_{f,t}}{P_t} \right)^{-\epsilon} y_t, \]

\[ P_t^{1-\epsilon} = \int_0^1 P_{f,t}^{1-\epsilon} df. \]

### A.2.5 Aggregation

Substituting \( y_{f,t} = y_{t,t} = y_t \left( \frac{P_{f,t}}{P_t} \right)^{-\epsilon} \) into the factor demands derived earlier yields:

\[ h_{i,t} = (1 - \alpha) m_{i,t} y_{f,t}/w_t, \]

\[ k_{i,t-1} = \alpha m_{i,t} y_{f,t}/[q_{k,t-1}(1 + r_k^t) - q_k^t(1 - \delta)\xi_t]. \]

Aggregation over all firms \( i \) gives us aggregate labor and capital:

\[ h_t = (1 - \alpha) m_{t} y_t D_t/w_t, \]

\[ k_{t-1} = \alpha m_{t} y_t D_t/[q_{k,t-1}(1 + r_k^t) - q_k^t(1 - \delta)\xi_t], \]

where \( D_t = \int_0^1 (P_{f,t}/P_t)^{-\epsilon} df \) denotes the price dispersion. It is given by the following recursive form:

\[ D_t = (1 - \psi)(\pi_t^*)^{-\epsilon} + \psi \pi_t \xi_t D_{t-1}. \] (85)

The aggregate capital-labor ratio is equal to the individual capital-labor ratio:

\[ k_{t-1}/h_t = \alpha(1 - \alpha)^{-1} w_t/[q_{k,t-1}(1 + r_k^t) - q_k^t(1 - \delta)\xi_t] = k_{i,t-1}/h_{i,t}. \] (86)
Now calculate aggregate supply by aggregating \( y_{i,t} = a_t(\xi_t k_{i,t-1})^{\alpha} h_{i,t}^{1-\alpha} \):

\[
\int_0^1 a_t(\xi_t k_{i,t-1})^{\alpha} h_{i,t}^{1-\alpha} \, di = a_t \xi_t^{\alpha} \left( \frac{k_{t-1}}{h_t} \right)^{\alpha} \int_0^1 h_{i,t} \, di = a_t(\xi_t k_{t-1})^{\alpha} h_t^{1-\alpha},
\]

while aggregation over \( y_{i,t} \) gives:

\[
\int_0^1 y_{i,t} \, df = y_t \int_0^1 \left( \frac{P_{f,t}}{P_t} \right)^{-\epsilon} \, df = y_t D_t.
\]

So we get the following relation for aggregate supply \( y_t \):

\[
y_t D_t = a_t(\xi_t k_{t-1})^{\alpha} h_t^{1-\alpha}.
\]

(87)
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Households</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$h_{1/3}$</td>
<td>1/3</td>
<td>Labor supply</td>
</tr>
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<td><strong>Financial intermediaries</strong></td>
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<td></td>
</tr>
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<td>$\phi$</td>
<td>6</td>
<td>Leverage ratio</td>
</tr>
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<td>$\Gamma_k$</td>
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<td>Quarterly credit spread $E[r^k - r^d]$</td>
</tr>
<tr>
<td>$\Gamma_{cb}$</td>
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<td>Quarterly credit spread $E[r^d - r^{cb}]$</td>
</tr>
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<td>Haircut parameter</td>
</tr>
<tr>
<td>$\lambda_b/\lambda_k$</td>
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<td>Diversion rate bonds over private loans</td>
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<td><strong>Government policy</strong></td>
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<tr>
<td>$i/y$</td>
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<td>Investment-output ratio</td>
</tr>
<tr>
<td>$g/y$</td>
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<td>Gov’t spending-output ratio</td>
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<tr>
<td>$q_{b}/y$</td>
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<td>Gov’t liabilities-output ratio (quarterly)</td>
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<tr>
<td>$s_{b}/b$</td>
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<td>Fraction of gov’t financing by banks</td>
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</tbody>
</table>

Table 1: Calibration targets for the NK version of the model.

B Calibration
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Definition</th>
</tr>
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<tbody>
<tr>
<td><strong>Households</strong></td>
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</tr>
<tr>
<td>β</td>
<td>0.990</td>
<td>Discount rate</td>
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<td>ν</td>
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<td>Degree of habit formation</td>
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<td>Ψ</td>
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<td>Relative utility weight of labor</td>
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<td>ϕ</td>
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<td>Inverse Frisch elasticity of labor supply</td>
</tr>
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<td>κs,b,h</td>
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<td>Constant portfolio adjustment cost function</td>
</tr>
<tr>
<td>̂s,b,h</td>
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<td>Reference level portfolio adjustment cost function</td>
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<td><strong>Financial Intermediaries</strong></td>
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<td>λk</td>
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<td>Fraction of private loans that can be diverted</td>
</tr>
<tr>
<td>λb</td>
<td>0.1930</td>
<td>Fraction of govt bonds that can be diverted</td>
</tr>
<tr>
<td>χ</td>
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<td>Proportional transfer to entering bankers</td>
</tr>
<tr>
<td>θ</td>
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<td>Survival rate of the bankers</td>
</tr>
<tr>
<td><strong>Intermediate good firms</strong></td>
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</tr>
<tr>
<td>ε</td>
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<td>Elasticity of substitution</td>
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<td><strong>Capital good firms</strong></td>
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<td>ρz</td>
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<td>Autoregressive component of productivity</td>
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<td>ρξ</td>
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<td>Autoregressive component of capital quality</td>
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<td>ρr</td>
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<td>Real payment to government bondholder</td>
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<td>Parameter government debt duration (5 yrs)</td>
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<td>Tax feedback parameter from government debt</td>
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<td>Inflation feedback on nominal interest rate</td>
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<tr>
<td>κy</td>
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<td>Output feedback on nominal interest rate</td>
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<tr>
<td><strong>Shocks</strong></td>
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<td>σz</td>
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<td>Standard deviation productivity shock</td>
</tr>
<tr>
<td>σξ</td>
<td>0.050</td>
<td>Standard deviation capital quality shock</td>
</tr>
<tr>
<td>σr</td>
<td>0.0025</td>
<td>Standard deviation interest rate surprise shock</td>
</tr>
</tbody>
</table>

Table 2: Model parameters for the NK version.
C Additional figures

Money market rates

Figure 7: Money market rates at which commercial banks can obtain unsecured funding in the European interbank market. The EONIA rate is charged on loans with a maturity of 1 month, while 1-year Euribor interest rates are charged on loans with a maturity of 1 year. Source: European Central Bank (2015).

D Alternative financial sector setup

D.1 Gertler Kiyotaki setup

In this section I will derive a slightly modified financial sector setup, very similar to Gertler and Kiyotaki (2010), but with the collateral constraint added. I redo the calculations in appendix A.1.

In the main text, the collateral constraint is given by $d_{j,t}^{b_h} \leq \theta_t q_t^{b_h} s_{j,t}^{b,p}$. In this appendix I will apply a more general formulation, namely $d_{j,t}^{b_h} \leq \theta_t \kappa_t s_{j,t}^{b,p}$, where $\kappa_t$ can be equal to:

$$
\kappa_t = \begin{cases} 
q_t^{b_h} & \text{“Regular collateral constraint”;}
1 & \text{“No risk-adjustment collateral constraint”}.
\end{cases}
$$

The law of motion for net worth, which includes recapitalizations by the government and financial
sector repayments is given by:

\[
\begin{align*}
    n_{j,t+1} &= (1 + r_{t+1}^b) q_{t,s_{j,t}}^k + (1 + r_{t+1}^b) \mu_{t,s_{j,t}}^b \\
    &= (1 + r_{t+1}^d) d_{j,t} - (1 + r_{t+1}^c) d_{j,t}^b + n_{j,t}^n - \tilde{n}_{j,t+1}^n \\
    &= (1 + r_{t+1}^b) q_{t,s_{j,t}}^k + (1 + r_{t+1}^b) \mu_{t,s_{j,t}}^b \\
    &= (1 + r_{t+1}^d) d_{j,t} - (1 + r_{t+1}^c) d_{j,t}^b + \tilde{n}_{t+1}^n n_{j,t} \\
    &= (1 + r_{t+1}^b) q_{t,s_{j,t}}^k + (1 + r_{t+1}^b) \mu_{t,s_{j,t}}^b \\
    &= (1 + r_{t+1}^d) d_{j,t} - (1 + r_{t+1}^c) d_{j,t}^b + \tilde{n}_{t+1}^n n_{j,t} \\
    &= (1 + r_{t+1}^d) d_{j,t} - (1 + r_{t+1}^c) d_{j,t}^b + \tau_{t+1}^n - \tilde{n}_{t+1}^n \\
    &= (1 + r_{t+1}^d + \tau_{t+1}^n - \tilde{n}_{t+1}^n) q_{t,s_{j,t}}^k + (1 + r_{t+1}^b + \tau_{t+1}^n - \tilde{n}_{t+1}^n) \mu_{t,s_{j,t}}^b \\
    &= (1 + r_{t+1}^d + \tau_{t+1}^n - \tilde{n}_{t+1}^n) d_{j,t} - (1 + r_{t+1}^c + \tau_{t+1}^n - \tilde{n}_{t+1}^n) d_{j,t}^b,
\end{align*}
\]

Now we remember the optimization problem of the financial intermediary:

\[
\begin{align*}
    V_{j,t} &= \max_{\{s_{j,t}, d_{j,t}^b, d_{j,t}^d\}} E_t \left[ \beta \Lambda_{t+1} \left\{ (1 - \theta) n_{j,t+1} + \theta V_{j,t+1} \right\} \right], \\
    s.t. \quad n_{j,t} + d_{j,t} + d_{j,t}^b &\geq \sum_{n_{j,t} + d_{j,t} + d_{j,t}^b} q_{s_{j,t}}^k + \mu_{s_{j,t}}^b, \\
    n_{j,t} + d_{j,t} + d_{j,t}^b &\geq \sum_{n_{j,t} + d_{j,t} + d_{j,t}^b} q_{s_{j,t}}^k + \mu_{s_{j,t}}^b, \\
    n_{j,t} &= (1 + r_{t+1}^d + \tau_{t+1}^n - \tilde{n}_{t+1}^n) q_{s_{j,t}}^k + (1 + r_{t+1}^b + \tau_{t+1}^n - \tilde{n}_{t+1}^n) \mu_{s_{j,t}}^b, \\
    \theta_{t,k\beta} s_{j,t}^b &\geq \sum_{n_{j,t} + d_{j,t} + d_{j,t}^b} \sum_{n_{j,t} + d_{j,t} + d_{j,t}^b} q_{s_{j,t}}^k + \mu_{s_{j,t}}^b,
\end{align*}
\]

where we have abbreviated the value function of the financial intermediary by \( V_{j,t} = V \left( s_{j,t}^{k,b}, d_{j,t}^b, d_{j,t}^d \right) \).

We set up the accompanying Lagrangian of the problem:

\[
\begin{align*}
    \mathcal{L} &= (1 + \mu_{\tau}) E_t \left[ \beta \Lambda_{t+1} \left\{ (1 - \theta) \left( 1 + r_{t+1}^d + \tau_{t+1}^n - \tilde{n}_{t+1}^n \right) q_{t,s_{j,t}}^k + (1 + r_{t+1}^b + \tau_{t+1}^n - \tilde{n}_{t+1}^n) \mu_{t,s_{j,t}}^b \right\} \\
    &\quad - (1 + r_{t+1}^d + \tau_{t+1}^n - \tilde{n}_{t+1}^n) d_{j,t} - (1 + r_{t+1}^c + \tau_{t+1}^n - \tilde{n}_{t+1}^n) d_{j,t}^b + \theta V \left( s_{j,t}^{k,b}, d_{j,t}^b, d_{j,t}^d \right) \right] \\
    &\quad - \mu_{\tau} q_{s_{j,t}}^k - \mu_{\tau} q_{s_{j,t}}^b + \chi_t \left( 1 + r_{t+1}^d + \tau_{t+1}^n - \tilde{n}_{t+1}^n \right) q_{t-s_{j,t-1}}^k + (1 + r_{t+1}^b + \tau_{t+1}^n - \tilde{n}_{t+1}^n) \mu_{t-s_{j,t-1}}^b \\
    &\quad - (1 + r_{t+1}^d + \tau_{t+1}^n - \tilde{n}_{t+1}^n) d_{j,t-1} - (1 + r_{t+1}^c + \tau_{t+1}^n - \tilde{n}_{t+1}^n) d_{j,t-1}^b - q_{s_{j,t}}^k - \mu_{t,s_{j,t}}^b + d_{j,t} + d_{j,t}^b \\
    &\quad + \omega_t \left( \theta_{t,k\beta} s_{j,t}^b - d_{j,t}^b \right).
\end{align*}
\]
This gives rise to the following first order conditions:

\[ s_{j,t}^{k,p} : (1 + \mu_t) E_t \left[ \beta \Lambda_{t,t+1} \left( (1 - \theta) \left( 1 + r_{t+1}^k + \tau_{t+1}^n - \tau_{t+1}^p \right) q_t^k + \theta \frac{\partial}{\partial s_{j,t}^{k,p}} \left[ V \left( s_{j,t-1}^{k,p}, s_{j,t-1}^{b,p}, d_{j,t-1}^{b}, d_{j,t}^{b} \right) \right] \right) \right] \]

\[ = \mu_t \lambda_b q_t^b - \chi t q_t^b = 0, \]  

\[ s_{j,t}^{b,p} : (1 + \mu_t) E_t \left[ \beta \Lambda_{t,t+1} \left( (1 - \theta) \left( 1 + r_{t+1}^b + \tau_{t+1}^n - \tau_{t+1}^p \right) q_t^b + \theta \frac{\partial}{\partial s_{j,t}^{b,p}} \left[ V \left( s_{j,t-1}^{k,p}, s_{j,t-1}^{b,p}, d_{j,t-1}^{b}, d_{j,t}^{b} \right) \right] \right) \right] \]

\[ = \mu_t \lambda_b q_t^b - \chi t q_t^b + \omega t \theta = 0, \]  

\[ d_{j,t} : (1 + \mu_t) E_t \left[ \beta \Lambda_{t,t+1} \left( (1 - \theta) \left( 1 + r_{t+1}^d + \tau_{t+1}^n - \tau_{t+1}^p \right) (1 - 1) \right) + \theta \frac{\partial}{\partial d_{j,t}} \left[ V \left( s_{j,t-1}^{k,p}, s_{j,t-1}^{b,p}, d_{j,t-1}^{b}, d_{j,t}^{b} \right) \right] \right] \]

\[ + \chi_t = 0, \]  

\[ d_{j,t}^{d} : (1 + \mu_t) E_t \left[ \beta \Lambda_{t,t+1} \left( (1 - \theta) \left( 1 + r_{t+1}^d + \tau_{t+1}^n - \tau_{t+1}^p \right) (1 - 1) \right) + \theta \frac{\partial}{\partial d_{j,t}^{d}} \left[ V \left( s_{j,t-1}^{k,p}, s_{j,t-1}^{b,p}, d_{j,t-1}^{b}, d_{j,t}^{b} \right) \right] \right] \]

\[ + \chi_t - \omega_t = 0, \]

with complementary slackness conditions:

\[ \mu_t : \left( V \left( s_{j,t-1}^{k,p}, s_{j,t-1}^{b,p}, d_{j,t-1}^{b}, d_{j,t}^{b} \right) - \lambda_b q_t^b s_{j,t}^{b,p} - \lambda_b q_t^b s_{j,t}^{b,p} \right) \mu_t = 0, \]  

\[ \chi_t : \left( (1 + r_{t}^k + \tau_{t}^n - \tau_{t}^p) q_t^k s_{j,t}^{b,p} + (1 + r_{t}^b + \tau_{t}^n - \tau_{t}^p) q_t^b s_{j,t}^{b,p} - (1 + r_{t}^d + \tau_{t}^n - \tau_{t}^p) d_{j,t-1}^{d,b} - q_t^k s_{j,t}^{b,p} - q_t^b s_{j,t}^{b,p} + d_{j,t}^{d} + d_{j,t}^{b} \right) \chi_t = 0, \]

\[ \omega_t : \left( \theta t \kappa_t s_{j,t}^{b,p} - d_{j,t}^{b} \right) \omega_t = 0. \]

Now we apply the envelope theorem to find the derivatives:

\[ \frac{\partial}{\partial s_{j,t-1}^{k,p}} \left[ V \left( s_{j,t-1}^{k,p}, s_{j,t-1}^{b,p}, d_{j,t-1}^{b}, d_{j,t}^{b} \right) \right] = \chi_t \left( 1 + r_{t}^k + \tau_{t}^n - \tau_{t}^p \right) q_t^k, \]

\[ \frac{\partial}{\partial s_{j,t-1}^{b,p}} \left[ V \left( s_{j,t-1}^{k,p}, s_{j,t-1}^{b,p}, d_{j,t-1}^{b}, d_{j,t}^{b} \right) \right] = \chi_t \left( 1 + r_{t}^b + \tau_{t}^n - \tau_{t}^p \right) q_t^b, \]

\[ \frac{\partial}{\partial d_{j,t-1}^{d,b}} \left[ V \left( s_{j,t-1}^{k,p}, s_{j,t-1}^{b,p}, d_{j,t-1}^{b}, d_{j,t}^{b} \right) \right] = -\chi_t \left( 1 + r_{t}^d + \tau_{t}^n - \tau_{t}^p \right), \]

\[ \frac{\partial}{\partial d_{j,t}^{b}} \left[ V \left( s_{j,t-1}^{k,p}, s_{j,t-1}^{b,p}, d_{j,t-1}^{b}, d_{j,t}^{b} \right) \right] = -\chi_t \left( 1 + r_{t}^b + \tau_{t}^n - \tau_{t}^p \right). \]

Subsititution of the envelope conditions [95] - [98] with [88] - [91], we find the following relation...
between the different assets:

\[
s_{j,t}^{k,p} = \lambda_k \left( \frac{\mu_k}{1 + \mu_k} \right) + \frac{\chi_t}{1 + \mu_t} = E_t \left[ \beta \Lambda_{t,t+1} \{(1 - \theta) + \theta \chi_{t+1}\} (1 + r_{t+1} + \tau_t^n - \tilde{\tau}_t^n) \right],
\]

(99)

\[
s_{b,t}^{s,b} = \lambda_b \left( \frac{\mu_b}{1 + \mu_b} \right) + \frac{\chi_t}{1 + \mu_t} = E_t \left[ \beta \Lambda_{t,t+1} \{(1 - \theta) + \theta \chi_{t+1}\} (1 + r_{t+1} + \tau_t^n - \tilde{\tau}_t^n) \right],
\]

(100)

\[
d_{j,t} = \frac{\chi_t}{1 + \mu_t} = E_t \left[ \beta \Lambda_{t,t+1} \{(1 - \theta) + \theta \chi_{t+1}\} (1 + r_{t+1} + \tau_t^n - \tilde{\tau}_t^n) \right],
\]

(101)

\[
d_{b,t}^{c,b} = \frac{\chi_t}{1 + \mu_t} - \frac{\omega_t}{1 + \mu_t} = E_t \left[ \beta \Lambda_{t,t+1} \{(1 - \theta) + \theta \chi_{t+1}\} (1 + r_{t+1} + \tau_t^n - \tilde{\tau}_t^n) \right].
\]

(102)

Now we define the following variables:

\[
\eta_t = \frac{\chi_t}{1 + \mu_t},
\]

(103)

\[
\nu_t^k = \lambda_k \left( \frac{\mu_k}{1 + \mu_k} \right) + \frac{\chi_t}{1 + \mu_t} = \lambda_k \left( \frac{\mu_k}{1 + \mu_k} \right) + \eta_t,
\]

(104)

\[
\nu_t^b = \lambda_b \left( \frac{\mu_b}{1 + \mu_b} \right) + \frac{\chi_t}{1 + \mu_t} = \lambda_b \left( \frac{\mu_b}{1 + \mu_b} \right) + \eta_t - \left( \frac{\chi_t}{1 + \mu_t} \right) \theta_t \left( \frac{\omega_t}{1 + \mu_t} \right)
\]

(105)

\[
\eta_t^c = \frac{\chi_t}{1 + \mu_t} - \frac{\omega_t}{1 + \mu_t} = \eta_t - \frac{\omega_t}{1 + \mu_t}
\]

(106)

We remember that we can rewrite (103) into \( \chi_t = (1 + \mu_t) \eta_t \). Substitution of (103) - (106). This gives rise to the following first order conditions:

\[
\nu_t^k = E_t \left[ \beta \Lambda_{t,t+1} \{(1 - \theta) + \theta (1 + \mu_{t+1}) \eta_{t+1}\} (1 + r_{t+1} + \tau_t^n - \tilde{\tau}_t^n) \right],
\]

(107)

\[
\nu_t^b = E_t \left[ \beta \Lambda_{t,t+1} \{(1 - \theta) + \theta (1 + \mu_{t+1}) \eta_{t+1}\} (1 + r_{t+1} + \tau_t^n - \tilde{\tau}_t^n) \right],
\]

(108)

\[
\eta_t = E_t \left[ \beta \Lambda_{t,t+1} \{(1 - \theta) + \theta (1 + \mu_{t+1}) \eta_{t+1}\} (1 + r_{t+1} + \tau_t^n - \tilde{\tau}_t^n) \right],
\]

(109)

\[
\eta_t^c = E_t \left[ \beta \Lambda_{t,t+1} \{(1 - \theta) + \theta (1 + \mu_{t+1}) \eta_{t+1}\} (1 + r_{t+1} + \tau_t^n - \tilde{\tau}_t^n) \right].
\]

(110)

Now we assume a particular function for the value function, and will later check whether our first order conditions are consistent with it:

\[
V_{j,t} = V \left( s_{j,t-1}^{k,p}, s_{j,t-1}^{b,p}, d_{j,t-1}, d_{j,t-1}^{c,b} \right) = \nu_t^k s_{j,t}^{k,p} + \nu_t^b s_{j,t}^{b,p} - \eta_t d_{j,t} - \eta_t^c d_{j,t}^{c,b}
\]

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Substitution of the first order conditions (104) - (106) in the value function of the typical financial intermediary gives the following expression:

\[
V_{j,t} = \nu_t^k q_t^k s_{j,t} + \nu_t^b q_t^b s_{j,t} - \eta_t d_{j,t} - \eta_t^b d_{j,t} \\
= \left( \lambda_k \left( \frac{\mu_t}{1 + \mu_t} \right) + \eta_t \right) q_t^k s_{j,t} + \left( \lambda_b \left( \frac{\mu_t}{1 + \mu_t} \right) + \eta_t \right) q_t^b s_{j,t} \\
- \eta_t d_{j,t} - \left( \eta_t - \frac{\omega_t}{1 + \mu_t} \right) d_{j,t} \\
= \eta_t \left( q_t^k s_{j,t} + q_t^b s_{j,t} - d_{j,t} - d_{j,t} \right) \\
+ \frac{\mu_t}{1 + \mu_t} \left( \lambda_k q_t^k s_{j,t} + \lambda_b q_t^b s_{j,t} \right) - \frac{\omega_t}{1 + \mu_t} \left( \theta_t \kappa_t s_{j,t} - d_{j,t} \right) \\
= \eta_t n_{j,t} + \frac{\mu_t}{1 + \mu_t} \left( \lambda_k q_t^k s_{j,t} + \lambda_b q_t^b s_{j,t} \right),
\]

where the term with \( \omega_t \) drops out because of the slackness condition (104). We can now rewrite the leverage constraint:

\[
V_{j,t} \geq \lambda_k q_t^k s_{j,t} + \lambda_b q_t^b s_{j,t} \implies \eta_t n_{j,t} \geq \frac{\mu_t}{1 + \mu_t} \left( \lambda_k q_t^k s_{j,t} + \lambda_b q_t^b s_{j,t} \right) \implies \left( 1 - \frac{\mu_t}{1 + \mu_t} \right) \left( \lambda_k q_t^k s_{j,t} + \lambda_b q_t^b s_{j,t} \right) \leq \eta_t n_{j,t} \implies \lambda_k q_t^k s_{j,t} + \lambda_b q_t^b s_{j,t} \leq (1 + \mu_t) \eta_t n_{j,t}.
\]

Now we substitute the expressions for the shadow values of the different asset classes in the expression for the expected discounted profits of the financial intermediary to obtain the following expression:

\[
V_{j,t} = \max_{\theta} E_t [\beta \Lambda_{t+1} \{(1 - \theta) n_{j,t+1} + \theta V_{j,t+1}\}] \\
= E_t \left[ \beta \Lambda_{t+1} \left\{(1 - \theta) n_{j,t+1} + \theta \left( n_{t+1} n_{j,t+1} + \frac{\mu_t}{1 + \mu_t} \left( \lambda_k q_{t+1}^k s_{j,t+1} + \lambda_b q_{t+1}^b s_{j,t+1} \right) \right) \right\} \right] \\
= E_t \left[ \beta \Lambda_{t+1} \left\{(1 - \theta) n_{j,t+1} + \theta \left( n_{t+1} n_{j,t+1} + \frac{\mu_t}{1 + \mu_t} \left( \lambda_k q_{t+1}^k s_{j,t+1} + \lambda_b q_{t+1}^b s_{j,t+1} \right) \right) \right\} \right] \\
= E_t [\beta \Lambda_{t+1} \{(1 - \theta) n_{j,t+1} + \theta (1 + \mu_t) n_{j,t+1}\}] \\
= E_t [\beta \Lambda_{t+1} \{(1 - \theta) + \theta (1 + \mu_t)\} n_{j,t+1} ] \\
= E_t \left[ \beta \Lambda_{t+1} \{(1 - \theta) + \theta (1 + \mu_t)\} n_{j,t+1} \right] \\
= E_t \left[ \beta \Lambda_{t+1} \left\{(1 - \theta) + \theta (1 + \mu_t)\right\} \left( 1 + r_{t+1}^b + \tau_{t+1}^n - \tilde{\tau}_{t+1}^n \right) q_t^b s_{j,t} \right] \\
+ \left( 1 + r_{t+1}^b + \tau_{t+1}^n - \tilde{\tau}_{t+1}^n \right) q_t^b s_{j,t} - \left( 1 + r_{t+1}^d + \tau_{t+1}^n - \tilde{\tau}_{t+1}^n \right) d_{j,t} \right) \right) \right] \\
= \left( 1 + r_{t+1}^b + \tau_{t+1}^n - \tilde{\tau}_{t+1}^n \right) q_t^b s_{j,t} - \left( 1 + r_{t+1}^d + \tau_{t+1}^n - \tilde{\tau}_{t+1}^n \right) d_{j,t} \right) \right).
Comparing with the initial guess for the solution, we obtain the following first order conditions:

\[ \nu^k_t = E_t \left[ \beta \Lambda_{t,t+1} \left\{ (1 - \theta) + \theta (1 + \mu_{t+1}) \eta_{t+1} \right\} \left( 1 + r^k_{t+1} + \tau^n_{t+1} - \tilde{\tau}^n_{t+1} \right) \right], \]

\[ \nu^b_t = E_t \left[ \beta \Lambda_{t,t+1} \left\{ (1 - \theta) + \theta (1 + \mu_{t+1}) \eta_{t+1} \right\} \left( 1 + r^b_{t+1} + \tau^n_{t+1} - \tilde{\tau}^n_{t+1} \right) \right], \]

\[ \eta_t = E_t \left[ \beta \Lambda_{t,t+1} \left\{ (1 - \theta) + \theta (1 + \mu_{t+1}) \eta_{t+1} \right\} \left( 1 + r^d_{t+1} + \tau^n_{t+1} - \tilde{\tau}^n_{t+1} \right) \right], \]

\[ \eta^{cb}_t = E_t \left[ \beta \Lambda_{t,t+1} \left\{ (1 - \theta) + \theta (1 + \mu_{t+1}) \eta_{t+1} \right\} \left( 1 + r^{cb}_{t+1} + \tau^n_{t+1} - \tilde{\tau}^n_{t+1} \right) \right]. \]

We see that the solutions (107) - (110) and (111) - (114) coincide, and hence that our initial guess for the value function is correct. The law of motion for aggregate net worth is given by:

\[ n_t = \theta \left[ (r^k_t - r^d_t) q^k_{t-1} s^{k,p}_{t-1} + (r^b_t - r^d_t) q^b_{t-1} s^{b,p}_{t-1} \right. \]

\[ + \left. (r^d_t - r^{cb}_t) q^{cb}_{t-1} + \left( 1 + r^d_t \right) n_{t-1} \right] + \omega p_{t-1} + \tilde{n}^p_t - \tilde{n}^q_t \]

(115)
D.1.1 Financial Sector First Order Conditions

The resulting first order conditions for the financial sector are now given by:

\[ \nu_t^k = \lambda_k \left( \frac{\mu_t}{1 + \mu_t} \right) + \frac{\chi_t}{1 + \mu_t} \quad (116) \]

\[ \nu_t^b = \lambda_b \left( \frac{\mu_t}{1 + \mu_t} \right) + \frac{\chi_t}{1 + \mu_t} - \left( \frac{\kappa_t}{q_t^2} \right) \theta_t \left( \frac{\omega_t}{1 + \mu_t} \right) \quad (117) \]

\[ \eta_t = \frac{\chi_t}{1 + \mu_t} \quad (118) \]

\[ \eta_t^{cb} = \frac{\chi_t}{1 + \mu_t} - \frac{\omega_t}{1 + \mu_t} \quad (119) \]

\[ \nu_t^k = E_t \left[ \beta \Lambda_{t,t+1} \{(1 - \theta) + \theta (1 + \mu_{t+1}) \eta_{t+1}\} \left( 1 + r_{t+1}^b + \bar{r}_{t+1}^n - \tilde{r}_{t+1}^n \right) \right] \quad (120) \]

\[ \nu_t^b = E_t \left[ \beta \Lambda_{t,t+1} \{(1 - \theta) + \theta (1 + \mu_{t+1}) \eta_{t+1}\} \left( 1 + r_{t+1}^b + \bar{r}_{t+1}^n - \tilde{r}_{t+1}^n \right) \right] \quad (121) \]

\[ \eta_t = E_t \left[ \beta \Lambda_{t,t+1} \{(1 - \theta) + \theta (1 + \mu_{t+1}) \eta_{t+1}\} \left( 1 + r_{t+1}^b + \bar{r}_{t+1}^n - \tilde{r}_{t+1}^n \right) \right] \quad (122) \]

\[ \eta_t^{cb} = E_t \left[ \beta \Lambda_{t,t+1} \{(1 - \theta) + \theta (1 + \mu_{t+1}) \eta_{t+1}\} \left( 1 + r_{t+1}^b + \bar{r}_{t+1}^n - \tilde{r}_{t+1}^n \right) \right] \quad (123) \]

\[ (1 + \mu_t) \eta_{n, cb} \geq \lambda_k q_{s, j, t}^k + \lambda_b q_{s, j, t}^b, \quad (124) \]

\[ n_t = \theta \left[ (r_t^k - r_t^d) q_{t-1}^k s_{t-1}^{cb} + (r_t^b - r_t^d) q_{t-1}^b s_{t-1}^{cb} \right. \]
\[ + \left. (r_t^d - r_t^{cb}) q_{t-1}^d s_{t-1}^{cb} + (1 + r_t^d) n_{t-1} \right] + \omega_p_{t-1} + n_t^d - \bar{n}_t^d \quad (125) \]

\[ d_t^{cb} = \theta_t \kappa_t s_{t}^{cb} \quad (126) \]

D.1.2 Further simplification of the F.O.C.’s for mathematical proofs

Now we combine some of the F.O.C.’s found in section D.1.1 to obtain a better economic understanding and more intuition. We start by combining (116) and (117), while substituting (118) for \( \chi_t/(1 + \mu_t) \) to obtain:

\[ \nu_t^b - \eta_t = \frac{\lambda_b}{\lambda_k} \left( \nu_t^k - \eta_t \right) - \left( \frac{\kappa_t}{q_t^2} \right) \theta_t \left( \frac{\omega_t}{1 + \mu_t} \right) \]

Substitution of the expressions (120), (121) and (122) results in the following equation:

\[ \lambda_k E_t \left[ \Omega_{t,t+1} \left( r_{t+1}^k - r_{t+1}^d \right) \right] + E_t \left[ \Omega_{t,t+1} \left( r_{t+1}^b - r_{t+1}^d \right) \right] + \left( \frac{\kappa_t}{q_t^2} \right) \theta_t \left( \frac{\omega_t}{1 + \mu_t} \right), \quad (127) \]

where \( \Omega_{t,t+1} = \beta \Lambda_{t,t+1} \{(1 - \theta) + \theta (1 + \mu_{t+1}) \eta_{t+1}\} \) refers to the stochastic discount factor of the financial intermediaries, which is equal to the household’s stochastic discount factor, augmented
to incorporate the financial frictions.

Now we combine \(118\) and \(119\) to obtain the following relation between the shadow value on deposit funding and CB funding:

\[
\frac{\omega_t}{1 + \mu_t} = \eta_t - \eta^{cb}_t.
\]

Substitution of \(122\) and \(123\) gives rise to the following relation:

\[
\frac{\omega_t}{1 + \mu_t} = E_t \left[ \Omega_{t,t+1} \left( r^{d}_{t+1} - r^{cb}_{t+1} \right) \right].
\]

By these substitutions, we have removed the four shadow values \(\nu^k_t, \nu^b_t, \eta^{cb}_t, \text{ and } \chi_t\). Thus we get the following set of F.O.C.’s:

\[
\frac{\lambda^b}{\lambda^k} E_t \left[ \Omega_{t,t+1} \left( r^{k}_{t+1} - r^{d}_{t+1} \right) \right] = E_t \left[ \Omega_{t,t+1} \left( r^{b}_{t+1} - r^{d}_{t+1} \right) \right] + \left( \frac{\kappa_t}{\eta_t} \right) \theta_t \left( \frac{\omega_t}{1 + \mu_t} \right),
\]

\[
\frac{\lambda^b}{1 + \mu_t} = E_t \left[ \Omega_{t,t+1} \left( r^{b}_{t+1} - r^{cb}_{t+1} \right) \right],
\]

\[
\frac{\omega_t}{1 + \mu_t} = E_t \left[ \Omega_{t,t+1} \left( r^{d}_{t+1} - r^{cb}_{t+1} \right) \right],
\]

\[
\eta_t = E_t \left[ \Omega_{t,t+1} \left( 1 + r^{d}_{t+1} + r^{n}_{t+1} - \tilde{r}^{n}_{t+1} \right) \right],
\]

\[
(1 + \mu_t) \eta_{n,t} \geq \lambda^k q^k_t s^k_{j,t} + \lambda^b q^b_t s^b_{j,t},
\]

\[
n_t = \theta \left[ \left( r^{k}_{t} - r^{d}_{t} \right) q^k_{t-1} s^k_{j,t-1} + \left( r^{b}_{t} - r^{d}_{t} \right) q^b_{t-1} s^b_{j,t-1} \right] + \left( r^{d}_{t} - r^{cb}_{t} \right) d^b_{t-1} + \left( 1 + r^{d}_{t} \right) n_{t-1} + \omega p_{t-1} + n^g_{t} - \tilde{n}^g_{t},
\]

\[
\theta_t = \theta_t \kappa_t s^b_{j,t},
\]

\[
\Omega_{t,t+1} = \beta \Lambda_{t,t+1} \left\{ (1 - \theta) + \theta (1 + \mu_{t+1}) \eta_{t+1} \right\}.
\]

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E Household financing of private loans

Now we introduce the possibility for households to finance private loans with return \( r^k_t \) on their holdings \( q^k_{t-1} s^k_{t-1} \), where \( s^k_{t-1} \) is the number of private loans purchased in period \( t-1 \). Households are less efficient in financial intermediation than commercial banks, hence they incur financial intermediation costs, which is quadratic in the deviation from the number of private loans \( \hat{s}^k_{t,h} \).

The household's problem changes into:

\[
\max_{\{c^t_{i+1}, h_{t+i}, W_{t+i}, s^k_{t,h}, s^b_{t,h}\}} \quad E_t \left[ \sum_{i=0}^{\infty} \beta^i \left\{ \log (c^t_{i+1} - v c^t_{t+1}) - \frac{h^{1+\varphi}_t}{1 + \varphi} \right\} \right]
\]

\[
\text{s.t.} \quad c^t_t + \tau^t_t + W^t_t + q^k_t s^k_t + q^b_t s^b_t + \frac{\kappa_{s^k,h}}{2} \left( s^k_t - \hat{s}^k_{t,h} \right)^2 + \frac{\kappa_{s^b,h}}{2} \left( s^b_t - \hat{s}^b_{t,h} \right)^2 = w^t h^t_t + (1 + r^d_t) W^t_{t-1} + (1 + r^k_t) q^k_{t-1} s^k_{t-1} + (1 + r^b_t) q^b_{t-1} s^b_{t-1} + \Pi^t_t,
\]

which give rise to an additional first order condition for private loans \( s^k_t \), next to the first order conditions from the main text:

\[
s^k_t : \quad E_t \left[ \beta^2 \lambda^t_{t+1} \left( \frac{1 + r^k_{t+1}}{q^k_t + \kappa_{s^k,h}} \right) \frac{(1 + r^k_{t+1}) q^k_t}{\lambda^t} \left( s^k_t - \hat{s}^k_{t,h} \right) \right] = 1. \tag{137}
\]

This introduces a new parameter, namely \( \kappa_{s^k,h} \), into the model. It is reasonable to assume that transaction costs for households are larger for private credit intermediation than for sovereign debt intermediation. We therefore set \( \kappa_{s^k,h} = 0.1 \), which is four times larger than \( \kappa_{s^b,h} \). The market clearing condition \( \text{(26)} \) for private loans changes into:

\[
k^t_t = s^k_{t,p} + s^k_{t,h}. \tag{138}
\]
F Robustness checks
Financial crisis impact, no additional policy intervention vs. CB liquidity facilities, including households financing of private loans.

Figure 8: Impulse response functions for the case with no additional policy (blue, solid) vs. a decrease in the nominal interest rate on CB liquidity facilities on impact with 50 basis points (red, slotted). The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state.
Figure 9: Impulse response functions for the case with no additional policy (blue, solid) vs. a decrease in the nominal interest rate on CB liquidity facilities on impact with 50 basis points (red, slotted). The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state.
Financial crisis impact, no additional policy intervention vs. CB liquidity facilities, 
\( \lambda_b/\lambda_k = 0.25 \)

Figure 10: Impulse response functions for the case with no additional policy (blue, solid) vs. a decrease in the nominal interest rate on CB liquidity facilities on impact with 50 basis points (red, slotted). The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state.
Financial crisis impact, no additional policy intervention vs. CB liquidity facilities,

\( \varphi = 1 \)

Figure 11: Impulse response functions for the case with no additional policy (blue, solid) vs. a decrease in the nominal interest rate on CB liquidity facilities on impact with 50 basis points (red, slotted). The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state.