A General Equilibrium Theory of Firm Formation under Optimal Expectations

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Abstract

We extend Lucas' (1978) by assuming that a fraction of individuals in an economy derive anticipatory utility from entrepreneurship. We show that if these individuals are able to bias their beliefs to inflate the anticipatory benefits they endogenously become optimists. Optimism has six main effects. First, there is a misallocation of talent which lowers output. Second, optimists are more likely to become entrepreneurs than realists. Third, entrepreneurs are more optimistic than workers. Fourth, when the fraction optimists is high, the majority of entrepreneurs are optimists. Fifth, optimism drives up the wage which makes workers better off. Sixth, optimism lowers the returns to entrepreneurship.

JEL Codes: D50; H21; J24; L26.

Keywords: General Equilibrium; Occupational Choice; Entrepreneurship; Anticipatory Utility, Optimism.

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1 Introduction

Six stylized facts stand out in the literature on entrepreneurship. First, the returns to entrepreneurship are found to be highly variable, more than wages, and more than the returns on public equity–see Borjas and Bronars (1989), Hamilton (2000), and Moskovitz and Vissing-Jorgensen (2002). Second, the average return to entrepreneurship is not significantly higher than average wages or average return to public equity.[1] Third, the majority of entrepreneurs are optimistic about the chances that their firms will succeed–see Cooper et al. (1988), Wu and Knott (2006), Landier and Thesmar (2009), Cassar (2010, 2012), and Hyytinen et al. (2014). Fourth, optimistic individuals are more likely to become entrepreneurs–see Gentry and Hubbard (2000), Hurst and Lusardi (2004), and Cassar and Friedman (2009). Fifth, entrepreneurs are more likely to be optimists than regular wage earners–see Arabsheibani et al. (2000), Fraser and Greene (2006), Puri and Robinson (2007), and Dawson et al. (2014).[2] Sixth, realistic entrepreneurs earn more than optimistic ones–see Dawson et al. (2015).

This paper presents a fully specified general equilibrium model of occupational choice that generates new qualitative predictions about the impact of optimism on labor, capital, and output markets. We illustrate quantitatively the general equilib-

[1] Measurement issues make it hard to compare the mean return to entrepreneurship to that of wage work or to public equity. Some empirical evidence shows that enterpreneurs are not deterred by the evidence of unfavorable returns to entrepreneurship. For example, Dunne et al. (1988) show that most businesses fail within a few years. Hamilton (2000) finds that after 10 years in business, median entrepreneurial earnings are 35% less than those on a paid job of the same duration. Moskovitz and Vissing-Jorgensen (2002) find that the returns from entrepreneurship are, on average, not different from the return on a diversified publicly traded portfolio (private equity puzzle) during the 1989–1998 period. In contrast, Kartashova (2014) shows that Moskowitz and Vissing-Jorgensen (2002)'s finding does not extend to the whole 1989–2010 period.

[2] For example, using data from the British Household Panel Survey for the period 1990-1996, Arabsheibani et al. (2000) find that the self-employed are 4.6 times as likely to forecast an improved financial position but experience a deterioration than to forecast a deterioration but experience an improvement. In contrast, for employees the ratio was only 2.9.

rium effects of optimism by calibrating the model to match salient features of US manufacturing data. The calibration shows that the model is able to explain the six motivating stylized facts.

Following Lucas (1978) we model a closed economy with a given workforce which is homogeneous with respect to productivity as an employee. Each member of the workforce is endowed with an entrepreneurial ability which varies across individuals. Individuals are risk neutral and maximize their expected income by choosing occupations. A firm in this economy is one entrepreneur together with the labor and capital under his control. The technology of the firm is as follows. Output is an increasing function of entrepreneurial ability, labor, and capital. Entrepreneurial ability is complementary to labor and capital. Decreasing returns to scale in labor and capital ensure that the competitive equilibrium exhibits a non-degenerate distribution of firm sizes.

We depart from Lucas (1978) by assuming that a fraction $\lambda \in (0, 1]$ of individuals in the economy derive non-pecuniary anticipatory utility from entrepreneurship and are able to bias their beliefs to inflate these anticipatory benefits. This assumption captures the idea that the anticipation of future profits–and of how enjoyable these will be–plays a major role in the decision to become an entrepreneur.[3] The remaining fraction $1 - \lambda$ of individuals in the economy has standard preferences (i.e., do not derive anticipatory utility from entrepreneurship) and hence has no reason to distort beliefs. There is a continuum of both types of individuals ranked by their entrepreneurial ability θ_0 which is distributed on $[0, 1]$.

Our model of beliefs follows the optimal expectations framework of Brunnermeier and Parker (2005).[4] Individuals who derive anticipatory utility from entrepreneur-

[3] There could be other non-pecuniary benefits from entrepreneurship. For example, an intrinsic taste for being an entrepreneur or a distaste to have a boss. We ignore other non-pecuniary benefits but the model could be extended to incorporate them.

[4] The assumption that individuals may distort their beliefs is not new in the literature. Other proeminent models of distorted beliefs due to anticipatory utility are Loewenstein (1987), Caplin and Leahy (2001), Koszegi (2006), Brunnermeier et al. (2007), Koszegi (2010), and Bénabou (2013).

ship choose their expectations of ability so as to maximize the sum of the material and anticipatory payoffs of entrepreneurship. This choice involves a trade-off between optimism, which raises the anticipatory payoff, and realism, which raises the material payoff by promoting efficient input choices. Optimal beliefs balance the anticipatory benefits of optimism with its efficiency cost. From now on, we refer to individuals who do not distort beliefs as realists and to those who do as individuals with optimal expectations.

The timing of the model is as follows. At $t = 0$ individuals with optimal expectations choose their beliefs of entrepreneurial ability for all future periods. At $t = 1$ individuals choose, given their ability expectations, between entrepreneurship and wage-earning. At $t = 1$ an individual with optimal expectations becomes an entrepreneur if the sum of the material and anticipatory payoffs from entrepreneurship is greater than the wage. At $t = 1$ a realist becomes an entrepreneur if the material payoff from entrepreneurship is greater than the wage. At $t = 2$ entrepreneurs choose, given their ability expectations, how much labor and capital to hire to maximize the perceived material payoff from entrepreneurship. At $t = 2$ entrepreneurs with optimal expectations realize the anticipatory payoff from entrepreneurship. At $t = 3$ entrepreneurs realize the material payoffs from running their firms.

Section 3 shows that individuals with optimal expectations endogenously choose to be optimists about their entrepreneurial ability. When the weight of anticipatory utility s is not too high, being optimist about entrepreneurial ability leads to first-order gains due to increased anticipatory utility from entrepreneurship and to second-order costs in realized profits due to distorted input choices. With a Cobb-Douglas technology with decreasing returns to scale $\eta \in (0, 1)$, the optimistic bias in beliefs is increasing with the weight of anticipatory utility s and decreasing with η. Hence, whether an individual with optimal expectations becomes an entrepreneur or a worker depends on his entrepreneurial ability θ_0 and on his optimistic bias in beliefs which is a function of tastes and technology.[5]

[5]In our model, like in Lucas' (1978), individuals have a choice between operating a firm or

Section 4 shows that with a Cobb-Douglas technology and a uniform distribution of ability there exists a unique optimal expectations competitive equilibrium. The equilibrium is characterized by: (i) a cut-off ability level $\hat{\theta}_R$ such that realists with ability less than $\hat{\theta}_R$ become workers and those with ability greater than $\hat{\theta}_R$ become entrepreneurs, (ii) a cut-off ability level $\hat{\theta}_O$ such that optimists with ability less than $\hat{\theta}_O$ become workers and those with ability greater than $\hat{\theta}_O$ become entrepreneurs, (iii) a market clearing wage that equates labor demand to supply, and (iv) a rental cost of capital that equates capital demand to supply.

In equilibrium the marginal optimistic entrepreneur has lower ability than the marginal realistic entrepreneur, i.e., the cut-off ability level $\hat{\theta}_O$ is less than $\hat{\theta}_R$. This means that in equilibrium there is a misallocation of talent which lowers output. The ablest people do not necessarily select into entrepreneurship: the lowest ability entrepreneurs are less talented at running a firm than the highest ability workers. In addition, we find that optimists are more likely to become entrepreneurs than realists and that entrepreneurs are more likely to be optimists than workers. Finally, if the fraction of optimists is high, the majority of entrepreneurs are optimists.

Section 5 shows how a change in the fraction of optimists affects the labor and capital markets. An increase in the fraction of optimists makes workers better off since it raises the wage. We show that an increase in the fraction of optimists can increase the fraction of workers and lower the fraction of entrepreneurs. In contrast, when We also provide conditions under which an increase in the fraction of optimists raises the rental rate of capital.

Section 6 calibrates the model to illustrate quantitatively the general equilibrium effects of optimism. The calibration matches salient features of US manufacturing data on labor's income share and the capital-output ratio. It shows that the presence of optimists may significantly change the distribution of income by driving up the

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working for a wage. We focus on ability and optimistic biases in beliefs as the main determinants which explain who becomes an entrepreneur and who works as an employee. There are of course many other factors which could influence this choice. For example, risk aversion, the disutility of exerting entrepreneurial effort, and access to funds needed to create a firm.

wage and lowering the returns to entrepreneurship. The mean return to entrepreneurship is 1.656 times higher than the wage in the absence of optimists but only 1.181 times higher in the presence of optimists. The calibration also shows that even though optimism has a large impact on the returns to entrepreneurship it has only a modest impact on output, the fraction of workers, and the fraction of entrepreneurs.

Our paper contributes to the literature on occupational choice using general equilibrium models (Lucas, 1978, Kanbur, 1979, Kihlstrom and Laffont, 1979, Bewley, 1989, and Lazear, 2005). We generalize Lucas (1978) by assuming that a fraction of individuals in the economy derive anticipatory utility from entrepreneurship and are able to bias their beliefs of entrepreneurial skill. More narrowly, our paper contributes to the literature that uses the general equilibrium approach to study the impact of optimism on market outcomes (de Meza and Southey, 1996, Manove, 2000, Fraser and Greene, 2006, and Rigotti et al. 2011). Our paper differs from these studies since optimism arises endogenously instead of being exogenous. Moreover, the optimal expectations framework constrains the optimistic bias in beliefs by tastes–the weight of anticipatory utility–and technology–the extent of decreasing returns to scale. This enables us to make novel qualitative and quantitative predictions about the impact of optimism on labor, capital, and output markets.

The reminder of the paper proceeds as follows. Section 2 sets up the model. Section 3 derives the optimal expectations under a Cobb-Douglas technology. Section 4 characterizes the competitive equilibrium under a Cobb-Douglas technology and a uniform distribution of ability. Section 5 contains comparative statics results. The model is calibrated to fit US manufacturing data in Section 6. Section 7 explains our contribution to the literature. Section 8 concludes the paper. All proofs can be found in the Appendix.

2 Set-up

The economy consists of a continuum of risk-neutral individuals of measure 1. They derive utility from consumption, and can earn income either as workers or by running their own firm. Individuals are ranked by their entrepreneurial ability, θ_0, which is distributed on $[0, 1]$ according to the cumulative distribution function $G(\theta_0)$. Each individual has one unit of labor. If an individual becomes a worker he supplies his unit of labor on the labor market and receives the competitive wage w. Thus, we assume all individuals have the same productivity (or ability) as workers. If an individual becomes an entrepreneur he can use without cost a technology defined by the continuous production function

$$y = f(l, k, \theta_0),$$

where y is output, l is labor, and k is capital. Any individual can run at most one firm. We assume that f is twice continuously differentiable with $f_l > 0$, $f_k > 0$, $f_{\theta_0} > 0$, $f_{ll} < 0$, $f_{kk} < 0$, $f_{l\theta_0} > 0$, $f_{k\theta_0} > 0$, and $f(0, k, \theta_0) = f(l, 0, \theta_0) = 0$. This production function combines as inputs one manager/owner, who is essential to operate the firm, with a labor input of l units and a capital input of k units. The stock of capital in the economy is fixed and equal to $\bar{K}$. Entrepreneurs rent capital in the capital market at the competitive rental cost of capital r.

Production exhibits decreasing returns to scale in the variable inputs, labor and capital, so that the competitive equilibrium exhibits a non-degenerate firm size distribution.[6] The assumption that entrepreneurial ability and labor are complements in production, i.e. $f_{l\theta_0} > 0$, is a critical one. This assumption implies that an optimistic entrepreneur will demand more labor than a realist with the same ability. If entrepreneurial ability and labor were substitutes in production, i.e. $f_{l\theta_0} < 0$, the opposite result would hold, namely, an optimistic entrepreneur would demand

[6] This assumption implies that the size of firms is finite. This could be due for instance to limits in entrepreneurs' span of control (Lucas 1978): as activity expands, it becomes more difficult to control, and the marginal product of the variable inputs diminishes.

less labor than a realist with the same ability. The assumption that entrepreneurial ability and capital are complements in production, i.e. $f_{k\theta_0} > 0$, is also critical.

If an individual becomes an entrepreneur and employs l workers he receives a material payoff equal to

$$\pi = pf(l, k, \theta_0) - wl - rk.$$

From now on the price of output p is normalized to be 1. Individuals can belong to one of two types: those who have optimal expectations of entrepreneurial ability and those who have rational expectations. Fraction $\lambda \in (0, 1)$ of the population has optimal expectations and fraction $1 - \lambda$ has realistic expectations. The distributions of entrepreneurial abilities and types are independent.

At $t = 0$ an individual with optimal expectations observes θ_0 and chooses his expectation of entrepreneurial ability θ so as to maximize the undiscounted sum of $f(l, k, \theta_0) - wl - rk$, his material payoff of being an entrepreneur at $t = 3$; and $s[f(l, k, \theta) - wl - rk]$, his anticipatory payoff of being an entrepreneur at $t = 2$. At $t = 1$ the individual, given his expectation of entrepreneurial ability θ, decides whether to be an entrepreneur or a worker and receive the market wage w. The individual becomes an entrepreneur if the sum of the material and anticipatory payoffs of being an entrepreneur is higher than w. At $t = 2$ an entrepreneur chooses l and k to maximize his material payoff given his expectation of entrepreneurial ability θ. At $t = 2$ the entrepreneur receives anticipatory utility from his expectation of material payoffs evaluated with belief θ: $s[f(l, k, \theta) - wl - rk]$. At $t = 3$ an entrepreneur realizes the material payoff $f(l, k, \theta_0) - wl - rk$.

According to this approach the total payoff at $t = 0$ of an individual with optimal expectations who selects to be an entrepreneur is

$$f(l, k, \theta_0) - wl - rk + s[f(l, k, \theta) - wl - rk],$$

where the parameter $s > 0$ measures the weight the individual places on anticipatory utility relative to material payoffs. Note that the material payoff component depends on the individual's actual entrepreneurial ability θ_0, while the anticipatory utility depends on the individual's expectation of entrepreneurial ability θ.

An individual who becomes an entrepreneur will choose to employ $l(w, r, \theta)$ workers and $k(w, r, \theta)$ units of capital at $t = 2$ where $l(w, r, \theta)$ and $k(w, r, \theta)$ are the values of l and k that solve the following problem

$$\max_{l,k}[f(l, k, \theta) - wl - rk].$$

The first-order conditions to this problem are

$$f_l(l, k, \theta) = w. \tag{1}$$

and

$$f_k(l, k, \theta) = r. \tag{2}$$

It follows from (1), the assumption of decreasing returns to labor, $f_{ll} < 0$, and complementarity between entrepreneurial ability and labor, i.e., $f_{l\theta_0} > 0$, that entrepreneurs with a higher θ hire more workers: $\partial l(w, r, \theta)/\partial\theta = -f_{l\theta}/f_{ll} > 0$. Similarly, it follows from (2), the assumption of decreasing returns to capital, $f_{kk} < 0$, and complementarity between entrepreneurial ability and capital, i.e., $f_{k\theta_0} > 0$, that entrepreneurs with a higher θ hire more capital: $\partial k(w, r, \theta)/\partial\theta = -f_{k\theta}/f_{kk} > 0$. At $t = 1$ the optimal expectation of an individual with ability θ_0 is the θ that solves the following problem

$$\begin{aligned} \max_{\theta\in[0,1]} \{ & f(l(w, r, \theta), k(w, r, \theta), \theta_0) - wl(w, r, \theta) - rk(w, r, \theta) \\ & + s\left[f(l(w, r, \theta), k(w, r, \theta), \theta) - wl(w, r, \theta) - rk(w, r, \theta)\right]\}. \end{aligned}$$

If the wage is w, a realistic individual with entrepreneurial ability θ_0 chooses to become a worker at wage w when

$$f(l(w, r, \theta_0), k(w, r, \theta_0), \theta_0) - wl(w, r, \theta_0) - rk(w, r, \theta_0) \leq w. \tag{3}$$

He selects to be an entrepreneur if

$$f(l(w, r, \theta_0), k(w, r, \theta_0), \theta_0) - wl(w, r, \theta_0) - rk(w, r, \theta_0) \geq w, \tag{4}$$

and he is indifferent if the equality holds in (3) and (4). If the wage is w, an individual with optimal expectations of ability θ^* and with entrepreneurial ability θ_0 chooses to become a worker at wage w when

$$\begin{aligned} & f(l(w,r,\theta^*),k(w,r,\theta^*),\theta_0) - wl(w,r,\theta^*) - rk(w,r,\theta^*) \\ +s & \left[f(l(w,r,\theta^*),k(w,r,\theta^*),\theta^*) - wl(w,r,\theta^*) - rk(w,r,\theta^*) \right] \leq w. \end{aligned} \tag{5}$$

He selects to be an entrepreneur if

$$\begin{aligned} & f(l(w,r,\theta^*),k(w,r,\theta^*),\theta_0) - wl(w,r,\theta^*) - rk(w,r,\theta^*), \\ +s & \left[f(l(w,r,\theta^*),k(w,r,\theta^*),\theta^*) - wl(w,r,\theta^*) - rk(w,r,\theta^*) \right] \geq w \end{aligned} \tag{6}$$

and he is indifferent if the equality holds in (5) and (6). Since there are only three markets–output, labor, and capital–by Walras' Law, general equilibrium is realized when the labor and capital markets clear. At the equilibrium wage, the labor demanded by individuals who choose to become entrepreneurs equals that supplied by individuals who choose to become workers. At the equilibrium rental cost of capital, the capital demanded by individuals who choose to become entrepreneurs equals the exogenous capital stock of the economy, $\bar{K}$. Formally, an equilibrium is (i) a partition $\{[0,\hat{\theta}_R],[\hat{\theta}_R,1]\}$ of $[0,1]$ where for all $\theta_0 \in [0,\hat{\theta}_R]$ (3) holds and for all $\theta_0 \in [\hat{\theta}_R,1]$ (4) holds, (ii) a partition $\{[0,\hat{\theta}_O],[\hat{\theta}_O,1]\}$ of $[0,1]$ where for all $\theta_0 \in [0,\hat{\theta}_O]$ (5) holds and for all $\theta_0 \in [\hat{\theta}_O,1]$ (6) holds, (iii) a wage w for which labor demand equals labor supply

$$\begin{aligned} (1-\lambda)\int_{\hat{\theta}_R}^{1} l(w,r,\theta_0)dG(\theta_0) + \lambda \int_{\hat{\theta}_O}^{1} l(w,r,\theta^*)dG(\theta_0) \\ = (1-\lambda)\int_{0}^{\hat{\theta}_R} dG(\theta_0) + \lambda \int_{0}^{\hat{\theta}_O} dG(\theta_0), \end{aligned} \tag{7}$$

and (iv) a rental cost of capital r for which capital demand equals the exogenous capital supply

$$(1-\lambda)\int_{\hat{\theta}_R}^{1} k(w,r,\theta_0)dG(\theta_0) + \lambda \int_{\hat{\theta}_O}^{1} k(w,r,\theta^*)dG(\theta_0) = \bar{K}. \tag{8}$$

In equilibrium, realists with ability below $\hat{\theta}_R$ become workers whereas those with ability above $\hat{\theta}_R$ become entrepreneurs. Similarly, individuals with optimal expectations and ability below $\hat{\theta}_O$ become workers whereas those with ability above $\hat{\theta}_O$ become entrepreneurs. We refer to a realist with ability $\hat{\theta}_R$ as the *marginal realistic entrepreneur*. We refer to an individual with optimal expectations and ability $\hat{\theta}_O$ as the *marginal optimistic entrepreneur*.

3 Optimal Expectations

In this section we derive the optimal expectations. We consider a specialized version of the model with a production function given by

$$y = f(l, k, \theta_0) = \theta_0 g(l, k) = \theta_0 l^\alpha k^\beta,$$

where $\alpha + \beta \equiv \eta \in (0, 1)$. Hence, the variable inputs, labor and capital, are combined under a Cobb-Douglas production function with decreasing returns to scale and entrepreneurial skill enters into the production function as the total factor productivity (TFP).[7]

At $t = 3$ the material payoff of an entrepreneur is

$$\pi = \theta_0 l^\alpha k^\beta - wl - rk. \tag{9}$$

We see from (9) that this production function, the assumption that individuals are risk neutral, and the assumption that entrepreneurial skill θ_0 belongs to $[0, 1]$, imply that entrepreneurial skill can be interpreted as the true probability of success of the firm (the project either succeeds with probability θ_0 or fails with probability $1 - \theta_0$, in which case output is zero).

At $t = 2$ an individual with expectation of ability θ who becomes an entrepreneur chooses to employ l workers and k units of capital where l and k are the solution to

$$\max_{l,k} (\theta l^\alpha k^\beta - wl - rk).$$

[7] This is a standard assumption in models with heterogeneous skill. See, for example, Lucas (1978), Murphy et al. (1991), de Meza and Southey (1996), and Poschke (2013).

The first-order conditions are

$$\alpha\theta l^{\alpha-1}k^{\beta} = w,$$

and

$$\beta\theta l^{\alpha}k^{\beta-1} = r.$$

Solving for l and k we obtain the input demands of an entrepreneur with expectations of ability θ:

$$l(w,r,\theta) = \theta^{\frac{1}{1-\alpha-\beta}} \left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\alpha-\beta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\alpha-\beta}}, \tag{10}$$

and

$$k(w,r,\theta) = \theta^{\frac{1}{1-\alpha-\beta}} \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha-\beta}} \left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\alpha-\beta}}, \tag{11}$$

respectively.

At $t = 0$ the total payoff of an entrepreneur with expectations of ability θ is:

$$\theta_0 l^{\alpha}k^{\beta} - wl - rk + s(\theta l^{\alpha}k^{\beta} - wl - rk) = (\theta_0 + s\theta)l^{\alpha}k^{\beta} - (1+s)(wl + rk).$$

where l is given by (10) and k by (11). Hence, at $t = 0$, the optimal expectation of ability (or the optimal expectation of the probability of success of the firm) of an entrepreneur with ability θ_0 is the θ that solves

$$\max_{\theta\in[0,1]} \left\{(\theta_0 + s\theta)[l(w,r,\theta)]^{\alpha}[k(w,r,\theta)]^{\beta} - (1+s)[wl(w,r,\theta) + rk(w,r,\theta)]\right\}. \tag{12}$$

Our first result characterizes the solution to (12).

Proposition 1: *If* $f(l,k,\theta_0) = \theta_0 l^{\alpha}k^{\beta}$ *and the weight of anticipatory utility* s *is less than* $\eta/(1-\eta)$*, then optimal expectations of entrepreneurial ability are given by*

$$\theta^* = \begin{cases} \frac{\eta}{\eta - s(1-\eta)}\theta_0 & \text{if } \theta_0 < \frac{\eta - s(1-\eta)}{\eta} \\ 1 & \text{if } \theta_0 \geq \frac{\eta - s(1-\eta)}{\eta} \end{cases}. \tag{13}$$

This results tells us if the weight of anticipatory utility s is less than $\eta/(1-\eta)$, then individuals with optimal expectations choose to be optimists about their entrepreneurial ability since the belief of entrepreneurial ability θ^* is greater than the actual ability θ_0. The intuition behind this result is straightforward. Being optimist about entrepreneurial ability leads to first-order gains due to increased anticipatory utility from entrepreneurship and to second-order costs in realized profits due to distorted input choices.

We see from (13) that the optimal expectation of ability θ^* of individuals with ability θ_0 below $1 - s(1-\eta)/\eta$ is a function of ability θ_0, the weight of anticipatory utility s, and the degree of decreasing returns to scale η. We also see from (13) that the optimal expectation of ability θ^* of individuals with ability θ_0 above $1-s(1-\eta)/\eta$ is equal to the highest possible ability level, i.e., $\theta^* = 1$. Hence, for individuals with ability above $1 - s(1-\eta)/\eta$ the optimal expectation of ability does not depend on θ_0, s, and η.

Let the optimistic bias in beliefs of an individual with ability θ_0 be the gap between θ^* and θ_0, i.e., $b^* = \theta^* - \theta_0$. From (13) we have

$$b^* = \begin{cases} \frac{s(1-\eta)}{\eta - s(1-\eta)}\theta_0 & \text{if } \theta_0 < \frac{\eta - s(1-\eta)}{\eta} \\ 1 - \theta_0 & \text{if } \theta_0 \geq \frac{\eta - s(1-\eta)}{\eta} \end{cases}.$$

The optimistic bias in beliefs of individuals with ability θ_0 below $1 - s(1-\eta)/\eta$ is increasing with the weight of anticipatory utility s. Everything else constant, the higher s is, the more important the anticipated payoff of entrepreneurship, $\theta l^\alpha k^\beta - wl - rk$, becomes relative to the material payoff of entrepreneurship, $\theta_0 l^\alpha k^\beta - wl - rk$, and so the greater are the benefits from holding optimistic beliefs.

The optimistic bias in beliefs of individuals with ability θ_0 below $1 - s(1-\eta)/\eta$ is decreasing with a decrease in the degree of decreasing returns to scale (an increase in η). The intuition behind this result is as follows. Everything else constant, the higher η is, the smaller is the material payoff of entrepreneurship. Similarly, everything else constant, the higher η is, the smaller is the anticipated payoff of entrepreneurship.[8] However, as η increases the decrease in the anticipated payoff of entrepreneurship

[8] As η converges to 1 the technology converges to constant returns to scale and the material and anticipated payoffs of entrepreneurship converge to zero.

is steeper than the decrease in material payoff of entrepreneurship. Therefore, the higher η is, the smaller are the benefits from holding optimistic beliefs.

4 Optimal Expectations Equilibrium

In this section we determine the optimal expectations equilibrium when the weight of anticipatory utility s is not too high. We show that the lowest ability entrepreneurs are less talented at running a firm than the highest ability workers. We also show how the optimal expectations equilibrium can be determined when the weight of anticipatory utility s is high.

A realist with ability $\hat{\theta}_R$ is indifferent between being an entrepreneur and a worker when

$$\hat{\theta}_R[l(w,r,\hat{\theta}_R)]^\alpha[k(w,r,\hat{\theta}_R)]^\beta - wl(w,r,\hat{\theta}_R) - rk(w,r,\hat{\theta}_R) = w$$

Simplifying this equation we obtain

$$\alpha^\alpha\beta^\beta(1-\eta)^{1-\eta}\hat{\theta}_R = w^{1-\beta}r^\beta. \tag{14}$$

When the weight of anticipatory utility is not too high (this will be made precise further on), an optimist with optimal expectation of ability $\theta^* = \eta\hat{\theta}_O/[\eta - s(1-\eta)]$ and ability $\hat{\theta}_O$ is indifferent between being an entrepreneur and a worker when

$$(\hat{\theta}_O + s\theta^*)[l(w,r,\theta^*)]^\alpha[k(w,r,\theta^*)]^\beta - (1+s)\left[wl(w,r,\theta^*) + rk(w,r,\theta^*)\right] = w.$$

Simplifying this equation we obtain

$$\alpha^\alpha\beta^\beta(1-\eta)^{1-\eta}\left[\frac{\eta}{\eta - s(1-\eta)}\right]^\eta\hat{\theta}_O = w^{1-\beta}r^\beta. \tag{15}$$

In equilibrium, labor demand must equal labor supply. The assumption that entrepreneurial ability is uniformly distributed on $[0,1]$ implies that (7) becomes:

$$(1-\lambda)\int_{\hat{\theta}_R}^1 l(w,r,\theta_0)d\theta_0 + \lambda\int_{\hat{\theta}_O}^1 l[w,r,\theta^*(\theta_0)]d\theta_0 = (1-\lambda)\hat{\theta}_R + \lambda\hat{\theta}_O.$$

After substituting for the labor demands of the two types of entrepreneurs and integrating over θ_0 we obtain

$$\begin{aligned}&\frac{1-\eta}{2-\eta}\left\{1+\lambda\frac{s}{\eta}-(1-\lambda)\hat{\theta}_R^{\frac{2-\eta}{1-\eta}}-\lambda\left[\frac{\eta}{\eta-s(1-\eta)}\right]^{\frac{1}{1-\eta}}\hat{\theta}_O^{\frac{2-\eta}{1-\eta}}\right\}\left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}}\\ =\;&(1-\lambda)\hat{\theta}_R+\lambda\hat{\theta}_O. \qquad (16)\end{aligned}$$

In equilibrium, capital demand must equal the exogenous capital supply. The assumption that entrepreneurial ability is uniformly distributed on $[0,1]$ implies that (8) becomes:

$$(1-\lambda)\int_{\hat{\theta}_R}^{1}k(w,r,\theta_0)d\theta_0+\lambda\int_{\hat{\theta}_O}^{1}k[w,r,\theta^*(\theta_0)]d\theta_0=\bar{K}.$$

After substituting for the capital demands of the two types of entrepreneurs and integrating over θ_0 we obtain

$$\begin{aligned}&\frac{1-\eta}{2-\eta}\left\{1+\lambda\frac{s}{\eta}-(1-\lambda)\hat{\theta}_R^{\frac{2-\eta}{1-\eta}}-\lambda\left[\frac{\eta}{\eta-s(1-\eta)}\right]^{\frac{1}{1-\eta}}\hat{\theta}_O^{\frac{2-\eta}{1-\eta}}\right\}\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}}\\ =\;&\bar{K} \qquad (17)\end{aligned}$$

Equations (14), (15), (16), and (17) form a system of four equations and four unknowns ($\hat{\theta}_R$, $\hat{\theta}_O$, w, and r) which defines the optimal expectations equilibrium when the weight of anticipatory utility is not too high.[9] Our second result describes this equilibrium.

Proposition 2: *If $f(l,k,\theta_0)=\theta_0 l^\alpha k^\beta$, θ_0 is uniformly distributed on $[0,1]$, and $s<\bar{s}$, then there exists a unique optimal expectations equilibrium where the marginal realistic entrepreneur has ability*

$$\hat{\theta}_R=\left[\frac{\alpha}{2-\beta}\frac{1+\lambda\frac{s}{\eta}}{1-\lambda+\lambda\phi(\eta,\beta,s)}\right]^{\frac{1-\eta}{2-\eta}}, \qquad (18)$$

[9] Note that from $\hat{\theta}_R$ and $\hat{\theta}_O$ we obtain the equilibrium number of workers $L=(1-\lambda)\hat{\theta}_R+\lambda\hat{\theta}_O$.

the marginal optimistic entrepreneur has ability

$$\hat{\theta}_O = \psi(\eta, s)\left[\frac{\alpha}{2-\beta}\frac{1+\lambda\frac{s}{\eta}}{1-\lambda+\lambda\phi(\eta,\beta,s)}\right]^{\frac{1-\eta}{2-\eta}}, \tag{19}$$

the wage is

$$w^* = \frac{\alpha^{\eta}(1-\eta)^{1-\eta}\bar{K}^{\beta}}{[1-\lambda+\lambda\psi(\eta,s)]^{\beta}}\left[\frac{\alpha}{2-\beta}\frac{1+\lambda\frac{s}{\eta}}{1-\lambda+\lambda\phi(\eta,\beta,s)}\right]^{\frac{(1-\eta)(1-\beta)}{2-\eta}}, \tag{20}$$

the fraction of workers is

$$L^* = [1-\lambda+\lambda\psi(\eta,s)]\left[\frac{\alpha}{2-\beta}\frac{1+\lambda\frac{s}{\eta}}{1-\lambda+\lambda\phi(\eta,\beta,s)}\right]^{\frac{1-\eta}{2-\eta}}, \tag{21}$$

and the rental cost of capital is

$$r^* = \frac{\beta(1-\eta)^{1-\eta}}{\alpha^{1-\eta}\bar{K}^{1-\beta}}[1-\lambda+\lambda\psi(\eta,s)]^{1-\beta}\left[\frac{\alpha}{2-\beta}\frac{1+\lambda\frac{s}{\eta}}{1-\lambda+\lambda\phi(\eta,\beta,s)}\right]^{\frac{(1-\eta)(2-\beta)}{2-\eta}}, \tag{22}$$

where $\bar{s}$ is the solution to

$$\frac{\alpha}{2-\beta}\left(1+\lambda\frac{\bar{s}}{\eta}\right) = [1-\lambda+\lambda\phi(\eta,\beta,\bar{s})]\left[\frac{\eta-\bar{s}(1-\eta)}{\eta}\right]^{2-\eta}, \tag{23}$$

and

$$\phi(\eta,\beta,s) = \left[1-\frac{s(1-\eta)(2-\eta)}{\eta(2-\beta)}\right]\left[\frac{\eta}{\eta-s(1-\eta)}\right]^{1-\eta},$$

and

$$\psi(\eta,s) = \left[\frac{\eta-s(1-\eta)}{\eta}\right]^{\eta}.$$

Proposition 2 shows us that the existence of individuals with optimal expectations leads to a misallocation of talent. In the rational expectations equilibrium ($\lambda = 0$) the marginal entrepreneur has ability

$$\hat{\theta}_0 = \left(\frac{\alpha}{2-\beta}\right)^{\frac{1-\eta}{2-\eta}}, \tag{24}$$

which implies that individuals with ability $[0, \hat{\theta}_0]$ become workers and individuals with ability $[\hat{\theta}_0, 1]$ become entrepreneurs. Hence, in the rational expectations equilibrium the ablest people become entrepreneurs. In contrast, in the optimal expectations equilibrium realists with ability $[0, \hat{\theta}_R]$ become workers and those with ability $[\hat{\theta}_R, 1]$ become entrepreneurs.[10] Furthermore, optimists with ability $[0, \hat{\theta}_O]$ become workers and those with ability $[\hat{\theta}_O, 1]$ become entrepreneurs. It follows from (18) and (19) that the marginal optimistic entrepreneur has a lower ability than the marginal realistic entrepreneur:

$$\hat{\theta}_O < \hat{\theta}_R. \tag{25}$$

Hence, amongst individuals with ability $\theta_0 \in [\hat{\theta}_O, \hat{\theta}_R]$ those who are realists become workers and those who are optimists become entrepreneurs. Therefore, in the optimal expectations equilibrium, the ablest people do not necessarily become entrepreneurs. Moreover, the lowest ability entrepreneurs are less talented at running a firm than the highest ability workers. This is an empirically attractive implication of the model since, in reality, the income distributions of workers and entrepreneurs have overlapping supports.[11]

In the rational expectations equilibrium ($\lambda = 0$) output is equal to

$$\begin{aligned} Y_0^* &= \int_{\hat{\theta}_0}^{1} \theta_0 [l(w_0^*, r_0^*, \theta_0)]^\alpha [k((w_0^*, r_0^*, \theta_0)]^\beta d\theta_0 \\ &= \frac{1-\eta}{2-\eta} \left(\frac{\alpha}{w_0^*}\right)^{\frac{\alpha}{1-\eta}} \left(\frac{\beta}{r_0^*}\right)^{\frac{\beta}{1-\eta}} \left(1 - \frac{\alpha}{2-\beta}\right) \\ &= (1-\eta)^{1-\eta} \alpha^{\frac{\alpha(1-\eta)}{2-\eta}} (2-\beta)^{\frac{-(2-\beta)(1-\eta)}{2-\eta}} \bar{K}^\beta. \end{aligned}$$

[10] The proof of Proposition 2 shows that $\hat{\theta}_R \in (0, 1)$.

[11] This stands in contrast to models where occupational choice is only based on heterogeneity in ability and where it is assumed that one occupation rewards ability more than the other. This results in income distributions for the two occupations with non-overlapping intervals (see, e.g., Parker, 2009).

In the optimal expectations equilibrium output is equal to

$$Y^* = (1-\lambda)\int_{\hat{\theta}_R}^{1} \theta_0 [l(w^*, r^*, \theta_0)]^\alpha [k((w^*, r^*, \theta_0)]^\beta d\theta_0 + \lambda \int_{\hat{\theta}_O}^{\frac{\eta - s(1-\eta)}{\eta}} \theta_0 [l(w^*, r^*, \theta^*)]^\alpha [k((w^*, r^*, \theta^*)]^\beta d\theta_0 + \lambda \int_{\frac{\eta - s(1-\eta)}{\eta}}^{1} \theta_0 [l(w^*, r^*, 1)]^\alpha [k((w^*, r^*, 1)]^\beta d\theta_0.$$

Output in the presence of optimists is lower than output in the absence of optimists, i.e., $Y^* < Y_0^*$. We know from Lucas (1978) that the decentralized general equilibrium maximizes output. As we have seen above, the existence of optimists leads to a misallocation of talent which lowers output.

When the weight of anticipatory utility s belongs to the interval $[\bar{s}, \eta/(1-\eta)]$, individuals with optimal expectations who select to become entrepreneurs hold the highest possible belief of ability, i.e., $\theta^* = 1$. In addition, a positive mass of individuals with $\theta^* = 1$ choose to be workers since their entrepreneurial ability θ_0 is not high enough to make entrepreneurship more attractive than working as an employee. Proposition 3 characterizes the optimal expectations equilibrium when $s \in [\bar{s}, \eta/(1-\eta)]$.

Proposition 3: *If $f(l, k, \theta_0) = \theta_0 l^\alpha k^\beta$, θ_0 is uniformly distributed on $[0, 1]$, and $s \in [\bar{s}, \eta/(1-\eta)]$, then the optimal expectations equilibrium is the unique solution to the system of four equations and four unknowns ($\hat{\theta}_R$, $\hat{\theta}_O$, w, and r):*

$$\alpha^\alpha \beta^\beta (1-\eta)^{1-\eta} \hat{\theta}_R = w^{1-\beta} r^\beta,$$

$$\alpha^\alpha \beta^\beta \left[\hat{\theta}_O + s - (1+s)\eta\right]^{1-\eta} = w^{1-\beta} r^\beta,$$

$$\left[(1-\lambda)\frac{1-\eta}{2-\eta}\left(1 - \hat{\theta}_R^{\frac{2-\eta}{1-\eta}}\right) + \lambda(1-\hat{\theta}_O)\right]\left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}} = (1-\lambda)\hat{\theta}_R + \lambda\hat{\theta}_O,$$

$$\left[(1-\lambda)\frac{1-\eta}{2-\eta}\left(1 - \hat{\theta}_R^{\frac{2-\eta}{1-\eta}}\right) + \lambda(1-\hat{\theta}_O)\right]\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}} = \bar{K}.$$

As we have seen, when the weight of anticipatory utility is not too high, i.e., s is less than $\bar{s}$ given by (23), the equilibrium wage, fraction of workers, and rental cost of capital are given by (20), (21), and (22), respectively. In contrast, when $s \in [\bar{s}, \eta/(1-\eta)]$ we are unable to obtain closed form solutions for w^*, L^*, and r^*. Hence, the rest of the paper focuses on the case where s is not too high.

We now show that the model predicts that optimists are more likely to become entrepreneurs than realists, that entrepreneurs are more likely to be optimists than workers, and, that, provided the fraction of optimists is high enough, the majority of entrepreneurs are optimists. Before doing this we need to define the probability an entrepreneur is an optimist and the probability a worker is an optimist. The fraction of entrepreneurs is equal to

$$E^* = E_O^* + E_R^* = \lambda(1-\hat{\theta}_O) + (1-\lambda)(1-\hat{\theta}_R).$$

Hence, the probability an entrepreneur is an optimist is given by

$$\Pr(O|E^*) = \frac{E_O^*}{E^*} = \frac{\lambda(1-\hat{\theta}_O)}{\lambda(1-\hat{\theta}_O) + (1-\lambda)(1-\hat{\theta}_R)}. \tag{26}$$

The fraction of workers is equal to

$$L^* = L_O^* + L_R^* = \lambda\hat{\theta}_O + (1-\lambda)\hat{\theta}_R.$$

Hence, the probability a worker is an optimist is given by

$$\Pr(O|L^*) = \frac{L_O^*}{L^*} = \frac{\lambda\hat{\theta}_O}{\lambda\hat{\theta}_O + (1-\lambda)\hat{\theta}_R}. \tag{27}$$

Proposition 4: *Assume* $f(l,k,\theta_0) = \theta_0 l^\alpha k^\beta$, θ_0 *is uniformly distributed on* $[0,1]$, *and* $s < \bar{s}$.

(i) The probability an optimist becomes an entrepreneur is greater than the probability a realist becomes an entrepreneur, i.e., $\Pr(E^*|O) > \Pr(E^*|R)$*;*

(ii) The probability an entrepreneur is an optimist is greater than the probability a worker is an optimist, i.e., $\Pr(O|E^*) > \Pr(O|L^*)$*;*
(iii) If

$$\lambda > \left[1 + \frac{1 - \hat{\theta}_R \left[\frac{\eta - s(1-\eta)}{\eta} \right]^{\eta}}{1 - \hat{\theta}_R} \right]^{-1}, \quad (28)$$

where $\hat{\theta}_R$ *is given by (18), then the majority of entrepreneurs are optimists, i.e.,* $\Pr(O|E^*) > 1/2$.

Proposition 4-(i) tells us that optimists are more likely to become entrepreneurs than realists. This result follows directly from (25) and is in line with the empirical evidence in Gentry and Hubbard (2000), Hurst and Lusardi (2004), and Cassar and Friedman (2009). Proposition 4-(ii) tells us that entrepreneurs are more likely to be optimists than workers. This result follows from (25), (26), and (27). This is in line with the empirical evidence in Arabsheibani et al. (2000), Fraser and Greene (2006), Puri and Robinson (2007), and Dawson et al. (2014).

According to (26) the majority of entrepreneurs can be either optimists ($\Pr(O|E^*) > 1/2$) or realists ($\Pr(O|E^*) < 1/2$). However, the empirical evidence in Cooper et al. (1988), Wu and Knott (2006), Landier and Thesmar (2009), Cassar (2010, 2012), and Hyytinen et al. (2014) shows that entrepreneurs are overwhelmingly optimists. Proposition 4-(iii) shows if the fraction of optimists λ is higher than the lower bound in (28), then the majority of entrepreneurs are optimists.

5 Comparative Statics

In this section we perform comparative statics on the impact of optimism on equilibrium outcomes. There are two main parameters which can be used to perform this analysis: the fraction of optimists λ and the weight of anticipatory utility s. By increasing λ while keeping everything else fixed we analyze the impact that a increase in the fraction of optimists has on equilibrium outcomes. By increasing s

while keeping everything else fixed we analyze the impact that an increase in optimistic biases in beliefs have on equilibrium outcomes (we know from Proposition 1 that the higher s is, the greater are the optimistic biases in beliefs of individuals with ability below $1 - s(1-\eta)/\eta$). We focus on comparative statics with respect to the fraction of optimists. At the end we discuss briefly the comparative statics with respect to the weight of anticipatory utility.

We start by showing that an increase in the fraction of optimists raises the ability of the marginal realistic entrepreneur $\hat{\theta}_R$ and the ability of the marginal optimistic entrepreneur $\hat{\theta}_O$.

Proposition 5: *If $f(l, k, \theta_0) = \theta_0 l^\alpha k^\beta$, θ_0 is uniformly distributed on $[0, 1]$, and $s < \bar{s}$, then an increase in the fraction of optimists raises the ability of the marginal realistic entrepreneur and the ability of the marginal optimistic entrepreneur, i.e., $\partial\hat{\theta}_R/\partial\lambda > 0$ and $\partial\hat{\theta}_O/\partial\lambda > 0$, respectively.*

It follows from $\partial\hat{\theta}_R/\partial\lambda > 0$ that the ability of the marginal realistic entrepreneur $\hat{\theta}_R$ is higher than the ability of the marginal entrepreneur in the rational expectations equilibrium $\hat{\theta}_0$, i.e., $\hat{\theta}_R > \hat{\theta}_0$. It follows from $\partial\hat{\theta}_O/\partial\lambda > 0$, (19), and (24) that if λ is low, then the ability of the marginal optimistic entrepreneur $\hat{\theta}_O$ is lower than the ability of the marginal entrepreneur in the rational expectations equilibrium $\hat{\theta}_0$, i.e., $\hat{\theta}_O < \hat{\theta}_0$. Hence, when λ is low we have

$$\hat{\theta}_O < \hat{\theta}_0 < \hat{\theta}_R,$$

and when λ is high we have

$$\hat{\theta}_0 < \hat{\theta}_O < \hat{\theta}_R.$$

Thus, in the optimal expectations equilibrium the misallocation of talent depends on the fraction of optimists λ. A realist with ability $\theta_0 \in [\hat{\theta}_0, \hat{\theta}_R]$ chooses to become a worker in the optimal expectations equilibrium but would select to be an entrepreneur in the rational expectations equilibrium. When λ is low, an optimist with ability $\theta_0 \in [\hat{\theta}_O, \hat{\theta}_0]$ selects to be an entrepreneur in the optimal expectations equilibrium but would choose to become a worker in the rational expectations equilibrium. When

λ is high, an optimist with ability $\theta_0 \in [\hat{\theta}_0, \hat{\theta}_O]$ chooses to become a worker in the optimal expectations equilibrium but would select to be an entrepreneur in the rational expectations equilibrium.

We now show that an increase in the fraction of optimists raises the market clearing wage.

Proposition 6: *If* $f(l, k, \theta_0) = \theta_0 l^\alpha k^\beta$, θ_0 *is uniformly distributed on* $[0, 1]$, *and* $s < \bar{s}$, *then an increase in the fraction of optimists leads to an increase in the wage, i.e.,* $\partial w^*/\partial\lambda > 0$.

The intuition behind Proposition 6 is as follows. The assumption that entrepreneurial ability and labor are complements and the fact that individuals with optimal expectations are optimists implies that, for a given wage, the demand for labor of an individual with optimal expectations is higher than the demand for labor of a realist of the same ability. This leads to an expansion of labor demand. Since an individual with optimal expectations derives anticipatory utility from entrepreneurship and is optimist about his entrepreneurial ability he will be, for a given wage, more attracted to entrepreneurship than a realist who of the same ability. This leads to a contraction of labor supply. The expansion of labor demand and contraction of labor supply raise the market clearing wage.

Our next result summarizes the impact of an increase in the fraction of optimists on the fractions of realistic workers, optimistic workers, and realistic entrepreneurs.

Proposition 7: *If* $f(l, k, \theta_0) = \theta_0 l^\alpha k^\beta$, θ_0 *is uniformly distributed on* $[0, 1]$, $s < \bar{s}$, *then an increase in the fraction of optimists leads to (i) a decrease in the fraction of realistic workers, i.e.,* $\partial L_R^*/\partial\lambda < 0$, *(ii) an increase in the fraction of optimistic workers, i.e.,* $\partial L_O^*/\partial\lambda > 0$, *and (iii) a decrease in the fraction of realistic entrepreneurs, i.e.,* $\partial E_R^*/\partial\lambda < 0$.

Proposition 7-(i) shows that an increase in the fraction of optimists lowers the fraction of realistic workers. The intuition behind this result is the following. Amongst $1 - \lambda$ realists the fraction $\hat{\theta}_R$ become workers so the fraction of realistic workers is

$L_R^* = (1-\lambda)\hat{\theta}_R$. An increase in λ lowers the fraction of realists and raises the ability of the marginal realistic entrepreneur $\hat{\theta}_R$. The first effect lowers the fraction of realistic workers but the second effect raises it. Hence, at first sight, an increase in the fraction of optimists has an ambiguous effect on the fraction of realistic workers. However, the first effect always dominates the second and therefore an increase in the fraction of optimists lowers the fraction of realistic workers.

Proposition 7-(ii) shows that an increase in the fraction of optimists raises the fraction of optimistic workers. Amongst λ optimists the fraction $\hat{\theta}_O$ become workers so the fraction of optimistic workers is $L_O^* = \lambda\hat{\theta}_O$. An increase in λ raises the fraction of optimistic workers because is raises the fraction of optimists and the ability of the marginal optimistic entrepreneur $\hat{\theta}_O$.

Proposition 7-(iii) shows that an increase in the fraction of optimists lowers the fraction of realistic entrepreneurs. Amongst $1-\lambda$ realists the fraction $1-\hat{\theta}_R$ become entrepreneurs so the fraction of realistic entrepreneurs is $E_R^* = (1-\lambda)(1-\hat{\theta}_R)$. An increase in λ lowers the fraction of realistic entrepreneurs because it lowers the fraction of realists and raises the ability of the marginal realistic entrepreneur $\hat{\theta}_R$.

Note that an increase in the fraction of optimists has an ambiguous effect on the fraction of optimistic entrepreneurs. Amongst λ optimists the fraction $1-\hat{\theta}_O$ become entrepreneurs so the fraction of optimistic entrepreneurs is $E_O^* = \lambda(1-\hat{\theta}_O)$. An increase in λ raises the fraction of optimists and the ability of the marginal optimistic entrepreneur $\hat{\theta}_O$. The first effect raises the fraction of optimistic entrepreneurs but the second effect lowers it. Without imposed additional conditions on the parameters of the model it is unclear which effect dominates.

We know from Proposition 7 that, on the one hand, an increase in the fraction of optimists lowers the fraction of realistic workers, and, on the other hand, it raises the fraction of optimistic workers. Therefore, an increase in the fraction of optimists appears to have an ambiguous effect on the fraction of workers (and entrepreneurs since $E^* = 1 - L^*$). However, our next result shows how the fraction of workers (and entrepreneurs) varies with the fraction of optimists.

Proposition 8: *Assume* $f(l,k,\theta_0)=\theta_0 l^\alpha k^\beta$, θ_0 *is uniformly distributed on* $[0,1]$, *and* $s<\bar{s}$.

(i) The fraction of workers (entrepreneurs) is a concave (convex) function of the fraction of optimists, i.e., $\partial^2 L^*/\partial\lambda^2<0$ *(*$\partial^2 E^*/\partial\lambda^2>0$*).*

(ii) If

$$\left[1+\frac{s}{\eta}-\phi(\eta,\beta,s)\right]\frac{\psi(\eta,s)}{\phi(\eta,\beta,s)}>\frac{2-\eta}{1-\eta}[1-\psi(\eta,s)]\left(1+\frac{s}{\eta}\right), \tag{29}$$

then an increase in the fraction of optimists leads to an increase (decrease) in the fraction of workers (entrepreneurs), i.e., $\partial L^*/\partial\lambda>0$ *(*$\partial E^*/\partial\lambda<0$*).*

(iii) If (29) is violated and $\bar{\lambda}$ *is the solution to*

$$\frac{1-\bar{\lambda}+\bar{\lambda}\psi(\eta,s)}{1-\bar{\lambda}+\bar{\lambda}\phi(\eta,\beta,s)}=\frac{2-\eta}{1-\eta}\frac{1-\psi(\eta,s)}{1+\frac{s}{\eta}-\phi(\eta,\beta,s)}\left(1+\bar{\lambda}\frac{s}{\eta}\right), \tag{30}$$

then an increase in the fraction of optimists leads to (a) an increase (decrease) in the fraction of workers (entrepreneurs) when $\lambda<\bar{\lambda}$*, i.e.,* $\partial L^*/\partial\lambda>0$ *(*$\partial E^*/\partial\lambda<0$*), and (b) a decrease (increase) in the fraction of workers (entrepreneurs) when* $\lambda>\bar{\lambda}$*, i.e.,* $\partial L^*/\partial\lambda<0$ *(*$\partial E^*/\partial\lambda>0$*).*

Proposition 8 shows that an increase in the fraction of optimists does not necessarily lead to a decrease (an increase) in the fraction of workers (entrepreneurs). Moreover, one of two cases might arise. First, an increase in the fraction of optimists raises (lowers) the fraction of workers (entrepreneurs). This happens when either inequality (29) is satisfied or inequality (29) is violated and the fraction of optimists is small, i.e., $\lambda<\bar{\lambda}$. In this case an increase in the fraction of optimists raises the fraction of optimistic workers more than it lowers the fraction of realistic workers. Second, an increase in the fraction of optimists lowers (raises) the fraction of workers (entrepreneurs). This happens when inequality (29) is violated and the fraction of optimists is high, i.e., $\lambda>\bar{\lambda}$. In this case an increase in the fraction of optimists raises the fraction of optimistic workers less than it lowers the fraction of realistic workers.

We now provide conditions under which an increase in the fraction of optimists raises the rental cost of capital.

Proposition 9: *If* $f(l,k,\theta_0) = \theta_0 l^\alpha k^\beta$, θ_0 *is uniformly distributed on* $[0,1]$, $s < \bar{s}$, *and either inequality (29) is satisfied or inequality (29) is violated and the fraction of optimists is small, i.e.,* $\lambda < \bar{\lambda}$, *then an increase in the fraction of optimists increases in the rental cost of capital, i.e.,* $\partial r^*/\partial\lambda > 0$.

To close this section we discuss informally the comparative statics with respect to the weight of anticipatory utility. An increase in the weight of anticipatory utility s raises the ability of the marginal realistic entrepreneur $\hat{\theta}_R$ and lowers the ability of the marginal optimistic entrepreneur $\hat{\theta}_O$. Hence, an increase in s lowers the fraction of realistic entrepreneurs E_R^* and raises the fraction of optimistic entrepreneurs E_O^*. In addition, an increase in s raises the fraction of realistic workers L_R^* and lowers the fraction of optimistic workers L_O^*. Finally, an increase in the weight of anticipatory utility s raises the wage w^*, lowers the number of workers L^* and raises the number of entrepreneurs E^*.

6 Calibration

This section calibrates the model to illustrate quantitatively the general equilibrium effects of entrepreneurial optimism. The calibration parameterizes the economy to match salient features of US manufacturing data, it follows Atkeson and Kehoe (2005) and Adler (2016). The calibration is summarized in Table I.

The degree of decreasing returns to scale η is an important parameter in the model since it simultaneously characterizes the firms' technology and constrains the optimistic bias in beliefs. Following Atkeson and Kehoe (2005) and Adler (2016) we set η to 0.85.

Given η equal to 0.85, a value of 0.612 for α matches labor's average income share (including managerial compensation) in manufacturing between 1998 and 2005. Again, following Atkeson and Kehoe (2005) and Adler (2016) we assume a capital-output ratio $\bar{K}/Y$ of 1.46 which together with a value for Y of 0.620 in the benchmark Lucas' model implies a capital stock $\bar{K}$ of 0.906.

Table I

Parameters of the Model

Parameter	Value	Description
Standard parameters		
η	0.85	decreasing returns to scale
α	0.612	labor's average income share
β	0.238	capital's average income share
$\bar{K}$	0.906	capital stock
Additional parameters		
λ	0.5	fraction with anticipatory utility
s	0.5	weight of anticipatory utility

We are left with the parameters λ and s to calibrate. The parameter λ measures the fraction of individuals with anticipatory utility and can vary between 0 and 1. Someone who experiences positive anticipatory benefits while waiting for the resolution of risk should display a preference for delayed resolution of risk. In addition, if such a person is offered several identical lottery tickets for different drawing dates with the same expected value, then he or she should have a preference for spreading the days of drawing. Kocher et al. (2014) test these two predictions using a laboratory experiment and find that, although 41.5 percent of participants prefer real lottery tickets for an immediate drawing rather than one for the subsequent day, 21.5 percent actually prefers delayed resolution, and the remaining 37 percent is indifferent. In addition, they find that around 70 percent of participants want tickets on two days rather than on one. Based on these two findings we set λ to 0.5.

The weight of anticipatory utility s is also an important parameter in the model since it constrains the optimistic bias in beliefs. We are not aware of empirical work that tries to quantify s. However, for the competitive equilibrium to be well defined, s must be smaller than the upper bound $\bar{s}$ in (23). Setting $\eta = 0.85$, $\alpha = 0.612$, $\beta = 0.238$, and $\lambda = 0.5$ in (23) and solving for $\bar{s}$ we obtain $\bar{s} = 1.5$. Hence, s can vary between 0 and 1.5. We believe that the weight of anticipatory utility s should be less than the weight of material payoffs therefore we set s to 0.5, the middle of the range between 0 and 1.

Table II summarizes the results of the calibration.

Table II

Comparing General Equilibrium Outcomes with and without Optimism

	Benchmark Lucas' model	Model with $\lambda = s = 0.5$	Percent change
Output (Y^*)	0.620	0.616	−0.73
Wage (w^*)	0.436	0.452	3.77
Rental cost of capital (r^*)	0.163	0.169	3.57
Mean returns to entrepreneurship	0.722	0.534	−26.08
Mean returns of realistic entrep	-	0.650	-
Mean returns of optimistic entrep	-	0.466	-
Fraction of workers (L^*)	0.871	0.870	−0.19
Ability of mg realistic entrep ($\hat{\theta}_R$)	-	0.904	-
Ability of mg optimistic entrep ($\hat{\theta}_O$)	-	0.835	-
Fraction of entrep (E^*)	0.129	0.130	1.30
Fraction of entrep optimists	-	0.631	-
Fraction of workers optimists	-	0.480	-

The first column in Table II lists the variables. The second column reports the equilibrium values in the benchmark Lucas' model where $\lambda = 0$. The third column the equilibrium values in the model with $\lambda = s = 0.5$. The fourth column reports the percent change in the equilibrium values of the variables common to both models.

In the benchmark Lucas' model the mean return to entrepreneurship is 0.722 and the wage is 0.436. Hence, in the absence of optimism, the mean return to entrepreneurship is 1.656 times higher than the wage. In the model with $\lambda = s = 0.5$ the mean return to entrepreneurship is 0.534 and the wage is 0.452. Hence, when $\lambda = s = 0.5$ the mean return to entrepreneurship is only 1.181 times higher than the wage. Moreover, an increase in the fraction of optimists from 0 to 50 percent leads to a 26.08 percent decline in the mean return to entrepreneurship, a 3.77 percent increase in the wage, and a 3.57 increase in the rental cost of capital.

The calibration shows that optimism can lead to a sharp decline in the mean return to entrepreneurship. This happens for three reasons. First, optimism lowers the mean return to entrepreneurship of realists by raising input prices. The mean return to entrepreneurship of a realist is 0.650 which is 9.97 percent less than 0.722. Second, optimism lowers the mean return to entrepreneurship of optimists by raising input prices and distorting input choices. The mean return to entrepreneurship of an optimist is only 0.466 which is 35.46 percent less than 0.722. Third, optimism lowers the fraction of realistic entrepreneurs–the ones with higher returns–and raises the fraction of optimistic entrepreneurs–the ones with lower returns. Note that the fact that realistic entrepreneurs earn, on average, more than optimistic ones is in line with the empirical evidence reported in Dawson et al. (2015).

In both models the fraction of workers is approximately 87 percent and the fraction of entrepreneurs 13 percent. This is not far from the fractions reported in Cagetti and de Nardi (2006), Blanchflower (2010), Hipple (2010), and Poschke (2013). Moreover, the calibration shows that an increase in the fraction of optimists from 0 to 50 percent only leads to a 0.73 percent decline in output, a 0.19 percent decline in the fraction of workers, and a 1.3 percent increase in the fraction of entrepreneurs.[12] Furthermore, the misallocation of talent affects less than 3.5 percent of the population:

$$\lambda(\hat{\theta}_0 - \hat{\theta}_O) + (1-\lambda)(\hat{\theta}_R - \hat{\theta}_0) = 0.5(0.871 - 0.835) + 0.5(0.904 - 0.871) = 0.0345.$$

Finally, when $\lambda = s = 0.5$, 63 percent of entrepreneurs are optimists but only 48 percent of workers are optimists. Hence, entrepreneurs are more optimistic than

[12]Under the calibrated parameters inequality (29) is violated and the solution to equation (30) is $\bar{\lambda} = 0.149$. Hence, an increase in the fraction of optimists lowers (raises) the fraction of workers (entrepreneurs) when $\lambda > 0.149$.

workers and the majority of entrepreneurs are optimists. This is in line with Arabsheibani et al. (2000), Fraser and Greene (2006), Puri and Robinson (2007), and Dawson et al. (2014).

What are we to take away from these results? As we have seen, the calibration shows that optimism can have a sharp negative impact on the returns to entrepreneurship and a positive impact on the wage. So, the model is a step in the right direction in terms of explaining why the mean returns from entrepreneurship are not found to be significantly higher than mean wages. However, the calibration also shows that having a large fraction of optimistic individuals in the economy can have a relatively modest impact on output and on occupational choices. As we have seen, output drops by 0.73 percent, there is 0.19 percent decline in the fraction of workers, and a 1.3 percent increase in the fraction of entrepreneurs. This is an interesting finding since it tells us that not all general equilibrium effects of biased beliefs are as large as one could have imagined a priori.

Finally, are there any policy implications one can take away from these results? Given their preferences, individuals in this economy are maximizing their utility and so the economy is in a first-best situation which implies that welfare is maximized. However, if the goal of a policymaker is to raise the output of the economy, then it is possible to do so with a revenue-neutral tax-subsidy scheme. The scheme consists of a lump-sum tax to (optimistic) entrepreneurs with profits below the market clearing wage and a lump-sum subsidy to workers. The tax revenues come only from low ability optimistic entrepreneurs and induces them to stay in the labor force. The tax revenues are redistributed to workers as a lump-sum subsidy which further induces low ability optimists to stay in the labor force. The results on this tax-subsidy scheme are available upon request.

7 Contribution to the Literature

Our paper contributes to the literature on occupational choice using general equilibrium models. In this broad line of research, Lucas (1978), Kanbur (1979), Kihlstrom and Laffont (1979), Bewley (1989) and Lazear (2005) are five prominent papers.

Lucas (1978) proposes a general equilibrium of occupational choice where differences in entrepreneurial ability determine who becomes a worker or an entrepreneur. He considers a closed economy with a workforce of size N and K units of homogeneous capital. Individuals are risk neutral and the output of a firm is an increasing function of entrepreneurial ability, labor and capital. Lucas shows that the most talented individuals become entrepreneurs and the less talented ones become workers. He also studies a dynamic version of the model which allows him to analyze the impact that an increase in the capital stock has on the evolution of firm size distribution. We extend Lucas (1978) by allowing for a fraction of individuals to form optimal expectations of entrepreneurial ability.[13]

Kanbur (1979) considers a general equilibrium model of occupational choice where individuals learn their ability as entrepreneurs by entering entrepreneurship. Those who become entrepreneurs can therefore make an informed future occupational choice. The cost of becoming an entrepreneur is the risk exposure relative to the safe alternative of being employed. Indeed, the payoff of an entrepreneur's project depends on his ability which is unknown to new entrepreneurs. Hence, entrepreneurship has an immediate cost (risk taking) and a postponed gain (informed future occupational choice). For this reason, more patient societies have a larger share of new entrepreneurs.

Kihlstrom and Laffont (1979) study the role of risk aversion in a general equilibrium model of occupational choice. Agents choose between operating a risky firm or working for a riskless wage. Agents with low risk aversion become entrepreneurs while those with high risk aversion become workers. Among entrepreneurs, the less risk averse are found to operate larger firms. So far there is very little empirical evidence

[13] Chapter 2 of Parker (2009) discusses in detail the main extensions of Lucas' (1978) model.

supporting this explanation. Moreover, risk attitudes cannot explain occupational choice when risks can be perfectly insured.

Jovanovic (1994) generalizes Lucas (1978) by allowing for individuals to have two dimensions of ability: managerial and working. He shows that when managerial skills are positively correlated with working skills the best potential managers could end up as wage workers. Cagetti and De Nardi (2006) study the effect of borrowing constraints on wealth distribution and occupational choices in a model where individuals have managerial and working ability (assumed to be uncorrelated).

Knight (1921) distinguishes risk from uncertainty. A project is risky when the probability of outcomes are known. If the probabilities are unknown, the project is uncertain. According to Knight, entrepreneurs start uncertain projects. Building on Knight's work, Bewley (1989) models the link between uncertainty aversion and the choice to become an entrepreneur. The main finding is that entrepreneurs are individuals with low uncertainty aversion.

Lazear (2005) considers an occupational choice model where individuals are endowed with two skills. An individual can be a specialist, in which case he receives income associated with his best skill, or he can be an entrepreneur, in which case he is limited by his weakest attribute. He shows that individuals endowed with more balanced ability sets are found to be more likely to become entrepreneurs.

Campanale (2010) considers a life-cycle occupational and portfolio choice model with learning. The key assumption is that the quality of a business project is not precisely known upon entry and is learned over time. The model shows that entry and private equity allocation for the majority of entrepreneurs can be rationalized even with negative expected premia on individual business investment. Since individuals can switch back to paid-employment, they find it worthwhile experimenting with entrepreneurship to find out if the project is good even if initially the expected return is low.

Poschke (2013), like Campanale (2010), also studies a dynamic occupational choice model with learning. Individuals differ in their efficiency as workers and in the productivity of the firms they start. Whereas efficiency as a worker is known, the productivity of entrepreneurial projects can only be found after implementing them. He shows that the option to abandon bad projects attracts low-ability agents into entrepreneurship.

More narrowly, our paper contributes to the literature that studies the implications of optimism using a general equilibrium framework. We are aware of at least four studies that do so: de Meza and Southey (1996), Manove (2000), Fraser and Greene (2006) and Rigotti et al. (2011).

In de Meza and Southey (1996) risk neutral individuals must choose between becoming entrepreneurs or employees. The output of a firm is an increasing function of entrepreneurial ability and capital. There are two types of individuals: realists and optimists. An individual who chooses to become an entrepreneur must select the right mix of self-finance and debt-finance from risk neutral banks to develop his project. Banks and realistic individuals know a project's true probability of success but optimists overestimate it. They find that optimists select maximum self-finance and any external finance is in the form of a standard debt contract. They also find that, though not all optimists necessarily become entrepreneurs, all entrepreneurs will be optimists.

Manove (2000) considers a competitive economy where individuals with different productivities as entrepreneurs choose to become entrepreneurs or employees. Entrepreneurs use their own capital, effort and labor provided by employees to produce. All individuals consume the good produced. There are two types of individuals: realists and optimists. Optimists overestimate their productivity as entrepreneurs. Manove shows that optimism increases the savings rates and work effort of optimists, which can have a positive effect on steady-state income (though the optimist's utility will be reduced). However, optimism may also tend to reduce income through a negative effect on economic efficiency. The sources of the two effects on income are distinct: the negative efficiency effect is associated with the overuse of external resources, such as hired labor and borrowed capital, whereas the positive incentive effect is associated with the overuse of resources internal to the entrepreneur, such as his personal savings and effort.

Fraser and Greene (2006) consider an occupational choice model in which entrepreneurs are uncertain about their true talent but learn from experience. It follows that optimism in talent lowers with experience. As a consequence, the impact of optimism on the decision to be an entrepreneur lowers with experience.

In Rigotti et al. (2011) individuals choose to be entrepreneurs or employees and between employing a traditional technology or a new one about which little is known. A firm is an entrepreneur-employee pair operating a particular technology. Individuals face ambiguity about firms' return and are either optimistic or pessimistic. Optimists are more likely to become entrepreneurs. Moreover, firms employing new and highly ambiguous technologies are run by optimistic entrepreneurs and employ optimistic employees.

Our paper contains two main innovations relative to these studies. First, the assumption that optimistic beliefs arise endogenously. This assumption constrains the degree of optimistic biases in beliefs by tastes–the weight of anticipatory utility–and technology–the extend of decreasing returns to scale. Second, we calibrate the model and provide quantitative estimates for the impact of optimism on equilibrium outcomes. The calibration shows that optimism can have a large negative effect on the returns to entrepreneurship and a moderate positive effect on the returns from paid work. In contrast, optimism has a relatively small effect on aggregate output and on the fraction of entrepreneurs in the economy.

8 Conclusion

We extend Lucas' (1978) general equilibrium model of occupational choice by assuming that fraction of the workforce has optimal expectations of entrepreneurial ability. Optimal expectations are modeled according to Brunnermeier and Parker (2005). We show that individuals with optimal expectations choose to be optimists about their entrepreneurial ability.

We find that optimism has six main effects on general equilibrium outcomes. First, there is a misallocation of talent which lowers output. Second, optimists are more likely to become entrepreneurs than realists. Third, entrepreneurs are more optimistic than workers. Fourth, when the fraction optimists is high, the majority of entrepreneurs are optimists. Fifth, optimism drives up the wage which makes workers better off. Sixth, optimism lowers the returns to entrepreneurship.

We calibrate the model to match salient features of US manufacturing data. We find that the presence of optimists may significantly change the distribution of income by driving up the wage and lowering the returns to entrepreneurship. The calibration also shows that even though optimism has a large impact on the returns to entrepreneurship it has only a modest impact on output, the fraction of workers, and of entrepreneurs.

9 Appendix

Proof of Proposition 1: Consider an individual with entrepreneurial ability θ_0 and expectation of entrepreneurial ability θ. Assume that this individual decides to be an entrepreneur at $t = 1$. At $t = 2$ this individual solves the following problem

$$\max_{l,k}(\theta l^{\alpha} k^{\beta} - wl - rk)$$

The first-order conditions are

$$\alpha\theta l^{\alpha-1} k^{\beta} = w,$$

and

$$\beta\theta l^{\alpha} k^{\beta-1} = r.$$

Solving for l and k we obtain

$$l(w, r, \theta) = \theta^{\frac{1}{1-\eta}} \left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}}, \tag{31}$$

and

$$k(w,r,\theta) = \theta^{\frac{1}{1-\eta}} \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}}, \tag{32}$$

where $\eta = \alpha + \beta$. At $t = 0$ this individual solves the problem:

$$\max_{\theta \in [0,1]} \left\{ (\theta_0 + s\theta)[l(w,r,\theta)]^{\alpha}[k(w,r,\theta)]^{\beta} - (1+s)\left[wl(w,r,\theta) + rk(w,r,\theta)\right] \right\}.$$

Substituting $l(w,r,\theta)$ by (31) and $k(w,r,\theta)$ by (32) and simplifying terms we obtain

$$\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}} \max_{\theta \in [0,1]} \left[(\theta_0 + s\theta)\theta^{\frac{\eta}{1-\eta}} - (1+s)\eta\theta^{\frac{1}{1-\eta}} \right].$$

The first-order condition is

$$(\theta_0 + s\theta)\frac{\eta}{1-\eta}\theta^{\frac{\eta}{1-\eta}-1} + s\theta^{\frac{\eta}{1-\eta}} = (1+s)\frac{\eta}{1-\eta}\theta^{\frac{1}{1-\eta}-1}. \tag{33}$$

Solving for θ we obtain

$$\theta^* = \frac{\eta}{\eta - s(1-\eta)}\theta_0. \tag{34}$$

This solution implies that all individuals with optimal expectations hold optimistic beliefs since $\theta^* > \theta_0$. For (34) to be an interior solution, i.e., $\theta^* \in (0,1)$, at least two conditions must be satisfied. First, it must be that

$$\eta > s(1-\eta),$$

or

$$s < \frac{\eta}{1-\eta}. \tag{35}$$

This condition places an upper bound on the weight of anticipatory utility. Second, it must be that

$$\frac{\eta}{\eta - s(1-\eta)}\theta_0 < 1,$$

or

$$\theta_0 < \frac{\eta - s(1-\eta)}{\eta}. \tag{36}$$

This condition says that there is an interior solution only for individuals whose ability is below an upper bound. We see from (36) that the upper bound depends on s and η. Hence the optimal expectations are as follows:

$$\theta^* = \begin{cases} \frac{\eta}{\eta - s(1-\eta)}\theta_0 & \text{if } \theta_0 < \frac{\eta - s(1-\eta)}{\eta} \\ 1 & \text{if } \theta_0 \geq \frac{\eta - s(1-\eta)}{\eta} \end{cases}.$$

To complete the proof we need to show that the second-order condition is satisfied. The first-order condition (33) is equivalent to

$$\frac{\eta}{1-\eta}\left[\theta_0 \theta^{\frac{\eta}{1-\eta}-1} + s\theta^{\frac{\eta}{1-\eta}} + \frac{1-\eta}{\eta} s\theta^{\frac{\eta}{1-\eta}} - (1+s)\theta^{\frac{1}{1-\eta}-1}\right] = 0,$$

or

$$\frac{\eta}{1-\eta}\left[\theta_0 \theta^{\frac{2\eta-1}{1-\eta}} + \left(\frac{s}{\eta} - 1 - s\right)\theta^{\frac{\eta}{1-\eta}}\right] = 0. \tag{37}$$

Taking the derivative of (37) with respect to θ, the second-order condition is given by

$$\frac{2\eta-1}{1-\eta}\theta_0 \theta^{\frac{2\eta-1}{1-\eta}-1} + \left(\frac{s}{\eta} - 1 - s\right)\frac{\eta}{1-\eta}\theta^{\frac{\eta}{1-\eta}-1} < 0,$$

or

$$(2\eta - 1)\theta_0 - [\eta - s(1-\eta)]\,\theta < 0.$$

Since $s < \eta/(1-\eta)$, the second-order condition is satisfied for any $\eta \leq 0.5$. When $0.5 < \eta < 1$ the second-order condition is satisfied as long as

$$(2\eta - 1)\theta_0 < [\eta - s(1-\eta)]\,\theta.$$

Replacing θ by $\theta^* = \eta\theta_0/[\eta - s(1-\eta)]$ we have

$$(2\eta - 1)\theta_0 < \eta\theta_0,$$

or

$$\eta < 1,$$

which is true. *Q.E.D.*

Proof of Proposition 2: Assume $s < \bar{s}$. The first step to determine the optimal expectations competitive equilibrium is to find out the labor market equilibrium condition. The labor demand from realistic entrepreneurs is

$$\begin{aligned} L_R^D &= (1-\lambda)\int_{\hat{\theta}_R}^{1} l(w,r,\theta_0)d\theta_0 \\ &= (1-\lambda)\left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}}\int_{\hat{\theta}_R}^{1}\theta_0^{\frac{1}{1-\eta}}d\theta_0 \\ &= (1-\lambda)\frac{1-\eta}{2-\eta}\left(1-\hat{\theta}_R^{\frac{2-\eta}{1-\eta}}\right)\left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}}. \end{aligned} \tag{38}$$

Note that for L_R^D to be well defined it must be that $\hat{\theta}_R < 1$. Recall that $\hat{\theta}_O$ is the ability threshold that determines the marginal optimistic entrepreneur. If $\hat{\theta}_O < [\eta - s(1-\eta)]/\eta$, then labor demand from optimistic entrepreneurs is the sum of the demand for labor coming from the mass of entrepreneurs with heterogeneous optimistic expectations, i.e., those with $\theta^* \in (\theta_0, 1)$, to the demand for labor coming from the mass of entrepreneurs with homogeneous optimistic expectations, i.e., those with $\theta^* = 1$:

$$\begin{aligned} L_O^D &= \lambda\left\{\int_{\hat{\theta}_O}^{\frac{\eta-s(1-\eta)}{\eta}} l(w,r,\theta^*)d\theta_0 + \int_{\frac{\eta-s(1-\eta)}{\eta}}^{1} l(w,r,1)d\theta_0\right\} \\ &= \lambda\left\{\int_{\hat{\theta}_O}^{\frac{\eta-s(1-\eta)}{\eta}} (\theta^*)^{\frac{1}{1-\eta}}d\theta_0 + \int_{\frac{\eta-s(1-\eta)}{\eta}}^{1} d\theta_0\right\}\left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}} \\ &= \lambda\left\{\left[\frac{\eta}{\eta-s(1-\eta)}\right]^{\frac{1}{1-\eta}}\int_{\hat{\theta}_O}^{\frac{\eta-s(1-\eta)}{\eta}}\theta_0^{\frac{1}{1-\eta}}d\theta_0 + s\frac{1-\eta}{\eta}\right\}\left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}} \\ &= \lambda\left\{\frac{1-\eta}{2-\eta}\left[\frac{\eta}{\eta-s(1-\eta)}\right]^{\frac{1}{1-\eta}}\left[\theta_0^{\frac{2-\eta}{1-\eta}}\right]_{\hat{\theta}_O}^{\frac{\eta-s(1-\eta)}{\eta}} + s\frac{1-\eta}{\eta}\right\}\left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}} \\ &= \lambda\frac{1-\eta}{2-\eta}\left\{1+\frac{s}{\eta}-\left[\frac{\eta}{\eta-s(1-\eta)}\right]^{\frac{1}{1-\eta}}\hat{\theta}_O^{\frac{2-\eta}{1-\eta}}\right\}\left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}}. \end{aligned} \tag{39}$$

Note that for L_O^D to be well defined it must be that $\hat{\theta}_O < [\eta - s(1-\eta)]/\eta$. From (38) and (39), labor demand is equal to

$$
\begin{aligned}
L^D &= L_R^D + L_O^D \\
&= (1-\lambda)\frac{1-\eta}{2-\eta}\left(1-\hat{\theta}_R^{\frac{2-\eta}{1-\eta}}\right)\left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}} \\
&\quad +\lambda\frac{1-\eta}{2-\eta}\left\{1+\frac{s}{\eta}-\left[\frac{\eta}{\eta-s(1-\eta)}\right]^{\frac{1}{1-\eta}}\hat{\theta}_O^{\frac{2-\eta}{1-\eta}}\right\}\left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}} \\
&= \frac{1-\eta}{2-\eta}\left\{1+\lambda\frac{s}{\eta}-(1-\lambda)\hat{\theta}_R^{\frac{2-\eta}{1-\eta}}-\lambda\left[\frac{\eta}{\eta-s(1-\eta)}\right]^{\frac{1}{1-\eta}}\hat{\theta}_O^{\frac{2-\eta}{1-\eta}}\right\}\left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}}
\end{aligned}
$$

Since each worker provides a unit of labor, labor supply is

$$
\begin{aligned}
L^S &= (1-\lambda)L_R^S + \lambda L_O^S \\
&= (1-\lambda)\int_0^{\hat{\theta}_R} d\theta_0 + \lambda\int_0^{\hat{\theta}_O} d\theta_0 \\
&= (1-\lambda)\hat{\theta}_R + \lambda\hat{\theta}_O. \qquad (40)
\end{aligned}
$$

In equilibrium, labor demand must equal labor supply:

$$
\begin{aligned}
&\frac{1-\eta}{2-\eta}\left\{1+\lambda\frac{s}{\eta}-(1-\lambda)\hat{\theta}_R^{\frac{2-\eta}{1-\eta}}-\lambda\left[\frac{\eta}{\eta-s(1-\eta)}\right]^{\frac{1}{1-\eta}}\hat{\theta}_O^{\frac{2-\eta}{1-\eta}}\right\}\left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}} \\
= \;& (1-\lambda)\hat{\theta}_R + \lambda\hat{\theta}_O, \qquad (41)
\end{aligned}
$$

The second step to determine the optimal expectations competitive equilibrium is to find out the capital market equilibrium condition. The capital demand from realistic entrepreneurs is

$$
\begin{aligned}
K_R^D &= (1-\lambda)\int_{\hat{\theta}_R}^1 k(w,r,\theta_0)d\theta_0 \\
&= (1-\lambda)\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}}\int_{\hat{\theta}_R}^1 \theta_0^{\frac{1}{1-\eta}}d\theta_0 \\
&= (1-\lambda)\frac{1-\eta}{2-\eta}\left(1-\hat{\theta}_R^{\frac{2-\eta}{1-\eta}}\right)\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}}. \qquad (42)
\end{aligned}
$$

Note that for K_R^D to be well defined it must be that $\hat{\theta}_R < 1$. Recall that $\hat{\theta}_O$ is the ability threshold that determines the marginal optimistic entrepreneur. If $\hat{\theta}_O < [\eta - s(1-\eta)]/\eta$, then capital demand from optimistic entrepreneurs is the sum of the demand for capital coming from the mass of entrepreneurs with heterogeneous optimistic expectations, i.e., those with $\theta^* \in (\theta_0, 1)$, to the demand for capital coming from the mass of entrepreneurs with homogeneous optimistic expectations, i.e., those with $\theta^* = 1$:

$$\begin{aligned} K_O^D &= \lambda \left\{ \int_{\hat{\theta}_O}^{\frac{\eta - s(1-\eta)}{\eta}} k(w, r, \theta^*) d\theta_0 + \int_{\frac{\eta - s(1-\eta)}{\eta}}^{1} k(w, r, 1) d\theta_0 \right\} \\ &= \lambda \left\{ \int_{\hat{\theta}_O}^{\frac{\eta - s(1-\eta)}{\eta}} (\theta^*)^{\frac{1}{1-\eta}} d\theta_0 + \int_{\frac{\eta - s(1-\eta)}{\eta}}^{1} d\theta_0 \right\} \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}} \\ &= \lambda \frac{1-\eta}{2-\eta} \left\{ 1 + \frac{s}{\eta} - \left[\frac{\eta}{\eta - s(1-\eta)}\right]^{\frac{1}{1-\eta}} \hat{\theta}_O^{\frac{2-\eta}{1-\eta}} \right\} \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}}. \end{aligned} \tag{43}$$

Note that for K_O^D to be well defined it must be that $\hat{\theta}_O < [\eta - s(1-\eta)]/\eta$. From (42) and (43), capital demand is equal to

$$\begin{aligned} K^D &= K_R^D + K_O^D \\ &= (1-\lambda) \frac{1-\eta}{2-\eta} \left(1 - \hat{\theta}_R^{\frac{2-\eta}{1-\eta}}\right) \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}} \\ &\quad + \lambda \frac{1-\eta}{2-\eta} \left\{ 1 + \frac{s}{\eta} - \left[\frac{\eta}{\eta - s(1-\eta)}\right]^{\frac{1}{1-\eta}} \hat{\theta}_O^{\frac{2-\eta}{1-\eta}} \right\} \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}} \\ &= \frac{1-\eta}{2-\eta} \left\{ 1 + \lambda \frac{s}{\eta} - (1-\lambda) \hat{\theta}_R^{\frac{2-\eta}{1-\eta}} - \lambda \left[\frac{\eta}{\eta - s(1-\eta)}\right]^{\frac{1}{1-\eta}} \hat{\theta}_O^{\frac{2-\eta}{1-\eta}} \right\} \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}} \end{aligned}$$

In equilibrium, capital demand must equal the exogenous capital supply:

$$\begin{aligned} & \frac{1-\eta}{2-\eta} \left\{ 1 + \lambda \frac{s}{\eta} - (1-\lambda) \hat{\theta}_R^{\frac{2-\eta}{1-\eta}} - \lambda \left[\frac{\eta}{\eta - s(1-\eta)}\right]^{\frac{1}{1-\eta}} \hat{\theta}_O^{\frac{2-\eta}{1-\eta}} \right\} \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}} \\ &= \bar{K}. \end{aligned} \tag{44}$$

The third step to determine the optimal expectations competitive equilibrium is to find out the ability level of the marginal realistic entrepreneur $\hat{\theta}_R$ and of the marginal optimistic entrepreneur $\hat{\theta}_O$. At $t = 1$ a realist with ability $\hat{\theta}_R$ is indifferent between being an entrepreneur and a worker when

$$\hat{\theta}_R[l(w,r,\hat{\theta}_R)]^\alpha[k(w,r,\hat{\theta}_R)]^\beta - wl(w,r,\hat{\theta}_R) - rk(w,r,\hat{\theta}_R) = w,$$

or

$$\hat{\theta}_R \left[\hat{\theta}_R{}^{\frac{1}{1-\eta}} \left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}}\right]^\alpha \left[\hat{\theta}_R{}^{\frac{1}{1-\eta}} \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}}\right]^\beta$$
$$-w\hat{\theta}_R{}^{\frac{1}{1-\eta}} \left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}} - r\hat{\theta}_R{}^{\frac{1}{1-\eta}} \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}} = w,$$

or

$$\hat{\theta}_R{}^{\frac{1}{1-\eta}} \left[\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}} - w\left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}} - r\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}}\right] = w,$$

or

$$\hat{\theta}_R{}^{\frac{1}{1-\eta}} \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}} \left[1 - w\left(\frac{\alpha}{w}\right) - r\left(\frac{\beta}{r}\right)\right] = w,$$

or

$$\hat{\theta}_R{}^{\frac{1}{1-\eta}} \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}} (1-\eta) = w,$$

or

$$\hat{\theta}_R{}^{\frac{1}{1-\eta}} \alpha^{\frac{\alpha}{1-\eta}} \beta^{\frac{\beta}{1-\eta}} (1-\eta) = w^{\frac{1-\beta}{1-\eta}} r^{\frac{\beta}{1-\eta}},$$

or

$$\alpha^\alpha \beta^\beta (1-\eta)^{1-\eta} \hat{\theta}_R = w^{1-\beta} r^\beta. \tag{45}$$

At $t = 1$ an individual with optimal expectations of ability $\theta^* = \eta\hat{\theta}_O/[\eta - s(1-\eta)]$ and ability $\hat{\theta}_O$ is indifferent between being an entrepreneur and a worker when

$$(\hat{\theta}_O + s\theta^*)[l(w,r,\theta^*)]^\alpha[k(w,r,\theta^*)]^\beta - (1+s)\left[wl(w,r,\theta^*) + rk(w,r,\theta^*)\right] = w,$$

or

$$(\hat{\theta}_O + s\theta^*) \left[(\theta^*)^{\frac{1}{1-\eta}} \left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}}\right]^\alpha \left[(\theta^*)^{\frac{1}{1-\eta}} \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}}\right]^\beta$$
$$-(1+s)\left[w(\theta^*)^{\frac{1}{1-\eta}} \left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}} + r(\theta^*)^{\frac{1}{1-\eta}} \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}}\right] = w,$$

or

$$\left[\frac{\eta - s(1-\eta)}{\eta}\theta^* + s\theta^*\right]\left[(\theta^*)^{\frac{\eta}{1-\eta}}\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}}\right]$$

$$-(1+s)(\theta^*)^{\frac{1}{1-\eta}}\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}}\left[w\left(\frac{\alpha}{w}\right)+r\left(\frac{\beta}{r}\right)\right] = w,$$

or

$$(\theta^*)^{\frac{1}{1-\eta}}\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}}\left[\frac{\eta - s(1-\eta)}{\eta} + s - (1+s)\eta\right] = w,$$

or

$$(\theta^*)^{\frac{1}{1-\eta}}\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}}\left[1 - \frac{s}{\eta} + 2s - (1+s)\eta\right] = w$$

or

$$\left[\frac{\eta}{\eta - s(1-\eta)}\right]^{\frac{1}{1-\eta}}\hat{\theta}_O^{\frac{1}{1-\eta}}\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}}\left[1 - \frac{s}{\eta} + 2s - (1+s)\eta\right] = w$$

or

$$\hat{\theta}_O\left[\frac{\eta}{\eta - s(1-\eta)}\right]\alpha^\alpha\beta^\beta\left[1 - \eta - \frac{s}{\eta}(1-\eta)^2\right]^{1-\eta} = w^{1-\beta}r^\beta,$$

or

$$\hat{\theta}_O\left[\frac{\eta}{\eta - s(1-\eta)}\right]\alpha^\alpha\beta^\beta(1-\eta)^{1-\eta}\left[\frac{\eta - s(1-\eta)}{\eta}\right]^{1-\eta} = w^{1-\beta}r^\beta,$$

or

$$\alpha^\alpha\beta^\beta(1-\eta)^{1-\eta}\left[\frac{\eta}{\eta - s(1-\eta)}\right]^\eta\hat{\theta}_O = w^{1-\beta}r^\beta. \tag{46}$$

It follows from (45) and (46) that

$$\alpha^\alpha\beta^\beta(1-\alpha-\beta)^{1-\alpha-\beta}\left[\frac{\eta}{\eta - s(1-\eta)}\right]^\eta\hat{\theta}_O = \alpha^\alpha\beta^\beta(1-\alpha-\beta)^{1-\alpha-\beta}\hat{\theta}_R,$$

or

$$\hat{\theta}_O = \left[\frac{\eta - s(1-\eta)}{\eta}\right]^\eta\hat{\theta}_R. \tag{47}$$

Substituting (45) and (47) into (41) we obtain

$$\begin{aligned} & 1 + \lambda\frac{s}{\eta} - (1-\lambda)\hat{\theta}_R^{\frac{2-\eta}{1-\eta}} - \lambda\left[\frac{\eta}{\eta - s(1-\eta)}\right]^{1-\eta}\hat{\theta}_R^{\frac{2-\eta}{1-\eta}} \\ = \ & \frac{2-\eta}{1-\eta}\frac{w^{\frac{1-\beta}{1-\eta}}r^{\frac{\beta}{1-\eta}}}{\alpha^{\frac{1-\beta}{1-\eta}}\beta^{\frac{\beta}{1-\eta}}}\left\{(1-\lambda)\hat{\theta}_R + \lambda\left[\frac{\eta - s(1-\eta)}{\eta}\right]^\eta\hat{\theta}_R\right\}, \end{aligned}$$

or

$$
\begin{aligned}
& 1+\lambda\frac{s}{\eta}-(1-\lambda)\hat{\theta}_R^{\frac{2-\eta}{1-\eta}}-\lambda\left[\frac{\eta}{\eta-s(1-\eta)}\right]^{1-\eta}\hat{\theta}_R^{\frac{2-\eta}{1-\eta}} \\
= \quad & \frac{2-\eta}{1-\eta}\frac{\alpha^{\frac{\alpha}{1-\eta}}\beta^{\frac{\beta}{1-\eta}}(1-\eta)\hat{\theta}_R^{\frac{1}{1-\eta}}}{\alpha^{\frac{1-\beta}{1-\eta}}\beta^{\frac{\beta}{1-\eta}}}\left\{(1-\lambda)\hat{\theta}_R+\lambda\left[\frac{\eta-s(1-\eta)}{\eta}\right]^{\eta}\hat{\theta}_R\right\},
\end{aligned}
$$

or

$$
\begin{aligned}
& 1+\lambda\frac{s}{\eta}+(1-\lambda)\hat{\theta}_R^{\frac{2-\eta}{1-\eta}}-\lambda\left[\frac{\eta}{\eta-s(1-\eta)}\right]^{1-\eta}\hat{\theta}_R^{\frac{2-\eta}{1-\eta}} \\
= \quad & \frac{2-\eta}{1-\eta}\frac{1-\eta}{\alpha}\left\{(1-\lambda)+\lambda\left[\frac{\eta-s(1-\eta)}{\eta}\right]^{\eta}\right\}\hat{\theta}_R^{\frac{2-\eta}{1-\eta}},
\end{aligned}
$$

or

$$
1+\lambda\frac{s}{\eta}=\left\{1-\lambda+\lambda\left[\frac{\eta}{\eta-s(1-\eta)}\right]^{1-\eta}+\frac{2-\eta}{\alpha}(1-\lambda)+\frac{2-\eta}{\alpha}\lambda\left[\frac{\eta-s(1-\eta)}{\eta}\right]^{\eta}\right\}\hat{\theta}_R^{\frac{2-\eta}{1-\eta}},
$$

or

$$
1+\lambda\frac{s}{\eta}=\left\{\frac{(2-\beta)(1-\lambda)}{\alpha}+\lambda\left[\frac{\eta}{\eta-s(1-\eta)}\right]^{1-\eta}+\lambda\frac{2-\eta}{\alpha}\left[\frac{\eta-s(1-\eta)}{\eta}\right]^{\eta}\right\}\hat{\theta}_R^{\frac{2-\eta}{1-\eta}}
$$

or

$$
\hat{\theta}_R^{\frac{2-\eta}{1-\eta}}=\frac{1+\lambda\frac{s}{\eta}}{\frac{(2-\beta)(1-\lambda)}{\alpha}+\lambda\left[\frac{\eta}{\eta-s(1-\eta)}\right]^{1-\eta}+\lambda\frac{2-\eta}{\alpha}\left[\frac{\eta-s(1-\eta)}{\eta}\right]^{\eta}}.
$$

Hence, the ability of the marginal realistic entrepreneur is

$$
\begin{aligned}
\hat{\theta}_R &= \left\{ \frac{1+\lambda\frac{s}{\eta}}{\frac{(2-\beta)(1-\lambda)}{\alpha} + \lambda\left[\frac{\eta}{\eta-s(1-\eta)}\right]^{1-\eta} + \lambda\frac{2-\eta}{\alpha}\left[\frac{\eta-s(1-\eta)}{\eta}\right]^{\eta}} \right\}^{\frac{1-\eta}{2-\eta}} \\
&= \left\{ \frac{\alpha}{2-\beta} \frac{1+\lambda\frac{s}{\eta}}{1-\lambda+\lambda\frac{\alpha}{2-\beta}\left[\frac{\eta}{\eta-s(1-\eta)}\right]^{1-\eta} + \lambda\frac{2-\eta}{2-\beta}\left[\frac{\eta-s(1-\eta)}{\eta}\right]^{\eta}} \right\}^{\frac{1-\eta}{2-\eta}} \\
&= \left\{ \frac{\alpha}{2-\beta} \frac{1+\lambda\frac{s}{\eta}}{1-\lambda+\lambda\frac{2-\beta-s/\eta(2-3\eta+\eta^2)}{2-\beta}\left[\frac{\eta}{\eta-s(1-\eta)}\right]^{1-\eta}} \right\}^{\frac{1-\eta}{2-\eta}} \\
&= \left\{ \frac{\alpha}{2-\beta} \frac{1+\lambda\frac{s}{\eta}}{1-\lambda+\lambda\left[1-\frac{s(1-\eta)(2-\eta)}{\eta(2-\beta)}\right]\left[\frac{\eta}{\eta-s(1-\eta)}\right]^{1-\eta}} \right\}^{\frac{1-\eta}{2-\eta}} \\
&= \left[\frac{\alpha}{2-\beta} \frac{1+\lambda\frac{s}{\eta}}{1-\lambda+\lambda\phi(\eta,\beta,s)} \right]^{\frac{1-\eta}{2-\eta}}, \qquad (48)
\end{aligned}
$$

where

$$
\begin{aligned}
\phi(\eta,\beta,s) &= \left[1-\frac{s(1-\eta)(2-\eta)}{\eta(2-\beta)}\right]\left[\frac{\eta}{\eta-s(1-\eta)}\right]^{1-\eta} \\
&= \left[1-\frac{s(1-\eta)(2-\eta)}{\eta(2-\beta)}\right]\left[\frac{\eta}{\eta-s(1-\eta)}\right]\left[\frac{\eta-s(1-\eta)}{\eta}\right]^{\eta} \\
&= \frac{\eta(2-\beta)-s(1-\eta)(2-\eta)}{[\eta-s(1-\eta)](2-\beta)}\left[\frac{\eta-s(1-\eta)}{\eta}\right]^{\eta} \\
&= \frac{\eta(2-\beta)-s(1-\eta)(2-\beta)(2-\eta)/(2-\beta)}{[\eta-s(1-\eta)](2-\beta)}\left[\frac{\eta-s(1-\eta)}{\eta}\right]^{\eta} \\
&= \frac{\eta-s(1-\eta)\frac{2-\eta}{2-\beta}}{\eta-s(1-\eta)}\left[\frac{\eta-s(1-\eta)}{\eta}\right]^{\eta}. \qquad (49)
\end{aligned}
$$

From (47) and (48) the ability of the marginal optimistic entrepreneur is

$$
\hat{\theta}_O = \left[\frac{\eta-s(1-\eta)}{\eta}\right]^{\eta}\left[\frac{\alpha}{2-\beta}\frac{1+\lambda\frac{s}{\eta}}{1-\lambda+\lambda\phi(\eta,\beta,s)}\right]^{\frac{1-\eta}{2-\eta}}.
$$

From (41) and (44) we have

$$\left[(1-\lambda)\hat{\theta}_R + \lambda\hat{\theta}_O\right]\left(\frac{w}{\alpha}\right)^{\frac{1-\beta}{1-\eta}}\left(\frac{r}{\beta}\right)^{\frac{\beta}{1-\eta}} = \bar{K}\left(\frac{w}{\alpha}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{r}{\beta}\right)^{\frac{1-\alpha}{1-\eta}},$$

or

$$\alpha r\bar{K} = \beta w\left[(1-\lambda)\hat{\theta}_R + \lambda\hat{\theta}_O\right],$$

or

$$r = \frac{\beta w}{\alpha\bar{K}}\left[1-\lambda+\lambda\psi(\alpha,\beta,s)\right]\hat{\theta}_R, \tag{50}$$

where

$$\psi(\eta,s) = \left[\frac{\eta - s(1-\eta)}{\eta}\right]^{\eta}. \tag{51}$$

Note that (49) and (51) together with $2-\eta \leq 2-\beta$ imply

$$\phi(\eta,\beta,s) \geq \psi(\eta,s). \tag{52}$$

Substituting (50) into (45) we obtain

$$\alpha^{\alpha}\beta^{\beta}(1-\eta)^{1-\eta}\hat{\theta}_R = w^{1-\beta}\left(\frac{\beta}{\alpha}\right)^{\beta} w^{\beta}\left[1-\lambda+\lambda\psi(\eta,s)\right]^{\beta}\hat{\theta}_R^{\beta}\bar{K}^{-\beta}.$$

Solving this equality with respect to w we obtain the equilibrium wage:

$$\begin{aligned} w^* &= \frac{\alpha^{\eta}(1-\eta)^{1-\eta}\bar{K}^{\beta}}{\left[1-\lambda+\lambda\psi(\eta,s)\right]^{\beta}}\hat{\theta}_R^{1-\beta} \\ &= \frac{\alpha^{\eta}(1-\eta)^{1-\eta}\bar{K}^{\beta}}{\left[1-\lambda+\lambda\psi(\eta,s)\right]^{\beta}}\left[\frac{\alpha}{2-\beta}\frac{1+\lambda\frac{s}{\eta}}{1-\lambda+\lambda\phi(\eta,\beta,s)}\right]^{\frac{(1-\eta)(1-\beta)}{2-\eta}}. \end{aligned}$$

The equilibrium rental cost of capital is equal to

$$\begin{aligned} r^* &= \frac{\beta w^*}{\alpha\bar{K}}\left[1-\lambda+\lambda\psi(\eta,s)\right]\hat{\theta}_R \\ &= \frac{\beta\alpha^{\eta}(1-\eta)^{1-\eta}\bar{K}^{\beta}\hat{\theta}_R^{1-\beta}}{\alpha\bar{K}\left[1-\lambda+\lambda\psi(\eta,s)\right]^{\beta}}\left[1-\lambda+\lambda\psi(\eta,s)\right]\hat{\theta}_R \\ &= \frac{\beta(1-\eta)^{1-\eta}}{\alpha^{1-\eta}\bar{K}^{1-\beta}}\left[1-\lambda+\lambda\psi(\eta,s)\right]^{1-\beta}\hat{\theta}_R^{2-\beta} \\ &= \frac{\beta(1-\eta)^{1-\eta}}{\alpha^{1-\eta}\bar{K}^{1-\beta}}\left[1-\lambda+\lambda\psi(\eta,s)\right]^{1-\beta}\left[\frac{\alpha}{2-\beta}\frac{1+\lambda\frac{s}{\eta}}{1-\lambda+\lambda\phi(\eta,\beta,s)}\right]^{\frac{(1-\eta)(2-\beta)}{2-\eta}}. \end{aligned}$$

The equilibrium labor force is equal to

$$\begin{aligned} L^* &= (1-\lambda)\hat{\theta}_R + \lambda\hat{\theta}_O = \left\{1-\lambda+\lambda\left[\frac{\eta-s(1-\eta)}{\eta}\right]^{\eta}\right\}\hat{\theta}_R \\ &= [1-\lambda+\lambda\psi(\eta,s)]\left[\frac{\alpha}{2-\beta}\frac{1+\lambda\frac{s}{\eta}}{1-\lambda+\lambda\phi(\eta,\beta,s)}\right]^{\frac{1-\eta}{2-\eta}}. \end{aligned}$$

For the equilibrium to be well defined we need to make sure that $\hat{\theta}_O$ is less than $\frac{\eta-s(1-\eta)}{\eta}$, i.e.,

$$\left[\frac{\eta-s(1-\eta)}{\eta}\right]^{\eta}\left[\frac{\alpha}{2-\beta}\frac{1+\lambda\frac{s}{\eta}}{1-\lambda+\lambda\phi(\eta,\beta,s)}\right]^{\frac{1-\eta}{2-\eta}} < \frac{\eta-s(1-\eta)}{\eta}$$

or

$$\left[\frac{\alpha}{2-\beta}\frac{1+\lambda\frac{s}{\eta}}{1-\lambda+\lambda\phi(\eta,\beta,s)}\right]^{\frac{1-\eta}{2-\eta}} < \left[\frac{\eta-s(1-\eta)}{\eta}\right]^{1-\eta}$$

or

$$\frac{\alpha}{2-\beta}\frac{1+\lambda\frac{s}{\eta}}{1-\lambda+\lambda\phi(\eta,\beta,s)} < \left[\frac{\eta-s(1-\eta)}{\eta}\right]^{2-\eta}$$

or

$$\frac{\alpha}{2-\beta}\left(1+\lambda\frac{s}{\eta}\right) < (1-\lambda)\left[\frac{\eta-s(1-\eta)}{\eta}\right]^{2-\eta}+\lambda\left[1-\frac{s(1-\eta)(2-\eta)}{\eta(2-\beta)}\right]\left[\frac{\eta-s(1-\eta)}{\eta}\right]. \tag{53}$$

The LHS of (53) is increasing in s whereas the RHS of (53) is decreasing in s. When s is equal to 0 the LSH of (53) is equal to $\alpha/(2-\beta) < 1$ and the RHS of (53) is equal to 1. When s is equal to $\eta/(1-\eta)$ the LSH of (53) is equal to $\alpha(1-\eta+\lambda)/(2-\beta)(1-\eta)$ and the RHS of (53) is equal to 0. Hence, there exists a unique $s \in (0, \eta/(1-\eta))$ such that the LHS and RHS of (53) are the same, which is given by

$$\frac{\alpha}{2-\beta}\left(1+\lambda\frac{\bar{s}}{\eta}\right) = (1-\lambda)\left[\frac{\eta-\bar{s}(1-\eta)}{\eta}\right]^{2-\eta}+\lambda\left[1-\frac{\bar{s}(1-\eta)(2-\eta)}{\eta(2-\beta)}\right]\left[\frac{\eta-\bar{s}(1-\eta)}{\eta}\right].$$

or

$$\frac{\alpha}{2-\beta}\left(1+\lambda\frac{\bar{s}}{\eta}\right) = [1-\lambda+\lambda\phi(\eta,\beta,\bar{s})]\left[\frac{\eta-\bar{s}(1-\eta)}{\eta}\right]^{2-\eta},$$

which is (23). Hence, inequality (53) is satisfied as long as $s < \bar{s}$. For the equilibrium to be well defined we also need to make sure that $\hat{\theta}_R$ is less than 1. From (47) we have

$$\begin{aligned}\hat{\theta}_R &= \left[\frac{\eta}{\eta - s(1-\eta)}\right]^{\eta} \hat{\theta}_O \\ &< \left[\frac{\eta}{\eta - s(1-\eta)}\right]^{\eta} \frac{\eta - s(1-\eta)}{\eta} = \left[\frac{\eta - s(1-\eta)}{\eta}\right]^{1-\eta} < 1.\end{aligned}$$

where the first inequality follows $s \leq \bar{s}$. *Q.E.D.*

Proof of Proposition 3: Assume $s \in (\bar{s}, \eta/(1-\eta))$. The first step to determine the optimal expectations equilibrium is to find out the labor market equilibrium condition. The labor demand from entrepreneurs with rational expectations is given by (38). If $s > \bar{s}$, then labor demand from entrepreneurs with optimal expectations is

$$L_O^D = \lambda \int_{\hat{\theta}_O}^{1} l(w, r, 1) d\theta_0 = \lambda (1 - \hat{\theta}_O) \left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}}. \tag{54}$$

From (38) and (54), labor demand is equal to

$$\begin{aligned}L^D &= (1-\lambda)\frac{1-\eta}{2-\eta}\left(1 - \hat{\theta}_R^{\frac{2-\eta}{1-\eta}}\right)\left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}} + \lambda(1-\hat{\theta}_O)\left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}} \\ &= \left[(1-\lambda)\frac{1-\eta}{2-\eta}\left(1 - \hat{\theta}_R^{\frac{2-\eta}{1-\eta}}\right) + \lambda(1-\hat{\theta}_O)\right]\left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}}.\end{aligned}$$

Labor supply is given by

$$L^S = (1-\lambda)\int_0^{\hat{\theta}_R} d\theta_0 + \lambda \int_0^{\hat{\theta}_O} d\theta_0 = (1-\lambda)\hat{\theta}_R + \lambda\hat{\theta}_O.$$

In equilibrium, labor demand must equal labor supply

$$\left[(1-\lambda)\frac{1-\eta}{2-\eta}\left(1 - \hat{\theta}_R^{\frac{2-\eta}{1-\eta}}\right) + \lambda(1-\hat{\theta}_O)\right]\left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}} = (1-\lambda)\hat{\theta}_R + \lambda\hat{\theta}_O. \tag{55}$$

The second step to determine the optimal expectations equilibrium is to find out the capital market equilibrium condition. The capital demand from entrepreneurs with rational expectations is given by (42). If $s > \bar{s}$, then capital demand from entrepreneurs with optimal expectations is

$$K_O^D = \lambda \int_{\hat{\theta}_O}^{1} k(w, r, 1) d\theta_0 = \lambda(1 - \hat{\theta}_O) \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}} \left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}}. \quad (56)$$

From (42) and (56), capital demand is equal to

$$\begin{aligned} K^D &= (1-\lambda)\frac{1-\eta}{2-\eta}\left(1 - \hat{\theta}_R^{\frac{2-\eta}{1-\eta}}\right)\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}} + \lambda(1-\hat{\theta}_O)\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}} \\ &= \left[(1-\lambda)\frac{1-\eta}{2-\eta}\left(1 - \hat{\theta}_R^{\frac{2-\eta}{1-\eta}}\right) + \lambda(1-\hat{\theta}_O)\right]\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}}. \end{aligned}$$

In equilibrium, capital demand must equal the exogenous capital supply

$$\left[(1-\lambda)\frac{1-\eta}{2-\eta}\left(1 - \hat{\theta}_R^{\frac{2-\eta}{1-\eta}}\right) + \lambda(1-\hat{\theta}_O)\right]\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}} = \bar{K}. \quad (57)$$

The third step to determine the optimal expectations competitive equilibrium is to find out $\hat{\theta}_R$ and $\hat{\theta}_O$. An individual with entrepreneurial ability $\hat{\theta}_R$ and rational expectation of ability is indifferent between being an entrepreneur and a worker at $t = 1$ when (45) holds:

$$\alpha^\alpha \beta^\beta (1-\eta)^{1-\eta} \hat{\theta}_R = w^{1-\beta} r^\beta. \quad (58)$$

An individual with entrepreneurial ability $\hat{\theta}_O$ and optimal expectation of entrepreneurial ability $\theta^* = 1$ is indifferent between being an entrepreneur and a worker at $t = 1$ when

$$(\hat{\theta}_O + s\theta^*)[l(w, r, \theta^*)]^\alpha [k(w, r, \theta^*)]^\beta - (1+s)\left[wl(w, r, \theta^*) + rk(w, r, \theta^*)\right] = w,$$

or

$$\begin{aligned} &(\hat{\theta}_O + s)\left[\left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}}\right]^\alpha \left[\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}}\right]^\beta \\ &-(1+s)\left[w\left(\frac{\alpha}{w}\right)^{\frac{1-\beta}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}} + r\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\eta}}\right] = w, \end{aligned}$$

or

$$(\hat{\theta}_O + s)\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}} - (1+s)\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}}\left[w\left(\frac{\alpha}{w}\right) + r\left(\frac{\beta}{r}\right)\right] = w,$$

or

$$\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\eta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\eta}}\left[\hat{\theta}_O + s - (1+s)\eta\right] = w,$$

or

$$\alpha^{\frac{\alpha}{1-\eta}}\beta^{\frac{\beta}{1-\eta}}\left[\hat{\theta}_O + s - (1+s)\eta\right] = w^{\frac{1-\beta}{1-\eta}} r^{\frac{\beta}{1-\eta}},$$

or

$$\alpha^{\alpha}\beta^{\beta}\left[\hat{\theta}_O + s - (1+s)\eta\right]^{1-\eta} = w^{1-\beta} r^{\beta}. \tag{59}$$

Equations (55), (57), (58), and (59) define the optimal expectations equilibrium when $s \in (\bar{s}, \eta/(1-\eta))$. *Q.E.D.*

Proof of Proposition 4:

(i) The probability an optimist becomes an entrepreneur is equal to

$$\Pr(E^*|O) = \frac{E_O^*}{\lambda} = \frac{\lambda(1-\hat{\theta}_O)}{\lambda} = 1 - \hat{\theta}_O,$$

and the probability a realist becomes an entrepreneur to

$$\Pr(E^*|R) = \frac{E_R^*}{1-\lambda} = \frac{(1-\lambda)(1-\hat{\theta}_R)}{1-\lambda} = 1 - \hat{\theta}_R.$$

Hence, optimists are more likely to become entrepreneurs than realists as long as

$$\Pr(E^*|O) > \Pr(E^*|R),$$

or

$$1 - \hat{\theta}_O > 1 - \hat{\theta}_R,$$

or

$$\hat{\theta}_O < \hat{\theta}_R,$$

which is true by (25).

(ii) Entrepreneurs are more likely to be optimists than workers if

$$\Pr(O|E^*) > \Pr(O|L^*),$$

or

$$\frac{\lambda(1-\hat{\theta}_O)}{\lambda(1-\hat{\theta}_O)+(1-\lambda)(1-\hat{\theta}_R)} > \frac{\lambda\hat{\theta}_O}{\lambda\hat{\theta}_O+(1-\lambda)\hat{\theta}_R},$$

or

$$\lambda\hat{\theta}_O(1-\hat{\theta}_O)+(1-\lambda)(1-\hat{\theta}_O)\hat{\theta}_R > \lambda(1-\hat{\theta}_O)\hat{\theta}_O+(1-\lambda)(1-\hat{\theta}_R)\hat{\theta}_O,$$

or

$$(1-\hat{\theta}_O)\hat{\theta}_R > (1-\hat{\theta}_R)\hat{\theta}_O,$$

or

$$\hat{\theta}_R > \hat{\theta}_O,$$

which is true by (25).

(iii) The majority of entrepreneurs are optimists if

$$\gamma_E^* = \frac{\lambda(1-\hat{\theta}_O)}{\lambda(1-\hat{\theta}_O)+(1-\lambda)(1-\hat{\theta}_R)} > \frac{1}{2},$$

or

$$2\lambda(1-\hat{\theta}_O) > \lambda(1-\hat{\theta}_O)+(1-\lambda)(1-\hat{\theta}_R),$$

or

$$\lambda(1-\hat{\theta}_O)+\lambda(1-\hat{\theta}_R) > 1-\hat{\theta}_R,$$

or

$$\lambda > \frac{1-\hat{\theta}_R}{2-\hat{\theta}_R-\hat{\theta}_O} = \frac{1-\hat{\theta}_R}{1-\hat{\theta}_R+1-\hat{\theta}_R\left[\frac{\eta-s(1-\eta)}{\eta}\right]^\eta} = \left[1+\frac{1-\hat{\theta}_R\left[\frac{\eta-s(1-\eta)}{\eta}\right]^\eta}{1-\hat{\theta}_R}\right]^{-1}.$$

Q.E.D.

Proof of Proposition 5: Assume $s < \bar{s}$. We wish to show that

$$\frac{\partial \hat{\theta}_R}{\partial \lambda} > 0.$$

From the definition of $\hat{\theta}_R$ we have

$$\begin{aligned}
\frac{\partial \hat{\theta}_R}{\partial \lambda} &= \frac{1-\eta}{2-\eta}\left(\frac{\alpha}{2-\beta}\right)^{\frac{1-\eta}{2-\eta}}\left(\frac{1+\frac{\lambda s}{\eta}}{1-\lambda+\lambda\phi}\right)^{-\frac{1}{2-\eta}}\frac{\partial}{\partial\lambda}\left(\frac{1+\frac{\lambda s}{\eta}}{1-\lambda+\lambda\phi}\right) \\
&= \frac{1-\eta}{2-\eta}\left(\frac{\alpha}{2-\beta}\right)^{\frac{1-\eta}{2-\eta}}\left(\frac{1+\frac{\lambda s}{\eta}}{1-\lambda+\lambda\phi}\right)^{-\frac{1}{2-\eta}}\frac{1+\frac{s}{\eta}-\phi}{(1-\lambda+\lambda\phi)^2} \\
&= \frac{1-\eta}{2-\eta}\hat{\theta}_R\frac{1+\frac{s}{\eta}-\phi}{(1+\frac{\lambda s}{\eta})(1-\lambda+\lambda\phi)}. \qquad (60)
\end{aligned}$$

Hence, $\partial\hat{\theta}_R/\partial\lambda > 0$ as long as

$$1+\frac{s}{\eta} > \phi = \left[1-\frac{s(1-\eta)(2-\eta)}{\eta(2-\beta)}\right]\left[\frac{\eta}{\eta-s(1-\eta)}\right]^{1-\eta}. \qquad (61)$$

Note that $\phi > 0$ and $\phi(\eta,\beta,0) = 1$. The derivative of ϕ with respect to s is equal to

$$\begin{aligned}
\frac{\partial\phi}{\partial s} &= -\frac{(1-\eta)(2-\eta)}{\eta(2-\beta)}\left[\frac{\eta}{\eta-s(1-\eta)}\right]^{1-\eta} \\
&\quad + \left[1-\frac{s(1-\eta)(2-\eta)}{\eta(2-\beta)}\right]\left[\frac{\eta}{\eta-s(1-\eta)}\right]^{1-\eta}\frac{(1-\eta)^2}{\eta-s(1-\eta)} \\
&= \frac{(1-\eta)\left[(2-\beta)(1-\eta)-(2-\eta)+s(1-\eta)(2-\eta)\right]}{[\eta-s(1-\eta)](2-\beta)}\left[\frac{\eta}{\eta-s(1-\eta)}\right]^{1-\eta} \\
&= \frac{(1-\eta)\left[-\eta-\beta(1-\eta)+s(1-\eta)(2-\eta)\right]}{[\eta-s(1-\eta)](2-\beta)}\left[\frac{\eta}{\eta-s(1-\eta)}\right]^{1-\eta}. \qquad (62)
\end{aligned}$$

It follows from (62) that: (i) ϕ decreases with s when $s \in [0,\breve{s})$, (ii) ϕ increases with s when $s \in (\breve{s}, \eta/(1-\eta))$, and (iii) ϕ attains a minimum at $\breve{s} = \frac{\eta+\beta(1-\eta)}{(1-\eta)(2-\eta)} < \frac{\eta}{1-\eta}$

which is given by

$$
\begin{aligned}
\phi(\breve{s}) &= \left[1-\frac{\frac{\eta+\beta(1-\eta)}{(1-\eta)(2-\eta)}(1-\eta)(2-\eta)}{\eta(2-\beta)}\right]\left[\frac{\eta}{\eta-\frac{\eta+\beta(1-\eta)}{(1-\eta)(2-\eta)}(1-\eta)}\right]^{1-\eta}\\
&= \left[1-\frac{\eta+\beta(1-\eta)}{\eta(2-\beta)}\right]\left[\frac{\eta}{\eta-\frac{\eta+\beta(1-\eta)}{2-\eta}}\right]^{1-\eta}\\
&= \frac{\eta(2-\beta)-\eta-\beta(1-\eta)}{\eta(2-\beta)}\left[\frac{(2-\eta)\eta}{(2-\eta)\eta-\eta-\beta(1-\eta)}\right]^{1-\eta}\\
&= \frac{\eta-\beta}{\eta(2-\beta)}\left[\frac{(2-\eta)\eta}{(1-\eta)(\eta-\beta)}\right]^{1-\eta} = \frac{\alpha}{\eta(2-\beta)}\left[\frac{(2-\eta)\eta}{(1-\eta)\alpha}\right]^{1-\eta}.
\end{aligned}
$$

This implies that

$$\max_{s\in[0,\bar{s}]}\phi(s) = \max\{1,\phi(\bar{s})\}.$$

We know from the definition of $\bar{s}$ that

$$\phi(\bar{s}) = \frac{\frac{\alpha}{2-\beta}\left(1+\frac{\lambda\bar{s}}{\eta}\right)}{\lambda\left[\frac{\eta-\bar{s}(1-\eta)}{\eta}\right]^{2-\eta}} - \frac{1-\lambda}{\lambda}.$$

If we can show that $1+\frac{\bar{s}}{\eta} > \phi(\bar{s})$ we are done:

$$1+\frac{\bar{s}}{\eta} > \frac{\frac{\alpha}{2-\beta}\left(1+\frac{\lambda\bar{s}}{\eta}\right)}{\lambda\left[\frac{\eta-\bar{s}(1-\eta)}{\eta}\right]^{2-\eta}} - \frac{1-\lambda}{\lambda},$$

or

$$1+\frac{\lambda\bar{s}}{\eta} > \frac{\frac{\alpha}{2-\beta}\left(1+\frac{\lambda\bar{s}}{\eta}\right)}{\left[\frac{\eta-\bar{s}(1-\eta)}{\eta}\right]^{2-\eta}},$$

or

$$\left[\frac{\eta-\bar{s}(1-\eta)}{\eta}\right]^{2-\eta} > \frac{\alpha}{2-\beta},$$

or

$$\eta-\bar{s}(1-\eta) > \eta\left(\frac{\alpha}{2-\beta}\right)^{\frac{1}{2-\eta}},$$

or

$$\bar{s} < \frac{\eta}{1-\eta}\left[1-\left(\frac{\alpha}{2-\beta}\right)^{\frac{1}{2-\eta}}\right].$$

From the definition of $\bar{s}$ this inequality is satisfied if

$$\frac{\alpha}{2-\beta}\left[1+\frac{\lambda}{1-\eta}-\frac{\lambda}{1-\eta}\left(\frac{\alpha}{2-\beta}\right)^{\frac{1}{2-\eta}}\right] >$$
$$(1-\lambda)\left(\frac{\alpha}{2-\beta}\right)+\lambda\left\{1-\left[1-\left(\frac{\alpha}{2-\beta}\right)^{\frac{1}{2-\eta}}\right]\frac{2-\eta}{2-\beta}\right\}\left(\frac{\alpha}{2-\beta}\right)^{\frac{1}{2-\eta}},$$

or

$$\left[1+\frac{1}{1-\eta}-\frac{1}{1-\eta}\left(\frac{\alpha}{2-\beta}\right)^{\frac{1}{2-\eta}}\right]\left(\frac{\alpha}{2-\beta}\right)^{\frac{1-\eta}{2-\eta}} >$$
$$1-\left[1-\left(\frac{\alpha}{2-\beta}\right)^{\frac{1}{2-\eta}}\right]\frac{2-\eta}{2-\beta},$$

or

$$\left[1+\frac{1}{1-\eta}-\frac{1}{1-\eta}\left(\frac{\alpha}{2-\beta}\right)^{\frac{1}{2-\eta}}\right]\left(\frac{\alpha}{2-\beta}\right)^{\frac{1-\eta}{2-\eta}} >$$
$$1-\frac{2-\eta}{2-\beta}+\frac{2-\eta}{2-\beta}\left(\frac{\alpha}{2-\beta}\right)^{\frac{1}{2-\eta}},$$

or

$$\left[1+\frac{1}{1-\eta}-\frac{1}{1-\eta}\left(\frac{\alpha}{2-\beta}\right)^{\frac{1}{2-\eta}}\right]\left(\frac{\alpha}{2-\beta}\right)^{\frac{1-\eta}{2-\eta}} > \frac{\alpha}{2-\beta}+\frac{2-\eta}{2-\beta}\left(\frac{\alpha}{2-\beta}\right)^{\frac{1}{2-\eta}},$$

or

$$1+\frac{1}{1-\eta}-\frac{1}{1-\eta}\left(\frac{\alpha}{2-\beta}\right)^{\frac{1}{2-\eta}} > \left[\frac{\alpha}{2-\beta}+\frac{2-\eta}{2-\beta}\left(\frac{\alpha}{2-\beta}\right)^{\frac{1}{2-\eta}}\right]\left(\frac{\alpha}{2-\beta}\right)^{-\frac{1-\eta}{2-\eta}},$$

or

$$1+\frac{1}{1-\eta}-\frac{1}{1-\eta}\left(\frac{\alpha}{2-\beta}\right)^{\frac{1}{2-\eta}} > \left(\frac{\alpha}{2-\beta}\right)^{\frac{1}{2-\eta}}+\frac{2-\eta}{2-\beta}\left(\frac{\alpha}{2-\beta}\right)^{\frac{\eta}{2-\eta}},$$

or

$$\frac{2-\eta}{1-\eta}\left[1-\left(\frac{\alpha}{2-\beta}\right)^{\frac{1}{2-\eta}}\right] > \frac{2-\eta}{2-\beta}\left(\frac{\alpha}{2-\beta}\right)^{\frac{\eta}{2-\eta}},$$

or

$$\frac{2-\eta}{1-\eta}\left[1-\left(\frac{\alpha}{2-\beta}\right)^{\frac{1}{2-\eta}}\right] > \frac{2-\alpha-\beta}{2-\beta}\left(\frac{\alpha}{2-\beta}\right)^{\frac{\eta}{2-\eta}},$$

or

$$\frac{2-\eta}{1-\eta}\left[1-\left(\frac{\alpha}{2-\beta}\right)^{\frac{1}{2-\eta}}\right] > \left(\frac{\alpha}{2-\beta}\right)^{\frac{\eta}{2-\eta}} - \left(\frac{\alpha}{2-\beta}\right)^{\frac{2}{2-\eta}},$$

or

$$\frac{2-\eta}{1-\eta}\left[1-\left(\frac{\alpha}{2-\beta}\right)^{\frac{1}{2-\eta}}\right] - \left(\frac{\alpha}{2-\beta}\right)^{\frac{\eta}{2-\eta}} + \left(\frac{\alpha}{2-\beta}\right)^{\frac{2}{2-\eta}} > 0,$$

or

$$\left[1-\left(\frac{\alpha}{2-\beta}\right)^{\frac{1}{2-\eta}}\right] + \frac{1}{1-\eta}\left[1-\left(\frac{\alpha}{2-\beta}\right)^{\frac{1}{2-\eta}}\right] - \left(\frac{\alpha}{2-\beta}\right)^{\frac{\eta}{2-\eta}} + \left(\frac{\alpha}{2-\beta}\right)^{\frac{2}{2-\eta}} > 0,$$

or

$$\left[1-\left(\frac{\alpha}{2-\beta}\right)^{\frac{\eta}{2-\eta}}\right] + \left\{\frac{1}{1-\eta}\left[1-\left(\frac{\alpha}{2-\beta}\right)^{\frac{1}{2-\eta}}\right] - \left(\frac{\alpha}{2-\beta}\right)^{\frac{1}{2-\eta}}\right\} + \left(\frac{\alpha}{2-\beta}\right)^{\frac{2}{2-\eta}} > 0.$$

This inequality holds because the three terms inside brackets in the LHS are strictly positive. Hence, we have shown $\partial\hat{\theta}_R/\partial\lambda > 0$. Let us now show that

$$\frac{\partial\hat{\theta}_O}{\partial\lambda} > 0.$$

We know from Proposition 2 that

$$\hat{\theta}_O = \left[\frac{\eta - s(1-\eta)}{\eta}\right]^{\eta}\hat{\theta}_R.$$

Hence

$$\frac{\partial\hat{\theta}_O}{\partial\lambda} = \left[\frac{\eta - s(1-\eta)}{\eta}\right]^{\eta}\frac{\partial\hat{\theta}_R}{\partial\lambda} > 0,$$

where the inequality follows from the fact that $\partial\hat{\theta}_R/\partial\lambda > 0$. *Q.E.D.*

Proof of Proposition 6: Let $s < \bar{s}$. The wage is equal to

$$\begin{aligned} w^* &= \frac{\alpha^\eta (1-\eta)^{1-\eta} \bar{K}^\beta}{(1-\lambda+\lambda\psi)^\beta} \left(\frac{\alpha}{2-\beta} \frac{1+\lambda\frac{s}{\eta}}{1-\lambda+\lambda\phi} \right)^{\frac{(1-\eta)(1-\beta)}{2-\eta}} \\ &= \frac{\alpha^\eta (1-\eta)^{1-\eta} \bar{K}^\beta}{(1-\lambda+\lambda\psi)^\beta} \hat{\theta}_R^{1-\beta}. \end{aligned}$$

The impact of a change in λ on w^* is given by

$$\begin{aligned} \frac{\partial w^*}{\partial \lambda} &= \beta \alpha^\eta (1-\eta)^{1-\eta} \bar{K}^\beta (1-\lambda+\lambda\psi)^{-\beta-1} (1-\psi) \hat{\theta}_R^{1-\beta} \\ &+ (1-\beta) \frac{\alpha^\eta (1-\eta)^{1-\eta} \bar{K}^\beta}{(1-\lambda+\lambda\psi)^\beta} \hat{\theta}_R^{-\beta} \frac{\partial \hat{\theta}_R}{\partial \lambda} \\ &= \frac{\alpha^\eta (1-\eta)^{1-\eta} \bar{K}^\beta}{(1-\lambda+\lambda\psi)^\beta \hat{\theta}_R^\beta} \left[\beta \frac{1-\psi}{1-\lambda+\lambda\psi} \hat{\theta}_R + (1-\beta) \frac{\partial \hat{\theta}_R}{\partial \lambda} \right] > 0. \end{aligned}$$

Hence, an increase in λ raises the wage. *Q.E.D.*

Proof of Proposition 7: Let $s < \bar{s}$.

(i) The fraction of realistic workers is $L_R^* = (1-\lambda)\hat{\theta}_R$. Hence,

$$\begin{aligned} \frac{\partial L_R^*}{\partial \lambda} &= -\hat{\theta}_R + (1-\lambda) \frac{\partial \hat{\theta}_R}{\partial \lambda} \\ &= -\hat{\theta}_R + (1-\lambda) \frac{1-\eta}{2-\eta} \hat{\theta}_R \frac{1+\frac{s}{\eta}-\phi}{(1+\frac{\lambda s}{\eta})(1-\lambda+\lambda\phi)} \\ &= \left[-1 + (1-\lambda) \frac{1-\eta}{2-\eta} \frac{1+\frac{s}{\eta}-\phi}{(1+\frac{\lambda s}{\eta})(1-\lambda+\lambda\phi)} \right] \hat{\theta}_R, \end{aligned}$$

where the second equality follows from (60). Therefore $\partial L_R^* / \partial \lambda < 0$ when

$$(1-\lambda) \frac{1-\eta}{2-\eta} \frac{1+\frac{s}{\eta}-\phi}{\left(1+\frac{\lambda s}{\eta}\right)(1-\lambda+\lambda\phi)} < 1,$$

or

$$(1-\eta)(1-\lambda)\left(1+\frac{s}{\eta}-\phi\right) < (2-\eta)(1-\lambda+\lambda\phi)\left(1+\frac{\lambda s}{\eta}\right).$$

Simplifying and rearranging terms this inequality is equivalent to

$$\begin{aligned}0 \quad < \quad & -\lambda(1-\phi)+s\left(2\lambda-\lambda^2\right)\left(\frac{1}{\eta}-1\right)+s\left(\lambda-\lambda^2\right)\frac{1}{\eta} \\ & +s\lambda^2\left(\frac{2}{\eta}-1\right)\phi+\left[1-s\left(\frac{1}{\eta}-1\right)\right]+\phi(1-\eta).\end{aligned}$$

Since $s < \eta/(\eta-1)$ all terms in the RHS of the inequality are non-negative when $\phi \geq 1$. Hence the inequality is satisfied when $\phi \in [1, 1+s/\eta]$. We now show that the inequality is also satisfied when $\phi \in (0,1)$. When $\phi \in (0,1)$ the RHS is a concave function of λ (the second derivative of the RHS with respect to λ is equal to $-2(1-\phi)(2-\eta)\frac{s}{\eta}$). Hence, the RHS attains a minimum either at $\lambda = 0$ or at $\lambda = 1$. When $\lambda = 0$ the inequality becomes

$$0 < \left[1-s\left(\frac{1}{\eta}-1\right)\right]+\phi(1-\eta),$$

which is true. When $\lambda = 1$ the inequality becomes

$$0 < \phi+s\left(\frac{2}{\eta}-1\right)\phi+\phi(1-\eta),$$

which is true. Therefore, when $\phi \in (0,1)$ the inequality is satisfied. Hence, we have shown that $\partial L_R^*/\partial\lambda < 0$.

(ii) The fraction of optimistic workers is $L_O^* = \lambda\hat{\theta}_O$, which is an increasing function of λ since both terms increase with λ. Therefore, $\partial L_O^*/\partial\lambda > 0$.

(iii) The fraction of realistic entrepreneurs is $E_R^* = (1-\lambda)(1-\hat{\theta}_R)$, which is a decreasing function of λ since both terms inside brackets decrease with λ. Therefore, $\partial E_R^*/\partial\lambda < 0$. *Q.E.D.*

Proof of Proposition 8: Let $s < \bar{s}$.

(i) We wish to show that the equilibrium fraction of workers L^* is a concave function of the fraction of optimists λ. The equilibrium fraction of workers is equal to

$$L^* = (1-\lambda)\hat{\theta}_R+\lambda\hat{\theta}_O = (1-\lambda+\lambda\psi)\hat{\theta}_R.$$

The impact on L^* of a change in λ is given by

$$\frac{\partial L^*}{\partial \lambda} = -(1-\psi)\hat{\theta}_R + (1-\lambda+\lambda\psi)\frac{\partial \hat{\theta}_R}{\partial \lambda}. \tag{63}$$

From (63) it follows that

$$\frac{\partial^2 L^*}{\partial \lambda^2} = -2(1-\psi)\frac{\partial \hat{\theta}_R}{\partial \lambda} + (1-\lambda+\lambda\psi)\frac{\partial^2 \hat{\theta}_R}{\partial \lambda^2}. \tag{64}$$

We know from Proposition 5 that

$$\begin{aligned}
\frac{\partial \hat{\theta}_R}{\partial \lambda} &= \frac{\partial}{\partial \lambda}\left[\left(\frac{\alpha}{2-\beta}\frac{1+\lambda\frac{s}{\eta}}{1-\lambda+\lambda\phi}\right)^{\frac{1-\eta}{2-\eta}}\right] \\
&= \frac{1-\eta}{2-\eta}\left(\frac{\alpha}{2-\beta}\frac{1+\lambda\frac{s}{\eta}}{1-\lambda+\lambda\phi}\right)^{\frac{1-\eta}{2-\eta}-1}\frac{\alpha}{2-\beta}\frac{\partial}{\partial \lambda}\left(\frac{1+\lambda\frac{s}{\eta}}{1-\lambda+\lambda\phi}\right) \\
&= \frac{1-\eta}{2-\eta}\left(\frac{\alpha}{2-\beta}\right)^{\frac{1-\eta}{2-\eta}}\left(\frac{1+\frac{\lambda s}{\eta}}{1-\lambda+\lambda\phi}\right)^{-\frac{1}{2-\eta}}\frac{1+\frac{s}{\eta}-\phi}{(1-\lambda+\lambda\phi)^2}.
\end{aligned} \tag{65}$$

From (65) we obtain

$$\begin{aligned}
\frac{\partial^2 \hat{\theta}_R}{\partial \lambda^2} &= -\frac{z}{2-\eta}\left(\frac{1+\frac{\lambda s}{\eta}}{1-\lambda+\lambda\phi}\right)^{-\frac{1}{2-\eta}-1}\frac{(1+\frac{s}{\eta}-\phi)^2}{(1-\lambda+\lambda\phi)^4} \\
&\quad +2z\left(\frac{1+\frac{\lambda s}{\eta}}{1-\lambda+\lambda\phi}\right)^{-\frac{1}{2-\eta}}\frac{\left(1+\frac{s}{\eta}-\phi\right)(1-\phi)}{(1-\lambda+\lambda\phi)^3} \\
&= -\frac{z}{2-\eta}\left(\frac{1+\frac{\lambda s}{\eta}}{1-\lambda+\lambda\phi}\right)^{-\frac{1}{2-\eta}}\frac{1-\lambda+\lambda\phi}{1+\frac{\lambda s}{\eta}}\frac{(1+\frac{s}{\eta}-\phi)^2}{(1-\lambda+\lambda\phi)^4} \\
&\quad +2z(1-\phi)\left(\frac{1+\frac{\lambda s}{\eta}}{1-\lambda+\lambda\phi}\right)^{-\frac{1}{2-\eta}}\frac{1+\frac{s}{\eta}-\phi}{(1-\lambda+\lambda\phi)^3} \\
&= z\left(\frac{1+\frac{\lambda s}{\eta}}{1-\lambda+\lambda\phi}\right)^{-\frac{1}{2-\eta}}\frac{1+\frac{s}{\eta}-\phi}{(1-\lambda+\lambda\phi)^3}\left[-\frac{1}{2-\eta}\frac{1+\frac{s}{\eta}-\phi}{1+\frac{\lambda s}{\eta}}+2(1-\phi)\right] \\
&= \frac{z}{1-\lambda+\lambda\phi}\left[-\frac{1}{2-\eta}\frac{1+\frac{s}{\eta}-\phi}{1+\frac{\lambda s}{\eta}}+2(1-\phi)\right]\frac{\partial \hat{\theta}_R}{\partial \lambda},
\end{aligned} \tag{66}$$

where

$$z = \frac{1-\eta}{2-\eta}\left(\frac{\alpha}{2-\beta}\right)^{\frac{1-\eta}{2-\eta}}.$$

Substituting (65) and (66) into (64) we obtain

$$\frac{\partial^2 L^*}{\partial \lambda^2} = z\left\{-2(1-\psi) + \frac{1-\lambda+\lambda\psi}{1-\lambda+\lambda\phi}\left[-\frac{1}{2-\eta}\frac{1+\frac{s}{\eta}-\phi}{1+\frac{\lambda s}{\eta}} + 2(1-\phi)\right]\right\}\frac{\partial \hat{\theta}_R}{\partial \lambda}.$$

Since $\partial\hat{\theta}_R/\partial\lambda > 0$ it follows that $\partial^2 L^*/\partial\lambda^2 < 0$ as long as

$$-2(1-\psi) + \frac{1-\lambda+\lambda\psi}{1-\lambda+\lambda\phi}\left[-\frac{1}{2-\eta}\frac{1+\frac{s}{\eta}-\phi}{1+\frac{\lambda s}{\eta}} + 2(1-\phi)\right] < 0.$$

When $\phi \in [1, 1+s/\eta]$, the second term on the LHS is non-positive and $\partial^2 L^*/\partial\lambda^2 < 0$. When $\phi \in (0,1)$ a sufficient condition for $\partial^2 L^*/\partial\lambda^2 < 0$ is

$$\frac{1-\psi}{1-\phi} \geq \frac{1-\lambda+\lambda\psi}{1-\lambda+\lambda\phi}. \tag{67}$$

This inequality is satisfied since $\psi \leq \phi < 1$–see (52)–implies that the LHS of (67) is greater than or equal to 1 and the RHS of (67) is less than or equal to 1. Hence, L^* is a concave function of λ.

(ii) and (iii) We start by showing that

$$\left.\frac{\partial L^*}{\partial \lambda}\right|_{\lambda=0} > 0,$$

which implies that L^* is not a decreasing function of λ. From (63) we have

$$\begin{aligned}
\left.\frac{\partial L^*}{\partial \lambda}\right|_{\lambda=0} &= -(1-\psi)\left.\hat{\theta}_R\right|_{\lambda=0} + \left.(1-\lambda+\lambda\psi)\right|_{\lambda=0}\left.\frac{\partial\hat{\theta}_R}{\partial\lambda}\right|_{\lambda=0} \\
&= -(1-\psi)\hat{\theta}_0 + \frac{1-\eta}{2-\eta}\hat{\theta}_0\left(1+\frac{s}{\eta}-\phi\right) \\
&= \left[\frac{1-\eta}{2-\eta}\left(1+\frac{s}{\eta}-\phi\right) - (1-\psi)\right]\hat{\theta}_0.
\end{aligned}$$

This derivative is positive as long as

$$(1-\eta)\left(1+\frac{s}{\eta}-\phi\right) > (2-\eta)(1-\psi),$$

or

$$\phi\eta - \phi - \eta - s + \frac{s}{\eta} + 1 > \psi\eta - \eta - 2\psi + 2,$$

or

$$\phi\eta - \phi - s + \frac{s}{\eta} > \psi\eta - 2\psi + 1,$$

or

$$\phi\eta - \phi - s + \frac{s}{\eta} - \psi\eta + \psi + \psi - 1 > 0,$$

or

$$-s + \frac{s}{\eta} - \frac{s(1-\eta)^2}{\eta - s(1-\eta)}\frac{\alpha}{2-\beta}\psi + \psi - 1 > 0,$$

or

$$\frac{s(1-\eta)}{\eta} > \frac{s(1-\eta)^2}{\eta - s(1-\eta)}\frac{\alpha}{2-\beta}\psi + (1-\psi),$$

or

$$\psi > \frac{s(1-\eta)^2}{\eta - s(1-\eta)}\frac{\alpha}{2-\beta}\psi + \frac{\eta - s(1-\eta)}{\eta},$$

or

$$1 > \frac{s(1-\eta)^2}{\eta - s(1-\eta)}\frac{\alpha}{2-\beta} + \left[\frac{\eta - s(1-\eta)}{\eta}\right]^{1-\eta},$$

We know from Proposition 5 that

$$\frac{\alpha}{2-\beta} < \left[\frac{\eta - \bar{s}(1-\eta)}{\eta}\right]^{2-\eta} < \left[\frac{\eta - s(1-\eta)}{\eta}\right]^{2-\eta},$$

where the second inequality follows from $s < \bar{s}$. Therefore the inequality is satisfied as long as

$$1 > \frac{s(1-\eta)^2}{\eta}\left[\frac{\eta - s(1-\eta)}{\eta}\right]^{1-\eta} + \left[\frac{\eta - s(1-\eta)}{\eta}\right]^{1-\eta},$$

or

$$1 > \left[\frac{s(1-\eta)^2}{\eta} + 1\right]\left[\frac{\eta - s(1-\eta)}{\eta}\right]^{1-\eta}$$

Note that if $s = 0$ the RHS of the inequality is equal to 1. We now show that the RHS is decreasing with s which implies that the inequality is satisfied for any $s \in (0, \bar{s})$.

The derivative of the RHS with respect to s is:

$$\begin{aligned}
& \frac{(1-\eta)^2}{\eta}\left[\frac{\eta-s(1-\eta)}{\eta}\right]^{1-\eta}-\frac{(1-\eta)^2}{\eta}\left[\frac{s(1-\eta)^2}{\eta}+1\right]\left[\frac{\eta-s(1-\eta)}{\eta}\right]^{-\eta} \\
= & \frac{(1-\eta)^2}{\eta}\left[\frac{\eta-s(1-\eta)}{\eta}\right]^{-\eta}\left[\frac{\eta-s(1-\eta)}{\eta}-\frac{s(1-\eta)^2}{\eta}-1\right] \\
= & -\frac{(1-\eta)^2}{\eta}\left[\frac{\eta-s(1-\eta)}{\eta}\right]^{-\eta}\left[\frac{s(1-\eta)}{\eta}+\frac{s(1-\eta)^2}{\eta}\right]<0.
\end{aligned}$$

Therefore, L^* is not a decreasing function of λ. Thus, we are left with two cases: (1) L^* is an increasing and concave function of λ (and $L^*(\lambda)$ attains a maximum at $\lambda=1$), and (2) L^* is a concave function of λ which attains a maximum at $\bar{\lambda}\in(0,1)$. Case (1) happens when

$$\begin{aligned}
\left.\frac{\partial L^*}{\partial\lambda}\right|_{\lambda=1} = & -(1-\psi)\left.\hat{\theta}_R\right|_{\lambda=1}+\left.(1-\lambda+\lambda\psi)\right|_{\lambda=1}\left.\frac{\partial\hat{\theta}_R}{\partial\lambda}\right|_{\lambda=1} \\
= & -(1-\psi)\left(\frac{\alpha}{2-\beta}\frac{1+\frac{s}{\eta}}{\phi}\right)^{\frac{1-\eta}{2-\eta}}+\psi\frac{1-\eta}{2-\eta}\left(\frac{\alpha}{2-\beta}\right)^{\frac{1-\eta}{2-\eta}}\left(\frac{1+\frac{s}{\eta}}{\phi}\right)^{-\frac{1}{2-\eta}}\frac{1+\frac{s}{\eta}-\phi}{\phi^2} \\
= & \left[-(1-\psi)+\frac{\psi}{\phi}\frac{1-\eta}{2-\eta}\frac{1+\frac{s}{\eta}-\phi}{1+\frac{s}{\eta}}\right]\left(\frac{\alpha}{2-\beta}\frac{1+\frac{s}{\eta}}{\phi}\right)^{\frac{1-\eta}{2-\eta}}>0.
\end{aligned}$$

This condition is satisfied when

$$(1-\eta)\left(1+\frac{s}{\eta}-\phi\right)\psi>(2-\eta)(1-\psi)\left(1+\frac{s}{\eta}\right)\phi. \tag{68}$$

Case (2) happens when (68) is violated. From (63) and the fact that $\partial^2 L^*/\partial\lambda^2<0$ it follows that in case (2) L^* attains a maximum at the λ which solves

$$(1-\lambda+\lambda\psi)\frac{\partial\hat{\theta}_R}{\partial\lambda}=(1-\psi)\hat{\theta}_R. \tag{69}$$

Note that

$$\frac{\partial\hat{\theta}_R}{\partial\lambda}=\frac{1-\eta}{2-\eta}\hat{\theta}_R\frac{1}{1+\lambda\frac{s}{\eta}}\frac{1+\frac{s}{\eta}-\phi}{1-\lambda+\lambda\phi}. \tag{70}$$

Substituting (70) into (69) we obtain

$$(1-\lambda+\lambda\psi)\frac{1-\eta}{2-\eta}\hat{\theta}_R\frac{1}{1+\lambda\frac{s}{\eta}}\frac{1+\frac{s}{\eta}-\phi}{1-\lambda+\lambda\phi}=(1-\psi)\hat{\theta}_R,$$

or

$$\frac{1-\eta}{2-\eta}\frac{1+\frac{s}{\eta}-\phi}{1+\lambda\frac{s}{\eta}}\frac{1-\lambda+\lambda\psi}{1-\lambda+\lambda\phi}=1-\psi,$$

or

$$\frac{1-\lambda+\lambda\psi}{1-\lambda+\lambda\phi}=\frac{2-\eta}{1-\eta}\frac{1-\psi}{1+\frac{s}{\eta}-\phi}\left(1+\lambda\frac{s}{\eta}\right). \tag{71}$$

Hence, L^* attains a maximum at the $\lambda \in (0,1)$ which solves (71): $\bar{\lambda}$. *Q.E.D.*

Proof of Proposition 9: From (50) the equilibrium rental cost of capital is equal to

$$r^*=\frac{\beta w^*}{\alpha\bar{K}}\left[1-\lambda+\lambda\psi(\eta,s)\right]\hat{\theta}_R=\frac{\beta}{\alpha\bar{K}}w^*L^*.$$

The impact of a change in λ on r^* is given by

$$\frac{\partial r^*}{\partial\lambda}=\frac{\beta}{\alpha\bar{K}}\left[\frac{\partial w^*}{\partial\lambda}L^*+w^*\frac{\partial L^*}{\partial\lambda}\right].$$

We know from Proposition 6 that $\partial w^*/\partial\lambda > 0$. Hence, $\partial L^*/\partial\lambda > 0$ is a sufficient condition for $\partial r^*/\partial\lambda > 0$. Therefore, it follows from Proposition 8 that $\partial r^*/\partial\lambda > 0$ when either (1) inequality (68) is satisfied or (2) inequality (68) is violated and $\lambda \in (\bar{\lambda}, 1]$.

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