Exercise 2: Simulation of estimators

Loops, matrices

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This assignment concerns the simulation of estimators and test statistics in the simple linear regression model. The model we assume uses explanatory variables $x_i, i = 1, 2, 3$.

- Write an Ox program `SimRegStat.ox`. In this program you first generate n realisations for x_i. Ox provides random numbers for many distributions. Search e.g. for help on `rann()` and `ranchi()`. Generate such that the final X matrix is the combination of $x_1 = 1, x_2 \sim U(0, 5), x_3 \sim \chi^2(3)$.

  Hint: Many random number generators require the statement `#include <oxprob.h>` near the top of your program.

  The matrix containing $[x_1, x_2, x_3]$ is generated **only once** (for a fixed value of n that is). It is treated **as fixed** for the remainder of this assignment.

- We consider the regression model

$$y = X\beta + u \qquad\qquad u \sim \mathcal{N}(0, \sigma_u^2)$$

  Write a function `SimReg()` in Ox to generate a vector of n values y_i, given specific values for the parameters $\beta = (\beta_1, \beta_2, \beta_3)'$, σ_u^2 and a matrix of values for X.

- Test your program by generating y_i's using $\beta = (0\ 1\ 2)', \sigma_u^2 = 2$ for $n = 100$ and for $n = 10$.

- Add a routine which returns $b, s(b)$ and s^2, the OLS estimates of the corresponding parameters and the standard deviations of the estimates b. Print once the estimates of β, their standard deviations, and the t-statistics.

  Hint: Adapt your previous program for these tasks

- Extend your Ox program `SimRegStat.ox` to a program `SimRegStatEx.ox`, in which you repeat the generation of $y_1, \dots, y_n$ for $M = 1000$ replications, keeping X, β, σ_u^2 fixed. Consider $n = 100$ and $n = 10$. For each sample of y, estimate β and σ_u^2 and store the estimates b and s^2 in the matrix `mOLS`:

$$\texttt{mOLS} = \begin{pmatrix} b_1 & b_2 & b_3 & s^2 \\ b_1 & b_2 & b_3 & s^2 \\ \vdots & \vdots & \vdots & \\ b_1 & b_2 & b_3 & s^2 \end{pmatrix} \begin{matrix} \text{for replication 1} \\ \text{for replication 2} \\ \vdots \\ \text{for replication } M. \end{matrix}$$

- Write a separate routine which takes `mOLS` as input and computes some useful descriptive statistics (minimum, maximum, mean, standard deviation, etc.) for the M values of b_1, b_2, b_3 and s^2.

- Compare the standard deviations of the M values of b to the standard deviations of the original estimates.