A large piece of a small pie: Minimum wages and unemployment benefits in an assignment model with search frictions.*

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Abstract

Most empirical studies on the minimum wage find a spike at the minimum wage, compression of wage differentials at a large interval above the minimum wage and small employment losses. This paper offers a search model which is consistent with these facts. We consider a continuum of worker and job types, Nash wage bargaining, and a production structure based on comparative advantages. The introduction of a minimum wage in this model makes some matches at the lower segments no longer profitable. In addition it leads to a redistribution of rents from firms to low skilled workers. A cluster of relatively simple vacancies, offering the minimum wage, is opened because the workers in this segment become relatively scarce. Finally, we give some numerical simulations to test for the validity of our approximations.

Keywords: assignment, search, minimum wage, unemployment

JEL codes J21, J23

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1 Introduction

Empirical studies on the employment effects of minimum wages have generated three stylized facts, of which some are still controversial. First, employment effects tend to be modest in the US, see e.g. Brown et al. (1982), Card and Krueger (1995) and can be substantial in other countries, see e.g. Baker et al.(1999) for Canada. Second, there is a mass of jobs offering exactly the minimum wage. Third, minimum wages tend to substantially compress the wage distribution, see Dinardo et al. (1996), Lee (1999) Teulings (2000).

The main goal of this paper is to construct a model that is consistent with those facts. Since the income and labor market status effects are very asymmetrically distributed over the population of workers and jobs, this model must explicitly take the heterogeneity of workers and jobs into account. Point of departure will be the assignment model with search frictions of Teulings and Gautier (2000). This model is characterized by a continuum of worker and job types, all searching for feasible job matches. The contact technology which describes how fast workers meet jobs is assumed to exhibit IRS. This implies that more contacts per period take place in a denser market. Part of those increasing returns will however be absorbed by workers and firms being more choosy as the scale of the market increases. We extend this model with a binding minimum wage. In the model, both unemployment benefits and a minimum wage can fulfill a potential useful role in the sense that they prevent workers from accepting jobs too easily. This can be understood as follows. Without search frictions, a worker will be matched with the job type that suits her best. When there are search frictions, this worker will be less choosy and also accept job types that suit her less. When her fallback position is extremely unattractive, she will accept virtually any job type. As a side effect, she reduces the expected rents of vacancies for those job types which are in her matching set but for which she is relatively ill-suited. One role for unemployment benefits and minimum wages is to improve the fall back position of the workers and make them more choosy. As a result, an equilibrium with little mismatch and high unemployment results.

Ideally, if the government could perfectly observe the skill of a worker, minimum wages and replacement rate levels should be differentiated over skill groups where workers with more skills need a larger search subsidy to prevent them from accepting the wrong jobs. This is consistent with the fact that in many countries unemployment benefits are a fraction of the last earned wage. In the Netherlands, the government even varies the minimum wage over different age groups. In general, it is however impossible to fully differentiate the minimum wage and replacement rate over skill groups. This prevents the economy from reaching a first best equilibrium. If benefits are the same for all workers, the replacement rate for low skilled workers will be relatively high and consequently low skilled unemployment as well. Moreover, when agents are risk averse, it might become optimal to give the low skilled workers higher benefits. For both types of policies, the trade-off is between match quality and unemployment.

Besides the similarity that both instruments lead to fewer mismatch and higher unemployment, there are also important differences. First, all unemployed workers are affected when unemployment benefits are increased (although low skilled workers are affected more strongly). While minimum wages are only binding for low worker-job type combinations. In that sense, unemployment benefits are a more powerful instrument to reduce mismatch. On the other hand, minimum wages are in general easier to implement and do not give rise to all sorts of moral hazard problems like reduced search intensities.

Another fact that needs to be explained is that almost all OECD countries have a binding minimum wage even though the aggregate welfare effects are often found to be modestly negative. Our model suggests that, even when unemployment benefits and the wage bargaining parameter are set at their optimal levels and agents are risk neutral, a majority of mainly middle class workers gains from a minimum wage while a minority of low and high skilled workers are worse off. Hence, a minimum wage with negative welfare effects can be supported by a voting equilibrium.

There are three models in the literature which are closely related to our benchmark model without a minimum wage. Shimer and Smith (2000) is probably most similar. Their model does however not have endogenous vacancy supply and a goods market. Moreover, they do not explicitly characterize the equilibrium. Sattinger (1995) also considers an assignment model with search frictions in a somewhat different setting (fixed per period interview rate and fixed matching rate). He also considers a fixed amount of jobs, which is an uncomfortable assumption when one is interested in the effects of a minimum wage and unemployment benefits. Finally, Marimon and Zilibotti (1999) allow for both ex ante heterogeneity of worker and job types and for free entry of vacancies. In their model, workers and jobs are both located on the same circle and the productivity of a match is fully determined by the distance between the worker and the job. Such a model cannot explain the asymmetric effects that a minimum wage has on low and high skilled workers. Moreover, they consider a CRS contact technology which implies that when the Hosios condition is satisfied, there is no role for unemployment benefits. In our model, there is always a role for unemployment benefits because due to the IRS contact technology, workers and firms will never receive a share of the surplus that fully rewards them for their contribution to the contact process.

The paper is organized as follows. Section 2 describes the basic model without a minimum wage. In section 3 we approximate the model by a second order Taylor expansion. In section 4 we derive the optimal replacement rate and in section 5 we show how a minimum wage can be implemented in the model. Section 6 gives numerical simulations and shows how a binding minimum wage can be supported by a voting equilibrium. Finally section 7 concludes.

2 The model

2.1 Basic assumptions

As in Gautier and Teulings (2000), we consider a continuum of worker (s) and job types (c):

$$\begin{aligned} s &\in [s^-, s^+] \\ c &\in [c^-, c^+],\ c^- > 0 \end{aligned}$$

The size of the labor force will be denoted by $\underline{L}$ and the density function of s by: $\underline{l}\,(s)$. Let $h(s)$ denote the number of unemployed workers of type s per unit of labor supply. Then the unemployment rate for type s workers is given by: $\frac{h(s)}{\underline{l}(s)}$ and the aggregate unemployment rate satisfies $u \equiv \int_{s^-}^{s^+} h(s)ds$. The supply of vacancies is determined by a free entry rule, which drives the asset value of a vacancy to zero in equilibrium. We will denote the number of vacancies of type c per unit of labor supply by $g(c)$. The total number of vacancies per unit of labor supply follows then from $v \equiv \int_{c^-}^{c^+} g(c)dc$ while the total number of vacancies is $\underline{L}v$. If vacancies could be opened costlessly, their stock would grow infinitely large. Therefore, assume that there are flow costs of keeping a vacancy open which are equal to K per period. Assume that those costs are independent of the job type. We can think of those costs as advertisement costs. In a richer model, those costs could be made dependent of the job type and interpreted as ex ante capital costs. Unemployed workers receive unemployment benefits $B(s)$, which are financed by a proportional wage tax. We will loosely refer to the ratio of B to reservation wages as the replacement rate. As stated in the introduction, one goal of this paper is to compare the redistribution and welfare effects of unemployment benefits with those of the minimum wage.

The productivity of a worker, s, at a job, c, is denoted by $F(s,c)$. Since each job type c produces its own commodity, we have to introduce a set of job type specific commodity prices $P(c)$. The gross per period value of a match between s and c is therefore equal to $P(c)F(s,c)$. Search frictions enter the model by a simple linear contact rate $\lambda_{i\to j}$ for worker (job) type i to run into job (worker) type j:

$$\begin{aligned} \lambda_{s\to c} &\equiv \lambda^* L g(c) \\ \lambda_{c\to s} &\equiv \lambda^* L h(s) \end{aligned} \tag{1}$$

where λ^* is a technology parameter which measures the efficiency of the matching process. For notational convenience we define $\lambda = \lambda^* \; L$. We can interpret λ then as the relevant scale of the labor market. Matches do not last forever. Shocks which destroy all match value, arrive at a Poisson rate δ. Finally, we assume for simplicity that both workers and firms are risk neutral. Since we allow for transferable utility, matches are formed if:

$$P(c)F(s,c) - R(s) > 0 \tag{2}$$

Finally we have to specify the production function: $F(s,c)$. As in Teulings and Gautier (2000) we assume it to be log supermodular with the following functional form:[1]

$$F(s,c) \equiv e^{s\,c} \tag{3}$$

This function captures the idea of absolute and comparative advantage of high skilled workers on complex jobs.

[1] Shimer and Smith (2000) show that log supermodularity is a sufficient condition for an assortative matching equilibrium to occur.

Since we consider a stationary economy, the number of workers finding a job must be equal to the number loosing their job. Hence:

$$\delta\left[\underline{l}(s)-h(s)\right]=\lambda\, h(s)\int_{m_c(s)} g(c)\,dc \tag{4}$$

Physical output per job type can be derived from the inflow of new workers of type s, times their productivity in job type c times the expected duration of the employment relation $1/\delta$:

$$Y(c)=\frac{\lambda}{\delta}g(c)\int_{m_s(c)} h(s)e^{sc}ds \tag{5}$$

2.2 The value of search in the benchmark case

In the benchmark case, the value of search for a worker and an employer respectively, can be expressed in terms of the following Bellman equations:

$$R(s)=B(s)+\frac{\lambda}{\rho+\delta}\int_{m_c(s)} g(c)\left[W(s,c)-R(s)\right]\,dc \tag{6}$$

$$K=\frac{\lambda}{\rho+\delta}\int_{m_s(c)} h(s)\left[P(c)F(s,c)-W(s,c)\right]\,ds \tag{7}$$

where $R(s)$ is the reservation wage of worker type s, $W(s,c)$ is the wage of a worker of type s who is employed at a job type c, and ρ is the discount rate. Since we allow for bargaining over the match surplus, any match with a value that exceeds the sum of the outside options of firm and worker is acceptable. Wages are set by a simple Nash bargaining rule. Hence:

$$W(s,c)=\beta P(c)F(s,c)+(1-\beta)R(s) \tag{8}$$

where β denotes the workers' bargaining power. Not surprisingly, β will be a crucial factor regarding the welfare loss in the decentralized economy. The functions $m_c(s)$ and $m_s(c)$ define the subsets of c and s respectively with whom type s and c are willing to form a match with. These subsets are determined by the condition that the log match surplus is positive.

2.3 The commodity market

Assume that each job c produces 1 unit of commodity c which is traded on a commodity market where consumers have "love for variety "utility. We consider two extreme cases, perfect and zero substitutability of goods. The demand for the various commodities is given by a continuous type CES utility function :

$$(1-\eta)p = \ln\left[\int_{c^-}^{c^+} \exp\left[(\eta+1)\ \underline{q}\ (c) + (1-\eta)p(c))\right] dc\right] \tag{9}$$

where p is the log price index of consumption (or alternatively, the price of the composite consumption good), $\underline{q}(c)$ is the weight of type c in consumption and η is the elasticity of substitution. From (9) we can derive that the demand for commodity c satisfies:

$$y(c) - y - \underline{q}(c) = \eta[p - p(c) + \underline{q}(c)]$$

where y denotes log aggregate consumption. In the first case, prices are effectively exogenous: $\eta = \infty : p - p(c) + \underline{q}(c) = 0$. Let p be the numeraire, hence, $p = 0$, implies $p(c) = \underline{q}(c)$. In the second case, the distribution of output per job type is exogenous: $\eta = 0 :\ y(c) = \underline{q}(c)$. We will focus in this paper on the zero substitution case.

$$\begin{aligned} \eta &= \infty : p(c) = \underline{q}(c), \forall c : \underline{q}"(c) < 0,\ \underline{q}'(c^-) = s^-,\ \underline{q}'(c^+) = s^+ \\ \eta &= 0 : y(c) = \underline{q}(c), \forall c : \underline{q}(c) > 0, \end{aligned} \tag{10}$$

The assumption $\underline{q}(.)$ being positive for the case that $\eta = 0$ implies that there is positive demand for all job types c. The assumption $\underline{q}"(.)$ being negative has the same effect, see Teulings and Gautier (2000).

3 Approximating the equilibrium

In this section we use the method of Teulings and Gautier (2000) to approximate the surplus for a given worker type (s) by a parabola around the job for which he reaches the maximal surplus, $c(s)$. Similarly, for a given job type, we approximate the surplus around the worker type for which it reaches a maximal surplus, $s(c)$. In a Walrasian world, $c(s) = s(c)$. In a world with search frictions this is not necessarily the case but as an approximation, we assume that this condition holds. As we will show in section 6, this turns out to be a harmless assumption but it simplifies the analysis a lot.

For a given job type c, the surplus is maximized if:

$$r'\left[s(c)\right] = c \tag{11}$$

where $s(c)$ is the optimal assignment for job c. Differentiating this first order condition with respect to c yields: $r''[s(c)]s'(c) = 1$. Since our supermodular production function implies that $s'(c) > 0$, it must also be that $r''(s) > 0$. This is the second order condition of the cost minimization problem: the (log) costs of hiring a worker with an additional unit of skill are increasing while the log returns are constant, namely equal to the value of c for that firm.

The zero profit condition of firms reads: $p(c) + s(c)c - r\left[s(c)\right] = 0$. Since this condition applies identically for all c, its first difference must also apply. Hence (the effect via $s(c)$ drops out by the envelope theorem):

$$-p'(c) = s(c) \tag{12}$$

Differentiating a second time yields: $-p''(c) = s'(c) = 1/r''(s)$. Hence: $p''\left(c\right) < 0$.

A crucial variable in this model is the second derivative of the wage function. Since $r''\left[s(c)\right] = 1/s'(c)$, it is a measure of job heterogeneity. The higher $r''(s)$, the more variation there exists in job complexity per unit of s.

We solve the model by a second order Taylor expansion around $c(s)$. For this purpose, we benefit from the characteristics of the surplus $x[s, c(s)]$, which has a negative second derivative in both its arguments. In Teulings and Gautier (2000), we give a formal approximation. Here, we give an intuitive description. First, we apply an approximation for the integrand: $e^{x(s,c)} - 1 \simeq x\left(s, c\right)$. Next, the maximum of the integrand of the domain of integration can be approximated by a second order Taylor expansion: $x\left[s, c(s)\right] \simeq -\frac{1}{2}x_{cc}\left[s, c\left(s\right)\right]\Delta^2$, where $\Delta = c^{+}(s) - c(s) = c(s) - c^{-}(s)$. This equation can easily be solved for Δ. We apply $x_{cc}\left[s, c(s)\right] = p''\left[c(s)\right] = -c'(s)^{-1}$. Hence, the surface of the rectangle (A+B) in Figure 1 equals $2\Delta x\left[s, c(s)\right] = 2\sqrt{2c'(s)x\left[s, c(s)\right]^3}$. Integration over a parabola yields the result that two thirds of this surface is below the parabola (A:B=2:1). The above argument implies that the average surplus of a worker of type s above her log reservation wage $r(s)$ can be approximated by $\frac{2}{3}x\left[s, c(s)\right]$,

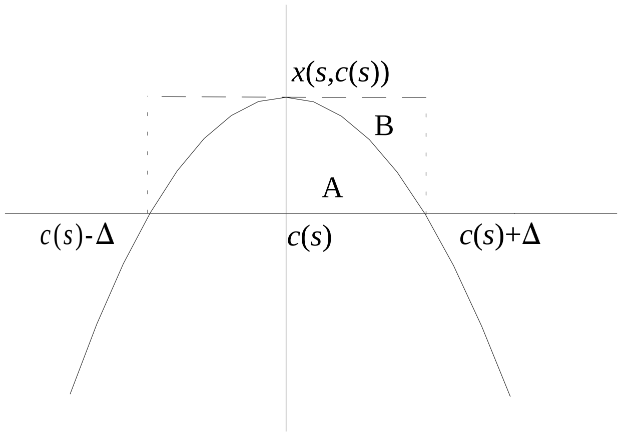

Figure 1: Taylor approximation of the surplus for a given worker type s

or by a complementary argument, the average loss relative to the Walrasian optimum is: $\frac{1}{3}x\left[s,c(s)\right]$.

Under those assumptions, we derived in Teulings and Gautier (2000) that:

$$x^*(s)^5 = \frac{81}{128}Q^2B^*\left(s\right)^2K^*\left(s\right)^2\underline{l}(s)^{-2}c'(s) \tag{13}$$

$$\frac{h(s)}{\underline{l}(s)} = \frac{2}{3}\frac{\delta\beta}{\rho+\delta}B^*\left(s\right)^{-1}x^*(s) \tag{14}$$

$$g\left[c(s)\right] = \frac{2}{3}\frac{\delta(1-\beta)}{(\rho+\delta)}K^*(s)^{-1}x^*(s)c'(s)^{-1} \tag{15}$$

where $x^*(s) \equiv x\left[s,c(s)\right]$, $Q \equiv \frac{(\rho+\delta)^2}{\delta\beta(1-\beta)\lambda}$, $B^*(s) \equiv 1-\frac{B}{R(s)}$, and $K^*(s) \equiv \frac{K}{R(s)}$. The constant terms are a direct result of integrating the Taylor approximation. In the next section we use this analytical approximation to derive the optimal replacement rate.

4 The optimal replacement rate

Before we will analyze the welfare effects of changes in the replacement rate, it is useful to know what the optimal replacement rate in the decentralized equilibrium is. Therefore, we first derive the replacement rate which minimizes the welfare loss due to search frictions. This loss as a fraction of total output consists of three parts: the lost production due to unemployment ($\frac{h(s)}{l(s)}$ times the cost of unemployment[2]), the costs of maintaining vacancies and the cost of mismatch (recall from Section 3 that the average loss relative to the optimal Walrasian allocation is ${}^{1}/_{3}x^{*}(s)$). This gives us:

$$Loss_{Wal} \simeq \frac{1}{3}x^{*}(s)\left[\frac{2\delta}{\rho+\delta}\left(\frac{\beta}{B^{*}(s)}+(1-\beta)\right)+1\right]= \quad (16)$$

$$\frac{1}{3}AB^{*}(s)^{0.4}\left[\frac{2\delta}{\rho+\delta}\left(\frac{\beta}{B^{*}(s)}+(1-\beta)\right)+1\right] \quad (17)$$

where $A = x^{*}(s)/B^{2/5}$. The optimal benefit level ($B^{o}(s)$) follows then from d Loss/d$B^{*}(s) = 0$:

$$B^{o}(s) = \left(1-\frac{3\delta\beta}{\rho+\delta+2\delta\,(1-\beta)}\right)R(s)$$

First note that the optimal benefit level is decreasing in β. This makes sense since there is no need to improve the outside option of workers when they already have a good bargaining position.

The fact that the optimal replacement rate is positive does not simply follow from inefficient bargaining and cannot be undone by introducing market makers as in the efficient bargaining models of Shimer (1995) and Moen (1997). In this respect our model differs from Marimon and Zilibotti (1999) where $B^{o}(s) = 0$ when the Hosios condition is met. As mentioned in the introduction, our IRS matching technology prevents the decentralized market economy to settle down at an efficient equilibrium.

This can be illustrated as follows. Consider a social planner who is able to set all incentives right. The only thing he cannot control is the contact technology. Loosely speaking this implies that workers and firms behave as if their contribution to the search process is equal to their share of the surplus. Workers behave as if $\beta = 1$ and firms behave as if $(1-\beta) = 1$. The surplus equation (13) therefore goes down by a factor: $\beta^{0.4}(1-\beta)^{0.4}$, because the denominator in Q, becomes: $\delta\lambda$ instead of $\delta\lambda\ \beta(1-\beta)$ (and the elasticity of Q is 0.4). The planner's loss for a given $B(s)$ is then equal to $\left(1-\beta^{0.4}(1-\beta)^{0.4}\right) Loss_{Wal}$ or:

[2] Here (and below, when discussing the cost of a vacancy), the proper normalization would be to use the actual wage $W(s,c)$ instead of $R(s)$. From section 4: $\mathrm{E}_{m_c(s)}\left[\frac{W(s,c)}{R(s)}-1\right] \simeq \frac{1}{3}x\left[s,c(s)\right]$. Hence, the proper cost are: $B^{*}(s)+\frac{1}{3}x\left[s,c(s)\right]$. This is however a second order effect in $x\left[s,c(s)\right]$.

$$Loss_{SP} \simeq \frac{1}{3}x^*(s)\left[\left(1-\beta^{0.4}(1-\beta)^{0.4}\right)\left(\frac{2\delta\beta}{(\rho+\delta)B^*(s)}+\frac{2\delta(1-\beta)+\rho+\delta}{(\rho+\delta)}\right)\right] \quad (18)$$

Welfare is maximized when $B(s) = R(s) - \varepsilon$ and $\beta = \varepsilon$. In that case, workers will be extremely choosy with respect to the jobs they accept while firms are stimulated to offer the "right" amount of vacancies since they receive almost the full share of the surplus. In Teulings and Gautier (2000) we show that for a given B, the loss of search is minimized when $\beta = 0.5$. The intuition for this result is the following. In a world with a constant returns to scale contact technology, the Hosios-Pissarides condition tells us that a worker's share of the match surplus should be equal to his contribution to the matching process ($\beta = \eta$). In our model, neither the firms nor the workers can possibly receive their full contribution because $\eta = 1$. However, since they do contribute equally in the contact process, they should receive the same share. Hence, by setting $\beta = 0.5$, we eliminate one of the disturbances. This implies that when $\rho = \delta$, $B^o(s) = 0.5R(s)$ and that when $\rho > (<)\ \delta$, $B^o(s) < (>) 0.5R(s)$.

The effects of increasing the replacement rate are depicted in Figure 2 for workers and Figure 3 for firms. The parabola with the solid line gives the second order Taylor approximation of the expected match surplus for worker type s. The worker receives a share β of this surplus (the area under the dotted curve). When the replacement rate $B(s)/R(s)$ increases to $B_1(s)/R(s)$ for this worker type, his surplus becomes equal to the area under the bold curve. The worker basically receives a larger part of a smaller pie. His expected wealth change is given by area $(A - B)$. Firms are never better off. Their expected surplus reduces by an amount equal to area A in Figure (3). In response to the higher replacement rate, they will restore profitability by opening fewer vacancies. At the aggregate level this leads to a higher equilibrium unemployment rate.

5 A minimum wage

Under a binding minimum wage, the value of search for a worker and an employer respectively, can be expressed in terms of the following Bellman equations:

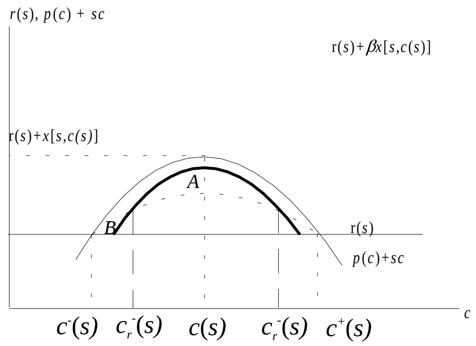

Figure 2: The effect of increasing the replacement rate on the position of workers

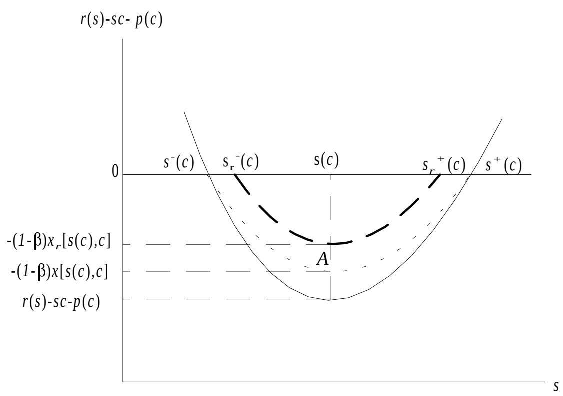

Figure 3: The effects of increasing the replacement rate on jobs

$$R(s) = B_1 + \frac{\lambda}{\rho + \delta} \int_{m_c(s)} g(c) \left[\max(W(s,c), W_{\min}) - R(s) \right] dc$$

$$K = \frac{\lambda}{\rho + \delta} \int_{m_s(c)} h(s) \left[P(c) F(s,c) - \max(W(s,c), W_{\min}) \right] ds$$

where $R(s)$ is the reservation wage of worker type s, $W(s,c)$ is the wage that would result without the introduction of a minimum wage for a worker of type s who is employed at a job type c, $W_{\min}$ is the minimum wage, and ρ is the discount rate. A contact results in a match if :

$$P(c)F\,(s,c) - \max[W(s,c), W_{\min}] > 0 \tag{19}$$

When the minimum wage is binding, the worker receives the minimum wage and the firm gets the rest of the surplus: $P(c)F(s,c) - W_{\min}$. The reason for this is that the fallback position of the worker is still unemployment. As a consequence, the alternative of not accepting a job is receiving unemployment benefits plus the option value of search.

5.1 The effects of a minimum wage on workers

We can distinguish between four situations. In Figure 4, the minimum wage is non binding and the expected match surplus for worker type s is the same as in a world where no minimum wage exists. Figure 5 shows the case where the minimum wage is partially binding. The matching set for worker type s is smaller. For some jobs in his matching set, he can bargain a higher wage than in the market equilibrium without a minimum wage while for other jobs close to $c(s)$, he gets a higher wage than the minimum wage. The expected wealth change for worker type s is therefore equal to $2[B - D]$, while the job suppliers loose $2[A + B]$. Figure 6 shows the case where the minimum wage is strictly larger than the wage that would result without the introduction of a minimum wage $(r(s) + \beta x(s,c))$ Again, the expected wealth change for worker type s equals $2[B - D]$, while the firms loose $2[A + B]$. Finally, Figure 7 shows the case where the minimum wage is strictly binding for worker type s. No job types c are profitable enough for this worker to form a match. The worker looses $2D$ while the firms loose $2A$.

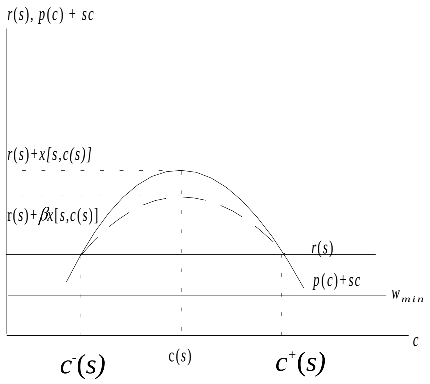

Figure 4: A non binding minimum wage for worker type s

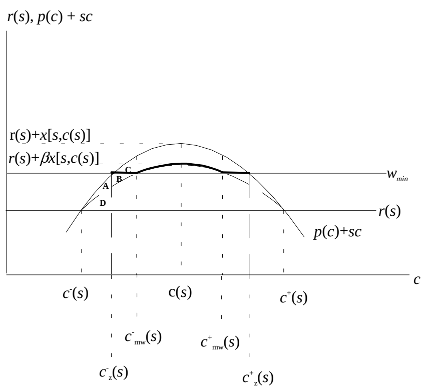

Figure 5: A partially binding minimum wage for worker type s

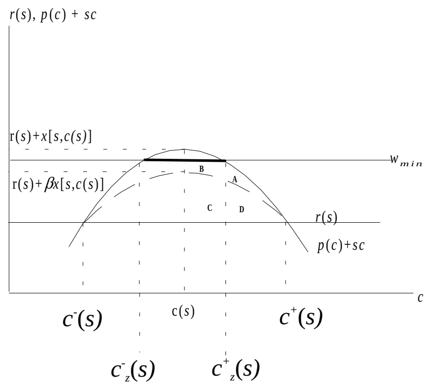

Figure 6: Partially binding minimum wage which is strictly larger than $r(s)+\beta x(s,c)$ for worker type s.

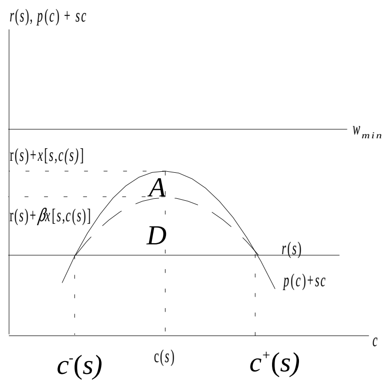

Figure 7: Strictly binding minimum wage for worker type s

5.2 The effect of a minimum wage on jobs

Figure 8 shows how the surplus is approximated by a parabola. For a given job type, the minimum wage "eats" part of the surplus. The lower the worker skill type, the more the surplus is reduced by the minimum wage. Figure 9 shows the losses of the firm in greater detail. In this Figure, we see the firm share (area A) of the surplus and 3 different worker types which are important, $s^{-}(s)$, $s^{+}(s)$, $s_z^{-}(c)$, $s_{mw}^{-}(c)$ to calculate area A, $s^{+}(c)$ and $s^{-}(c)$ are defined as the worker types for which the match surplus is zero: $x(s^{+,-}(c), c) = 0$, $s^{+}(c) > s^{-}(c)$, $s_z^{-}(c)$ is the worker type for which $x_{\min}\left[s_z^{-}(c), c\right] = p(c) + s_z(c)c - w_{\min} = 0$ and $s_{mw}^{-}(c)$ is the job type for which: $w_{\min} - s_{mw}(c)c = p(c) - \beta x\left[s_{mw}(c), c\right] - r(s_{mw})$ or $w_{\min} = (1-\beta)x\left[s_{mw}(c), c\right]$, s_v^{-} is the worker type for which: $w_{\min} = r(s_v)$.

The firm looses area $(B + C)$ while the worker gains $(B - D)$ when a minimum wage is introduced. The losses are all due to the fact that certain job-worker matches are no longer profitable when a minimum wage is introduced.

In principle it is possible to derive analytical expressions of area A, but again they become very messy and are not particularly insightful. We will therefore turn to numerical simulations of the model in the next section.

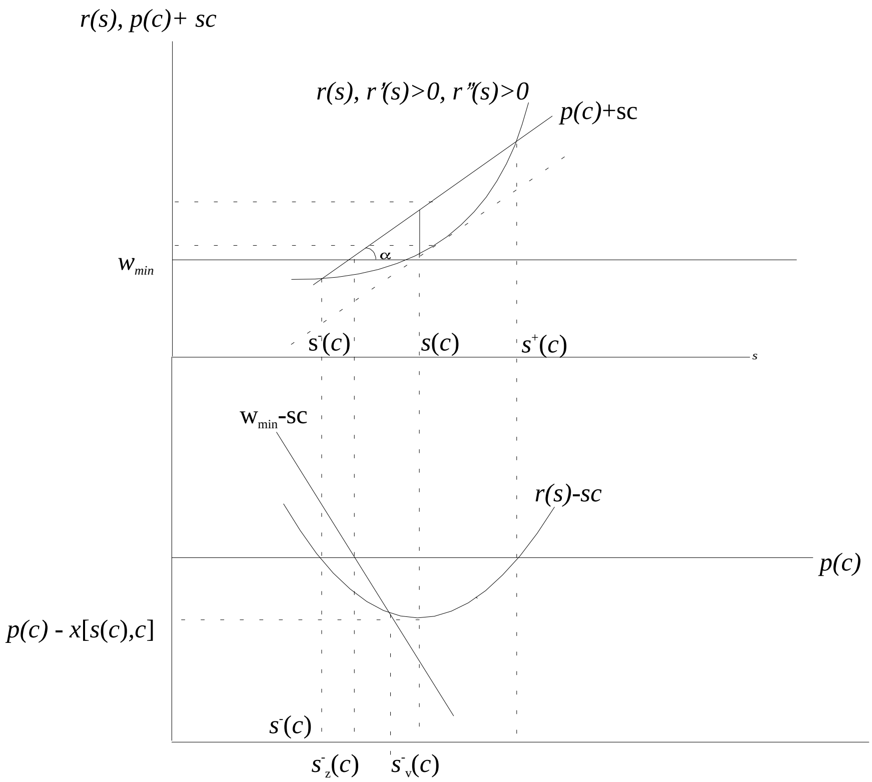

Figure 8: The effect of the minimum wage on the match surplus

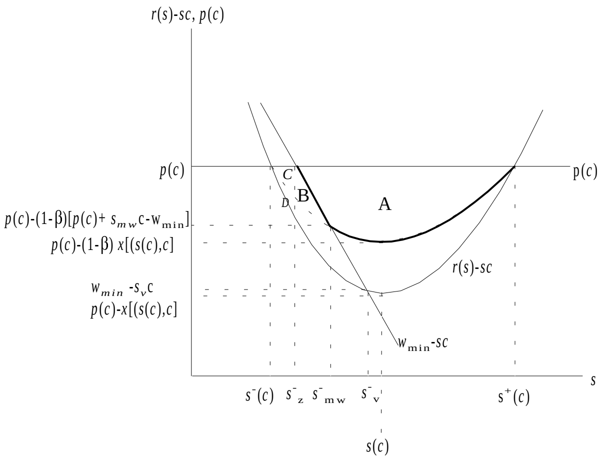

Figure 9: The expected firm's share of the surplus under a partially binding minimum wage

6 Simulations

6.1 Simulation results

In this section we present numerical simulations of the model. For matters of convenience, we rescaled s, and c, such that: $s^* \equiv -\ln(1-s)$ and $c^* \equiv \ln c$. The simulations are based on a 601*601 grid (both s^* and c^* range from minus three till plus three times their standard deviation and the domain of both variables is divided in 100 intervals per standard deviation). The distribution of log wages is reasonably described by a log normal distribution. By the transformation of s, $r^*_{Wal}(s^*)$ is linear in s^*. Therefore, s^* is also distributed normally. This distribution is conveniently specified as: $s^* \tilde{} N\left[0, (\gamma\sigma)^2\right]$.

The baseline parameter values of our simulations are: $\delta = 0.15$, $\beta = 0.5$, $\rho = 0.10$, $\theta = 4.0$, $\gamma = 5.0$, $\alpha = 0.0$, $K = 0.50$, $\sigma = 0.60, \eta = 0$. [3]Where $\gamma(s) = \gamma = \frac{r''(s)}{r'(s)^2}\frac{c'(s)}{c(s)^2}$[4]

[3]See Teulings and Gautier (2000) for a full discussion of the numerical simulations.

[4]Those values are in line with what is usually found for the US. For an extensive discussion of the complexity dispersion parameter in a model with search frictions, see Teulings and Gautier (2000). The constant complexity dispersion parameter, γ, was estimated to

First, we have to make some remarks about our assumption that $\eta = 0$ (no substitution on the goods market). In Teulings and Gautier (2000) we also considered the case of $\eta = \infty$. We showed that in that case, when search frictions are large, firms are reluctant to open vacancies close to the corners. Instead they offer a cluster of vacancies somewhat above the corner. The reason for this is that labor is relatively scarce there. As a result of this cluster, neighboring vacancies will be crowded out and one or more additional clusters can appear. Now imagine what happens when a minimum wage is introduced in such a model. The minimum wage defines the new corners and consequently influences the position of the first cluster of vacancies which on its turn determines the locations of the other clusters. This can lead to all sorts of exotic equilibria and complicated nonlinear dynamics. Since these processes are driven by the extreme assumption of perfect substitution on the goods market and are not particularly insightful for the questions at hand, we devote our attention to the $\eta = 0$ case, where prices are adjusted such that there is demand for all job types.

6.2 Unemployment benefit simulations

Table 1 shows how unemployment, the reservation wage and the expected match surplus varies over worker types (for B equal to respectively 0.0, 0.1, and 0.2). We have set β at 0.5, so that we can abstract from inefficient bargaining effects. The loss (compared to Walras) for the economy as a whole is lowest for $B = 0.1$. When $B = 0$, the negative mismatch effects dominate the positive employment effects while when $B = 0.2$, the negative unemployment effects dominate. We see that the highest worker types loose while the lowest worker types gain from an increase in B. There is a clear trade-off between mismatch and unemployment since a large increase in B from 0.0 to 0.2 which is accompanied by a large increase in unemployment (from 5.9% to 8.3%) leads to a small welfare loss from 0.162 to 0.163. Apparently, the improved average match quality almost fully compensates for the foregone production due to the higher unemployment rate.

TABLE 1 ABOUT HERE

be close to 5, see Teulings (1999b) for an estimate for the US, Teulings (1995) for the Netherlands, and Teulings and Vieira (1999) for Portugal.

6.3 Minimum wage simulations

Tables 2 and 3 give simulation results for $w_{\min} = \infty$, $w_{\min} = -1.76$ and $w_{\min} = -1.36$ for β is respectively 0.5 and 0.3. In both cases, we see that the minimum wage never improves aggregate welfare, and sometimes decreases aggregate welfare, although the losses are in general small. When workers have a weak bargaining position ($\beta = 0.3$), the minimum wage does a good job in reducing income inequality while when $\beta = 0.5$ the equalizing effect is much smaller. The mass of workers earning the minimum wage is substantial (2.4% for $\beta = 0.5$) when $w_{\min} = -1.36$. It may seem puzzling why almost all OECD countries have a binding minimum wage when the welfare effects are unfavorable. Tables 2 and 3 suggest a possible answer, namely that a majority of the worker skill types (in the middle of the distribution) favors a minimum wage.[5] In section 6.5 we will exactly derive how much each worker type gains or looses and when a minimum wage is supported by a voting equilibrium.

TABLE 2 ABOUT HERE

TABLE 3 ABOUT HERE

In Table 4, the interaction effects between the minimum wage, unemployment benefits and β are shown. The main lesson that can be drawn from this Table is that treating those effects separately can be misleading. When $B = 0.1$ and $\beta = 0.5$, the losses relative to Walras are minimized. In section 6.5 we show that even when B and β are set at those welfare maximizing values, a majority of workers supports a positive minimum wage, when skills are distributed normally.

TABLE 4 ABOUT HERE

Finally, Table 5 shows how unemployment, income inequality and the welfare loss compared to Walras vary with β. First, we see that the loss is minimized when $\beta = 0.5$, with and without a minimum wage. This was already analytically derived by Teulings and Gautier (2000). Next, we see that for all values of β, a minimum wage compresses the reservation wage distribution. The compression effect is largest when β is small.

TABLE 5 ABOUT HERE

[5] The welfare for skill type s is measured by $r(s)$, which captures both the expected income and the expected probability to find a job. Strictly speaking, we should have taken a weighted (by employment shares) average of wages and reservation wages because employed workers value the job find probability less than the unemployed workers. This would however add another dimension to the problem, since w depends on both s and c.

Figure 10 shows the shift of the reservation wage distribution due to a change of β from 0.2 to 0.5. In this picture, we also plotted the reservation wage distribution under a minimum wage of -1.5. Both reduce the standard deviation of $r(s)$ from 0.76 to 0.72. In section 6.5, we show that the compression due to the minimum wage of the actual wage distribution is much larger because the position of the unemployed workers is not taken into account.

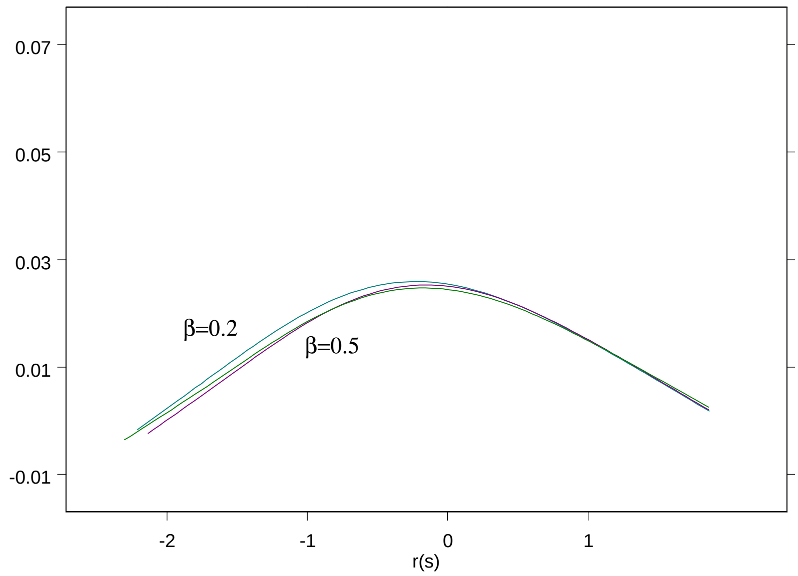

Figure 10: Reservation wage distribution for $\beta = 0.3$ and $\beta = 0.2$, with and without a minimum wage of -1.5

6.4 The aggregate outcome

Figure 11 shows the aggregate effect of the introduction of a minimum wage on the acceptable matching sets. The black area represents job and worker type combinations that disappear when a minimum wage is introduced. The dark grey area reflects the mass of workers that earn exactly the minimum wage. To understand why for given job types only matches with high worker types survive, note the following. First note that if $(s_{1,}c_1)$ is a feasible match under minimum wage $w_{\min}$ and if $(s_2(> s_1), c_1)$ is a feasible match when there is no minimum wage than $(s_{2,}c_1)$ is also a feasible match under minimum wage

$w_{\min}$. This simply follows from: $p(c_1) + s_2c_1 - w_{\min} > p(c_1) + s_1c_1 - w_{\min}$. However, if $(s_{1,}c_1)$ is a feasible match under minimum wage $w_{\min}$ and if $(s_{1,}c_2$ $(> c_1))$ is a feasible match without a minimum wage, this does *not* imply that $(s_{1,}c_2)$ is a feasible match under a minimum wage $w_{\min}$. The reason for this is that $p(c_2) + s_1c_2 - w_{\min} \lesseqqgtr (c_1) + s_1c_1 - w_{\min}$. Since $p(c_2) < p(c_1)$ but $s_1c_2 > s_1c_1$.

The dotted lines in the graph reflect $c(s)$ and $s(c)$. In our approximation we assumed them to be equal, which turns out to be a reasonable assumption.

6.5 Who are the winners and who are the losers?

From a political economy perspective it is interesting to know who wins and who looses when a minimum wage is introduced and in particular whether there exists a majority of voters for a particular minimum wage. In general, the lowest skill types loose most expected wealth while the skill types with a productivity just above the minimum wage, gain most. First, we set $\beta = 0.5$ and $B = 0.1$, so that given the existence of search frictions, welfare is maximized.

Next, consider Figure 12(a) which plots $[r(s)|w_{\min} = -1.5) - r(s)]$ for different worker skill types. This picture shows that the lowest skill types face huge losses when a minimum wage is introduced. Obviously, this is because the set of jobs which is available for them becomes much smaller, see Figure 12(d). The lower middle class workers with somewhat more skills, gain most. The minimum wage enables them to appropriate almost the entire match surplus. Moreover, since many of their competitors have become unemployed, they become relatively scarce. Finally, the high skilled workers face relatively small losses. The reason that their position worsens is caused by the fact that the prices of the products they produce fall relatively to the prices of the goods produced at the bottom segment of the labor market.

This picture also suggests that the wage distribution will be strongly compressed. Of course focussing on the wage distribution is misleading because it does not take the welfare loss of the unemployed workers into account. Therefore, from a welfare point of view, the distribution of $r(s)$ is more relevant than the distribution of $w(s)$.

For a voting maximizing political party, it is only relevant to know who gains and who looses from a minimum wage. Figure 12(b) portrays this. We assume that all gainers vote pro (1) and that all losers vote contra (-1) and that the worker types which are indifferent do not vote. For this particular

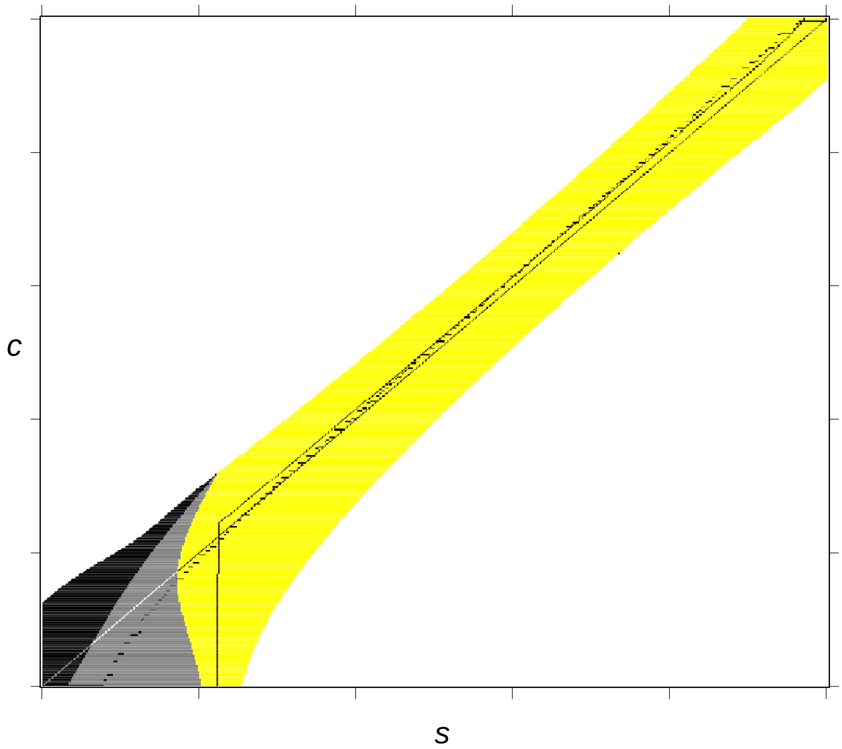

Figure 11: The effects of a minimum wage ($w_{\min} = -1.36$) on matching sets for $\lambda = 156$

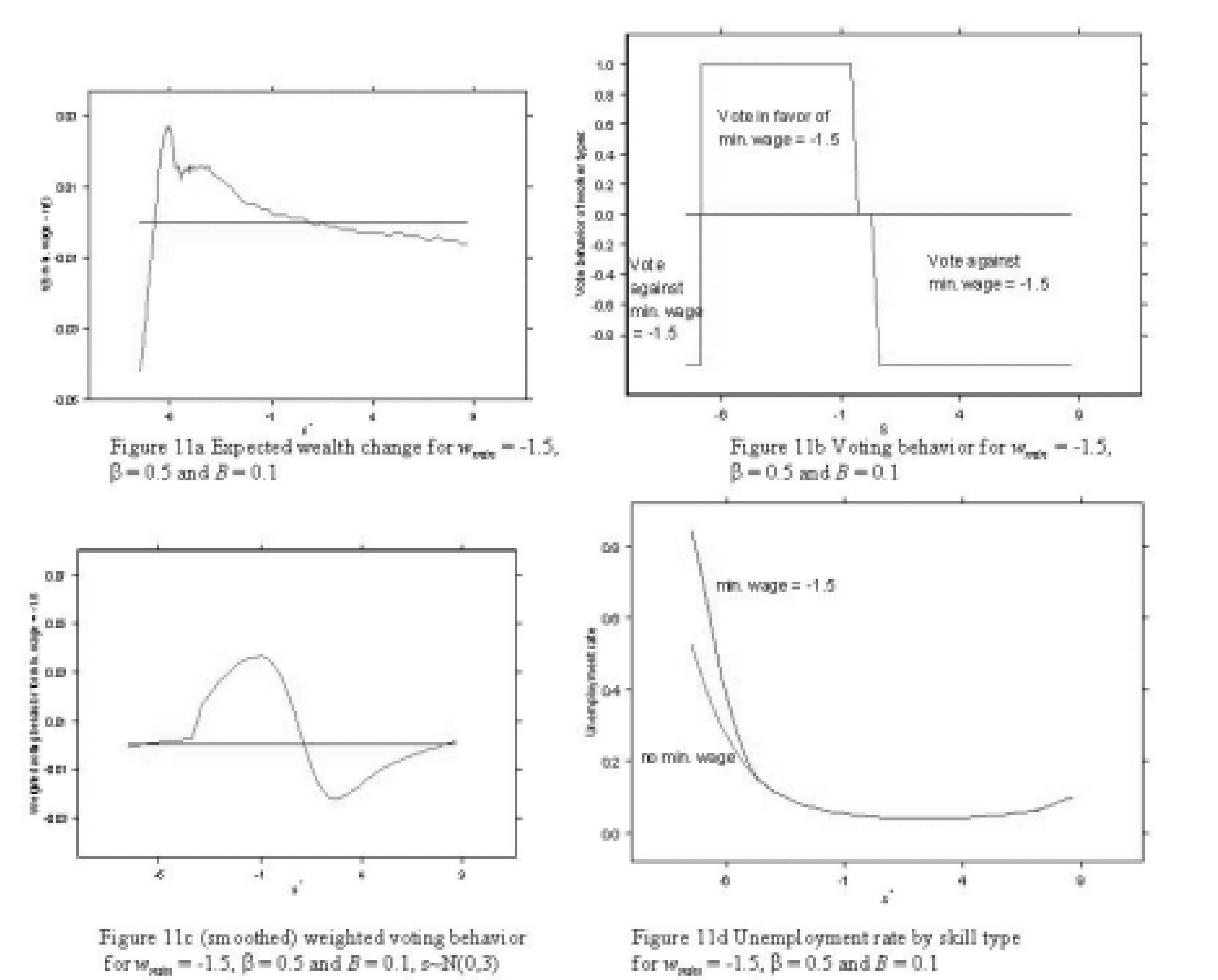

Figure 12: Who gains and who looses from a minimum wage?

example, 46.3% of the voting skill types would be in favor and 54.7% would be against a minimum wage. This would also be the election outcome if skills where uniformly distributed over the population. In the model we assumed however that skills were distributed normally ($\sim$N(0,3)). Figure 12(c) shows the skill weighted voting behavior. The mass above the zero-line represents pro voters while the mass below the zero line reflects contra voters. Since there are relatively few low skilled workers and relatively many middle class workers, there are relatively more gainers from a minimum wage. We calculated the fraction of favorable votes to be 61.4% in this case. To sum up. We showed that even though a minimum wage of -1.5 has negative effects for the economy as a whole, it can still be supported by a majority of voters.

7 Final Remarks

The main goal of this paper was to construct a model that can explain the stylized facts on minimum wages by taking the ex ante heterogeneity of workers and jobs into account. Our model is consistent with relatively small unemployment effects, the compression of the wage distribution and the mass of workers receiving a minimum wage. We found that the workers with the lowest skills experience the largest welfare loss while workers with an expected productivity close to the minimum wage gain the most. This welfare loss of the low skilled workers is not picked up when one focuses solely on the compression of the wage distribution which is only defined for employed workers. In addition, we have shown that unemployment benefits can be a more efficient way to redistribute income and that unemployment benefits and minimum wages are most useful when workers have a weak bargaining position. Empirical research on the bargaining power of workers therefore deserves a high place on the research agenda since it is crucial information for policy makers.

There are three important assumptions in this model which influenced the results and which are worth discussing. First, throughout the paper we have assumed that β is the same for all worker types but it is conceivable that low skilled workers have a substantial weaker bargaining position than high skilled workers for example because they have fewer possibilities to ex post appropriate all sorts of ex ante investments. This remains however also an important empirical question. If it is the case, the welfare effects of unemployment benefits and to a lesser extent of a minimum wage are much more favorable. Second, we assumed risk neutral agents. Under risk aversion, unemployment benefits play an even more important role. Third, on the job search reduces the usefulness of minimum wages and unemployment benefits in the sense that mismatch will last shorter. Of course, on the job search is not costless. Whereas unemployed workers can be available the next day, it often takes two or more months before one can move to a new job. Our assumption of no on the job search can be viewed as the extreme case where it is infinitely costly to switch jobs. In general, the easier it is to search on the job, the less need there is for search subsidies. Finally, we abstracted from all sorts of transaction cost issues. One advantage of a minimum wage as redistribution tool is that its enforcement is relatively cheap and straightforward relatively to a system of unemployment benefits.

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A Tables

Table 1: Simulation results for B for different worker skill groups when $\beta = 0.5$, $\eta = 0$ and $\lambda = 156$

	s^*	-7.5	-6.3	-5.1	-3.0	0.0	3.0	6.0
$\mathbf{B = 0.00}$	$u(\%)$	34.7	22.4	14.1	7.1	4.3	3.9	5.2
stdev $r(s) = 0.79$	$r(s)$	-3.00	-2.33	-1.81	-1.12	-0.32	0.43	1.21
loss $= 0.162$	$x(s)$	1.12	0.75	0.52	0.32	0.20	0.19	0.26
$u = 5.9\%$								
$\mathbf{B = 0.10}$	$u(\%)$	52.4	31.3	18.2	8.4	4.6	4.0	5.3
stdev $r(s) = 0.72$	$r(s)$	-2.13	-1.90	-1.60	-1.06	-0.31	0.43	1.20
loss $= 0.161$	$x(s)$	0.67	0.53	0.41	0.28	0.19	0.18	0.26
$u = 6.9\%$								
$\mathbf{B = 0.20}$	$u(\%)$	73.6	46.1	25.4	10.1	5.0	4.2	5.4
stdev $r(s) = 0.65$	$r(s)$	-1.58	-1.51	-1.37	-0.97	-0.29	0.43	1.19
loss $= 0.163$	$x(s)$	0.35	0.32	0.29	0.23	0.18	0.18	0.25
$u = 8.3\%$								

Table 2: Minimum wage simulation results for different worker skill groups when $B = 0.1$, $\beta = 0.5$, $\eta = 0$ and $\lambda = 156$

	s^*	-7.5	-6.3	-5.1	-3.0	0.0	3.0	6.0
$\mathbf{w}_{\min} = -\infty$	$u(\%)$	50.5	30.3	17.6	8.2	4.5	4.0	5.3
stdev $r(s) = 0.72$	$r(s)$	-2.20	-1.94	-1.63	-1.07	-0.31	0.43	1.20
loss $= 0.161$	$x(s)$	0.716	0.56	0.42	0.28	0.19	0.18	0.26
$u = 6.8\%$								
spike $= 0.0\%$								
$\mathbf{w}_{\min} = \mathbf{-1.76}$	$u(\%)$	63.8	33.8	17.6	8.2	4.5	4.0	5.3
stdev $r(s) = 0.72$	$r(s)$	-2.20	-1.94	-1.62	-1.06	-0.31	0.43	1.20
loss $= 0.162$	$x(s)$	0.72	0.56	0.42	0.28	0.19	0.18	0.26
$u = 7.0\%$								
spike $= 0.4\%$								
$\mathbf{w}_{\min} = \mathbf{-1.36}$	$u(\%)$	90.9	57.7	23.2	8.1	4.6	4.0	5.3
stdev $r(s) = 0.70$	$r(s)$	-2.30	-1.96	-1.60	-1.05	-0.31	0.43	1.20
loss $= 0.163$	$x(s)$	0.96	0.50	0.41	0.28	0.19	0.18	0.26
$u = 7.8\%$								
spike $= 2.4\%$								

Table 3: Minimum wage simulation results for different worker skill groups when $\beta = 0.3$, $B = 0.1$, $\eta = 0$ and $\lambda = 156$

	s^*	-7.5	-6.3	-5.1	-3.0	0.0	3.0	6.0
$\mathbf{w}_{\min} = -\infty$	$u(\%)$	42.5	23.2	12.8	5.6	3.0	2.6	3.5
stdev $r(s) = 0.75$	$r(s)$	-2.24	-2.01	-1.70	-1.11	-0.33	0.43	1.20
loss $= 0.168$	$x(s)$	0.70	0.57	0.45	0.30	0.21	0.20	0.27
$u = 4.7\%$								
spike $= 0.0\%$								
$\mathbf{w}_{\min} = \mathbf{-1.76}$	$u(\%)$	59.1	27.5	12.7	5.6	3.0	2.6	3.5
stdev $r(s) = 0.74$	$r(s)$	-2.17	-1.97	-1.69	-1.11	-0.33	0.43	1.20
loss $= 0.169$	$x(s)$	0.61	0.54	0.45	0.30	0.21	0.20	0.27
$u = 4.9\%$								
spike $= 0.3\%$								
$\mathbf{w}_{\min} = \mathbf{-1.36}$	$u(\%)$	1.0	56.6	17.7	5.5	3.0	2.6	3.5
stdev $r(s) = 0.71$	$r(s)$	-2.40	-1.95	-1.56	-1.10	-0.33	0.42	1.20
loss $= 0.170$	$x(s)$	1.03	0.49	0.36	0.30	0.21	0.20	0.27
$u = 6.0\%$								
spike $= 1.6\%$								

Table 4: Simulation results for different values of B*100 ($\eta = 0$ and $\lambda = 156$)

$w_{\min}$	β	B	0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35
-∞	**0.3**	$u(\%)$	4.0	4.4	4.8	5.3	6.0	6.8	7.9	8.9
		sd $r(s)$	0.82	0.78	0.74	0.71	0.67	0.64	0.61	0.58
		loss	0.172	0.170	0.169	0.167	0.166	**0.166**	0.167	0.169
	0.5	$u(\%)$	5.9	6.4	6.9	7.5	8.3	9.1	10.1	11.2
		sd $r(s)$	0.79	0.75	0.72	0.68	0.65	0.62	0.59	0.56
		loss	0.163	0.162	**0.161**	0.162	0.163	0.163	0.166	0.169
-1.40	**0.3**	$u(\%)$	5.7	5.8	5.9	6.1	6.3	6.9	7.8	9.0
		sd $r(s)$	0.74	0.72	0.71	0.69	0.67	0.64	0.61	0.58
		loss	0.174	0.172	0.170	0.168	0.167	**0.166**	0.166	0.169
		spike(%)	2.1	1.8	1.4	0.9	0.3	0.0	0.0	0.0
	0.5	$u(\%)$	7.3	7.5	7.8	8.1	8.4	9.1	10.0	11.1
		sd $r(s)$	0.75	0.72	0.70	0.68	0.65	0.62	0.59	0.56
		loss	0.164	0.163	**0.162**	0.162	0.163	0.164	0.166	0.169
		spike(%)	3.3	2.7	2.1	1.3	0.4	0.0	0.0	0.0

Table 5: Simulation results for different values of β for $\eta = 0$, $B = 0.1$ and $\lambda = 156$

$w_{\min}$	β	0.2	0.3	0.4	0.5	0.6	0.7
$-\infty$	$u(\%)$	3.9	4.8	5.8	6.9	8.2	9.9
	stdev $r(s)$	0.76	0.74	0.73	0.72	0.71	0.71
	loss	0.184	0.168	0.162	0.161	0.166	0.177
-1.40	$u(\%)$	5.2	5.9	6.7	7.7	9.0	10.8
	stdev $r(s)$	0.72	0.71	0.70	0.69	0.69	0.69
	loss	0.186	0.170	0.164	0.162	0.167	0.188
	spike(%)	1.0	1.4	1.7	2.0	2.2	2.4