Supplements to "Equilibrium Directed Search with Multiple Applications"

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1 Shortlisting Derivations

1.1 The Matching Probability

We first derive $m(\theta)$ for a worker (call her A). Let q_1 be the probability that an application leads to a first-round offer. Let q_2 be the probability that an application would lead to a second round offer given it does not generate a first-round offer. Recall that we are assuming that workers make two applications. We then have

$$m(\theta) = 1 - (1 - q_1)^2 + (1 - q_1)^2(1 - (1 - q_2)^2) = 1 - (1 - q_1)^2(1 - q_2)^2. \quad (1)$$

The probability that A gets an offer in the first round is $1 - (1 - q_1)^2$. The probability that she gets an offer in the second round is the probability that she fails to get a first-round offer, $(1 - q_1)^2$, times the probability of getting a second-round offer conditional on not having received an offer in the first round, $1 - (1 - q_2)^2$.

The calculation of q_1 is as before. Suppose A applies to vacancy V. Let Y be the number of other applications to V. Y is Poisson $(2/\theta)$. Then

$$q_1 = \sum_{y=0}^{\infty} \frac{1}{y+1} P[Y = y] = \frac{\theta}{2}(1 - e^{-2/\theta}). \quad (2)$$

Now suppose A applies to V and doesn't get a first-round offer (neither from V nor from the other vacancy to which she applies). The probability that A gets a second-round offer from V is q_2.

To compute q_2 some notation is useful. Let $C_1 = 1$ if A gets a 1st-round offer from V; 0 otherwise. Thus, $P[C_1 = 1] = q_1$. Similarly, let $C_2 = 1$ if A gets a second-round offer from V; 0 otherwise. Assuming that A did not get a first-round offer from the other vacancy to which she applied (in which case, the following computations are not relevant), we have

$$q_2 = P[C_2 = 1|C_1 = 0].$$

Suppose $C_1 = 0$. Then V made a first-round offer to some other worker – call him B. In order for A to get a second-round offer from V, it must be that V failed to hire B in the first round. This can occur in two ways. First, B gets a second first-round offer, and the vacancy (call it V*) making this other offer has no second-round candidate. This occurs with probability $e^{-2/\theta}$.[1] Second, B gets a second first-round offer, the vacancy making the offer has a second-round candidate, and B chooses the other vacancy. This occurs with probability

$$q_1 \frac{1 - \frac{e^{-2/\theta}}{q_1}}{2} = (q_1 - e^{-2/\theta})/2.$$

The probability that V fails to hire in the first round is thus

$$e^{-2/\theta} + \frac{q_1 - e^{-2/\theta}}{2} = \frac{q_1 + e^{-2/\theta}}{2}.$$

Next, given that A is not first on V's short list, what is the probability that she is second? If y applicants other than A applied to V, and if one of those applicants was chosen to be first on V's short list, then there are $y-1$ remaining applicants with whom A is competing for second place. Given y, the probability that A is second is thus $1/y$. To find the probability that A is second on V's short list, given that she is not first, we need to sum this conditional probability against the probability mass function for Y. We know that unconditionally, Y is Poisson $(2/\theta)$. We also know that V did not make an offer to A in the first round, i.e., $C_1 = 0$. So,

$$\begin{aligned} P[Y &= y|C_1 = 0] = \frac{P[C_1 = 0|Y = y]P[Y = y]}{P[C_1 = 0]} \\ &= \frac{\frac{y}{y+1} e^{-2/\theta} (\frac{2}{\theta})^y / y!}{1 - q_1} = \frac{y e^{-2/\theta} (\frac{2}{\theta})^y / (y+1)!}{1 - q_1} \text{ for } y = 0, 1, \ldots \end{aligned}$$

[1] B gets the other first-round offer with probability q_1. Let $C_1 = 1$ if B gets a first-round offer from V*, and let Y be the number of workers in addition to B who applied to V*. Then V* has no second candidate on its short list if $Y = 0$. Using Bayes Law,

$$P[Y = 0|C_1 = 1] = \frac{P[C_1 = 1|Y = 0]P[Y = 0]}{P[C_1 = 1]} = \frac{e^{-2/\theta}}{q_1}.$$

We thus have

$$P[C_1 = 1 \text{ and } Y = 0] = q_1 \frac{e^{-2/\theta}}{q_1} = e^{-2/\theta}.$$

The probability that A is second on V's short list given that she was not first is then

$$\sum_{y=1}^{\infty} \frac{1}{y} P[Y = y | C_1 = 0] = \sum_{y=1}^{\infty} \frac{e^{-2/\theta}(\frac{2}{\theta})^y/(y+1)!}{1-q_1} = \frac{q_1 - e^{-2/\theta}}{1-q_1}$$

and

$$q_2 = P[C_2 = 1 | C_1 = 0] = \left(\frac{q_1 + e^{-2/\theta}}{2}\right)\left(\frac{q_1 - e^{-2/\theta}}{1-q_1}\right). \tag{3}$$

Substitution then gives $m(\theta)$.

1.2 Expected Profit for a Vacancy

The efficient level of labor market tightness is determined as before by $c_v = m_\theta(\theta^*)$. That is, from the social planner's perspective, the only effect of allowing for shortlisting is to change the form of $m(\theta)$. We want to compare the efficient level of labor market tightness, θ^*, with the corresponding free-entry equilibrium value, θ^{**}. Assuming for now the existence of a symmetric equilibrium posted wage, the free-entry value of labor market tightness is determined by $c_v = \pi(w(\theta^{**}))$, where $w(\theta)$ is the symmetric equilibrium posted wage given labor market tightness θ and $\pi(w(\theta))$ is expected profit net of the cost of vacancy creation for a vacancy that posts the equilibrium wage in a market with labor market tightness θ. In this subsection, we derive the general form of $\pi(\cdot)$.

Suppose all vacancies post w. The number of applications that any one vacancy receives is Poisson with parameter $2/\theta$. Vacancy V gets no applications (and thus no profit) with probability $e^{-2/\theta}$; it receives one application (and thus has only one applicant on its short list) with probability $\frac{2}{\theta}e^{-2/\theta}$; it receives two or more applications (and thus has two applicants on its short list) with probability $1 - e^{-2/\theta} - \frac{2}{\theta}e^{-2/\theta}$.

Suppose V has only one applicant (again, call her A) on its short list. With probability $1 - q_1$, A does not receive a competing offer in the first round, in which case V's profit is $1 - w$.[2] With probability q_1, A has a competing first-round offer. The other vacancy (V^*) has only this applicant, i.e., no one in second place on its short list, with probability $\frac{e^{-2/\theta}}{q_1}$.[3] In this case, the two vacancies drive the wage to 1 (and profit to zero) through Bertrand competition. With probability $1-\frac{e^{-2/\theta}}{q_1}$, however, V^* has a second-round choice. In this case, Bertrand competition pushes the wage to s, the maximum wage V^* is willing to

[2] A accepts any $w \geq 0$. Were she instead to hold out in hopes of receiving a second-round offer from the other vacancy to which she applied, she could not do better than w. The reason is that there cannot be competition for A's services in the second round. Of course, if workers each make $a > 2$ applications, then there is a nontrivial first-round reservation wage problem for workers. It would be straightforward, but algebraically tedious, to add this feature.

[3] The derivation is given in footnote 1.

pay rather than dropping out to proceed to the second round, and V realizes a profit of $1 - s$.

This highest wage, s, that a vacancy with two applicants on its short list is willing to pay in the first round is determined by

$$1 - s = (1 - q_1)(1 - q_2)(1 - w). \tag{4}$$

The right-hand side can be understood as follows. With probability $1 - q_1$, a vacancy's second-place candidate is still available after the first round. Conditional on still being available, she fails to get a competing second-round offer with probability $1 - q_2$. The vacancy then realizes a profit of $1 - w$.

Summarizing, a vacancy has only one applicant on its short list with probability $\frac{2}{\theta}e^{-2/\theta}$. In this case, the vacancy's expected profit is

$$(1 - q_1)(1 - w) + (q_1 - e^{-2/\theta})(1 - s).$$

Now suppose V receives two or more applications. V's first-round choice (again, call her A) fails to get a competing first-round offer with probability $1 - q_1$, in which case V's profit is $1 - w$. With probability q_1, A does receive a competing first-round offer. The other vacancy competing for A (call it V*) has no second-round candidate with probability $\frac{e^{-2/\theta}}{q_1}$. In this case, V is outbid and proceeds to the second round. V's second-round choice (call him B) is still available with probability $1 - q_1$. Given that he is still available, B receives no competing second round offer with probability $1 - q_2$, and V's profit is $1 - w$. If B does receive a competing second-round offer, then Bertrand competition drives profit to zero. Alternatively, with probability $1 - \frac{e^{-2/\theta}}{q_1}$, V* does have a second applicant on its short list. Both V and V* are willing to bid the wage up to s. V then gets A with probability $\frac{1}{2}$ and realizes profit $1 - s$. With probability $\frac{1}{2}$, V fails to get A and proceeds to its second-round choice (again, call him B). As before, B is still available in the second round with probability $1 - q_1$; given that he is still available, B receives no competing second round offer with probability $1 - q_2$; and V gets profit $1 - w$.

Summarizing, a vacancy has two applicants on its short list with probability $1 - e^{-2/\theta} - \frac{2}{\theta}e^{-2/\theta}$. In this case, the vacancy's expected profit is

$$(1-q_1)(1-w)+q_1\left[\frac{e^{-2/\theta}}{q_1}(1 - q_1)(1 - q_2)(1 - w) + (1 - \frac{e^{-2/\theta}}{q_1})\left(\frac{1-s}{2} + \frac{(1-q_1)(1-q_2)(1-w)}{2}\right)\right].$$

Using equation (4) gives an expected profit of

$$(1 - q_1)(1 - w) + q_1(1 - s).$$

We can now compute the expected profit for a vacancy that posts the same wage w as all other vacancies:

$$\begin{aligned}\pi(w) &= \tfrac{2}{\theta}e^{-2/\theta}\Big((1 - q_1)(1 - w) + (q_1 - e^{-2/\theta})(1 - s)\Big) \\ &\quad + \Big(1 - e^{-2/\theta} - \tfrac{2}{\theta}e^{-2/\theta}\Big)\left((1 - q_1)(1 - w) + q_1(1 - s)\right) \\ &= \Big(1 - e^{-2/\theta}\Big)\left[(1 - q_1)(1 - w) + q_1(1 - s)\right] - \tfrac{2}{\theta}e^{-4/\theta}(1 - s). \qquad (5)\end{aligned}$$

1.3 The Equilibrium Wage

1.3.1 Deviations

Suppose all vacancies, save possibly one, post w. Suppose a deviant (D) posts w'. A deviation to w' changes the worker application intensity to D from $2/\theta$ to ξ. The indifference condition giving $\xi = \xi(w'; w)$ is given below.

Consider the deviant posting w' and receiving applications at rate ξ. D receives exactly one application with probability $\xi e^{-\xi}$. With probability $1 - q_1$, D's applicant (again, call her A) does not have a competing first-round offer. In this case, D's profit is $1 - w'$. With probability q_1, A has a competing first-round offer. With probability $\frac{e^{-2/\theta}}{q_1}$, the competing vacancy (V*) has no second-round candidate, and Bertrand competition drives profit to zero. With probability $1 - \frac{e^{-2/\theta}}{q_1}$, V* has a second-round candidate, and D realizes profit $1 - s$. Summarizing, D receives expected profit

$$(1 - q_1)(1 - w') + (q_1 - e^{-2/\theta})(1 - s)$$

with probability $\xi e^{-\xi}$.

D receives 2 or more applications with probability $1 - e^{-\xi} - \xi e^{-\xi}$. D's first-round choice fails to get a competing first-round offer with probability $1 - q_1$, in which case D's profit is again $1 - w'$. With probability q_1, A has another first-round offer. V* has no second-round candidate with probability $\frac{e^{-2/\theta}}{q_1}$, and D is thus outbid and proceeds to the second round. D's second-round choice (B) is still available with probability $1 - q_1$. Given that B is still available, he does not receive a competing second-round offer with probability $1 - q_2$, and D gets profit $1 - w'$. If B does receive a second-round offer, Bertrand competition drives profit to zero.

Now suppose V* has a second-round choice. This occurs with probability $1 - \frac{e^{-2/\theta}}{q_1}$. In this case, D wins the race for A ($s' > s$) if $w' > w$. D's profit is then $1 - s$. If $w' < w$, D loses the race and turns to its second-round candidate (B). B is still available with probability $1 - q_1$; given that he is still available, he receives no competing second-round offer with probability $1 - q_2$; and D gets profit $1 - w'$.

Note that with 2 or more applicants, D's expected profit (as a function of w') depends on whether w' is greater or less than w. Specifically, if $w' > w$, D receives expected profit

$$\begin{aligned} & (1 - q_1)(1 - w') + q_1 \left[\frac{e^{-2/\theta}}{q_1}(1 - q_1)(1 - q_2)(1 - w') + (1 - \frac{e^{-2/\theta}}{q_1})(1 - s) \right] \\ = \; & (1 - q_1)(1 - w') + e^{-2/\theta}(1 - s') + (q_1 - e^{-2/\theta})(1 - s), \end{aligned}$$

while if $w' < w$, D receives expected profit

$$\begin{aligned} & (1 - q_1)(1 - w') + q_1 \left[\frac{e^{-2/\theta}}{q_1}(1 - q_1)(1 - q_2)(1 - w') + (1 - \frac{e^{-2/\theta}}{q_1})(1 - q_1)(1 - q_2)(1 - w') \right] \\ = \; & (1 - q_1)(1 - w') + q_1(1 - s'). \end{aligned}$$

Summarizing, if $w' > w$,

$$
\begin{aligned}
\pi(w';w) &= \xi e^{-\xi}\left[(1-q_1)(1-w') + (q_1 - e^{-2/\theta})(1-s)\right] \\
&\quad + \left(1 - e^{-\xi} - \xi e^{-\xi}\right)\left[(1-q_1)(1-w') + e^{-2/\theta}(1-s') + (q_1 - e^{-2/\theta})(1-s)\right] \\
&= \left(1-e^{-\xi}\right)\left[(1-q_1)(1-w') + (q_1 - e^{-2/\theta})(1-s)\right] + \left(1 - e^{-\xi} - \xi e^{-\xi}\right) e^{-2/\theta}(1-s').
\end{aligned}
$$

If $w' < w$,

$$
\begin{aligned}
\pi(w';w) &= \xi e^{-\xi}\left[(1-q_1)(1-w') + (q_1 - e^{-2/\theta})(1-s)\right] \\
&\quad + \left(1 - e^{-\xi} - \xi e^{-\xi}\right)\left[(1-q_1)(1-w') + q_1(1-s').\right] \\
&= \left(1-e^{-\xi}\right)(1-q_1)(1-w') + \xi e^{-\xi}(q_1 - e^{-2/\theta})(1-s) + \left(1 - e^{-\xi} - \xi e^{-\xi}\right) q_1(1-s').
\end{aligned}
$$

To derive $\xi = \xi(w';w)$, we now turn to the indifference condition.

1.3.2 Indifference Condition

An applicant (A) should be indifferent between sending both applications to nondeviant (N) vacancies versus sending one application to N and the other to D when the arrival intensity is $2/\theta$ at any N vacancy and ξ at D. Consider an application to an N vacancy. A is first on N's short list with probability q_1. She is second on N's short list with probability $q_1 - e^{-2/\theta}$. (A is not first on N's short list with probability $1 - q_1$. Conditional on not being first, she is second with probability $\frac{q_1 - e^{-2/\theta}}{1-q_1}$.) Finally, she is out of the running at N with probability $1 - 2q_1 + e^{-2/\theta}$. Similarly, if A applies to D, she is first on D's short list with probability $q_1^D = \frac{1}{\xi}(1 - e^{-\xi})$, she is second on D's short list with probability $q_1^D - e^{-\xi}$, and she is out of the running at D with probability $1 - 2q_1^D + e^{-\xi}$.

Suppose A sends one application to D and one to N. There are 9 possibilities to consider.

1. A is first on both short lists. This occurs with probability $q_1^D q_1$. If neither D nor N has a second candidate, A's payoff is 1. Given that A is first on both short lists, this occurs with probability $\frac{e^{-\xi}e^{-2/\theta}}{q_1^D q_1}$. If D has a second candidate but N does not, A's payoff is s'. This occurs with probability $\frac{(q_1^D - e^{-\xi})e^{-2/\theta}}{q_1^D q_1}$. If N has a second candidate, but D does not, A's payoff is s. This occurs with probability $\frac{e^{-\xi}(q_1 - e^{-2/\theta})}{q_1^D q_1}$. If both vacancies have second candidates, A's payoff is s if $w' > w$ and s' if $w > w'$. The probability that both D and N have second candidates is $\frac{(q_1^D - e^{-\xi})(q_1 - e^{-2/\theta})}{q_1^D q_1}$.

2. A is first on D's short list and second on N's. This occurs with probability $q_1^D(q_1 - e^{-2/\theta})$, and A's payoff is w'.[4]

3. A is first on D's short list and out of the running at N. This occurs with probability $q_1^D(1 - 2q_1 + e^{-2/\theta})$, and A's payoff is again w'.

4. A is second on D's short list and first on N's. This occurs with probability $(q_1^D - e^{-\xi})q_1$, and A's payoff is w.

5. A is second on both short lists. This occurs with probability $(q_1^D - e^{-\xi})(q_1 - e^{-2/\theta})$.

   a. $w' > w$. The probability that A gets a second-round offer from D is $e^{-2/\theta}$. This follows because D hires its first-round candidate if that person has no other offer (probability $1-q_1$) or if that person has another offer and the competing vacancy has a second applicant (probability $q_1 - e^{-2/\theta}$). Thus D fails to hire its first-round candidate and makes a second-round offer to A with probability $1 - (1 - q_1 + q_1 - e^{-2/\theta}) = e^{-2/\theta}$. The probability that A gets a second-round offer from N is $\frac{q_1+e^{-2/\theta}}{2}$. N hires its first-round candidate if that person does not have another first-round offer (probability $1 - q_1$) or if that person has another offer, the other vacancy has a second- round candidate, and the applicant chooses N (probability $\dfrac{1}{2} \times (q_1 - e^{-2/\theta})$). There are now 4 possibilities. First, A receives a second-round offer neither from D nor from N. In this case, A's payoff is zero. Second, A receives a second-round offer from D but not from N. This occurs with probability $e^{-2/\theta}(1 - \frac{q_1+e^{-2/\theta}}{2})$, and A receives payoff w'. Third, A receives a second-round offer from N but not from D. This occurs with probability $\frac{q_1+e^{-2/\theta}}{2}(1-e^{-2/\theta})$, and A receives payoff w. Finally, A receives second-round offers from both D and N. This occurs with probability $\frac{e^{-2/\theta}(q_1+e^{-2/\theta})}{2}$, and A receives payoff 1. Thus, when $w' > w$, A's expected payoff in the event that she is second on both short lists is $e^{-2/\theta}(1 - \frac{q_1+e^{-2/\theta}}{2})w' + \frac{q_1+e^{-2/\theta}}{2}(1 - e^{-2/\theta})w + \frac{e^{-2/\theta}(q_1+e^{-2/\theta})}{2}$.

   b. $w' < w$. In this case, the probability that D makes a second-round offer is q_1 since the only way that D can succeed in the first round is if its candidate does not have another offer (probability $1-q_1$). The probability that A gets a second-round offer from N is again $\frac{q_1+e^{-2/\theta}}{2}$. With probability $q_1(1 - \frac{q_1+e^{-2/\theta}}{2})$, A gets a second-round offer from D but not from N. In this case, A's payoff is w'. With probability $(1 - q_1)\frac{q_1+e^{-2/\theta}}{2}$, D hires in the first round, but N does not. In this case, A's payoff is w. Finally, with probability $\frac{q_1(q_1+e^{-2/\theta})}{2}$, both D and N make second-round offers to A and A's payoff is 1. Summarizing, if $w' < w$, A's expected payoff is $q_1(1 - \frac{q_1+e^{-2/\theta}}{2})w' + (1 - q_1)\frac{q_1+e^{-2/\theta}}{2}w + \frac{q_1(q_1+e^{-2/\theta})}{2}$.

[4] We evaluate the derivative of $\xi(w'; w)$ at $w' = w$, so we need not consider the case in which w' is "considerably less than" w. Were that the case, A might prefer to reject w' in hopes of getting a second round offer from N.

6. A is second on D's short list and out of the running at N. This occurs with probability $(q_1^D - e^{-\xi})(1 - 2q_1 + e^{-2/\theta})$. If $w' > w$, D hires in the first round with probability $1 - e^{-2/\theta}$ and A's payoff is zero. With probability $e^{-2/\theta}$, A's payoff is w'. If $w' < w$, D fails to hire in the first round with probability q_1. In this case, A's payoff is w'.

7. A is out of the running at D and first on N's short list. This occurs with probability $(1 - 2q_1^D + e^{-\xi})q_1$. In this case, A's payoff is w.

8. A is out of the running at D and second on N's short list. This occurs with probability $(1 - 2q_1^D + e^{-\xi})(q_1 - e^{-2/\theta})$. N hires its first-round candidate with probability $1 - \frac{(q_1 + e^{-2/\theta})}{2}$ and A's payoff is zero. Alternatively, N fails to hire on the first round with probability $\frac{(q_1 + e^{-2/\theta})}{2}$, in which case A's payoff is w.

9. Finally, A is out of the running at both D and N. This occurs with probability $(1 - 2q_1^D + e^{-\xi})(1 - 2q_1 + e^{-2/\theta})$, and in this case, A's payoff is zero.

The discussion above is summarized in the following table, which presents the expected payoff for a worker who sends one application to D and one to N for each of the nine possible outcomes associated with that application strategy.

D	N	Probability	Expected Payoff ($w' > w$)
1	1	$q_1^D q_1$	$\frac{e^{-\xi} e^{-2/\theta}}{q_1^D q_1} + \frac{(q_1^D - e^{-\xi}) e^{-2/\theta}}{q_1^D q_1} s' + \frac{q_1^D (q_1 - e^{-2/\theta})}{q_1^D q_1} s$
1	2	$q_1^D(q_1 - e^{-2/\theta})$	w'
1	x	$q_1^D(1 - 2q_1 + e^{-2/\theta})$	w'
2	1	$(q_1^D - e^{-\xi})q_1$	w
2	2	$(q_1^D - e^{-\xi})(q_1 - e^{-2/\theta})$	$\frac{e^{-2/\theta}(2 - q_1 - e^{-2/\theta})w'}{2} + \frac{(q_1 + e^{-2/\theta})(1 - e^{-2/\theta})w}{2} + \frac{e^{-2/\theta}(q_1 + e^{-2/\theta})}{2}$
2	x	$(q_1^D - e^{-\xi})(1 - 2q_1 + e^{-2/\theta})$	$w' e^{-2/\theta}$
x	1	$(1 - 2q_1^D + e^{-\xi})q_1$	w
x	2	$(1 - 2q_1^D + e^{-\xi})(q_1 - e^{-2/\theta})$	$\frac{(q_1 + e^{-2/\theta})}{2} w$
x	x	$(1 - 2q_1^D + e^{-\xi})(1 - 2q_1 + e^{-2/\theta})$	0

D	N	Probability	Expected Payoff ($w' < w$)
1	1	$q_1^D q_1$	$\frac{e^{-\xi} e^{-2/\theta}}{q_1^D q_1} + \frac{(q_1^D - e^{-\xi}) q_1}{q_1^D q_1} s' + \frac{e^{-\xi} (q_1 - e^{-2/\theta})}{q_1^D q_1} s$
1	2	$q_1^D(q_1 - e^{-2/\theta})$	w'
1	x	$q_1^D(1 - 2q_1 + e^{-2/\theta})$	w'
2	1	$(q_1^D - e^{-\xi})q_1$	w
2	2	$(q_1^D - e^{-\xi})(q_1 - e^{-2/\theta})$	$q_1(1 - \frac{q_1 + e^{-2/\theta}}{2})w' + (1 - q_1)\frac{q_1 + e^{-2/\theta}}{2} w + \frac{q_1(q_1 + e^{-2/\theta})}{2}$
2	x	$(q_1^D - e^{-\xi})(1 - 2q_1 + e^{-2/\theta})$	$q_1 w'$
x	1	$(1 - 2q_1^D + e^{-\xi})q_1$	w
x	2	$(1 - 2q_1^D + e^{-\xi})(q_1 - e^{-2/\theta})$	$\frac{(q_1 + e^{-2/\theta})}{2} w$
x	x	$(1 - 2q_1^D + e^{-\xi})(1 - 2q_1 + e^{-2/\theta})$	0

We can now compute the value of sending one application to D and one to N, i.e., a (D,N) strategy, for any w', w pair. The table indicates that the form of this value differs according to whether $w' > w$ or vice versa.

Indifference between sending one application to D and one to N versus sending both applications to N vacancies defines $\xi(w', w)$. We want to find how the application intensity to D varies with small deviations from w, first for the case in which the deviant's wage is above the wage offered by the N vacancies and then for the case of $w' < w$. That is, we want to find $\frac{\partial \xi^+(w',w)}{\partial w'}|_{w'=w}$ and $\frac{\partial \xi^-(w',w)}{\partial w'}|_{w'=w}$, the right-hand and left-hand side derivatives of the application intensity, evaluated at $w' = w$.

We begin with $\frac{\partial \xi^+(w',w)}{\partial w'}|_{w'=w}$. The expected payoff from a (D,N) strategy when $w' > w$ is found using the figures in the top panel of the table and can be written as:

$$\begin{aligned}
&e^{-\xi}e^{-2/\theta} + (q_1^D - e^{-\xi})e^{-2/\theta}s' + q_1^D(q_1 - e^{-2/\theta})s + (q_1^D - e^{-\xi})(1-q_1)e^{-2/\theta}q_2 \\
&+w'\{q_1^D(1-q_1)(1+e^{-2/\theta}(1-q_2)) - e^{-\xi}e^{-2/\theta}(1-q_1)(1-q_2)\} \\
&+w\{q_1(1-q_1^D) + (1 - q_1^D - e^{-2/\theta}(q_1^D - e^{-\xi}))(1-q_1)q_2\}
\end{aligned}$$

The application intensity ξ is found by equating the individual's expected payoff from a (D,N) strategy to the expected payoff from applying to two N vacancies. We find $\frac{\partial \xi^+(w',w)}{\partial w'}$ by taking the derivatives of both sides with respect to w'. Since the expected payoff from applying to two N vacancies does not depend on w', this entails equating the derivative of the expected payoff from a (D,N) strategy with respect to w' to zero and solving for $\frac{\partial \xi^+(w',w)}{\partial w'}$.

This gives: $\frac{\partial \xi^+(w',w)}{\partial w'}|_{w'=w} =$

$$\frac{(1-q_1)[q_1+2e^{-2/\theta}(1-q_2)(q_1-e^{-2/\theta})]}{e^{-4/\theta}(1-q_1)[(1-2q_2)-w(2-3q_2)]-\frac{\partial q_1^D}{\partial \xi}[q_1(q_2+q_1(1-q_2))+(1-q_1)q_2e^{-2/\theta}+w((1-q_1)e^{-2/\theta}-q_1+(1-q_1^2)(1-q_2))]}$$

Next, we find $\frac{\partial \xi^-(w',w)}{\partial w'}|_{w'=w}$. The procedure is the same, but we must take into account the differences in the expected payoff a (D,N) strategy when $w > w'$. The expected payoff is now found using the figures in the bottom panel of the table and can be written as:

$$\begin{aligned}
&e^{-\xi}e^{-2/\theta} + e^{-\xi}(q_1 - e^{-2/\theta})s + q_1(q_1^D - e^{-\xi})s' + q_1(q_1^D - e^{-\xi})(1-q_1)q_2] \\
&+w'(1-q_1)[q_1^D + q_1(q_1^D - e^{-\xi})(1-q_2)] \\
&+w[q_1(1-q_1^D) + q_2(1-q_1)(1 - q_1^D - q_1(q_1^D - e^{-\xi}))].
\end{aligned}$$

Setting the derivative of this expression with respect to w' equal to zero allows us to find
$\frac{\partial \xi^-(w',w)}{\partial w'}|_{w'=w} =$

$$\frac{(1-q_1)q_1(1+2(q_1-e^{-2/\theta})(1-q_2))}{e^{-2/\theta}(1-q_1)(e^{-2/\theta}-q_2(q_1+e^{-2/\theta})-w((q_1+e^{-2/\theta})(1-q_2)-q_1q_2))-\frac{\partial q_1^D}{\partial \xi}(q_1^2+2q_1q_2(1-q_1)+w((3q_1-2)q_1q_2-2q_1^2+1-q_2))}$$

1.3.3 Equilibrium with Shortlisting

We seek a symmetric pure-strategy Nash equilibrium posted wage. That is, we seek a posted wage w with the property that if all other vacancies post w, an individual vacancy neither has an incentive to post a higher wage nor a lower wage. If all vacancies post w, then there will be three wages paid in equilibrium, namely, w, s, and 1.

Recall that for $w' > w$,

$$\pi^{+}(w';w) = \left(1 - e^{-\xi}\right)[(1-q_1)(1-w')+(q_1-e^{-2/\theta})(1-s)]+\left(1 - e^{-\xi} - \xi e^{-\xi}\right)e^{-2/\theta}(1-s').$$

The right-hand side derivative of profit is

$$\begin{aligned}\frac{\partial\pi^{+}(w',w)}{\partial w'} &= \left(e^{-\xi}\left((1-q_1)(1-w') + (q_1 - e^{-2/\theta})(1-s)\right) + \xi e^{-\xi}e^{-2/\theta}(1-s')\right)\frac{\partial\xi^{+}(w';w)}{\partial w'}\\ &\quad - \left(1-e^{-\xi}\right)(1-q_1) - \left(1 - e^{-\xi} - \xi e^{-\xi}\right)e^{-2/\theta}(1-q_1)(1-q_2).\end{aligned}$$

Evaluating at $w' = w$ gives

$$\begin{aligned}\frac{\partial\pi^{+}(w',w)}{\partial w'} &= \left(e^{-2/\theta}\left((1-q_1)(1-w) + (q_1 - e^{-2/\theta})(1-s)\right) + \frac{2}{\theta}e^{-4/\theta}(1-s)\right)\frac{\partial\xi^{+}(w;w)}{\partial w'}\\ &\quad - \left(1-e^{-2/\theta}\right)(1-q_1) - \left(1 - e^{-2/\theta} - \frac{2}{\theta}e^{-2/\theta}\right)e^{-2/\theta}(1-q_1)(1-q_2).\end{aligned}$$

We find the left-hand side derivative in a similar fashion. For $w' < w$,

$$\pi^{-}(w';w) = \left(1 - e^{-\xi}\right)(1-q_1)(1-w')+\xi e^{-\xi}(q_1-e^{-2/\theta})(1-s)+\left(1 - e^{-\xi} - \xi e^{-\xi}\right)q_1(1-s'),$$

so

$$\begin{aligned}\frac{\partial\pi^{-}(w',w)}{\partial w'} &= \begin{pmatrix} e^{-\xi}((1-q_1)(1-w') + (q_1 - e^{-2/\theta})(1-s)) \\ -\xi e^{-\xi}\left((q_1 - e^{-2/\theta})(1-s) - q_1(1-s')\right)\end{pmatrix}\frac{\partial\xi^{-}(w';w)}{\partial w'}\\ &\quad - \left(1-e^{-\xi}\right)((1-q_1) + q_1(1-q_1)(1-q_2)) + \xi e^{-\xi}q_1(1-q_1)(1-q_2).\end{aligned}$$

Evaluating at $w' = w$ gives

$$\begin{aligned}\frac{\partial\pi^{-}(w,w)}{\partial w'} &= \left(e^{-2/\theta}\left((1-q_1)(1-w) + (q_1 - e^{-2/\theta})(1-s)\right) + \frac{2}{\theta}e^{-4/\theta}(1-s)\right)\frac{\partial\xi^{-}(w;w)}{\partial w'}\\ &\quad -(1-e^{-2/\theta})(1-q_1) - q_1(1 - e^{-2/\theta} - \frac{2}{\theta}e^{-2/\theta})(1-q_1)(1-q_2).\end{aligned}$$

Given θ, a posted wage w is a symmetric Nash equilibrium if $\frac{\partial\pi^{+}(w',w)}{\partial w'}|_{w'=w} \leq 0$ and $\frac{\partial\pi^{-}(w',w)}{\partial w'}|_{w'=w} \geq 0$.

We investigate the nature of equilibrium numerically. For θ below approximately 0.42, both derivatives are negative for all $w \in [0,1]$. Thus, for these values of θ, the unique pure-strategy symmetric Nash equilibrium is $w = 0$. For θ above this cutoff level, there exists a range of w such that both inequalities are satisfied. The range of equilibrium posted wages goes from about 0.01 to about 0.04 when $\theta = 0.5$. When $\theta = 2$, there is again a range of equilibrium posted wages, this time from about $w = 0.36$ to about $w = 0.71$. We have repeated this exercise for many values of θ, and the result is always qualitatively the same. The left-hand side derivative of profit with respect to the deviant wage, evaluated at the common wage, is always greater than the corresponding right-hand side derivative. Both derivatives are positive at $w = 0$ and both are negative (and equal to each other) at $w = 1$. Thus, given θ above the cutoff level, there is a continuum of equilibria, ranging from the wage at which $\frac{\partial \pi^+(w',w)}{\partial w'}|_{w'=w} = 0$ to the one at which $\frac{\partial \pi^-(w',w)}{\partial w'}|_{w'=w} = 0$.

1.4 Efficiency

The final step is to investigate the relationship between the equilibrium and efficient levels of θ. We show numerically that there is excessive vacancy creation in equilibrium; that is, $\theta^{**} > \theta^*$.

As in Section 3 of the paper, θ^* is defined by $c_v = m_\theta(\theta^*)$, where the derivative $m_\theta(\theta)$ is now computed using equation (1) and the definitions of q_1 and q_2, which are given in equations (2) and (3). The equilibrium value, θ^{**} is defined by the free-entry condition, $c_v = \pi(w(\theta^{**}))$, where $w(\theta)$ is an equilibrium wage given θ. As noted above, for θ below the cutoff level, $w(\theta) = 0$. For θ above the cutoff level, we focus on $w^-(\theta)$, that is, the wage that, given θ, solves $\frac{\partial \pi^-(w',w)}{\partial w'}|_{w'=w} = 0$. Given θ, this is the highest possible equilibrium wage.

In Figure 1, we plot $m_\theta(\theta)$ and $\pi(w^-(\theta))$ against θ. As in Section 3 of the paper, $\pi(w^-(\theta)) > m_\theta(\theta)$ for each value of θ. Equivalently, $\theta^{**} > \theta^*$.

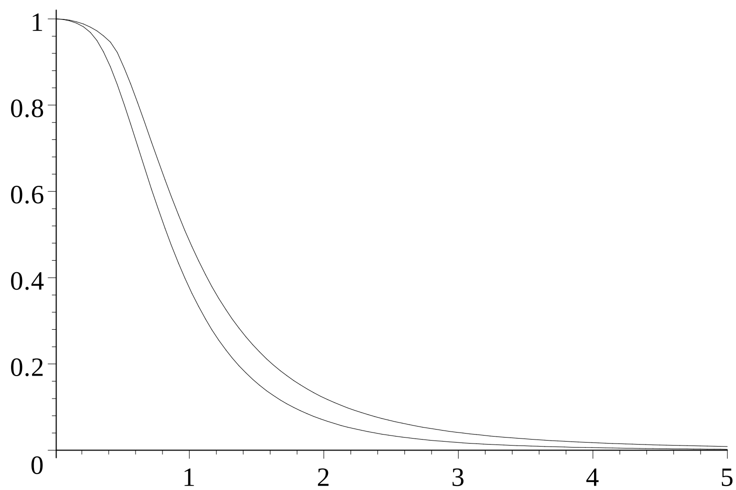
2 Offer-Beating Strategies

2.1 Proof of Proposition 5

Proposition 5 *Let* $\overline{w}(\theta; a) = \dfrac{\frac{a}{\theta} e^{-a/\theta}}{1 - e^{-a/\theta}}$. *There exists a continuum of symmetric offer-beating Nash equilibria indexed by* $w \in [0, \overline{w}(\theta; a)]$.

The strategy of proof is simple. We first show that if all vacancies follow an offer-beating strategy at any common posted wage w, it is never in the interest of any one vacancy to post a higher wage, w'. Posting a higher wage increases the probability of attracting an applicant. This is beneficial only if that applicant receives no other offers. We place an upper bound on the expected benefit associated with an upward deviation in the posted wage by supposing that an arbitrarily small increase in the posted wage above w attracts one or more applicants with probability one. Nevertheless, it is not profitable to post $w' > w$. The increase in the probability of attracting an applicant is outweighed by the decrease in the probability that the vacancy will receive a positive profit

Figure 1: $\pi(w(\theta))$ (upper curve) and $m_\theta(\theta)$ (lower curve)



from that worker. Second, we check that a downward deviation, i.e., $w' < w$, is not profitable. This is the case for all $w \in [0, \overline{w}(\theta; a)]$. The argument is essentially the same as the one we used for the case of $a = 1$ in the proof of Proposition 2.

Expected profit in a symmetric offer-beating equilibrium in which all vacancies post w is

$$\pi(w) = (1-w)(1-e^{-a/\theta})(\frac{1-(1-q)^a}{aq}), \qquad \text{where } q = \frac{\theta}{a}(1-e^{-a/\theta}).$$

The first term in $\pi(w)$ is profit for a vacancy that hires a worker at w, the second term is the probability the vacancy receives at least one application, and the third term is the probability that the vacancy hires conditional on receiving at least one application. The derivation of the third term is as follows. Consider an applicant selected by a particular vacancy. The number of other offers this applicant has is $bin(a-1, q)$. Given that all vacancies follow the offer-beating strategy, i.e., do not engage in Bertrand competition, the probability that the vacancy in question succeeds in hiring the applicant is then

$$\sum_{x=0}^{a-1} \frac{1}{x+1} \binom{a-1}{x} q^x (1-q)^{a-1-x} = \frac{1-(1-q)^a}{aq}.$$

We first consider the expected profit associated with an upward deviation, i.e., a posted wage of $w' > w$. We bound this expected profit, which we call

$\pi^+(w'; w)$, by noting that an upward deviation can increase the hiring probability to at most 1 and that profit conditional on hiring the worker, $1 - w'$, is less than $1 - w$. The deviant makes a profit on its applicant only if all the other applications that the applicant makes are rejected. This occurs with probability $(1 - q)^{a-1}$. If the applicant has one or more other offers, the offer-beating strategy followed by the other vacancies calls for Bertrand competition since $w' > w$. We thus have

$$\pi^+(w'; w) < (1 - w) \cdot 1 \cdot (1 - q)^{a-1}.$$

The fact that no vacancy wants to make an upward deviation then follows from

$$(1 - q)^{a-1} < (1 - e^{-a/\theta})(\frac{1 - (1 - q)^a}{aq}) = \frac{1 - (1 - q)^a}{\theta},$$

which holds for $a \geq 2$.

To verify this, rewrite the inequality as

$$y(a, q) = \frac{1 - (1 - q)^a}{\theta} - (1 - q)^{a-1} > 0.$$

Let $x = \frac{a}{\theta}$, so $q(x) = \frac{1 - e^{-x}}{x}$, and define $z(x, a) = ay(a, q)$ or

$$z(x, a) = x(1 - (1 - q)^a) - a(1 - q)^{a-1}.$$

We want to show that $z(x, a) > 0$ for all $x > 0$ and $a \geq 2$. This is done by induction. First,

$$z(x, 2) = xq(2 - q) + 2q - 2 = \frac{1 - e^{-2x} - 2xe^{-2x}}{x}.$$

Using L'Hôpital's Rule, $z(0, 2) = 0$. Since the numerator of $z(x, 2)$ is increasing in x, it follows that $z(x, 2) > 0$.

Now suppose $z(x, b) > 0$ for some integer $b > 0$. We have

$$\begin{aligned} z(x, b+1) &= x(1 - (1 - q)^{b+1}) - (b + 1)(1 - q)^b \\ &= \left(x(1 - (1 - q)^b) - b(1 - q)^{b-1}\right)(1 - q) + xq - (1 - q)^b \\ &= z(x, b)(1 - q) + xq - (1 - q)^b. \end{aligned}$$

Thus,

$$\begin{aligned} z(x, b+1) &> xq - (1 - q)^b = 1 - e^{-x} - (1 - q)^b \\ &> 1 - e^{-x} - (1 - q) = q - e^{-x} = \frac{1 - e^{-x} - xe^{-x}}{x}. \end{aligned}$$

Because the numerator equals the probability of two or more arrivals in a Poisson process with parameter x, this final term is positive for all $x > 0$. Thus, $z(x, b) > 0 => z(x, b + 1) > 0$, and our proof by induction is complete.

Next, we consider the expected profit associated with a downward deviation, i.e., a posted wage of $w' < w$. To develop an expression for this expectation, $\pi^-(w'; w)$, we mimic the argument given in the proof of Proposition 2. Specifically, suppose workers apply to the deviant (D) with Poisson intensity ξ, where ξ is determined by an indifference condition to be given below. Then

$$\pi^-(w'; w) = (1 - w')(1 - e^{-\xi})(1 - q)^{a-1}.$$

The second term is the probability that D gets at least one application, and the third term is the probability that D's chosen applicant has no other offers. Note that the final term is independent of w'.

The condition determining ξ is that each worker be indifferent between sending all a applications to nondeviants (N) versus $a-1$ applications to N and one application to D. The expected payoff to the first strategy depends on neither w' nor ξ. The expected payoff to the second strategy is

$$q^D(1 - q)^{a-1}w' + (1 - (1 - q)^{a-1})w,$$

where

$$q^D = \frac{1 - e^{-\xi}}{\xi}$$

is the probability that a worker's application to D is accepted. The first term in this expected payoff is the probability that the worker gets the offer from D but no offers from N; in this case, the payoff is w'. The second term is the probability of at least one offer from N; in this case the expected payoff is w. Equating these two expected payoffs defines ξ as a function of w'. Using

$$\frac{dq^D}{d\xi} = \frac{-\left(1 - e^{-\xi} - \xi e^{-\xi}\right)}{\xi^2},$$

it is straightforward to derive

$$\frac{d\xi}{dw'} = \frac{\xi(1 - e^{-\xi})}{w'\left(1 - e^{-\xi} - \xi e^{-\xi}\right)}.$$

Finally,

$$\frac{d\pi^-(w'; w)}{dw'} = \left[-(1 - e^{-\xi}) + (1 - w')e^{-\xi}\frac{d\xi}{dw'}\right](1 - q)^{a-1}.$$

This derivative is nonnegative, i.e., D has no incentive to post $w' < w$, so long as

$$\begin{aligned} w'\left(1 - e^{-\xi} - \xi e^{-\xi}\right) &\leq (1 - w')\xi e^{-\xi}, \text{ i.e.,} \\ w' &\leq \frac{\xi e^{-\xi}}{1 - e^{-\xi}}. \end{aligned}$$

Evaluating at $w' = w$, D has no incentive to undercut the common wage w so long as $w \leq \frac{a}{\theta}\frac{e^{-a/\theta}}{1 - e^{-a/\theta}}$. *QED*

2.2 Proof of Proposition 6

Proposition 6 *There is excessive vacancy creation in any symmetric offer-beating Nash equilibrium.*

Suppose all vacancies follow an offer-beating strategy with a posted wage of w. The equilibrium value of θ is then determined as usual by

$$c_v = \frac{m(\theta)}{\theta}(1-w).$$

Now, however, any $w \in [0, \overline{w}(\theta; a)]$ is consistent with symmetric Nash equilibrium, so there is a corresponding range of θ that is consistent with free-entry equilibrium. The lowest possible equilibrium level of labor market tightness is the one associated with $\overline{w}(\theta; a)$. Call this lowest possible equilibrium value of labor market tightness θ^{**}. Then θ^{**} solves

$$c_v = \frac{1 - \left(1 - \frac{\theta}{a}(1 - e^{-a/\theta})\right)^a}{\theta}\left(1 - \frac{a}{\theta}\frac{e^{-a/\theta}}{\left(1 - e^{-a/\theta}\right)}\right).$$

As above, let $q = \frac{\theta}{a}(1 - e^{-a/\theta})$. The free-entry condition is then

$$c_v = \frac{1-(1-q)^a}{aq}(1 - e^{-a/\theta} - \frac{a}{\theta}e^{-a/\theta}). \qquad (6)$$

The planner's problem is unchanged, so the efficient level of labor market tightness, θ^*, is again the solution to

$$c_v = (1 - \frac{\theta}{a}(1 - e^{-a/\theta}))^{a-1}(1 - e^{-a/\theta} - \frac{a}{\theta}e^{-a/\theta}),$$

cf., equation (9). This condition can be rewritten as

$$c_v = (1-q)^{a-1}(1 - e^{-a/\theta} - \frac{a}{\theta}e^{-a/\theta}). \qquad (7)$$

Since $1-(1-q)^a > aq(1-q)^{a-1}$ so long as $a \geq 2$ (by the properties of the binomial), the right hand side of (6) is greater than the right hand side of (7) for each $\theta > 0$. That is, $\theta^{**} > \theta^*$. *QED*