Multiproduct Pricing and The Diamond Paradox

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Abstract

I show that multiproduct firms suffer less than single-product firms from the "Diamond Paradox". Equilibrium prices are high because rational consumers understand that visiting a store exposes them to a hold-up problem when they have search costs. However a store with more products attracts more consumers with low valuations, and therefore charges lower prices. Advertising a few products at low prices enables the firm to credibly commit to low prices across the rest of the store. "Loss-leading" can therefore be optimal. If demands are subject to random shocks, prices are shown to move counter cyclically.

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1 Introduction

Typically retailers only advertise the prices of a few of their products (and many others do no advertising at all). Consequently consumers usually learn the price of a particular product only once they have arrived at the retailer. Despite all of the marketing and online price comparisons available to supermarket shoppers, even in that sector most price information is probably still gathered instore.[1] However shopping (especially for basics) can be time-consuming, so visiting stores and learning about prices is not the easy and costless activity which many papers assume it to be. I therefore present a simple model which captures this important feature. Consumers must pay a (small) shopping cost in order to travel to a multiproduct monopolist and discover its prices.

The first contribution of the paper is to show that multiproduct firms can resolve the Diamond Paradox [8]. The exact details of this Paradox depend upon whether a consumer has unit or downward-sloping demand. Most of the literature (implicitly or explicitly) assumes unit demand, and so does this paper. Suppose firms sell a single product, and consumers must pay a cost $s > 0$ to learn any retailer's price. If consumers expect each retailer to charge p^E, they only visit a firm if they value the product more than $p^E + s$. So once somebody has turned up to a store, they will buy the product there provided its actual price is less than $p^E + s$ (the consumer still gets positive surplus, and it is not worthwhile to pay another s and visit another retailer). Therefore firms always charge more than consumers expected. The only equilibrium outcome of the model is for consumers to expect very high prices, nobody to visit any retailer, and no trade to occur.

Unsurprisingly, many papers have suggested possible ways to overcome this 'no trade' result. Possible resolutions include advertising (Wernerfelt [24]), product differentiation and unknown match values (Anderson and Renault [1], Konishi and Sandfort [13]), as well as repeated interaction between consumers and producers (Bagwell and Ramey [4]). In Stahl [21], some consumers are 'shoppers' - meaning they have zero search costs and learn

[1] Simester [20] suggested that (in 1995) a typical supermarket stocked $25,000$ products and advertised 200 of them. Large UK supermarkets stock $30,000$ different products ('The rise and rise of Lidl Britain', The Daily Telegraph, September 10^{th} 2008). Tesco.com allows a comparison of some $10,000$ prices with its main rivals, but may be slightly out of date; searching the website is time-consuming; smaller outlets are excluded; and relatively few people may use such sites often.

every price in the market. The remaining consumers pay $s > 0$ for each price quotation that they learn. Firms cut prices to win business from the shoppers, and hence trade occurs.[2]

In this paper I interpret the Diamond Paradox as occurring because of a 'sample selection problem'. Only consumers with relatively high valuations find it worthwhile to go shopping. Therefore retailers always want to charge high prices, which ultimately causes the market to collapse. Multiproduct retailers can overcome the paradox, because they attract a broader selection of consumers. Somebody with a low valuation on one product may visit a store because they have a high valuation on something else. Consequently consumers within the store are more representative of the population. This restrains the firm's incentives to surprise consumers with high prices, and makes equilibria with trade possible. Although every consumer has the same strictly positive search cost, some consumers behave *as if* they are shoppers in the sense of Stahl.

The model also provides several insights into firm behaviour. It predicts that a store with a broader product selection should charge lower prices but earn more profit on each good. A simple rationale is also provided for why low advertised prices on a few products can credibly signal low prices on the rest of the store's (unadvertised) products. Prices are also predicted to be countercyclical.

In a recent survey, 86% of American consumers said they thought larger stores (with broader product selections) charge lower prices.[3] Consistent with this, Hoch *et al* [10] find that larger grocery stores have more elastic demand curves. There is also abundant anecdotal evidence that bigger retailers tend to charge less. For example although the major UK supermarkets use national pricing, this does not apply to their smaller stores, which are typically more expensive.[4] The main explanations for this are costs and convenience. Larger stores may enjoy economies of scale and buyer

[2] Burdett and Judd [6] present a similar idea. All consumers are ex ante identical, but the number of prices learnt upon paying s is stochastic and sometimes exceeds 1. Hence some consumers have better information, which encourages firms to cut price.

[3] 'American Consumer Institute Survey Finds Consumers Prefer Shopping at Larger Stores and Wholesale Clubs', PR Newswire, 13$^{\text{th}}$ April 2006. A Canadian price survey suggests that store size is indeed the biggest determinant of price ('Bigger grocery stores less expensive: study', The Gazette (Montreal), April 5$^{\text{th}}$ 2007).

[4] In 2006 Tesco was accused of charging up to 25% more in some smaller stores ('MPs demand tough retail regulator', The Sunday Telegraph, 12$^{\text{th}}$ February 2006).

power, which they pass on in lower prices. Smaller (convenience) stores may also attract time-poor but cash-rich consumers. The model in this paper presents a different interpretation. A retailer always prices to a select sample of relatively high-valuation consumers. However a store with more products attracts more consumers, and these consumers have on average lower valuations. Hence a larger retailer finds it optimal to charge lower prices. The recent drive towards one-stop shopping[5] can then be interpreted as an attempt by firms to commit to low prices across their whole product range.

Most retailers either do no price advertising, or they advertise a small number of products at very low prices. Occasionally products are even sold below cost, with examples ranging from books[6] to realtime share prices[7]. Supermarkets are also well-known for using loss-leaders to boost store traffic.[8] This raises an interesting question: what effect does price advertising have on overall price levels? There are two main viewpoints. On the one hand, retailers may increase the prices of unadvertised products, to make up lost margin on items that are sold cheaply. On the other hand, advertising a few products may enable efficient retailers to signal their low costs (and general low price level).[9] The papers by Lal and Matutes [14] and Simester [20] capture these ideas.

In Lal and Matutes [14] consumers have unit demands and identical willingnesses to pay. One product is unadvertised and consumers pay their reservation price for it. Another product is advertised, and used solely to compete for store traffic. In Simester [20] firms also stock two products, and advertise the price of only one. Consumers have identical and downward-sloping demand for the unadvertised good. A firm's production cost is correlated across products, and more efficient firms charge less for their unadvertised product. They may signal this by advertising a low price on the other product.

[5]See for example 'Post Office Stocks up for Fight', Marketing, September 14$^{\text{th}}$ 2005. Supermarkets also now stock a range of previously non-core products, such as electricals and garden furniture.

[6]'Harry Potter 7: wizard of a loss leader', The Vancouver Sun, July 7$^{\text{th}}$ 2007.

[7]'British operator to offer stock data free on Net', The Vancouver Sun, July 15$^{\text{th}}$ 1999

[8]See Appendix 5.6 of the Competition Commission's 2008 Groceries Final Report. All of the major supermarkets admitted to using loss-leaders, with these accounting for on average 3% of revenue. Increasing store traffic was one stated reason.

[9]See for example John Fingleton's argument in 'Remove the shackles from retail and distribution sectors', The Irish Times, May 30$^{\text{th}}$ 2003.

The crucial assumption in my paper is that consumers have heterogeneous willingnesses to pay. This implies that a retailer receives only a select sample of consumers who have relatively high product valuations. When a product is advertised at a low price, this draws new consumers into the store. These new visitors must have relatively low valuations for other products (otherwise they would have visited even without the advertising). The retailer optimally cuts prices on unadvertised product lines in an attempt to win more business from the new consumers. Hence the model provides an intuitive explanation for how low advertised prices on a few products can successfully signal a low store-wide price image. Loss-leadership pricing is also shown to sometimes be optimal.[10]

It is well-known that retailers tend to mark down items during periods of high general demand - such as weekends and holidays. Further, when individual items are in high demand, they are often advertised at low prices. (See Warner and Barsky [23] and MacDonald [15]) One possible explanation for countercyclical pricing is Rotemberg and Saloner's [19] model of collusion. Alternatively, Warner and Barsky argue that during weekends, consumers visit more shops and so become more responsive to prices. Bils [5] argues that new customers may have less brand attachment. When lots of new consumers enter the market, it is optimal to cut price and persuade them to try the product. In my model, weekends mean that more consumers are interested in buying a product. New consumers visit the store, and they tend to have low valuations on other products. Hence the retailer again has more incentives to charge lower prices across the whole of its product range.

The rest of the paper proceeds as follows. Section 2 sets out the main assumptions. In Section 3 I solve the model and demonstrate how a multi-product retailer can overcome the Diamond Paradox. Comparative statics results are provided in Section 4, whilst Sections 5 and 6 discuss the results and provide possible extensions.

[10] Empirical evidence is ambiguous. Milyo and Waldfogel [18] study the lifting of a ban on alcohol price advertising. They find little effect on overall alcohol prices, and no effect on the prices of unadvertised alcohol products. However their sample is small, and they cite other (cross-sectional) papers which do find that advertising results in lower prices. Collins *et al* [7] present some evidence that banning loss-leaders results in high prices.

2 Assumptions

There is a single firm that produces n goods, indexed by $j = 1, 2, ..., n$, at zero marginal cost. The products are neither substitutes nor complements, and consumers demand at most one unit of each. Consumer valuations for the n goods are denoted $(v_1, v_2, ..., v_n)$. For each consumer, v_l and v_k are independent whenever $l \neq k$. Each v_j is drawn from $[a_j, b_j]$ (where $b_j > 0$) using a distribution function $F_j(v_j)$ (with corresponding density $f_j(v_j)$). $f_j(v_j)$ is strictly positive, continuously differentiable, and logconcave. This ensures that the hazard rate $\frac{f_j(p)}{1-F_j(p)}$ is increasing, and holds for many common distributions.[11] In the textbook zero-search-cost model, each good's profit function is strictly quasiconcave and has a unique maximiser $p_j^m = \arg\max p[1 - F_j(p)]$.

The monopolist may advertise A prices, where A is strictly less than n and could be zero. The firm is legally obliged to honour all advertised prices. Consumers use advertisements to form rational *expectations* about all prices, denoted $\left(p_1^E, p_2^E, ...p_n^E\right)$. Consumers must pay a shopping cost $s > 0$ to visit the store[12]. Once incurred, this cost is sunk. Prior to visiting the store, consumers know their valuations $(v_1, v_2, ..., v_n)$. Using their *expectations* about price, they turn up if and only if they expect to earn a surplus greater than the shopping cost. After they have arrived at the store, consumers learn *actual* prices $(p_1, p_2, ...p_n)$ and make their purchases. All parties are risk neutral and rational. The move order can be summarised as:

1. The monopolist chooses which goods (if any) to advertise, and at what prices. It then chooses (but does not disclose) the prices of the remaining goods

2. Consumers form expectations about all prices, and decide whether or not to visit the store

3. Consumers who decided to turn up then learn actual prices, and make purchase decisions

[11] See Bagnoli and Bergstrom [3].

[12] I use the terms 'search cost' and 'shopping cost' interchangeably. s is a search cost because it must be paid to learn prices. But it is also a shopping cost because it must be paid even by a consumer who only buys advertised products.

3 Characterising Equilibrium Prices

Equilibrium in this model has three requirements. First, consumers only visit the store if they expect to earn enough surplus to cover the shopping cost (given their valuations and expected prices). Second, actual prices must maximise firm profit (given expected prices and therefore given the types of consumers who visit the store). Third, expected prices must equal actual prices (rational expectations).

Write the demand for an unadvertised good (call it good 1) as:

$$D_1 = \int_{p_1}^{b_1} f(v_1) \Pr\left(\sum_{j=1}^{n} \max\left(v_j - p_j^E, 0\right) \geq s\right) dv_1 \tag{1}$$

A consumer buys good 1 if (a) he values it more than its *actual* price, and (b) he turns up to the store. Turning up is only worthwhile if total expected surplus $\sum_{j=1}^{n} \max\left(v_j - p_j^E, 0\right)$ exceeds the shopping cost s. Through this turn-up decision, demand for any one product depends upon the expected prices of all goods. Nevertheless conditional upon visiting the store, demand for a product depends only upon its actual price. This is because goods are neither substitutes nor complements.[13]

In equilibrium, the firm must maximise its profit by setting p_1 equal to p_1^E. Given any vector of expected prices $\left(p_2^E, ..., p_n^E\right)$, there is at most one p_1^E where this is true, and it satisfies the following first order condition[14]

$$D_1|_{p_1=p_1^E} - p_1^E f_1\left(p_1^E\right) \underbrace{\Pr\left(\sum_{j=2}^{n} \max\left(v_j - p_j^E, 0\right) \geq s\right)}_{\text{Probability a consumer with } v_1=p_1^E \text{ turns up}} \leq 0 \tag{2}$$

To understand (2), consider a small increase in p_1 above the expected level p_1^E. The firm *gains* revenue on existing consumers, who have mass equal to

[13] Notice that when p_1 hits $p_1^E + s$, demand kinks and becomes more responsive to changes in price. Hence one can rationalise the finding that sometimes consumers are loss-averse with respect to price changes.

[14] Condition (2) only checks for small deviations in p_1 around p_1^E, and may have multiple solutions, but at most one is compatible with profit being globally maximised at $p_1 = p_1^E$. Profit is quasiconcave in p_1 if $\Pr\left(\sum_{j=2}^{n} \max\left(v_j - p_j^E, 0\right) \geq s\right) \geq \frac{1}{2}$ or if the standard monopoly profit function $p\left[1 - F_1(p)\right]$ is concave. See Claims 1 and 2 in the Appendix.

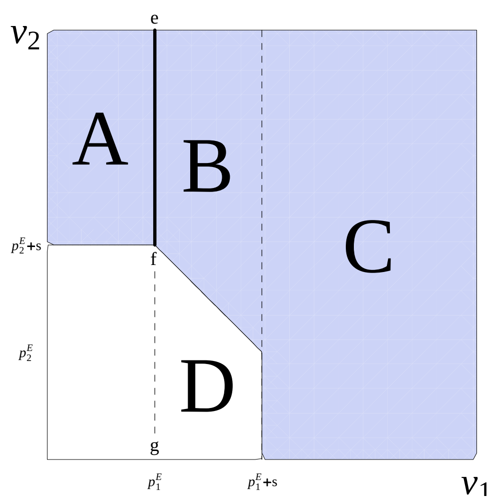

Figure 1: The firm attracts consumers with relatively high valuations

demand, but *loses* revenue on consumers who stop buying good 1 following the price rise. Each consumer who stops buying the good (a) has a marginal valuation for it, and (b) visits the store. Part (a) explains the $f\left(p_1^E\right)$ term, whilst part (b) explains the term $\Pr\left(\sum_{j=2}^n \max\left(v_j - p_j^E, 0\right) \geq s\right)$. Intuitively, since the consumer is marginal for good 1 but has turned up, he must have expected surplus of s or more on the other $n-1$ goods. Figure 1 provides a graphical intuition when $n = 2$. Only consumers in the shaded region $A + B + C$ visit the store, so the firm gets an 'adverse selection' of high-valuation types. When $p_1 = p_1^E$, demand for product 1 equals $B + C$. Small changes in p_1 around p_1^E only affect the behaviour of consumers on the thick line $e\ f$. All other marginal consumers have $v_2 < p_2^E + s$, so they do not visit the store and do not observe changes in p_1.

Figure 1 also helps draw a parallel with the Diamond Paradox. If only good 1 is sold and its price is unadvertised, the firm faces a strong sample selection problem, because only high-valuation consumers in region C turn up. In particular, no consumer with $v_1 = p_1^E$ visits the store. Hence the pricing condition (2) is only satisfied when $D_1|_{p_1=p_1^E} = 0$ - i.e. no trade.[15] A key result of the paper is that a multiproduct firm can attract marginal

[15] If $n = 1$ but $s = 0$, demand for good 1 is $B + C + D$ and all marginal consumers (on line $e\ f\ g$) visit the store. Hence (2) simplifies to $1 - F_1\left(p_1^E\right) - p_1^E f_1\left(p_1^E\right) \leq 0$ - the standard monopoly first order condition.

consumers, and thus make trade possible:

Proposition 1 *When n is sufficiently large, there exist equilibria in which the firm optimises, consumer expectations are fulfilled, and trade occurs*

When $n = 1$ trade breaks down because no marginal consumer ever visits the store. By contrast when n is sufficiently large, many consumers visit the store, and many of these have marginal valuations for each product. Demand curves become sufficiently sensitive to small price changes, so the firm is deterred from surprising consumers with price increases. Of course if there is no advertising, no-trade Diamond equilibria exist as well. (If consumers expect very high prices, nobody visits the store, so the firm is indifferent between all prices and is happy to charge whatever consumers expected) However the Diamond outcome is not a very compelling prediction of possible play. For example, suppose the monopolist must incur an ϵ entry cost. The firm would never enter if it expected a zero-profit Diamond outcome; by entering the market, the firm signals to consumers that it expects to play a non-Diamond equilibrium. Discussion of equilibrium multiplicity and (where required) selection is left till the next section.

Before concluding this section, rewrite the pricing condition (2) as

$$\underbrace{\Pr\left(\sum_{j=2}^{n} t_j \geq s\right)}_{\text{Shoppers}} \underbrace{\left[1 - F_1\left(p_1^E\right) - p_1^E f_1\left(p_1^E\right)\right]}_{\text{Standard first order condition}} + \underbrace{\Pr\left(\sum_{j=1}^{n} t_j \geq s > \sum_{j=2}^{n} t_j\right)}_{\text{Diamond consumers}} \leq 0 \tag{3}$$

where $t_j = \max\left(v_j - p_j^E, 0\right)$ is expected surplus on good j. Pricing is affected by the behaviour of three different groups. Consumers with $\sum_{j=1}^{n} t_j < s$ do not expect to cover the shopping cost, and therefore do not visit the store. They do not observe the actual price p_1 and so their demand for good 1 remains at zero. Consumers with $\sum_{j=1}^{n} t_j \geq s$ do visit the store, and subdivide into two groups:

- **Shoppers** for product 1 have $\sum_{j=2}^{n} t_j \geq s$
- **Diamond consumers** for product 1 have $\sum_{j=1}^{n} t_j \geq s > \sum_{j=2}^{n} t_j$

Shoppers for product 1 turn up at the store regardless of their valuation for the product. Consequently they reveal no information about their v_1, which continues to be distributed on $[a_1, b_1]$ with the usual density $f_1(v_1)$. The profit earned on these consumers is simply $p_1[1 - F_1(p_1)]$ (the same as in a standard zero-search-cost monopoly problem), so small changes in p_1 around p_1^E affect profits by $1 - F_1\left(p_1^E\right) - p_1^E f_1\left(p_1^E\right)$. The term 'shoppers for product 1' is used to describe these consumers because they act *as if* they have no shopping cost when it comes to buying good 1. In contrast, Diamond consumers for product 1 have $v_1 - p_1^E > 0$. They would all continue to buy product 1 even if the firm increased p_1 slightly above p_1^E. Their demand is locally perfectly inelastic - just like the consumers in a standard Diamond problem. *Therefore the multigood problem with positive shopping cost is really an average of the standard monopoly and Diamond problems.*

Lemma 2 *When trade occurs, $p_j^E \geq p_j^m$ for each unadvertised good j*

A small increase in p_1 above p_1^E always gains extra revenue on Diamond consumers; in equilibrium, this must be balanced by losses on shoppers. Consequently the equilibrium price of product 1 exceeds p_1^m. Since the firm's pricing problem is an average of the standard monopoly and Diamond problems, in general p_1^E strictly exceeds p_1^m.

Corollary 3 *The firm prefers equilibria with lower unadvertised prices*

Interestingly, both consumers and the firm itself benefit from a small reduction in unadvertised prices. Intuitively, a decrease in p_1^E benefits the firm in two ways. Firstly, it lowers the price of product 1 closer to p_1^m and therefore makes the product more profitable. Secondly, the price decrease draws more consumers into the store, expanding demand and therefore profits on other products in the store.

4 Comparative Statics

I first solve the model for small shopping costs. I then show that the results generalise to arbitrary s, provided two natural conditions are imposed.

Comparative static results are best understood using sample selection and shoppers/Diamond consumers. A shopper for product 1 has $\sum_{j=2}^{n} t_j \geq s$ whilst a Diamond consumer for the same product has $\sum_{j=1}^{n} t_j \geq s > \sum_{j=2}^{n} t_j$. Figure 2 illustrates this in $\left(v_1, \sum_{j=2}^{n} t_j\right)$ space. The distribution of $\sum_{j=2}^{n} t_j$

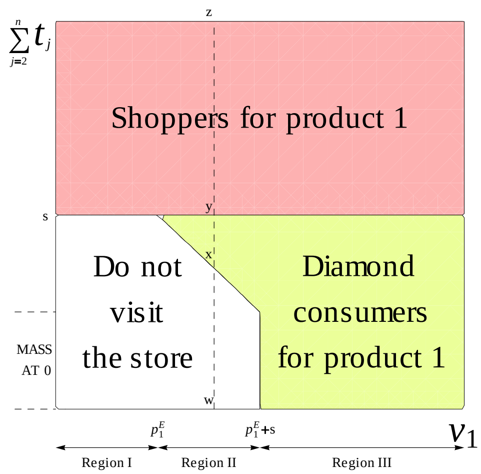

Figure 2: Shoppers and Diamond consumers

has mass at 0, which I represent using an area. It is useful to split the Figure into three regions and analyse a shift up in the distribution of $\sum_{j=2}^{n} t_j$. *Region I*: consumers with $v_1 \leq p_1^E$ only visit the store if $\sum_{j=2}^{n} t_j \geq s$. So if $\sum_{j=2}^{n} t_j$ increases, more consumers visit the store and become shoppers for product 1. *Region III*: consumers with $v_1 \geq p_1^E + s$ definitely visit the store, and are shoppers or Diamond consumers for product 1 depending upon whether $\sum_{j=2}^{n} t_j$ exceeds s. Clearly then, as $\sum_{j=2}^{n} t_j$ increases, the number of shoppers increases and the number of Diamond consumers falls. *Region II* deals with $v_1 \in \left(p_1^E, p_1^E + s\right)$. An increase in $\sum_{j=2}^{n} t_j$ converts some Diamond consumers to shoppers. It also attracts new people to the store, some of whom become shoppers and others who become Diamond consumers. Therefore in Regions I and III, an increase in $\sum_{j=2}^{n} t_j$ raises the ratio of shoppers to Diamond consumers, whilst Region II is ambiguous. I now propose two methods to overcome this ambiguity.

4.1 Small Shopping Costs

I begin by focusing on the special case $s \to 0_+$ (the shopping cost is strictly positive but arbitrarily small). The mass of consumers with $v_1 \in \left(p_1^E, p_1^E + s\right)$ - Region II in Figure 2 - becomes vanishingly small. Referring to the above discussion, it is clear that a shift up in the distribution of $\sum_{j=2}^{n} t_j$ will increase the probability of being a shopper and decrease the probability of being a Diamond consumer for product 1.[16] When p_1^E is interior, the pricing condition (3) can then be simplified to

$$|\epsilon_1| = \frac{1}{1 - \prod_{j=2}^{n} F_j\left(p_j^E\right)} \tag{4}$$

where ϵ_1 is the price elasticity of demand in a standard (zero-search-cost) monopoly problem. In equilibrium $|\epsilon_1|$ is equated with the inverse probability of a marginal consumer visiting the store. If the latter is equal to 1, $|\epsilon_1| = 1$ and $p_1^E = p_1^m$. (Note that $|\epsilon_1|$ is strictly increasing in p_1^E)

Small search frictions induce complementarities in the way different products are priced. Suppose for example that consumers suddenly expect p_2^E to be higher. The distribution of $\sum_{j=2}^{n} t_j$ is shifted down, so the probability of being a shopper for product 1 decreases whilst the probability of being a Diamond consumer increases. With more Diamond consumers to exploit through price increases, and fewer shoppers to tempt with price cuts, the firm wants to charge more than originally expected for good 1. Consequently its equilibrium price increases - even though the search friction is minimal and the two products are independent in both use and valuation.

Complementarity in pricing decisions opens up the possibility of multiple equilibria. If the firm commits to $\left(p_2^E, ..., p_n^E\right)$ via advertising, there is a single p_1^E that solves (4) and therefore (subject to profit being quasiconcave) a unique equilibrium. Multiplicity becomes possible whenever there is more than one unadvertised product. To illustrate, suppose that goods 1 and 2 are unadvertised and that any other prices are fixed (perhaps by advertising). If p_1^E is expected to be 'high', product 2 has few shoppers and therefore p_2^E is also expected to be high - in which case product 1 has few shoppers and the expectation that p_1^E is high can be justified. At the same time if p_1^E is expected to be 'low', p_2^E is also expected to be low which in turn can justify the original expectation about p_1^E. Expectations can therefore be self-reinforcing, as the following (striking but atypical) example demonstrates:

Example 4 *No advertising, and valuations are iid with $F(v) = \ln \frac{v}{a}$ ($\ln \frac{b}{a} = 1$ and $p^m = a$).*[17] *Equilibria are symmetric*

When $n = 2$, any $p^E \in [a, b]$ is an equilibrium

When $n \geq 3$, the only equilibria are a and b

[16] Limit probabilities are $1 - \prod_{j=2}^{n} F_j\left(p_j^E\right)$ and $\left[1 - F\left(p_1^E\right)\right] \prod_{j=2}^{n} F_j\left(p_j^E\right)$ respectively.

[17] Although $f(v) = v^{-1}$ is not logconcave, $p\left[1 - F(p)\right]$ is concave.

Complementarity ensures existence of a Pareto Dominant equilibrium. Hold constant any advertised prices and look for equilibrium vectors of unadvertised prices. Since the equilibrium price of product j is an increasing function of the expected prices of all other goods, Tarski's Fixed Point Theorem guarantees the existence of a lowest price vector. By Corollary 3, this is also be the Pareto Dominant equilibrium.

Assumption: *Agents play the Pareto Dominant equilibrium*

This assumption is probably less severe than it might first appear. There always exists a region of fixed entry costs such that the firm only enters if it expects to play the Pareto Dominant equilibrium. Hence by Forwards Induction Logic, entering coordinates expectations on the low-price equilibrium. Often there is one low-price equilibrium and several high-price (low-profit) equilibria, so the region of entry costs for which this works can be quite large. Burning money to artificially increase fixed costs, as well as cheap-talk in advertisements, can also help focus beliefs.[18]

Proposition 5 *A store with more products charges lower prices*

Precisely, if advertised prices are held constant and n is increased, unadvertised products become cheaper.[19] When a new product is introduced, some additional consumers are attracted to the store, and they act as shoppers for each existing product. Also some consumers who were previously turning up but were reliant on a particular product, are not any longer. Hence each existing product receives more shoppers and less Diamond consumers. The firm wants to lower its unadvertised prices and expand output sold to these shoppers. So consumer expectations adjust downwards and

[18] In equilibrium there would be no need to actually burn the money.

Forwards Induction can work in other ways too. For example, stocking n goods may give a unique equilibrium, but n' goods give both a low and a high equilibrium. Choosing n goods may be best if with n' the high equilibrium is played, but choosing n' goods may be best if the low equilibrium is played. By choosing n' goods the firm signals it expects the low equilibrium.

[19] Some restrictions are needed to ensure that profit is quasiconcave in the price of the new product. It is sufficient that valuations for the new product have a similar distribution to an existing product, and/or the conditions given earlier hold.

equilibrium prices are lower.[20] As n grows large, the price of each product gets close to its standard monopoly level and the firm can almost extract all monopoly rents. See also Armstrong [2].

When prices are unadvertised and distributions are identical, non-Diamond equilibria exist if and only if n exceeds a threshold $\tilde{n}$. In the case of $U[a,b]$ valuations, $\tilde{n} = 2 - \frac{a}{b}$ and any non-Diamond equilibrium is unique. Four examples are shown in Figure 3 (in each case $p^m = \frac{1}{2}$). As n increases the firm attracts a broader mix of consumers who have lower valuations. Hence the sample selection problem is less severe and equilibrium price decreases. With $U[0,1]$ valuations, price is very close to $1/2$ even when $n = 10$. Intuitively if consumers expect every product to have price $\frac{1}{2}$, the probability they turn up is $1 - \frac{1}{2}^{10} \approx 1$. There is almost no sample selection problem, so the firm prices almost as if there were no shopping cost. However as a is reduced, the tail of the distribution is pulled down and there are more low-valuation consumers for each product. Given any expected price, fewer people visit the store and the firm faces a more adverse selection. Therefore as a is reduced, equilibrium price increase. I now reintroduce advertising and consider its impact on equilibrium.

Proposition 6 *If the monopolist reduces an advertised price, the prices of unadvertised products fall as well*

Lower advertised prices - just like increased product variety - attract a broader mix of consumers to the store. The firm therefore faces a weaker sample selection problem, and responds by charging less on each unadvertised product. The implicit assumption behind the Proposition is that a firm must (for legal or other reasons) not renege on any price that it has committed to via advertising. Conditional upon that assumption, an advertised price on one product is informative about the prices of everything else. In particular, the model gives a theoretical justification behind the idea that low advertised prices on some (it need not be many) products can be very

[20] There is an an analogy with Keynesian multipliers. Suppose initial price expectations are q_0. Given expectations q_0, with the new product the firm wants to charge $q_1 < q_0$. If consumers then expect q_1, shoppers become yet more numerous and the firm wants to charge prices $q_2 < q_1$ instead. Consumers can then expect prices q_2 and so forth. The decreasing sequence $\{q_t\}_{t=0}^{\infty}$ converges to the rational expectations equilibrium. The initial increase in n gets successively multiplied up into lower and lower prices.

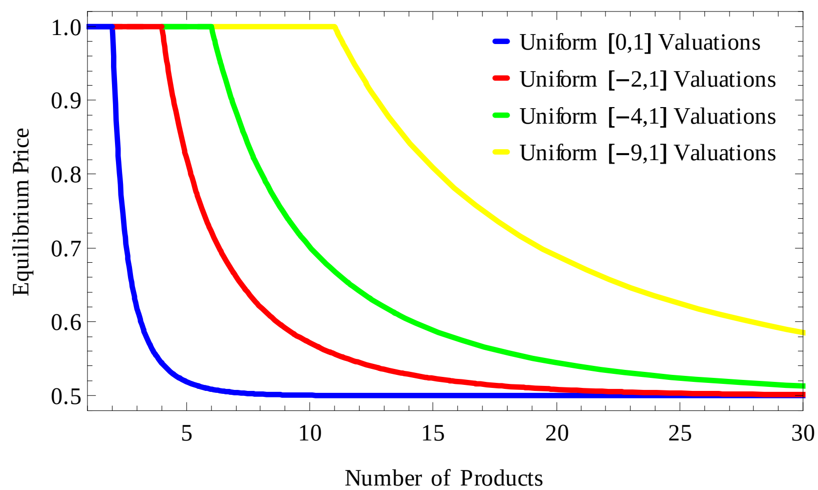

Figure 3: The effect of product range on price

effective at building a 'low-price' image on the rest of the store's product portfolio.

I now consider the effect of product-specific demand shocks. A simple way to model these is to place the following structure on consumer valuations:

$$v_j : \begin{cases} = -\infty & \text{with probability } 1 - \alpha_j < 1 \\ \overset{iid}{\sim} [a, b] \text{ via } F(v) & \text{with probability } \alpha_j \end{cases}$$

An increase in α_j means that product j is more popular, and ceteris paribus is demanded more often by consumers. Within a convenience store, for example, certain products like milk will have a higher α than other products such as shampoo. When any advertised prices are held constant, the following result obtains:

Proposition 7 *A positive demand shock on one product reduces all unadvertised prices. Within the store, more popular unadvertised products have higher prices*

Consider the first part of the Proposition. Higher demand for any one product brings extra people into the store, especially those with low valuations. Hence the firm faces a less adverse selection, and charges lower prices across the store. Prices are therefore countercyclical. Now consider the

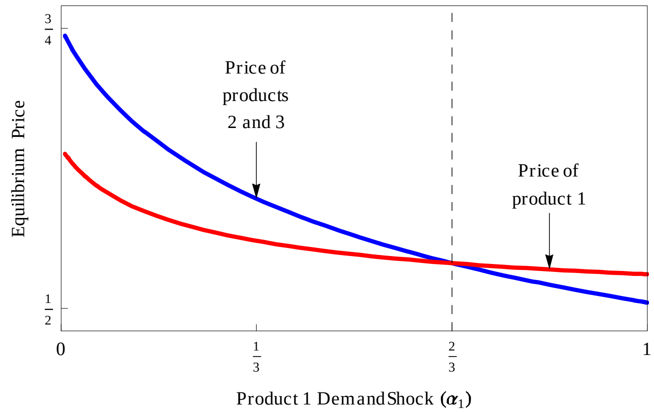

Figure 4: The effect of demand shocks on price

second part of the Proposition. A very popular product attracts many consumers to the store and therefore provides less-popular products with many shoppers. However these less-popular products are not an important factor in consumers' decision to visit the store, and therefore do not provide many shoppers for the popular products. This gives the firm a greater incentive to surprise consumers with price cuts on unpopular products (to attract the relatively large number of shoppers) and price rises on popular products (to attract the relatively large number of Diamond consumers).

Figure 4 illustrates graphically the effect of demand shocks. In the example there are three unadvertised products, $F(v) = 2v - 1$, and $\alpha_2 = \alpha_3 = \frac{2}{3}$. When α_1 increases above $\frac{2}{3}$, product 1 becomes the most important factor affecting whether consumers visit the store, and hence it becomes relatively more expensive. But the prices of all goods are strictly decreasing in α_1.

To summarise, increased product variety, low-price adverts, and positive demand shocks, all attract a broader mix of consumers to the store. This eases the firm's sample selection problem and reduces unadvertised prices. Although store traffic increases in each case, it is the broader mix of consumers that is crucial. For example suppose with probability $1 - \beta$ a consumer has $v_j = -\infty \forall j$, and with probability $\beta > 0$ product valuations are *iid* on interval $[a, b]$. Increases in β raise store traffic, but the mix is unchanged - new visitors have the same joint distribution over valuations as old visitors. Consequently the firm's pricing incentives are unchanged, and there is no effect on equilibrium prices.

4.2 General Shopping Costs

It is intuitive that the comparative statics results in the previous section continue to hold whenever s is 'sufficiently small'.[21] These results also hold for a general s in simple environments where the firm only stocks two products. They also extend to arbitrary numbers of products provided that two natural conditions are imposed on the distribution of consumer surplus.

Proposition 8 *Consider an arbitrary s*

- ***Product range*** *Assume identical distributions and no advertising*

  Equilibrium price falls when n is increased from 2 to 3

- ***Advertising*** *Assume $n = 2$ and one product is advertised*

  The equilibrium unadvertised price increases in the advertised price

- ***Demand shocks*** *When $n = 2$, Proposition 7 holds*

I now show how the comparative statics results can be extended beyond the two-product environment considered in Proposition 8. Let $T_{m,p}$ be the total expected surplus from m products when facing expected price vector $p = \left(p_1^E, p_2^E, ..., p_n^E\right)$. The two conditions on $T_{m,p}$ are as follows:

- **C1** Surplus decreases in price in the sense of hazard rate dominance

$$\frac{\Pr\left(T_{m,p} \geq z\right)}{\Pr\left(T_{m,q} \geq z\right)} \text{ increases in } z,\ z \in (0, s) \text{ and } q > p$$

- **C2** Surplus increases in m in the sense of hazard rate dominance

$$\frac{\Pr\left(T_{m,p} \geq z\right)}{\Pr\left(T_{m-1,p} \geq z\right)} \text{ increases in } z,\ z \in (0, s)$$

$T_{m,p}$ is always increasing in m and decreasing in p in the sense of first order stochastic dominance (FOSD). Conditions C1 and C2 are stronger, and require that conditional distributions satisfy FOSD as well. Hazard

[21] Region II in Figure 2 remains small, so increases in $\sum_{j=2}^{n} t_j$ still increase the ratio of shoppers to Diamond consumers for product 1.

rate dominance is a common assumption in problems with multidimensional types.[22]

C1 ensures that pricing problems are complementary. To illustrate this, suppose that the vector $\left(p_2^E, p_3^E, ..., p_n^E\right)$ increases. Product 1 becomes more expensive if and only if its ratio of Diamond consumers to shoppers increases (since the firm then has more incentives to surprise consumers with a high price). A sufficient condition for this is that region II in Figure 2 is well-behaved. Take a typical locus of points in this region $w\ x\ y\ z$. We require that the probability of being on $x\ y$ relative to the probability of being on $y\ z$, is increasing in the price vector $\left(p_2^E, p_3^E, ..., p_n^E\right)$. This is the same as requiring that $\Pr\left(\sum_{j=2}^n t_j \geq z\right) / \Pr\left(\sum_{j=2}^n t_j \geq s\right)$ is increasing in price, or that C1 holds. This ensures that a Pareto Dominant equilibrium exists, and that unadvertised prices are positively associated with all other prices.

C2 governs comparative statics in product range, search cost, and demand shocks. Assume either C1 is satisfied or that distributions are identical (in which case a Pareto Dominant equilibrium exists). C2 says that conditional on visiting the store (having $\sum_{j=1}^n t_j \geq s$), a consumer is less likely to be a shopper for a product (have $\sum_{j \neq k} t_j \geq s$) when s is larger. Equivalently when the search cost increases, Diamond consumers become more numerous relative to shoppers. It follows immediately that when s increases, the firm has incentives to charge more on each product than was previously expected - and hence equilibrium unadvertised prices increase. Stocking an additional product has the opposite effect. Some consumers expect to get positive surplus from the new product, and this effectively decreases their shopping cost. As a result shoppers become relatively more numerous, the firm has incentives to reduce prices, so in equilibrium unadvertised products become cheaper. A positive demand shock has the same effect: when α_j increases, extra surplus is randomly allocated across consumers so their effective search cost falls and the firm wants to charge less on each product.

4.3 Continuity of Equilibrium around $s = 0$

Equilibrium prices need not be discontinuous around $s = 0$, but usually are. If the firm stocks a *single* product and s increases from 0 to something slightly positive, the equilibrium price jumps up from p^m to b. This discontinuity

[22] Hazard rate dominance is weaker than (and implied by) the monotone likelihood ratio property. The latter is commonly used. See for example Milgrom [17].

does not happen in a multiproduct context if there exists some good l which is advertised at a price $p_l < a_l$. This is because every consumer visits the store to buy l and is a shopper for all unadvertised products - which are consequently priced at their respective standard monopoly levels. Ruling this out, a necessary condition for avoiding discontinuity is that there exists an unadvertised product j with $p_j^m = a_j$. A sufficient condition is that there exists a second unadvertised product k that also has $p_k^m = a_k$. The reasoning is best illustrated through an example. Suppose $n = 2$, $v_1 \sim U\left[\frac{2}{3}, 1\right]$ with $p_1^m = \frac{2}{3}$, and $v_2 \sim U\left[\frac{1}{3}, 1\right]$ with $p_2^m = \frac{1}{2}$. Let $s \to 0_+$ and look for an equilibrium in which $p_1^E \to \frac{2}{3}$ and $p_2^E \to \frac{1}{2}$. In the limit everybody visits the store to buy product 1 and is therefore a shopper for product 2 - rationalising the expectation $p_2^E \to p_2^m = \frac{1}{2}$. Only two-thirds of consumers (those with $v_2 > \frac{1}{2}$) are shoppers for product 1, the rest being Diamond consumers. Nevertheless using these probabilities, the lefthand side of pricing condition (3) is strictly negative when evaluated at $p_1^E = \frac{2}{3}$ so this is an equilibrium. Intuitively even marginal consumers are very valuable to the firm since the distribution is shifted up a lot relative to marginal cost. This explains why the firm strictly prefers to sell to everybody when $s = 0$. Even when $s \to 0_+$, provided there are sufficiently many valuable marginal consumers in the store, the firm still wishes to sell to them all.

5 Discussion

> "large volume operations create an impression of lower prices... lots of advertising, and a wide assortment... are the accepted cues for lower prices; a small store is considered the strongest indicator of high prices... [loss leaders are] associated with high volume operations." (Brown 1969)

This quotation from a consumer survey captures the essence of the model very well. It is often argued that retailers with high volumes enjoy lower costs, which they pass on in lower prices. This paper suggests another channel - sample selection. A retailer always receives relatively high-valuation consumers from the population. This selection problem is less severe whenever the store sells a broader product range, has high-demand goods, and uses low-price advertising. The model also has many interesting implications.

5.1 Informative advertising

Large retailers often advertise (very) low prices on a selection of their products. Simester [20] argues that low-price advertising can signal a low marginal cost, and hence signal low prices across the whole store. Lal and Matutes [14] argue instead that unadvertised prices are high and unrelated to advertised prices.

My model gives a different perspective - advertising is informative. I demonstrate that low-price advertising on a few products, can act as a credible signal of store-wide low prices. This happens even when products are independent in both use and valuation. A retailer should clearly never advertise a product at a price above its standard monopoly level. Hence there is a natural dispersion between cheap advertised products and expensive unadvertised items. Nevertheless consumers are sophisticated. They use advertisements to make inferences about other prices, whilst recognising that advertised prices are typically much lower. Selling certain products below cost can also be a profitable strategy, though examples usually require large s and/or discrete value distributions

Example 9 *Two products with valuations iid $U[0,1]$. If $s = \frac{4}{5}$ and only one product is advertised, it is sold at a loss*

According to the model, products should be advertised at low prices when their demand is exogenously high. We saw in Proposition 7 that the most popular products are also the most expensive when they are unadvertised. Advertising them therefore has two benefits. Firstly, their price can be reduced and their profitability greatly increased. Secondly, because they are so popular, advertising them cheaply helps bring lots of new consumers to the store, which helps commit to significantly lower prices on the rest of the product range. There is lots of empirical evidence that products tend to fall in price when their demand is high (MacDonald [15], Warner and Barsky [23]). This might seem puzzling, but the model provides an intuitive rationale for this behaviour. The model also says that prices in general should be lower when aggregate demand is higher. Empirical evidence suggests this is also true - retailers seem to mark down items at weekends and during holiday seasons (Warner and Barsky [23]).

Kaul and Wittink [12] report that advertised products usually have more elastic demand curves. The model presented in this paper provides a simple explanation. If product 1 is unadvertised, its demand is equal to

$\int_{p_1}^{b_1} f_1(v_1) \Pr\left(\sum_{j=1}^{n} \max\left(v_j - p_j^E, 0\right) \geq s\right) dv_1$. If instead it is advertised, its demand equals

$$\int_{p_1}^{b_1} f_1(v_1) \Pr\left(\max(v_1 - p_1, 0) + \sum_{j=2}^{n} \max\left(v_j - p_j^E, 0\right) \geq s\right) dv_1$$

Demand for good 1 is more responsive to changes in p_1 when the price is advertised. The reason is that turnup decisions now depend upon p_1 rather than on just a fixed expectation p_1^E.

5.2 Product Line Spillovers

The model also suggests a different way of thinking about product range. Convenience retailers (as well as specialist/niche suppliers) are interpreted as being 'trapped' into charging unprofitably high prices. These retailers attract a small subset of consumers who tend to be interested in only a few items and therefore have very inelastic demand curves. Expecting very high prices, only consumers with especially high valuations turn up to the store. By contrast, stores with a broader product range and/or higher-demand items charge lower prices but earn proportionately higher profits (so doubling each α_j more than doubles profits). This works solely through demand elasticity and has nothing directly to do with demand, although the resulting price decreases do of course expand demand.[23] Consequently introducing a new product brings both a direct benefit (its own profit) and an indirect one (higher profit on other items). Nevertheless (assuming introducing a new product has some fixed cost) it is unclear whether optimal product range increases or decreases when moving from $s = 0$ to $s \to 0_+$. On the one hand a new product brings indirect benefits and is therefore more valuable. But on the other hand prices are high and profit on the new product is therefore low, so it is less valuable. Either effect may dominate, depending upon the precise example. Nevertheless the model may partly explain the recent drive towards one-stop shopping.

[23] For instance the comparative statics results were easiest to prove when $s \to 0_+$. But in that case demand for one product was essentially totally independent of all other prices. The comparative statics in n worked only at the margin.

6 Extensions

6.1 Correlation

It is simple to argue that no trade can occur in equilibrium if valuations are perfectly positively correlated and all prices are unadvertised. However advertising is very effective in the presence of correlation. To illustrate, suppose there are n products and $v_j = v$. v is drawn from $[a, b]$ and the standard monopoly price is $p^m > a$. If the firm advertises one product at price $p^m - s$, the other $n - 1$ (unadvertised) products cost p^m. This is an equilibrium because all consumers with $v \in [p^m, b]$ visit the store. When s is small, advertising is then a very effective way of committing to low prices.

When valuations are affiliated and s is small, the comparative statics results in Section 4.1 continue to hold. Letting $s \to 0_+$, we can rewrite (4)

$$|\epsilon_1| = \frac{1}{\Pr\left(\sum_{j=2}^{n} \max\left(v_j - p_j^E, 0\right) \Big| \, v_1 = p_1^E\right)} \tag{4*}$$

Therefore fixing $\left(p_2^E, p_3^E, ..., p_n^E\right)$ there is a unique equilibrium p_1^E that is increasing in other expected prices. Adding new products (and/or advertising old ones at lower prices) brings in more consumers who are marginal for product 1. This again makes demand for product 1 more elastic and pushes down its equilibrium price.

It is interesting to consider whether the monopolist should choose a niche or an eclectic product range. Niche products are rated similarly (you either love or hate everything), but eclectic products can be valued very differently. To be concrete, suppose *eclectic* products have valuations independently and uniformly distributed on $[a, b]$. Further, consumer i's valuation for the *niche* product j is $v_{ij} = x_i + y_j$. x_i is a consumer-specific taste parameter that is $U[a, b]$; y_j is product-specific and is $U[-\epsilon, \epsilon]$ where ϵ is small but at least an order of magnitude larger than s.[24] I assume $\frac{b}{2} > a$ and look for symmetric equilibria.

Imagine the firm must choose between stocking $\bar{n}$ niche products or $\bar{n}$ eclectic products - which should it choose? If p_{ni} and p_{ec} denote equilibrium

[24] Within the population, each niche product's valuation is approximately uniformly distributed on $[a, b]$. Since ϵ is small, the distribution is exactly uniform everywhere except very close to a and b.

prices under the two arrangements, then it is simple to demonstrate that

$$\begin{aligned} \frac{1 - F\left(p_{ni}\right)}{p_{ni} f\left(p_{ni}\right)} - 1 + \frac{1}{n} &= 0 \\ \frac{1 - F\left(p_{ec}\right)}{p_{ec} f\left(p_{ec}\right)} - 1 + F\left(p_{ec}\right)^{n-1} &= 0 \end{aligned}$$

(Note that as $n \to \infty$, price tends towards the standard monopoly price in both cases[25]) Sometimes an eclectic mix always delivers a lower price (and higher profit) regardless of $\bar{n}$. Otherwise (and this is usually the case) the optimal choice is to pick a niche product range when $\bar{n}$ is small, and an eclectic mix when $\bar{n}$ is large. This is illustrated in Figure 5 when valuations are uniformly distributed on $[0, 1]$. The crucial factor is always how many marginal consumers visit the store. When $\bar{n}$ is small and products are eclectic, few marginal consumers turn up - valuations for other products are dispersed and prices relatively high. But if products are niche, the marginal consumer for product 1 has a high x and therefore high valuations for other products - making it quite likely he will visit the store. Consequently when $\bar{n}$ is small, a niche selection delivers more marginal consumers and therefore lower price. As shown in the figure, this reverses when $\bar{n}$ is sufficiently large. Intuitively some consumers who are marginal for product 1 had quite a low x_i but a high y_1 draw. To persuade them to visit the store requires a large y_j draw on some other product j - and to achieve this requires many extra products to be added. On the other hand, with an eclectic selection, each new product is totally different and valuations are completely random - therefore it is much easier to attract new marginal consumers.

6.2 Substitutes

Imagine a situation in which the retailer sells several products but they are all substitutes - meaning that a consumer wants to buy at most one of them. It is simple to argue that without advertising, only a no-trade Diamond equilibrium exists.

An advertised price on one product acts as a signal about the prices of other substitute products. To illustrate, suppose there are two goods with

[25] It may seem surprising that this happens when products are niche. As $n \to \infty$, many consumers do not visit the store. However they all have low valuations (for everything) so the firm never wants to sell to them anyway. The key is that as $n \to \infty$, all marginal consumers turn up.

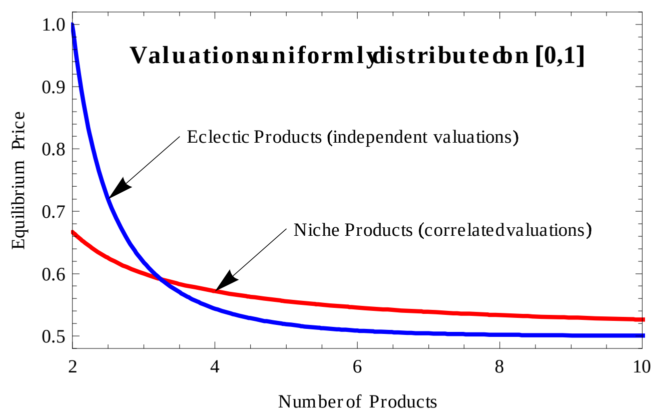

Figure 5: Niche versus eclectic product selections

valuations independently and uniformly distributed on $[0,1]$. Product 1 is unadvertised, and product 2 is advertised at price p_2^E. Suppose the firm charges slightly more for product 1 than consumers expect. Most consumers who intended to buy the product, still do. However some decide to buy product 2 instead. Clearly in equilibrium $p_1^E > p_2^E$ - if the reverse were true, the firm would benefit when people substituted from product 1 to product 2, and therefore definitely charge more for good 1 than expected. There is a unique equilibrium, with

$$p_1^E = \frac{1 + 4p_2^E + s}{3}$$

As expected $\frac{\partial p_1^E}{\partial p_2^E} > 0$ - an increase in p_2^E both increases the demand for product 1 *and* makes it less costly when consumers substitute from good 1 and good 2. So the firm wants to charge more for product 1. One problem is that since p_1^E is high, profit is only quasiconcave if $p_2^E \leq \frac{1}{3} - s$ (otherwise p_1^E is so high that the firm may benefit from charging less than expected). If we let $s \to 0$ and advertise product 2 at a price of $\frac{1}{3}$, then the unadvertised product's equilibrium price is $\frac{7}{9}$. This large difference occurs because otherwise the firm would have a very strong incentive to surprise consumers and charge more for good 1 than they were expecting. Nevertheless advertising has commitment value. This brief extension captures in a stylised way Footnote 34 (and surrounding discussion) of Ellison and Ellison [9].

6.3 Shoppers

Up to now I have assumed that everybody has the same strictly positive shopping cost. I also showed that the monopolist's problem could be rewritten in such a way that some consumers acted *as if* they were shoppers and had zero shopping cost (in the sense of Stahl [21]). I now briefly consider what happens if some consumers have an exogenously given zero search cost.

Suppose that a fraction γ of consumers have $s \equiv 0$, whilst the remaining $1-\gamma$ have $s \to 0_+$. Then the analogue of (4) is

$$|\epsilon_1| = \frac{1}{1-(1-\gamma)\prod_{j=2}^{n} F_j\left(p_j^E\right)} \tag{4**}$$

Intuitively a consumer is only not marginal if she is not a shopper in the exogenous sense (happens with probability $1-\gamma$) and also not a shopper endogenously (which happens with probability $\prod_{j=2}^{n} F_j\left(p_j^E\right)$). It is then apparent that the usual comparative statics continue to hold. In addition, if the proportion of shoppers increases, more of the population visits the store and the firm can infer less about valuations, and hence charges lower prices. These results generalise provided that the non-shoppers' s is not too large.

6.4 Competition

Fuller investigation of the effects of competition is deferred to another paper. In this short section I briefly demonstrate that in the absence of advertising, competition does not influence pricing decisions.

Suppose there are several retailers, and they each stock the same n products. There is no price advertising, and look for a symmetric equilibrium in which each firm charges the same prices. An individual retailer could make small adjustments to prices and not cause any consumers to search another retailer. Hence at the margin, a competitive firm has the same pricing incentives as the monopolist did in the earlier part of this paper. In particular there is always an equilibrium in which retailers charge the same prices as a multiproduct monopolit would when facing consumers with search cost s. This obviously has the flavour of the original Diamond [8] result.

7 Conclusion

Multiproduct firms help resolve the Diamond paradox, but the essential intuition about prices being high persists. Only consumers with high valuations turn up, and the store exploits them by raising prices. Consumers understand this, so in equilibrium price is high, and profits low. A firm with a broader and frequently-demanded product range has less incentive to hold consumers up, and so charges lower prices. The model therefore provides an intuitive explanation for why retailers have embraced one-stop shopping by expanding into (previously) non-core activities. Further, low-price advertising on a few products acts as a commitment to charging low prices across the whole store. Hence the model also provides a novel explanation for why firms sometimes use loss-leaders. In addition, prices may move countercyclically and products should be advertised at low prices when their demand is high.

A Appendix

Proof of equation (2): Let $T = \sum_{j=2}^{n} \max\left(v_j - p_j^E, 0\right)$. Differentiating $p_1 D_1$ with respect to p_1 gives $D_1 - p_1 f_1(p_1) \Pr\left(T + \max\left(p_1 - p_1^E, 0\right) \geq s\right)$ $(\star)$. This is continuous in p_1 for $p_1 < p_1^E + s$. Setting $p_1 = p_1^E$ gives (**2**). Now check that $p_1 = p_1^E$ is globally optimal. Charging $p_1^E + s$ (weakly) dominates any higher price, since when $p_1 \geq p_1^E + s$, $p_1 D_1 = p_1 [1 - F_1(p_1)]$ which is decreasing in p_1. We need to check that $p_1 D_1$ is strictly increasing in p_1 for $p_1 \in \left[a_1, p_1^E\right)$ and strictly decreasing in p_1 for $p_1 \in \left(p_1^E, p_1^E + s\right]$. There are two ways. (1) rewrite $(\star)$ as

$$[D_1 - p_1 f_1(p_1) \Pr(T \geq s)] + p_1 f_1(p_1) \left[\Pr(T \geq s) - \Pr\left(T + \max\left(p_1 - p_1^E, 0\right) \geq s\right)\right]$$

The second term is 0 when $p_1 \leq p_1^E$ and negative otherwise. Differentiating the first term with respect to p_1 gives $-f_1(p_1) \Pr\left(T + \max\left(p_1 - p_1^E, 0\right) \geq s\right) - [f_1(p_1) + p_1 f_1'(p_1)] \Pr(T \geq s)$. This is strictly negative provided $-2f_1(p_1) - p_1 f_1'(p_1) < 0$ (this is the second derivative of $p_1 [1 - F_1(p_1)]$). If that holds, the first term is strictly positive for $p_1 \in \left[a_1, p_1^E\right)$ and strictly negative for $p_1 \in \left(p_1^E, p_1^E + s\right]$. (2) $(\star)$ is proportional to

$$D_1 / f_1(p_1) - p_1 \Pr(T \geq s) + p_1 \left[\Pr(T \geq s) - \Pr\left(T + \max\left(p_1 - p_1^E, 0\right) \geq s\right)\right]$$

The second term is 0 when $p_1 \leq p_1^E$ and negative otherwise. Differentiating the first term with respect to p_1 gives $-\Pr\left(T + \max\left(p_1 - p_1^E, 0\right) \geq s\right) - \Pr(T \geq s) - \frac{D_1 f_1'(p_1)}{f_1(p_1)^2}$. Since $1 - F_1(v)$ is logconcave, this is weakly less than $-2\Pr(T \geq s) + \frac{D_1}{1 - F_1(p_1)}$, which is weakly less than $-2\Pr(T \geq s) + 1$, which is negative provided $\Pr(T \geq s) \geq 1/2$. When this holds, profit is strictly increasing in p_1 for $p_1 \in \left[a_1, p_1^E\right)$ and strictly decreasing in p_1 for $p_1 \in \left(p_1^E, p_1^E + s\right]$.

Proof of equation (3): Add and subtract $\Pr(T \geq s)\left[1 - F_1\left(p_1^E\right)\right]$ to the lefthand side of (2) and rearrange. ■

Claim 1: (3) has at most one equilibrium solution: (3) is continuous in p_1^E. Any *interior* equilibrium solution must be on the concave part of $p[1 - F_1(p)]$. (From part 1 of the first proof, if p_1^E is on a strictly convex part of $p[1 - F_1(p)]$ then the first order condition is negative for p_1 below but close to p_1^E. This means profit is falling in p_1 for p_1 close to p_1^E so the firm should charge less than p_1^E) On the interval $[a_1, b_1]$, $p[1 - F_1(p)]$ is either all strictly convex, all strictly concave, or there is a $\tilde{p}$ such that it is strictly concave for $p < \tilde{p}$ and strictly convex for $p > \tilde{p}$. (The second derivative is $p f_1(p)[-2/p - f'(p)/f(p)]$ and the term in square brackets strictly increases in p) When $p[1 - F_1(p)]$ is concave, $1 - F_1\left(p_1^E\right) - p_1^E f_1\left(p_1^E\right)$ is strictly decreasing in p_1^E. So if (**3**) has a corner solution, there are no other solutions where $p\left[1-F_1(p)\right]$ is concave. If (**3**) has an interior solution, at most one is where $p\left[1-F_1(p)\right]$ is concave. ■

Claim 2: quasiconcavity as $s \to 0$: If p_1^E is interior, it is clear from the first and previous proofs that a necessary and sufficient condition is that p_1^E be on a concave part of $p\left[1-F_1(p)\right]$. (If an increase in n reduces the equilibrium p_1^E, quasiconcavity then continues to hold) If $p_1^E = a_1$, generically $(\star)$ is strictly negative there. Since $(\star)$ is continuous, it is also strictly negative for $p_1 \in (a_1, a_1+s]$. ■

Claim 3: the same distribution implies the same price Suppose v_1 and v_2 have the same distribution $F(\cdot)$ but $p_1^E < p_2^E$. This implies:

$\Pr\left(\sum_{j=1}^n t_j \geq s \geq \sum_{j\neq 1} t_j\right) + \Pr\left(\sum_{j\neq 1} t_j \geq s\right)\left[1-F\left(p_1^E\right) - p_1^E f\left(p_1^E\right)\right] \leq 0$

$\Pr\left(\sum_{j=1}^n t_j \geq s \geq \sum_{j\neq 2} t_j\right) + \Pr\left(\sum_{j\neq 2} t_j \geq s\right)\left[1-F\left(p_2^E\right) - p_2^E f\left(p_2^E\right)\right] = 0$

If p_2^E is an equilibrium, then it and p_1^E are on the concave part of $p\left[1-F(p)\right]$. But then $1-F\left(p_2^E\right)-p_2^E f\left(p_2^E\right)$ is less than $1-F\left(p_1^E\right)-p_1^E f\left(p_1^E\right)$. Also $\Pr\left(\sum_{j\neq 2} t_j \geq s\right) > \Pr\left(\sum_{j\neq 1} t_j \geq s\right)$, which yields a contradiction. ■

Proof of Proposition 1: for simplicity ignore advertising. For each product j, consider the modified pricing inequality $1/\left|\epsilon_j\right| - \Pr\left(\sum_{k\neq j} t_k \geq s\right) \leq 0$ $(\diamondsuit)$, and call $\tilde{p}_j^E$ the solution when $\prod_{k\neq j} F_k\left(p_k^E\right) = 1/2$. Set $p_j^E = \tilde{p}_j^E \forall j$ and keep adding products until $\Pr\left(\sum_{k\neq j} \max\left(v_k - \tilde{p}_k^E, 0\right) \geq s\right) \geq \frac{1}{2}\forall j$.[26] Suppose this requires m products. The m inequalities of the form $(\diamondsuit)$ give a map $P \to P$ where $P = \times_{k=1}^m \left[p_k^m, \tilde{p}_k^E\right]$. We really care about m inequalities of the form (2), whose solutions are (weakly) below those that solve $(\diamond)$. So the former give a continuous map $P \to P$, therefore by Brouwer's fixed point theorem there exists an equilibrium (quasiconcavity is satisfied). This also holds if more products are added. ■

Proof of Corollary 3: Suppose $p_1^E > p_1^m$ and lower it slightly. This increases the number of visitors to the store, which strictly increases demand (and so profits) for all other goods. Now think about profits on good 1, and condition on a consumer's T. Show that for any T, profit on good 1 is decreasing in p_1^E. If $T \geq s$, profit on good 1 equals $p_1^E\left[1-F_1\left(p_1^E\right)\right]$

[26] Provided s is finite this happens in finite time. In particular, once it holds for an existing product, it holds for all new products as well.

which is strictly decreasing in p_1^E. If $T = z < s$ and $p_1^E + s - z \geq b_1$, profit on good 1 is zero. If $T = z < s$ but $p_1^E + s - z < b_1$, profit on good 1 is $p_1^E \left[1 - F_1\left(p_1^E + s - z\right)\right]$. The first derivative with respect to p_1^E is $1 - F_1\left(p_1^E + s - z\right) - p_1^E f_1\left(p_1^E + s - z\right)$. This is proportional to $\frac{1-F_1\left(p_1^E+s-z\right)}{f_1\left(p_1^E+s-z\right)} - p_1^E$ which is strictly decreasing in p_1^E. It is negative for $p_1^E = p_1^m$ and therefore negative for all higher p_1^E too. So profit on good 1 is strictly decreasing in p_1^E. ■

Proofs of Propositions 5-7: Let U be the set and cardinality of unadvertised products. p_k^E increases in p_j^E $\forall k \in U, \forall j \neq k$. So if $X = \times_{j=1}^{U}\left[p_j^m, b_j\right]$ the map $X \rightarrow X$ is increasing, so by Tarski's fixed point theorem there is a lowest (Pareto dominant) equilibrium. Suppose this equilibrium is non-Diamond, and denoted $\left(q_1^E, ..., q_U^E\right)$. If $q_j^E = p_j^m$ any j then q_j^E is still the equilibrium price when other prices are $\left(q_1^E, .., q_{j-1}^E, q_{j+1}^E, .., q_U^E\right)$ and when new products are added, advertised prices cut, or α parameters increased - so rule this out. *New product* - need to define q_{U+1}^E. If the new product is advertised, q_{U+1}^E is the advertised price; if it's unadvertised, use (2) and the vector $\left(q_1^E, ..., q_U^E\right)$ to solve for it. If $Y = \times_{j=1}^{U+1}\left[p_j^m, q_j^E\right]$ the map $Y \rightarrow Y$ is increasing so there is a lowest equilibrium, which is lower than before. This is because each of the U old first order conditions is strictly negative when evaluated at $\left(q_1^E, ..., q_U^E, q_{U+1}^E\right)$. *Advertised price* - lowering an advertised price reduces the U first order conditions and so produces a new lowest equilibrium. *Positive demand shocks* - an increase in α_1 just scales demand and thickness of demand for product 1, but it reduces the first order conditions on all (other) unadvertised products, exactly as in the other two proofs. At all times profit remains quasiconcave in the prices of existing unadvertised goods (see Claim 2).

Relative prices - suppose $\alpha_1 > \alpha_2$ but $p_1^E < p_2^E$. Equilibrium requires $\frac{1-F\left(p_1^E\right)}{p_1^E f\left(p_1^E\right)} + \prod_{j=2}^{n}\left[1 - \alpha_j + \alpha_j F\left(p_j^E\right)\right] \leq \frac{1-F\left(p_2^E\right)}{p_2^E f\left(p_2^E\right)} + \prod_{j\neq 2}^{n}\left[1 - \alpha_j + \alpha_j F\left(p_j^E\right)\right]$. But $\frac{1-F\left(p_1^E\right)}{p_1^E f\left(p_1^E\right)} > \frac{1-F\left(p_2^E\right)}{p_2^E f\left(p_2^E\right)}$ and $\alpha_2\left[F\left(p_2^E\right) - 1\right] > \alpha_1\left[F\left(p_1^E\right) - 1\right]$. ■

Proof of Section 4.2: Suppose all prices are interior, and focus on good 1. (2) slopes down at the equilibrium p_1^E; as when proving Propositions 5-7, we want to show that at the old p_1^E, the first order condition decreases in n and α_j $(j \neq 1)$ and increases in other prices. Divide (2) by $\Pr\left(\sum_{j=2}^{n} t_j \geq s\right)$; the ratio $\Pr\left(\sum_{j=1}^{n} t_j \geq s\right) / \Pr\left(\sum_{j=2}^{n} t_j \geq s\right)$ is crucial. C2 ensures it decreases in n. Next expand the top of the ratio by conditioning on each possible v_1. To show the ratio increases in p_2^E say, it is sufficient to show that terms of the form $\Pr\left(\sum_{j=2}^n t_j \geq z\right) / \Pr\left(\sum_{j=2}^n t_j \geq s\right)$ increase in p_2^E - which they do by condition C1. Now consider an increase in α_2. Take (2) and rewrite it as follows (D_z is the lefthand side of (2) given shopping cost z and the existence of all n products except product 2)

$$\left(1-\alpha_2+\alpha_2 \Pr\left(v_2 < p_2^E\right)\right) D_s + \alpha_2 \left[\Pr\left(v_2 \geq p_2^E + s\right) D_0 + \int_{p_2^E}^{p_2^E+s} f_2(z)\, D_{s+p_2^E-z} dz\right]$$

D_0 is negative; if D_s is negative, by C2 so is $C_z \forall z \in (0, s)$. So if p_1^E is interior and (2) is zero, $D_s > 0$ and $\Pr\left(v_2 \geq p_2^E + s\right) D_0 + \int_{p_2^E}^{p_2^E+s} f_2(z)\, D_{s+p_2^E-z} dz < 0$. Then an increase in α_2 decreases $\Pr\left(v_2 \geq p_2^E + s\right) D_0 + \int_{p_2^E}^{p_2^E+s} f_2(z)\, D_{s+p_2^E-z} dz$.[27]

Now for Proposition 8. When distributions are identical so are unadvertised prices (Claim 3), so there is a lowest equilibrium, and the equilibrium condition is decreasing in the single price. The relevant C2 condition also holds when moving from 2 to 3 products. When $n = 2$ the relevant C1 condition holds so prices are complementary. The α result for $n = 2$ is easiest to prove directly. (Assuming $b_1 - p_1^E > s$), $\Pr(t_1 + t_2 \geq s) / \Pr(t_2 \geq s)$ is

$$\frac{\alpha_1 \Pr(v_1 \geq s)}{\alpha_2 \Pr\left(v_2 \geq p_2^E + s\right)} + \alpha_1 \int_0^s h(v_1 = z) \frac{\Pr\left(v_2 \geq p_2^E + s - z\right)}{\Pr\left(v_2 \geq p_2^E + s\right)} + 1$$

which clearly decreases in α_2. Using techniques already used, it is also simple to show that very generally (not just for $n = 2$), more popular products are more expensive. ■

[27] Corner solutions require the proofs to be modified slightly. For example with α_2 comparative statics, if (2) is negative, then *potentially* $D_s < 0$. But the (2) is a weighted sum of two negative things, so stays negative when α_2 is increased.

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