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Strategic Pricing, Consumer Search and the Number of Firms

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We examine an oligopoly model where some consumers engage in costly non-sequential search to discover prices. There are three distinct price-dispersed equilibria characterized by low, moderate and high search intensity. The effects of an increase in the number of firms on search behaviour, expected prices, price dispersion and welfare are sensitive (i) to the equilibrium consumers' search intensity, and (ii) to the status quo number of firms. For instance, when consumers search with low intensity, an increase in the number of firms reduces search, does not affect expected price, leads to greater price dispersion and reduces welfare. In contrast, when consumers search with high intensity, increased competition results in more search and lower prices when the number of competitors in the market is low to begin with, but in less search and higher prices when the number of competitors is large. Duopoly yields identical expected price and price dispersion but higher welfare than an infinite number of firms.

1. INTRODUCTION

A question that has received much attention in economic theory is how the number of competitors and market outcomes are related. Despite the existence of work showing the contrary (see, *e.g.* Satterthwaite (1979), Rosenthal (1980)), the prediction that emerges from a market where firms interact in a Cournot fashion has come to dominate economic thought, namely, that an increase in the number of competitors leads to larger aggregate production, lower market price and improved market performance measured in terms of some social welfare criterion (Ruffin, 1971).

This paper challenges the generality of this belief by presenting an oligopoly model with consumer search where the equilibrium expected price may be constant, increasing or non-monotonic in the number of firms. We present an economy with two types of consumers: consumers who search for prices at no cost ("fully-informed"), and consumers who must pay a fixed search cost for each price quotation they observe ("less-informed").[1] All buyers have the same willingness to pay for the good and buy either a single unit or nothing at all. On the supply side of the market, there are N firms producing a homogeneous good at constant marginal cost. We analyse a one-shot simultaneous move game: firms set prices and consumers decide how many prices to search for at the same moment in time.

Our model is essentially an oligopolistic version of Burdett and Judd (1983) where some consumers search costlessly. In this model, consumers search using a fixed-sample-size search strategy, *i.e.* they choose the number of prices to observe before receiving any offer. Non-sequential search is appealing when consumers find it more advantageous to gather price information quickly (Morgan and Manning, 1985). This occurs when the search outcome is observed with delay and delay is costly. For instance, an MBA graduate looking for a job

1. The presence in the model of consumers who search costlessly captures the fact that some people have a negligible opportunity cost of time. Stahl (1989) argues that some consumers love shopping around. In the Internet age, fully-informed consumers can be seen as those who use electronic agents to search for prices on the Web (cf. Janssen and Moraga-González, 2000). In other contexts, one may think of these consumers as those who read consumer reports, magazines and newspapers.

typically applies to various firms instead of applying first to a single firm and deciding to apply to another firm only after getting an unacceptable offer.[2] A person moving his/her home overseas typically contacts several removal companies, which only quote prices after inspecting the house and ascertaining the technical aspects of the moving. A buyer searching for a mortgage to finance the purchase of a new house often applies to several banks for price quotations at once, rather than applying first to one bank and then to another bank in case the former quotes an unsatisfactory price. A manufacturer searching for another firm to subcontract the production of an input often asks subcontractors to submit offers together. Our work contributes to the understanding of how this type of oligopolistic markets function.

Irrespective of the number of firms, there is a maximum of three types of price-dispersed equilibria. These equilibria are characterized by the intensity with which less-informed consumers search. These consumers may search with high, moderate or low intensity in equilibrium. In the high search intensity equilibrium, less-informed consumers randomize between searching for one price and searching for two prices. This is the type of equilibrium that Burdett and Judd (1983) focused on most thoroughly. In the moderate search intensity equilibrium, less-informed consumers search once. This is similar to the situation analysed by Varian (1980), with the modification that search is endogenous here. Finally, in the low search intensity equilibrium, less-informed consumers randomize between searching for one price and not searching at all.[3] This equilibrium has been overlooked in previous research due to the fact that the first price quotation is typically assumed to be observed without incurring search cost. In this equilibrium, less-informed consumers expect prices to be so high that they are indifferent between searching for one price and not searching at all. In equilibrium, these expectations turn out to be correct.

For any type of search intensity, a symmetric price equilibrium exists only in mixed strategies.[4] The intuition is as follows. A firm has an incentive to charge low prices in an attempt to attract consumers who compare prices. We will refer to this force as a business-stealing effect. However, the fact that in any equilibrium some of the less-informed consumers search for only one price implies that firms always hold monopoly power over some of this type of consumers. This gives firms an incentive to charge high prices. We will refer to this force as a surplus-appropriation effect. A firm's equilibrium mixed strategy balances these two forces to maximize profits.

The three types of equilibria are studied in turn. For each type of equilibrium, we are interested in existence results and in the effects of an increase in the number of firms on expected price, price dispersion and welfare. Our first observation is that, in any equilibrium, the price distributions for N and $N+1$ firms cannot be ranked using the criterion of first-order stochastic dominance. The reason is that the two effects that shape the distribution of prices, *i.e.* the business-stealing and the surplus-appropriation effect, strengthen as N increases. In other words, firms respond to entry by decreasing the frequency with which they charge intermediate prices and by increasing the frequency with which they charge more extreme prices. Interestingly, the strengthening of the business-stealing effect weakens as the number of competitors in the market increases.

2. For a labour market model where workers search non-sequentially, see Fershtman and Fishman (1994).

3. We note that in all these possible equilibria, zero-cost consumers sample all the stores. Truly costly search only occurs if less-informed consumers search with high intensity because a fraction of these consumers trades-off the benefits from price comparison against the cost of an extra search. To keep the same terminology, though, we also speak of search equilibria in the cases of moderate and low search intensity even though less-informed consumers observe just one price in the former case and some of them may even not search at all in the latter case.

4. We concentrate on symmetric equilibria. Baye, Kovenock and de Vries (1992) study existence of asymmetric equilibria in Varian's model. They argue, however, that only the symmetric equilibrium survives an appealing equilibrium refinement.

We first study the moderate search intensity equilibrium. Here we obtain two results. Our first finding is that the equilibrium expected price increases in the number of firms. That is, the surplus-appropriation effect dominates the business-stealing effect for all parameters. Moreover, since in this equilibrium all less-informed consumers search for one price and acquire the product for sure, welfare turns out to be insensitive to N. Second, and more importantly, we note that, for given values of the other parameters, the moderate search intensity equilibrium fails to exist if the market accommodates a large enough number of competitors. Thus, under endogenous fixed-sample-size search, Varian's assumption that less-informed consumers search for exactly one price cannot be supported in equilibrium if there are sufficiently many firms in the market. When N becomes large, less-informed consumers find it beneficial to search less intensively, which motivates an examination of the low search intensity equilibrium.

When less-informed consumers search with low intensity in equilibrium, we obtain the following results. First, we find that, provided that the number of firms is sufficiently large, this type of equilibrium always exists, even if search costs are arbitrarily small. Hence, there are good reasons to investigate the properties of this type of equilibrium. Second, we find that if the number of competitors increases, the equilibrium expected price remains constant, price dispersion increases and welfare declines. The intuition is as follows. If less-informed consumers did not change their search intensity, expected price would increase in the number of firms, exactly for the same reasons as previously mentioned in the discussion of the moderate search intensity equilibrium. This would make it less attractive for these buyers to search for just one price, and thus consumers respond by economizing on search. As a result, fewer less-informed consumers remain in the market as N increases. Interestingly, when N rises consumers adjust their search behaviour in such a way that the business-stealing effect and the surplus-appropriation effect are equally strengthened. Consequently, an increase in the number of competitors does not alter the equilibrium expected price and only results in greater price dispersion. In the limit economy when N goes to infinity, firms randomize between marginal cost pricing and monopoly pricing.

We finally focus on the high search intensity equilibrium, where consumers randomize between obtaining one and two price quotations. We first show that if less-informed consumers did not change their search behaviour, the equilibrium expected price would increase in the number of firms. Once again, this partial result stems from the dominating influence of the surplus-appropriation effect. However, we find that in this case less-informed consumers' search incentives are non-monotonic in the number of firms. In particular, we note that less-informed consumers search more intensively when an additional firm enters the market and the number of competitors is small to begin with. By contrast, when the number of firms is already large enough, less-informed consumers search less as a result of entry. This pattern of consumer search arises because the strengthening of the business-stealing effect weakens as N increases, so the incentives to search more thoroughly are eventually eliminated and thereafter further entry discourages search. Interestingly, the incentives of the firms to increase prices and the incentives of the consumers to search initially more and then less interact in a manner such that the equilibrium expected price is non-monotonic with respect to the number of firms.

Another important result is that in an endogenous high search intensity equilibrium, expected price and price dispersion under duopoly are exactly identical to those under an infinite number of firms. We establish this result by showing that firms' incentives to set prices are similar under both market structures. The intuition is as follows. Note that an individual firm sells to a consumer either (i) because the consumer is less-informed and does not compare prices, or (ii) because he/she is less-informed, does compare prices and the firm's price is lower than the price of just one competitor, or (iii) because the consumer searches costlessly and the firm quotes the lowest price in the market. With only two firms in the market, being the lowest-price firm is, however, identical to charging a price lower than just one competitor. But this is also true when there are infinitely many firms in the market as the chance of being the lowest-price firm is negligible. Thus, in the limit economy a firm behaves as if it was competing with just one other firm for only a fraction of the less-informed consumers, which explains the similarity to the duopoly case. Consumers understand the firms' incentives to set prices and adjust their search behaviour in a way such that the equilibrium price distributions coincide in those two settings. The results derived when consumers search with high intensity suggest that two firms are enough for a competitive outcome and that three or more firms yield lower expected prices.[5]

The majority of the consumer search literature has analysed competitive models (*e.g.* Reinganum (1979), MacMinn (1980), Burdett and Judd (1983), Rob (1985), Bester (1988, 1994), Fershtman and Fishman (1992, 1994), Benabou (1993), McAfee (1995), Burdett and Coles (1997)). The main contribution of our paper to this body of work is to show how oligopolistic pricing and the intensity of consumer search are quite sensitive to the number of firms in the market. Some papers have dealt with consumer search in oligopolistic markets but they have used the unsatisfactory assumption that consumers know the realized market distribution of prices before they search, even though they cannot tell which firm charges a particular price (*e.g.* Braverman (1980), Salop and Stiglitz (1982), Stiglitz (1987)). As Stahl (1996) argues, the use of this assumption *à la* Stackelberg gives consumers quite a bit of information before they engage in actual search, without further theoretical justification.

The interaction between consumer search and firm pricing in an oligopoly setting is also examined in work by Stahl (1989, 1996).[6] The differences between the approach taken by Stahl and ours concern the search behaviour of the less-informed consumers and the structure of search cost. His papers consider sequential search and buyers obtain the first price quotation for free. In contrast, we deal with fixed-sample-size search in a model where every search (including the first one) is costly.[7] As it is well known, both sequential and non-sequential search rules have their own advantages and disadvantages (see Morgan and Manning, 1985). Compared to Stahl's work, we derive the following interesting and new results. First, changes in the parameter values lead to different degrees of search intensity in equilibrium; in this sense, our paper offers an explanation for why consumers search more in some markets than in other markets. For example, a movement from duopoly to triopoly leads to more actual search in an equilibrium with high search intensity. Second, for fixed values of the parameters multiple equilibria may exist; this can explain price differences across seemingly identical markets. Third, in Stahl's model the equilibrium expected price increases in the number of firms and approaches the monopoly price when N converges to infinity. This result, which is in line with Diamond (1971), does not arise in the moderate search intensity equilibrium of our model. The reason is that even though the equilibrium expected price increases initially in N, eventually a low search intensity equilibrium emerges and then expected price ceases to rise and further entry results only in greater price dispersion. This difference in results is due to the fact that in our setting also the first price quotation is costly to obtain. Moreover, in our high search intensity equilibrium, expected price is non-monotonic, does not converge to the monopoly price, and prices remain dispersed in the limit economy. The reason

5. As all consumers acquire the product in this type of equilibrium, welfare is higher the less intensive consumers' search activity is. Since consumers search more intensively under triopoly than under duopoly, and since they search more under an infinite number of firms than under duopoly, it follows that, among these three cases, welfare attains its maximum under duopoly.

6. Recently, Baye and Morgan (2001) have studied an oligopolistic search model to analyse the effects of information intermediation on the Internet.

7. On the basis of our analysis, we can say that the comparative statics results we obtain for the low and moderate search intensity equilibrium would remain valid in a sequential search model with costly initial search. However, a high search intensity equilibrium would not exist if consumers did search sequentially. The reason is that, under this type of consumer search, equilibrium firm pricing discourages consumers to search beyond the first price.

for this is that a fraction of less-informed consumers keeps comparing prices in equilibrium for any N.

The rest of the paper is organized as follows. Section 2 describes the model and shows that only the three consumer search strategies mentioned above can be part of a Nash equilibrium. The comparative statics analysis of an increase in the number of firms when consumers search with moderate, low and high intensity is given in Sections 3, 4 and 5, respectively. Section 6 characterizes existence of the three types of equilibria in terms of exogenous parameters. We conclude in Section 7. Proofs are in the Appendix.

2. THE MODEL AND PRELIMINARY RESULTS

Consider a market for a homogeneous good. On the demand side of the market, there is a mass of consumers, who wish to purchase one unit of the good at most. The mass of consumers is normalized to one without loss of generality. A fraction $\lambda \in (0, 1)$ of the consumers searches for prices costlessly. We will refer to these consumers as fully-informed consumers. The rest of the consumers, a fraction $1 - \lambda$, must pay search cost $c > 0$ to observe a price quotation. These consumers, referred to as less-informed consumers, may decide to obtain several price quotations, say n, in which case they incur a total search cost equal to nc. All consumers are fully rational, *i.e.* the informed consumers buy the good from the lowest-price store, while the less-informed consumers acquire it from the store with the lowest price in their sample, provided that they obtain a non-negative surplus.[8] The maximum price any consumer is willing to pay for the good is $v > c$.

On the supply side of the market there are $N \geq 2$ firms.[9] These firms produce the good at constant returns to scale and their identical unit production cost is normalized to zero, without loss of generality. In this economy, welfare W (total surplus) is simply measured by the multiplication of willingness-to-pay v by the number of buyers who acquire the product in equilibrium minus search costs. Thus, two types of inefficiencies may arise in our setting: one appears when consumers search excessively and the other when buyers exit the market.[10]

Firms and consumers play a simultaneous move game. An individual firm chooses its price taking price choices of the rivals as well as consumers' search behaviour as given. A firm i's strategy is denoted by a distribution of prices $F_i(p)$. Let $F_{-i}(p)$ denote the vector of prices charged by all the firms other than i. Then $\pi_i(p_i, F_{-i}(p))$ denotes the (expected) profit to firm i from charging price p_i given rivals' strategies. Less-informed consumers form conjectures about the distribution of prices in the market and decide how much to search. A less-informed consumer's strategy is thus a probability distribution over the set $\{0, 1, 2, \ldots, N\}$. Let μ_n denote the probability with which a less-informed consumer searches for n price quotations. We will only consider symmetric equilibria (cf. footnote 4). A symmetric equilibrium is a pair $\{F(p), \{\mu_n\}_{n=0}^N\}$ such that (a) $\pi_i(p_i, F_{-i}(p))$ is equal to a constant $\overline{\pi}$ for all p in the support of $F(p)$, $\forall i$, (b) $\pi_i(p_i, F_{-i}(p)) \leq \overline{\pi}$ for all p_i, $\forall i$ and (c) $\{\mu_n\}_{n=0}^N$ is an optimal search behaviour for the less-informed consumers given that their conjectures about the price distribution actually used by the firms are correct.

Our first result shows that only three possible consumer search strategies can be part of an equilibrium. Namely, less-informed consumers may search (i) with low intensity, (ii) with moderate intensity, or (iii) with high intensity. Low search intensity arises when $0 < \mu_0 < 1$ and $\mu_0 + \mu_1 = 1$. Moderate search intensity refers to the case where $\mu_1 = 1$. Finally, we say that

8. We assume that, once price information has been processed by the consumers, "ordering" the good involves no additional costs.

9. We shall refer to the case $N \to \infty$ as the "limit economy", or as the "fully competitive" case.

10. We note that if demand were downward sloping, further dead-weight losses would arise even if all consumers were served.

less-informed consumers search with high intensity when $0 < \mu_2 < 1$ and $\mu_1 + \mu_2 = 1$. Next we show that any other search strategy cannot be part of an equilibrium.

Lemma 1. *Symmetric equilibria where* (i) $\mu_0 = 1$, *or* (ii) $\mu_n = 1$, *for some* $n = 2, 3, \ldots, N$, *or* (iii) $\mu_n > 0$, *for some* $n = 3, 4, \ldots, N$, *or* (iv) $\mu_0 + \mu_1 + \mu_2 = 1$ *with* $\mu_0 > 0$, *and* $\mu_2 > 0$ *do not exist.*

This result shows that less-informed consumers never search for more than two prices in equilibrium. This is because all less-informed consumers incur the same cost to discover prices and search cost is linear in price quotations.

We are interested in the different symmetric equilibria that may emerge in this economy and in the comparative statics effects of an increase in the number of competitors on firms' prices, price dispersion and welfare. A feature that is common to all three possible equilibria of our model is that less-informed consumers search for one price with strictly positive probability, *i.e.* $\mu_1 > 0$. This implies that any equilibrium necessarily exhibits price dispersion.

Lemma 2. *Suppose less-informed consumers search with low, moderate or high intensity. Then, if* $F(p)$ *is the price distribution of a symmetric equilibrium, it must be atomless. Hence, there is no symmetric equilibrium where firms employ a pure strategy.*

The intuition behind the fact that firms randomize prices in any equilibrium is found in the observation that firms intend to extract surplus from different groups of consumers, resulting in a business-stealing effect and in a surplus-appropriation effect as discussed in the Introduction. A firm's mixed strategy is intended to balance the benefits accruing from charging low and high prices.

In the following three sections we will consider the cases of low, moderate and high search intensity separately. We will characterize when these equilibria exist and analyse the impact of an increase in the number of competitors on market variables. An increase in the number of firms has, presumably, two effects. First, it directly influences the supply side of the market by bringing more competitors together. Second, it may have an effect on the demand side of the market by possibly altering the consumer search intensity. As a presentation strategy, we have chosen to consider first a context where consumer search intensity is exogenous and to incorporate endogenous consumer search later. This strategy is useful for two reasons: First, it helps disentangle the different mechanisms at work when considering the effects of an increase in the number of firms on the relevant market variables. This allows us to show the detrimental effects of non-optimal consumer search behaviour. Second, it enables us to compare our results with two branches of the literature: earlier papers with exogenous consumer search rules and later articles with endogenous consumer search. Section 6 brings together the results of Sections 3 and 5 and discusses which equilibria exist in different parameter regions.

3. MODERATE SEARCH INTENSITY EQUILIBRIUM

We first consider the case of moderate search intensity, which is essentially Varian's model with endogenous non-sequential search. The main innovative result in this section is that the equilibrium of Varian's model disappears when there are sufficiently many competitors and consumer search is endogenous.

Exogenous consumer search

Given that in the moderate search intensity equilibrium less-informed consumers search for one price quotation, the expected pay-off to firm i from charging price p_i when competitors choose a random pricing strategy according to the cumulative distribution function $F(p)$ is

$$\pi_i(p_i, F(p)) = p_i\left[\frac{1-\lambda}{N} + \lambda(1-F(p_i))^{N-1}\right]. \tag{1}$$

Equation (1) can be interpreted as follows. Firm i obtains a per consumer profit of p_i. The expected demand faced by firm i stems from the different groups of buyers. Firm i attracts the fully-informed consumers only when it is the lowest-price store, which happens with probability $(1 - F(p_i))^{N-1}$. Less-informed consumers observe only one price. Firm i attracts these consumers when it happens that they observe firm i's price, which occurs with probability $1/N$.

In equilibrium, a firm must be indifferent between charging any price in the support of $F(\cdot)$. The maximum price a firm will ever charge is v since no buyer observing a price above his/her willingness-to-pay will acquire the good. Moreover, the upper bound of the price distribution cannot be lower than v because a firm charging a different upper bound would gain by slightly raising its price. Thus, it must be that $F(v) = 1$, and $F(p) < 1$, for all $p < v$. Characterization of a mixed strategy equilibrium requires that any price in the support of $F(\cdot)$ must satisfy $\pi_i(p_i, F(p)) = \pi_i(v)$, *i.e.*

$$p_i\left[\frac{(1-\lambda)}{N} + \lambda(1-F(p_i))^{N-1}\right] = \frac{(1-\lambda)v}{N}.$$

Lemma 3 solves for the equilibrium price distribution.

Lemma 3. *Suppose less-informed consumers search with moderate intensity. Then, there exists a unique symmetric equilibrium price distribution*

$$F(p; N) = 1 - \left(\frac{(1-\lambda)(v-p)}{N\lambda p}\right)^{\frac{1}{N-1}}$$

with support $[\underline{p}(N), v]$, *where* $\underline{p}(N) = (1-\lambda)v/(\lambda N + (1-\lambda))$.

Consider next the effects of an increase in the number of firms on the equilibrium price distribution. Our first observation is that the equilibrium price distributions $F(p; N)$ and $F(p; N+1)$ cannot be ranked using the criterion of first-order stochastic dominance. To establish this result the following claims about the price distribution given in Lemma 3 are useful.

Claim 3.1. *The lower bound of* $F(p; N)$ *is decreasing in* N, i.e. $\underline{p}(N) > \underline{p}(N+1)$.

This is because the business-stealing effect strengthens as N rises.

Claim 3.2. *There exists a single price* $\hat{p} = \frac{(1-\lambda)v}{\left(\frac{N}{N+1}\right)^{N-1}\lambda N + (1-\lambda)}$ *satisfying* $F(\hat{p}; N) = F(\hat{p}; N+1)$*; moreover, for all* N, $\underline{p}(N) < \hat{p} < v$.

Claim 3.3. $F(p; N) > F(p; N+1)$ *for all* $p \in (\hat{p}, v)$.

This is because the surplus-appropriation effect also strengthens as N increases.

These three claims are illustrated in Figure 1 where $v = 1$, $\lambda = 0{\cdot}5$ and $N = 2$. An implication of these claims is that $F(p; N)$ and $F(p; N+1)$ cannot be ranked using the first-order stochastic dominance criterion.[11] Firms respond to entry by decreasing the frequency with

11. This contrasts sharply with Rosenthal (1980) where an increase in N shifts the entire price distribution downwards. Rosenthal's model and Varian's model are similar in that both deal with competition in a market divided

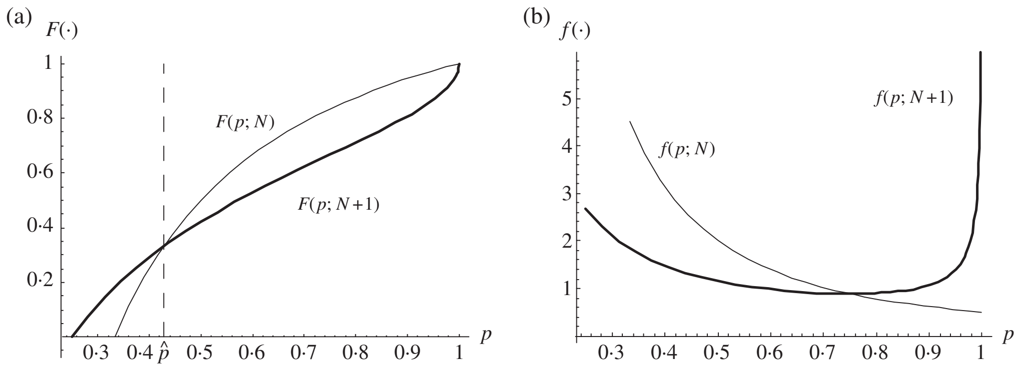

FIGURE 1

(a) Price distributions for N and $N+1$ firms; (b) frequency of prices for N and $N+1$ firms

which they charge intermediate prices and by increasing the frequency with which they charge low and high prices.

Once we have shown that a relationship of first-order stochastic dominance cannot be established between $F(p;N)$ and $F(p;N+1)$, a natural question to ask is whether expected price increases or decreases when the number of firms in the market rises. The next result answers this question:

Proposition 1. *Let $F(p;N)$ be the equilibrium price distribution under moderate search intensity, given in Lemma* 3. *Then, keeping the search behaviour of the consumers fixed, expected price increases in N and approaches v as N converges to infinity.*

From an economic point of view, Proposition 1 shows that the strength of the surplus-appropriation effect *vis-à-vis* the business-stealing effect increases as the number of firms in the market rises. In other words, as N increases, firms attach relatively less importance to the fully-informed consumers. This is because the probability of attracting a less-informed consumer falls less rapidly than the probability of attracting a fully-informed one. We note that this is true irrespective of the fraction of fully-informed consumers and the status quo number of firms in the market.

The question that arises now is whether less-informed consumers find it optimal to keep searching for one price when N increases. We shall see that the answer to this question is negative if N is large enough.

Endogenous consumer search

We now turn to consider optimal search behaviour of the less-informed consumers. A mixed strategy according to the cumulative distribution function $F(p;N)$ specified in Lemma 3 is an actual equilibrium with moderate search intensity if the following two additional conditions hold:

into a segment of well-informed buyers and a segment of less-informed buyers. The difference is that in Varian's model the size of the less-informed consumer segment is fixed so that the number of captive consumers per firm decreases as N rises. In contrast, in Rosenthal's model the size of the captive consumer segment per firm does not change when N rises and thus the size of the contestable market relative to the entire market decreases as N increases. This explains why the business-stealing effect is less important in Rosenthal's model and thus why the price distribution with $N+1$ firms dominates in a first-order stochastic sense the distribution with N firms in his analysis.

Condition 3.1. $v - E[p; N] - c > 0$.

Condition 3.2. $v - E[\min\{p_i, p_j\}; N] - 2c < v - E[p; N] - c$, for all $i, j \in N$.

Condition 3.1 simply states that less-informed consumers must obtain, *ex ante*, a positive expected surplus from searching for one price and acquiring the good. Condition 3.2 requires that these consumers find it unprofitable to search more intensively.[12]

Using the variable change

$$z = \left(\frac{(1-\lambda)(v-p)}{\lambda N p}\right)^{\frac{1}{N-1}}.$$

Condition 3.1 can be rewritten as

$$1 - \int_0^1 \frac{1}{1 + \frac{\lambda}{1-\lambda} N z^{N-1}} dz > \frac{c}{v}. \tag{2}$$

Let us denote the L.H.S. of (2) as $\Phi(1, N, \lambda)$.[13] Note that $\Phi(1, N, \lambda) = 1 - E[p; N]/v$.

Condition 3.2 requires that $E[p; N] - E[\min\{p_i, p_j\}; N] < c$. Or, in other words, that

$$\int_{\underline{p}(N)}^{v} 2pF(p; N) f(p; N) dp - E[p; N] < c.$$

This inequality can be rewritten as

$$E[p; N] - 2\int_{\underline{p}(N)}^{v} p f(p; N) \left(\frac{(1-\lambda)(v-p)}{\lambda N p}\right)^{\frac{1}{N-1}} dp < c.$$

We can introduce a similar variable change and proceed as in the proof of Proposition 1 to rewrite Condition 3.2 as follows:

$$\int_0^1 \frac{2z-1}{1 + \frac{\lambda}{1-\lambda} N (1-z)^{N-1}} dz < \frac{c}{v}. \tag{3}$$

Let $\rho(1, N, \lambda)$ denote the L.H.S. of (3).[14] Using equations (2) and (3) we can state the following result:

Proposition 2. *Let* $\Phi(1, N, \lambda) \geq \frac{c}{v} \geq \rho(1, N, \lambda)$. *Then an endogenous moderate search intensity equilibrium exists where less-informed consumers search for one price and firms randomly select prices from the set* $[(1-\lambda)v/(\lambda N + (1-\lambda)), v]$ *according to the cumulative distribution function*

$$F(p; N) = 1 - \left(\frac{(1-\lambda)(v-p)}{N\lambda p}\right)^{\frac{1}{N-1}}.$$

There is a unique symmetric endogenous moderate search intensity equilibrium.

We now provide a discussion on the existence of a moderate search intensity equilibrium. For this type of equilibrium to exist, the set of parameters given in Proposition 2 must be

12. We note that Condition 3.2 is sufficient to guarantee that less-informed buyers do not want to search for two, three or more prices. This is because the expectation of the minimum of a random sample of size *m* decreases in *m* at a decreasing rate.

13. The number 1 in the arguments of $\Phi(\cdot)$ stands for $\mu_1 = 1$.

14. Again, the number 1 in the arguments of $\rho(\cdot)$ stands for $\mu_1 = 1$.

non-empty. There are two alternative ways of looking at this question. First, one may wonder whether for any given number of firms N, there exist values of c, v and λ such that an equilibrium with moderate search intensity exists. To answer this question, it suffices to show that $\Phi(1, N, \lambda) \geq \rho(1, N, \lambda)$ and $\Phi(1, N, \lambda) > 0$. The first inequality holds if and only if

$$2\int_0^1 \frac{1-z}{1+\frac{\lambda}{1-\lambda}Nz^{N-1}}dz \leq 1, \tag{4}$$

which is always satisfied because the function $1-z$ is an upper bound of the integrand in (4), for all $N < \infty$. The second inequality follows directly from the definition of $\Phi(1, N, \lambda)$.

Second, one may wonder whether for given values of c, v and λ, there exists a moderate search intensity equilibrium for all N. It turns out that the answer to this question is negative: when N becomes sufficiently large, this type of equilibrium fails to exist. This follows from the proof of Proposition 1 where it is shown that $\Phi(1, N, \lambda)$ monotonically declines in N and converges to zero as N approaches infinity. What happens as N increases is that, if less-informed consumers keep searching for one price with probability one, expected prices tend to the monopoly price and, eventually, the condition $v - E[p; N] - c > 0$ is violated. The following result summarizes:

Proposition 3. (i) *Let* $\{F(p; N), \mu_1^* = 1\}$ *and* $\{F(p; N+1), \mu_1^* = 1\}$ *be equilibria with endogenous moderate search intensity as described in Proposition* 2. *Then,* $E[p; N+1] > E[p; N]$. (ii) *For any parameters c, v and λ there exists a critical number of firms $\widetilde{N}$ such that for all $N \geq \widetilde{N}$ an equilibrium with endogenous moderate search intensity does not exist.*

Notice that in our model every consumer who acquires the good at a firm generates a total surplus of v, which is somehow distributed between the firm and the consumer. Since in a moderate search intensity equilibrium all consumers acquire the product, and since their search intensity does not change when the number of firms increases, it follows that welfare is insensitive to the number of rivals.[15]

Proposition 3 illustrates that the equilibrium in Varian's model of sales fails to exist when the number of firms in the market is large and consumers search in an optimal fashion. In the next section, we shall see that when $N > \widetilde{N}$, consumers will find it optimal to search less intensively than in a moderate search intensity equilibrium. As a result, starting from a situation where consumers search with moderate intensity, expected price will never converge to the monopoly price as N approaches infinity due to consumers' economizing behaviour (see also Section 6).

4. LOW SEARCH INTENSITY EQUILIBRIUM

When less-informed consumers randomize between searching for one price quotation and not searching at all, the expected pay-off to firm i is

$$\pi_i(p_i, F(p)) = p_i\left[\frac{(1-\lambda)\mu_1}{N} + \lambda(1-F(p_i))^{N-1}\right]. \tag{5}$$

The economic interpretation of equation (5) is analogous to that of equation (1). The only difference is that the number of less-informed consumers who are now active is $(1-\lambda)\mu_1$, rather than $1-\lambda$.

15. We have been unable to analytically characterize the behaviour of price dispersion with respect to the number of firms in this setting. However, a numerical analysis reveals that the influence of N on price dispersion is sensitive to the size of the fraction of fully-informed consumers λ.

Exogenous consumer search

A similar analysis as in the case of moderate search yields the following result.

Lemma 4. *Suppose less-informed consumers search with low intensity. Then, for any fixed search intensity* $0 < \mu_1 < 1$ *there exists a unique symmetric equilibrium price distribution*

$$F(p; N, \mu_1) = 1 - \left(\frac{\mu_1(1-\lambda)(v-p)}{N\lambda p}\right)^{\frac{1}{N-1}}$$

with support $[\underline{p}(N, \mu_1), v]$ *where* $\underline{p}(N, \mu_1) = (1-\lambda)\mu_1 v/(\lambda N + (1-\lambda)\mu_1)$.

As before, $F(p; N, \mu_1)$ and $F(p; N+1, \mu_1)$ cannot be ranked using the first-order stochastic dominance criterion because both the business-stealing effect and the surplus-appropriation effect strengthen as N increases. However, holding μ_1 constant, it is readily seen that expected price increases in the number of firms and converges to the monopoly price.[16]

Endogenous consumer search

We now include optimal search behaviour of the less-informed consumers in the analysis. A mixed strategy according to the cumulative distribution function $F(p; N, \mu_1)$ specified in Lemma 4 is indeed an equilibrium with low search intensity if the following two additional conditions are met:

Condition 4.1. $v - E[p; N] - c = 0$.

Condition 4.2. $v - E[\min\{p_i, p_j\}; N] - 2c < 0$, for all $i, j \in N$.

Condition 4.1 states that the less-informed consumers must be indifferent between searching for one price and not searching at all. Condition 4.2 requires that no less-informed consumer finds it profitable to search more intensively. An argument similar to that given in the proof of Lemma 1(iv) can be used to show that if Condition 4.1 holds, then Condition 4.2 holds too. To show existence of a low search intensity equilibrium, note that for any $0 < \mu_1 < 1$ and for any N, the optimal strategy for the firms is given by $F(p; N, \mu_1)$ as defined in Lemma 4. Given this solution, Condition 4.1 can be rewritten as:

$$1 - \int_0^1 \frac{1}{1 + \frac{\lambda}{\mu_1(1-\lambda)} N z^{N-1}} dz = \frac{c}{v}. \tag{6}$$

Whenever equation (6) gives a solution for μ_1, we can substitute it into $F(p; N, \mu_1)$ to obtain the equilibrium mixed strategy for the firms. Denote the L.H.S. of equation (6) as $\Phi(\mu_1, N, \lambda)$.

16. The proof of this result is analogous to that of Proposition 1. Observe that expected price can now be written as

$$E[p; N, \mu_1] = \int_{\underline{p}(N,\mu_1)}^{v} \frac{\mu_1(1-\lambda)v}{\lambda p N(N-1)} \left(\frac{\mu_1(1-\lambda)(v-p)}{\lambda N p}\right)^{\frac{2-N}{N-1}} dp.$$

The result follows from using the change of variable

$$z = \left(\frac{\mu_1(1-\lambda)(v-p)}{\lambda N p}\right)^{\frac{1}{N-1}}$$

and taking the same steps as in the proof of Proposition 1.

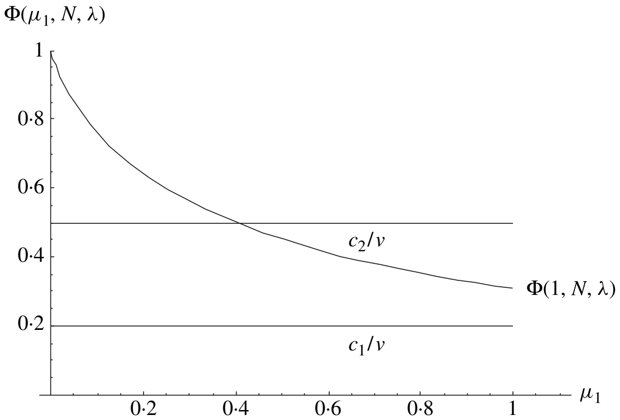

FIGURE 2
Low search intensity equilibrium ($N = 2, \lambda = 1/3$)

Note that setting $\mu_1 = 1$ in the L.H.S. of (6) yields the L.H.S. of (2). We can easily establish the following claims:

Claim 4.1. $\frac{d\Phi(\mu_1, N, \lambda)}{d\mu_1} < 0$.

Claim 4.2. $\lim_{\mu_1 \to 0} \Phi(\mu_1, N, \lambda) = 1$.

This follows from the observation that $F(p; N, \mu_1) \to 1$ as $\mu_1 \to 0$. The economic interpretation of this is that when $\mu_1 \to 0$, only fully-informed consumers are left over in the market and firms must employ marginal cost pricing.

These two claims imply that the L.H.S. of (6) is a smooth and declining function of μ_1 and, since $\Phi(1, N, \lambda) > 0$, it is always positive. In Figure 2 we have represented $\Phi(\cdot)$ as a function of μ_1, for particular parameter values. The flat lines represent the R.H.S. of (6) for different levels of search cost. For a given search cost, say c_2, an endogenous low search intensity equilibrium is given by some search intensity $\mu_1^* \in (0, 1)$ and a price distribution $F(p; N, \mu_1^*)$ where μ_1^* is determined by the intersection of the curve $\Phi(\cdot)$ and the line c_2/v. The claims above imply that if an equilibrium exists, it is unique. Note that if the search cost is sufficiently low, say c_1, then an equilibrium with low search intensity fails to exist.
The following proposition summarizes these findings:

Proposition 4. *Let* $1 > \frac{c}{v} > \Phi(1, N, \lambda)$. *Then an equilibrium exists where the less-informed consumers search with a low intensity* $\mu_1^* \in (0, 1)$ *given by the solution to* $\Phi(\mu_1, N, \lambda) - c/v = 0$, *and firms randomly select prices from the set* $\left[\frac{(1-\lambda)\mu_1^* v}{\lambda N + (1-\lambda)\mu_1^*}, v\right]$ *according to the cumulative distribution function*

$$F(p; N, \mu_1^*) = 1 - \left(\frac{\mu_1^*(1-\lambda)(v-p)}{N\lambda p}\right)^{\frac{1}{N-1}}.$$

There is a unique symmetric endogenous low search intensity equilibrium.

We are now in a position to examine the comparative statics effects of an increase in the number of firms when less-informed consumers search optimally. Our first observation pertains to the existence of a low search intensity equilibrium. The following claim proves useful in this respect:

Claim 4.3. $\Phi(\mu_1, N, \lambda) > \Phi(\mu_1, N+1, \lambda)$ *and* $\lim_{N\to\infty} \Phi(\mu_1, N, \lambda) = 0.$

Since $\Phi(\mu_1, N, \lambda) = 1 - E[p; N, \mu_1]/v$, this follows from the observation above that, holding constant the search intensity of the less-informed consumers, $E[p; N, \mu_1] < E[p; N+1, \mu_1]$ and $E[p; N, \mu_1] \to v$ as $N \to \infty$ (cf. footnote 16).

Claims 4.1–4.3 enable us to state that for any set of parameters v, c and λ, there exists a critical number of firms such that for all N beyond such a number an endogenous low search intensity equilibrium exists. We note that this critical number of firms is precisely $\widetilde{N}$, that is, the number of firms for which a moderate search intensity equilibrium fails to exist (see Proposition 3). We note that the low search intensity equilibrium has been disregarded by the search literature, and in the present setting it exists, even for arbitrarily low search costs, provided that N is large enough. This equilibrium is a natural extension of the model studied in Varian (1980) when consumers search optimally and all price quotations are costly to obtain.

Our second observation is that in an endogenous low search intensity equilibrium expected price is insensitive to changes in N. To see this, note that Condition 4.1 above must be met in equilibrium. This implies that expected price must be equal to $v - c$ for all N. To keep expected price constant as N increases, the probability μ_1^* must decrease, which offsets the upward pressure on prices caused by an increase in N. This is easily seen by looking at the expression for the expected price in this case

$$E[p; N, \mu_1] = \int_0^1 \frac{v}{1 + \frac{\lambda}{\mu_1(1-\lambda)} N z^{N-1}} dz$$

and noting that, $E[p; N, \mu_1]$ increases in μ_1.

Our third remark is that $F(p; N+1, \mu_1^*)$ is a mean-preserving spread of $F(p; N, \mu_1^*)$ under endogenous low search intensity. This follows from the following two remarks. First, since μ_1^* falls as a result of an increase in N, inspection of the lower bound of the equilibrium price distribution

$$\underline{p}(N) = \frac{v}{1 + \frac{\lambda N}{(1-\lambda)\mu_1^*}}$$

reveals that an increase in the number of firms enlarges the set of prices over which firms mix. Second, there exists a unique value of p, denoted by $\widetilde{p}$, such that $F(\widetilde{p}; N) = F(\widetilde{p}; N+1)$. This value is defined by the equality

$$\left(\frac{1-\lambda}{\lambda} \frac{v - \widetilde{p}}{\widetilde{p}}\right)^{\frac{1}{N(N-1)}} = \frac{\left(\frac{\mu_1^*(N+1)}{N+1}\right)^{\frac{1}{N}}}{\left(\frac{\mu_1^*(N)}{N}\right)^{\frac{1}{N-1}}},$$

where $\mu_1^*(N)$ and $\mu_1^*(N+1)$ are the equilibrium values of μ_1 when there are N and $N+1$ firms in the market, respectively. As expected price remains constant, these two observations imply that $F(p; N+1, \mu_1^*)$ is a mean-preserving spread of $F(p; N, \mu_1^*)$. A remark on the intuition behind this result is in line. We have noticed above that, for a given search intensity of the consumers, both the business-stealing effect and the surplus-appropriation effect strengthen as N increases but that, relative to the first effect, the second effect is stronger. The interesting issue about this result is that less-informed consumers' economizing behaviour makes the strengthening of the business-stealing effect proportional to the strengthening of the surplus-appropriation effect as N increases.

Finally, we note that the limiting equilibrium price distribution $\lim_{N\to\infty} F(p; N, \mu_1^*)$ is a discrete distribution where firms randomize between marginal cost pricing (with probability c/v) and monopoly pricing (with probability $(v-c)/v$). To see this, let us first investigate the behaviour of the density function when $N \to \infty$ for a fixed μ_1 and any $0 < p < v$. The density function can be written as

$$f(p; N, \mu_1) = \frac{1}{N-1}\left(\frac{\mu_1(1-\lambda)}{N\lambda}\right)^{\frac{1}{N-1}}\left(\frac{v-p}{p}\right)^{\frac{2-N}{N-1}}\frac{v}{p^2}.$$

It is easy to see that, for a fixed μ_1 and any $0 < p < v$, $f(p; N, \mu_1) \to 0$ as $N \to \infty$. This effect is strengthened by the fact that the probability μ_1^* with which less-informed consumers search for one price decreases as $N \to \infty$. Only for $p = 0$ and v, this argument does not hold. As we also know that $E[p; N] = v - c$ for all N, it follows that the probabilities with which the firms charge $p = 0$ and v must be equal to c/v and $(v-c)/v$, respectively. Interestingly, the probability that a firm charges marginal cost is increasing in search cost as more consumers leave the market when search cost is higher. These observations are summarized in the following result:

Proposition 5. (i) *For any parameters c, v and λ, there exists a critical number of firms $\widetilde{N}$ such that for all $N \geq \widetilde{N}$ an equilibrium with endogenous low search intensity as described in Proposition* 4 *exists.* (ii) *Let $\{F(p; N, \mu_1^*), \mu_1^*(N)\}$ and $\{F(p; N+1, \mu_1^*), \mu_1^*(N+1)\}$ be equilibria with endogenous low search intensity for N, and $N+1$ firms, respectively. Then, $\mu_1^*(N) > \mu_1^*(N+1)$ and $F(p; N+1, \mu_1^*)$ second-order stochastically dominates $F(p; N, \mu_1^*)$, for all N.* (iii) *The limiting equilibrium price distribution $\lim_{N\to\infty}\{F(p; N, \mu_1^*), \mu_1^*(N)\}$ is a discrete distribution where firms charge a price equal to* 0 *with probability c/v and a price equal to v with probability $(v-c)/v$.*

Proposition 5 highlights the beneficial effects of the optimizing behaviour of less-informed consumers. If these consumers did not search optimally, expected price would increase as a response to an increase in the number of competitors. When they search optimally, their behaviour fully offsets the firms' incentives to raise expected prices and entry only results in an increase in price dispersion.

To illustrate, Figure 3(a) below shows how the function $\Phi(\cdot) = (v - E[p; N])/v$ shifts downwards as the number of rivals increases (Claim 4.3). As a consequence, the equilibrium search intensity μ_1^* decreases with N. Figure 3(b) depicts the equilibrium price distributions when search intensity is endogenous for different values of N. This graph shows how an increase in the number of firms only results in greater price dispersion. It also shows that the price distribution for $N = 100$ is already close to the limiting price distribution.

Before closing this section, we would like to point out the detrimental effects of entry in a low search intensity equilibrium. To see this, observe that equilibrium welfare is $W(N) = \lambda v + \mu_1^*(N)(1-\lambda)(v-c)$. Since the equilibrium search intensity μ_1^* declines in N, it follows that welfare falls as the number of competitors increases. The fall in μ_1^* generates a dead-weight loss since socially desirable transactions are not realized.

5. HIGH SEARCH INTENSITY EQUILIBRIUM

Consider now the case where less-informed consumers randomize between searching for one price and searching for two prices. In this case, the expected pay-off to firm i is

$$\pi_i(p_i, F(p)) = p_i\left[\lambda(1-F(p_i))^{N-1} + \frac{2(1-\lambda)(1-\mu_1)}{N}(1-F(p_i)) + \frac{(1-\lambda)\mu_1}{N}\right].$$

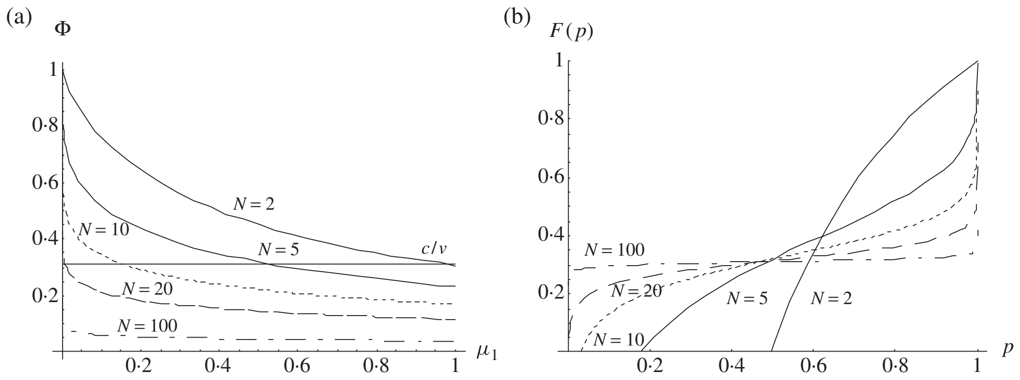

FIGURE 3

(a) ($\lambda = 1/3$; $c/v = 0{\cdot}31$); (b) ($\lambda = 1/3$; $c/v = 0{\cdot}31$)

Notice that, unlike before, $1 - \mu_1$ denotes now the probability with which less-informed consumers search for two prices. Again, this profit function is easily interpreted. Note first that consumers can now be grouped into three subsets: less-informed consumers who observe a single price, less-informed consumers who observe two prices, and fully-informed consumers. A firm i attracts the latter consumers only when it quotes the lowest price in the market, which happens with probability $(1 - F(p_i))^{N-1}$. There are $1 - \mu_1$ less-informed buyers who search twice; these consumers buy from firm i whenever they are aware of firm i's price, which occurs with probability $2/N$, and when firm i happens to be the lowest-price firm among the sampled ones, which occurs with probability $1 - F(p_i)$. Finally, there are μ_1 less-informed consumers who search for only one price; firm i attracts these consumers when it happens that they observe firm i's price, which occurs with probability $1/N$.

Exogenous consumer search

As before, the highest price a firm would ever charge is v; thus $F(v) = 1$ and $F(p) < 1$, $p < v$. In equilibrium, a firm must be indifferent between charging any price in the support of F and charging v. Hence, it must be the case that

$$p\left[\frac{(1-\lambda)\mu_1}{N} + \frac{2(1-\lambda)(1-\mu_1)}{N}(1-F(p)) + \lambda(1-F(p))^{N-1}\right] = \frac{\mu_1(1-\lambda)v}{N}. \qquad (7)$$

Unfortunately, an explicit solution to equation (7) for $F(p)$ does not exist for general values of N. However, by setting $F(\underline{p}) = 0$ and solving for $\underline{p}$, the lower bound of the price distribution can be found: $\underline{p}(N, \mu_1) = \mu_1(1-\lambda)v/[(1-\lambda)(2-\mu_1) + N\lambda]$. To show existence of an equilibrium price distribution it is convenient to rewrite equation (7) as

$$\lambda(1-F(p))^{N-1} + \frac{2(1-\lambda)(1-\mu_1)}{N}(1-F(p)) = \frac{\mu_1(1-\lambda)(v-p)}{Np}. \qquad (8)$$

Note that the L.H.S. of this equation is continuously increasing in $1 - F(p) \in [0, 1]$, while the R.H.S. is a positive constant with respect to $1 - F(p)$ and can take on values in $[0, \mu_1(1-\lambda)(v-\underline{p})/N\underline{p}]$. Therefore, for any $p \in [\underline{p}, v]$, $F(p)$ is uniquely defined as the solution to (8); moreover, as p increases $F(p)$ rises. The next result summarizes these findings:

Lemma 5. *Suppose less-informed consumers search with high intensity. Then, for any given search intensity* $0 < \mu_1 < 1$*, there exists a unique symmetric equilibrium price distribution*

$F(p, N, \mu_1)$ with support $[\underline{p}(N, \mu_1), v]$, where $\underline{p}(N, \mu_1) = \frac{\mu_1(1-\lambda)v}{N\lambda+(1-\lambda)(2-\mu_1)}$. Such a price distribution is given by the solution to equation (7).

Our next observations pertain to the comparative statics of an increase in the number of competitors, holding the search intensity of the consumers constant. We note that the behaviour of the price distribution with respect to N exhibited in Figure 1 above also holds in this case of high search intensity. To see this, note first that the lower bound of the equilibrium price distribution also decreases as N increases here. The reason is again that an increase in the number of firms strengthens the business-stealing effect. Second, observe that, using equation (7), it is easy to see that there exists some unique $\widetilde{p} \in (\underline{p}(N, \mu_1), v)$ for which $F(\widetilde{p}; N, \mu_1) = F(\widetilde{p}; N + 1, \mu_1) = 1/(N + 1)$. Thus, firms respond to entry by decreasing the frequency with which they set intermediate prices and by increasing the frequency of more extreme prices.

As earlier in the paper, these observations call for an examination of equilibrium expected prices under the price distributions $F(p; N, \mu_1)$ and $F(p; N + 1, \mu_1)$ when consumers search with high intensity. Our next result states that expected price increases monotonically in N. This again highlights the dominating influence of the surplus-appropriation effect. In addition, we show that expected price is bounded away from v for the limiting case $N \to \infty$, even if the search intensity of the less-informed consumers is held constant. This result is in contrast with the cases of moderate and low search intensity analysed above and follows from the fact that some less-informed consumers compare prices.

Proposition 6. *Let $F(p; N, \mu_1)$ be the equilibrium price distribution when less-informed consumers search with high intensity, given in Lemma* 5*. Then, holding the search behaviour of less-informed consumers constant, expected price increases in N and approaches*

$$E[p; \infty, \mu_1] = \frac{\mu_1 v \ln\left[\frac{2-\mu_1}{\mu_1}\right]}{2(1-\mu_1)} < v \qquad \textit{for all } \mu_1 \in (0, 1)$$

as N converges to infinity.

Endogenous consumer search

We now turn to consider optimal search behaviour of the less-informed consumers. The price distribution $F(p; N, \mu_1)$ and the search behaviour μ_1 constitute an endogenous equilibrium with high search intensity if $F(p; N, \mu_1)$ solves equation (7) and the following two additional conditions are satisfied:

Condition 5.1. $v - E[\min\{p_i, p_j\}; N] - 2c = v - E[p; N] - c$, for all $i, j \in N$.

Condition 5.2. $v - E[p; N] - c > 0$.

Condition 5.1 states that less-informed consumers must be indifferent between obtaining one price quotation and obtaining two price quotations. We note that if 5.1 holds, then buyers do not find it attractive to search for three or more price quotations. Condition 5.2 says that less-informed consumers' expected surplus must be positive, that is, less-informed consumers always enter the market.

Our first observation is that in any endogenous high search intensity equilibrium, Condition 5.2 is irrelevant. We show this by proving that if Condition 5.1 is satisfied, then

Condition 5.2 holds as well. To show this suppose, on the contrary, that $v - E[p; N] - c = 0$.[17] If this is so, then (using the derivations in Lemma 1)

$$0 = v - E[\min\{p_i, p_j\}; N] - 2c = v - 2E[p; N] + \int_{\underline{p}(N)}^{v} 2pF(p; N) f(p; N)dp - 2c$$

$$= \int_{\underline{p}(N)}^{v} 2pF(p; N) f(p; N)dp - E[p; N] - c$$

$$= \int_{\underline{p}(N)}^{v} 2pF(p; N) f(p; N)dp - v < \frac{v}{2} - v = -\frac{v}{2},$$

which constitutes a contradiction.

Our second observation pertains to the existence of an endogenous high search intensity equilibrium. Condition 5.1 requires that

$$\int_{\underline{p}(N,\mu_1)}^{v} pf(p; N, \mu_1)dp - 2\int_{\underline{p}(N,\mu_1)}^{v} pf(p; N, \mu_1)(1 - F(p; N, \mu_1))dp = c.$$

Integrating by parts yields

$$\int_{\underline{p}(N,\mu_1)}^{v} F(p; N, \mu_1)(1 - F(p; N, \mu_1)dp = c.$$

This equation can be rewritten as (see proof of Lemma 7)

$$\int_0^1 \frac{2z - 1}{1 + 2\left(\frac{1}{\mu_1} - 1\right)(1 - z) + \frac{1}{\mu_1}\frac{\lambda}{1-\lambda}N(1 - z)^{N-1}}dz = \frac{c}{v}. \tag{9}$$

Let $\rho(\mu_1, N, \lambda)$ denote the L.H.S. of equation (9). Note that setting $\mu_1 = 1$ in (9) yields the L.H.S. of (3). Denote $\overline{\rho}(\mu_1, N, \lambda) = \max_{\mu_1 \in (0,1)} \rho(\mu_1, N, \lambda)$.

Proposition 7. *Let $0 < \frac{c}{v} < \overline{\rho}(\mu_1, N, \lambda)$. Then, there exist at least one and at most two symmetric endogenous high search intensity equilibria. In equilibrium consumers search with a high intensity $\mu_1^* \in (0, 1)$ given by the solution to $\rho(\mu_1, N, \lambda) - c/v = 0$, and firms randomly select prices from the set $\left[\frac{\mu_1^*(1-\lambda)v}{N\lambda+(1-\lambda)(2-\mu_1^*)}, v\right]$ according to the price distribution that solves equation (7).*

We now provide a discussion on this existence result. As before, one can ask whether, for any given N, there exist parameter values c, v and λ such that a high search intensity equilibrium exists. This amounts to show that $\overline{\rho}(\mu_1, N, \lambda) > 0$ for all N. To prove that this is true, we shall make use of the envelope theorem and the proof of Lemma 6 presented later. The argument is as follows. Using the envelope theorem, it is easily seen that the behaviour of $\overline{\rho}(\mu_1, N, \lambda)$ with respect to N is just the behaviour of $\rho(\mu_1, N, \lambda)$ with respect to N. This is given in the proof of Lemma 6, where we prove three facts: (i) that $\rho(\mu_1, 2, \lambda) < \rho(\mu_1, 3, \lambda)$, (ii) that there exists a sufficiently large number of firms $\hat{N}$ such that $\rho(\mu_1, N, \lambda) > \rho(\mu_1, N + 1, \lambda)$ for all $N \geq \hat{N}$, and (iii) that in the limiting case $N \to \infty$, $\rho(\mu_1, N, \lambda)$ is bounded away from zero. These remarks imply that, for given values of v and λ, a high search intensity equilibrium exists for all N provided that c/v is small enough.

17. Note that the case $v - E[p; N] - c < 0$ is of no interest since no less-informed consumer would enter the market.

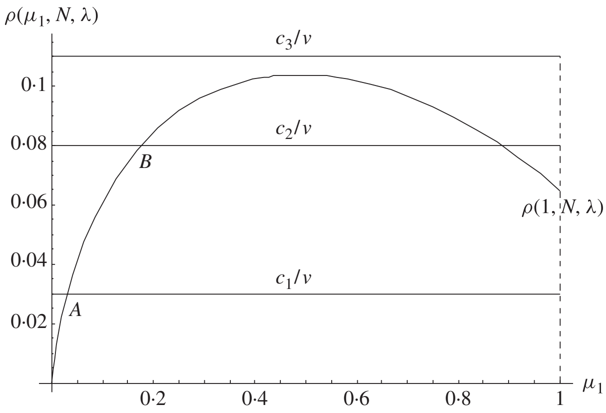

FIGURE 4

High search intensity equilibrium ($N = 2, \lambda = 0{\cdot}25$)

Proposition 7 also shows that there may be a single equilibrium or two equilibria, depending on the parameters. The result is illustrated in Figure 4. The curved schedule depicts the marginal gains from searching twice rather than once, *i.e.* the function $\rho(\mu_1, N, \lambda)$. For a sufficiently high search cost, for example c_3, a high search intensity equilibrium fails to exist. For a low enough search cost, for example c_1, there exists a unique equilibrium search intensity. The equilibrium value of μ_1 is given by the point at which the curve $\rho(N, \mu_1, \lambda)$ and the line c_1/v intersect (point A in the graph). By contrast, when the search cost is intermediate, say c_2, there are two equilibrium values (represented by the points B and C in the graph). We now argue (in line with Fershtman and Fishman, 1992) that if there are two equilibria, only the equilibrium with a higher search intensity (lower μ_1) is a stable equilibrium. In a neighbourhood to the left of point C, the expected gains to buyers from searching for two prices instead of searching for one price are larger than the cost of an extra search. Therefore, a small perturbation around point C so that $\mu_1 < \mu_1^*$ would lead consumers to search more intensively, a movement away from point C. Similarly, a small perturbation so that $\mu_1 > \mu_1^*$ would lead consumers to search less intensively. These observations suggest that the equilibrium represented by point C is not stable. A similar argument shows that the value of μ_1 corresponding to point B in the graph is a stable value of the equilibrium search intensity. In what follows, we will concentrate on this stable equilibrium with endogenous high search intensity.

We now focus on the comparative statics effects of an increase in the number of competitors on consumer search and firm pricing behaviour. We first observe that less-informed consumers' equilibrium search incentives are non-monotonic in the number of firms. In addition, we see that there exists a simple relationship between the incentives to search under duopoly and the same incentives under a fully competitive market. Later, we shall see the influence of consumer search behaviour on firm pricing.

Lemma 6. *Let c, v and λ be such that $0 < \frac{c}{v} < \overline{\rho}(\mu_1, 2, \lambda)$; then $\mu_1^*(2) > \mu_1^*(3)$, and there exists some $\hat{N}$ such that for all $N \geq \hat{N}$, $\mu_1^*(N) < \mu_1^*(N+1)$. Moreover, $\mu_1^*(\infty) = (1-\lambda)\mu_1^*(2)$.*

In the proof of this lemma we show that the function $\rho(\mu_1, N, \lambda)$ increases when we move from duopoly to triopoly. Thus, provided an equilibrium with high search intensity under duopoly exists, less-informed consumers search more intensively under triopoly than under duopoly.

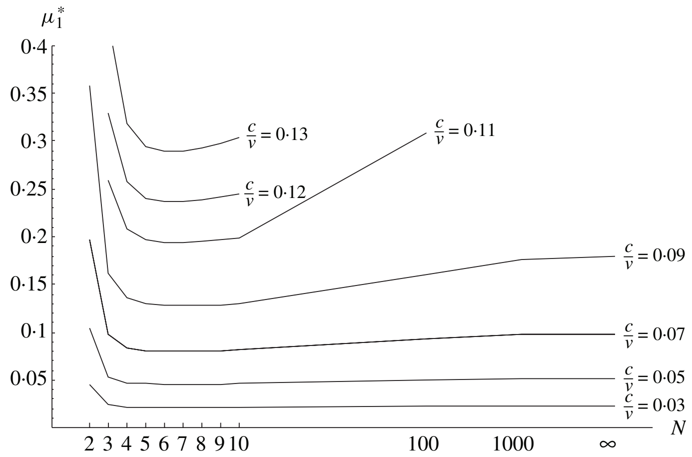

FIGURE 5

Equilibrium high search intensity and the number of firms ($\lambda = 0{\cdot}5$)

However, when the market accommodates already a large number of firms, we show that the function $\rho(\mu_1, N, \lambda)$ decreases in N; thus, further entry leads to less consumer search. The intuition behind this result is as follows. Recall that an increase in N strengthens both the business-stealing effect and the surplus-appropriation effect. When we move from duopoly to triopoly, the strengthening of the business-stealing effect turns out to be of sufficient magnitude compared with the strengthening of the surplus-appropriation effect so as to encourage consumer search. Recall also that the strengthening of the business-stealing effect weakens as N increases. This implies that when there are already many firms in the market, the strengthening of the business-stealing effect is of little magnitude and thus consumer search is discouraged.

We have conducted a number of simulations to examine the responsiveness of consumer search to entry for intermediate values of N. Figure 5 shows μ_1^* as a function of N, for different values of the other parameters. The graph reveals that consumers typically search more when N increases until there are six or seven firms in the market; from then on, consumers typically search less. The graph is also useful to illustrate the observation in the proof of Lemma 6 that the function $\rho(\mu_1, N, \lambda)$ is first increasing and then decreasing in N. For instance, if $c/v = 0{\cdot}12$, a high search intensity equilibrium does not exist when there are just two firms in the market. Entry of a third firm shifts the function $\rho(\mu_1, N, \lambda)$ upwards and a high search intensity equilibrium appears. Further entry continues shifting the function $\rho(\mu_1, N, \lambda)$ upwards, which encourages consumer search. This pattern changes when the number of firms reaches $N = 6$; from then on, marginal gains from searching twice rather than once fall in N, which discourages consumer search. The high search intensity equilibrium disappears when the number of firms reaches $N = 11$ in this case.

Lemma 6 also shows the existence of a simple relationship between consumers' search intensity when there are infinitely many firms and their search intensity when there are only two firms. We have noted above that when the number of firms in the market grows without limit, the probability that a firm is undercut by some other firm converges to 1. It is precisely for this reason that in the limit economy firms set their prices ignoring the fully-informed consumers altogether and concentrating on attracting the less-informed consumers. Note that the latter search for one price with probability μ_1 and for two prices with probability $1 - \mu_1$. So in effect, the limit economy can be seen as a duopoly competing for only $1-\lambda$ consumers. This is the reason behind the similarity between the two market structures. What consumers do in their search behaviour is to internalize the "practical" inexistence of the fully-informed consumers by searching more intensively in the limit economy, which explains the relationship $\mu_1^*(\infty) = (1-\lambda)\mu_1^*(2)$. These remarks together enable us to argue that, provided that consumers' optimal search intensity exhibits a smooth relationship with respect to the number of firms, consumers search activity attains its minimum under duopoly.

Our final and most important result borrows from these observations and brings together Proposition 6 and Lemma 6. We have seen above that holding the search intensity of the consumers constant, firms have an incentive to raise prices as a response to an increase in N (Proposition 6). Moreover, less-informed consumers respond to entry by searching more intensively when the number of firms is small to begin with, and less intensively otherwise (Lemma 6). Therefore, when the number of competitors increases and the status quo number of firms is large, both firm and consumer responses influence expected price in the same direction, which yields the result that expected price increases without ambiguity. By contrast, when the number of firms increases and the status quo number of competitors is small to begin with, firms' incentives to increase prices and consumers' incentives to search more influence expected price in opposite directions. Thus, the comparative statics analysis of entry in the case of an initially small number of firms requires to see which of these two effects is of greater magnitude. This task has proven to be very difficult because explicit solutions to equations (7) and (A.3) for arbitrary N do not exist. However, the next result shows that expected prices (and price dispersion) in the case of duopoly and in the case of a very large number of firms are equal, which, together with the observations above, suffices to show that expected price is non-monotonic in the number of firms when consumers search with high intensity in equilibrium.

Proposition 8. *Suppose less-informed consumers search with high intensity in equilibrium. Then, for large N, expected price is increasing in N. Moreover, duopoly yields the same expected price and price dispersion as the competitive case $N \to \infty$. Hence, expected price is non-monotonic in the number of firms.*

We would like to elaborate on three aspects of this result. The *first* is that expected prices and price dispersion (indeed the entire equilibrium price distributions) with two firms and with an infinite number of firms are equal. This result, which does not hold in related articles (see, *e.g.* Varian (1980), Stahl (1989)), arises because less-informed consumers' optimizing behaviour entirely internalizes the pressure that fully-informed consumers exert on the firms in a duopoly setting (Lemma 6).

The second question pertains to the behaviour of expected price when one moves from duopoly to triopoly. As we have not been able to get an analytic answer to this question, we have conducted a number of simulations to examine the behaviour of expected price with respect to N, for various levels of search cost c and percentages of informed consumers λ. Tables 1(a) and 1(b) report the results obtained when one compares duopoly and triopoly. It can be seen that expected prices are always lower under triopoly than under duopoly.[18] This illustrates the dominant influence of consumers' search behaviour when the market moves from duopoly to triopoly.

This dominant influence of consumer search on expected price also holds for markets with a larger number of firms. Figure 6 shows how expected price changes as N rises, for different

18. In Janssen and Moraga-González (2000) we show that when $N = 2$ expected price is constant when λ changes. The numerical results also show that price dispersion (measured by the variance of the equilibrium price distribution and omitted from Tables 1(a) and 1(b) to save on space) is greater under triopoly than under duopoly. This suggests that price dispersion exhibits a non-monotonic relationship with respect to the number of rivals.

TABLE 1(a)

Comparison of search intensities and expected prices under duopoly and triopoly

	$c = 0{\cdot}02$				$c = 0{\cdot}04$			
	$N = 2$		$N = 3$		$N = 2$		$N = 3$	
λ	μ_1	$E[p]$	μ_1	$E[p]$	μ_1	$E[p]$	μ_1	$E[p]$
0·05	0·0133175	0·032399	0·012642	0·0319147	0·0377135	0·074395	0·0357142	0·0729443
0·15	0·0148848	0·032399	0·0126536	0·0309735	0·0421507	0·074385	0·0356206	0·0701803
0·25	0·0168695	0·032399	0·0127142	0·0300587	0·0477708	0·074395	0·0357218	0·0675558
0·35	0·0194648	0·032399	0·0128375	0·0291591	0·0551193	0·074395	0·0360631	0·0650308
0·45	0·0230038	0·032399	0·0130476	0·0282625	0·065142	0·074395	0·0367291	0·0625661
0·55	0·0281157	0·032399	0·0133889	0·0273543	0·079618	0·074395	0·0378821	0·0601194
0·65	0·0361487	0·032399	0·0139534	0·0264139	0·102366	0·074395	0·0398653	0·0576378
0·75	0·0506076	0·032399	0·0149675	0·0254111	0·143312	0·074395	0·0435392	0·0550435
0·85	0·0843425	0·032399	0·0171903	0·0242782	0·238853	0·074395	0·051902	0·0521913
0·95	0·0253042	0·032399	0·0266565	0·0228223	0·716562	0·074395	0·0900745	0·0486904

TABLE 1(b)

Comparison of search intensities and expected prices under duopoly and triopoly

	$c = 0{\cdot}06$				$c = 0{\cdot}08$			
	$N = 2$		$N = 3$		$N = 2$		$N = 3$	
λ	μ_1	$E[p]$	μ_1	$E[p]$	μ_1	$E[p]$	μ_1	$E[p]$
0·05	0·0761624	0·128013	0·0717813	0·124774	0·138738	0·201257	0·129286	0·194051
0·15	0·0851227	0·128013	0·07104	0·118765	0·15506	0·201257	0·125723	0·181443
0·25	0·964723	0·128013	0·0708497	0·113248	0·175735	0·201257	0·123822	0·170569
0·35	0·111314	0·128013	0·071285	0·108093	0·202771	0·201257	0·123497	0·160894
0·45	0·131553	0·128013	0·072508	0·103193	0·239639	0·201257	0·124949	0·152057
0·55	0·160787	0·128013	0·0749205	0·0984449	0·292891	0·201257	0·128802	0·14379
0·65	0·206725	0·128013	0·079278	0·0937415	0·376564	0·201257	0·136529	0·135849
0·75	0·289417	0·128013	0·0876271	0·0889406	0·527205	0·201257	0·152047	0·12798
0·85	0·482362	0·128013	0·107198	0·0838095	0·878673	0·201257	0·189519	0·11983
0·95	Existence	Fails	0·200199	0·077776	Existence	Fails	0·372971	0·110658

values of the other parameters. The graph shows how expected price falls initially until there are approximately between four and six firms in the market and then increases monotonically, eventually converging to the duopoly level as N goes to infinity.

The last aspect we would like to mention is that in a high search intensity equilibrium, welfare is maximized when actual search intensity is lowest, since all consumers acquire the good in equilibrium. In this sense, Lemma 6 suffices to show that welfare is non-monotonic with respect to the number of firms, and that welfare under duopoly is higher than under triopoly and also higher than under an infinite number of firms because actual search is less intensive under the first market structure. Moreover, provided that search intensity is a smooth function of the number of firms, as suggested by our numerical analysis, welfare attains its maximum under duopoly.

6. OVERVIEW OF EQUILIBRIA

Having analysed the different types of equilibria in turn, we end our analysis by discussing which type of equilibrium exists for which set of exogenous parameters, and how we move from one type of equilibrium to another when N increases. We also show for which parameter constellations multiple equilibria exist. Finally, we present an overview of how expected price may vary with N.

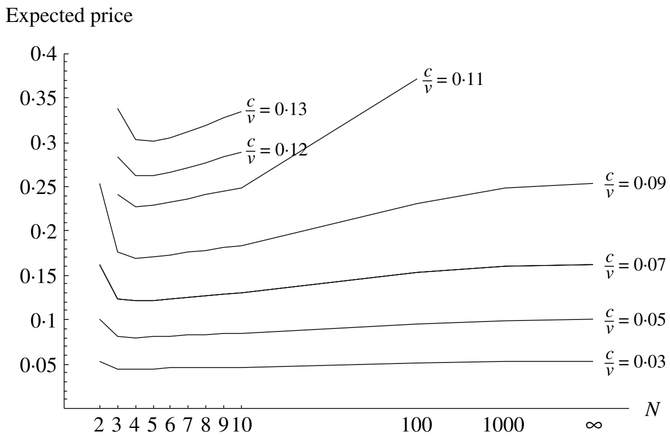

FIGURE 6

Expected price as a function of N under high search intensity ($\lambda = 0{\cdot}5$)

We note that whether the equilibrium exhibits high, moderate, or low search intensity depends on all parameters c/v, λ and N. Based on the results of Sections 3–5, we can provide the following characterization:

(i) Fix $c/v \geq \max_{\mu_1} \max_N \rho(\mu_1, N, \lambda)$. Then the equilibrium is unique; moreover, there exists a critical number of firms $\widetilde{N}(c/v, \lambda)$, given in Propositions 3 and 5, such that for all $N \geq \widetilde{N}(c/v, \lambda)$ a low search intensity equilibrium exists; otherwise, a moderate search intensity equilibrium exists.

(ii) Fix c/v such that $\max_{\mu_1} \max_N \rho(\mu_1, N, \lambda) \geq c/v > \max_{\mu_1} \rho(\mu_1, 2, \lambda)$. Then, there exist critical numbers of firms $N_1(c/v, \lambda)$ and $N_2(c/v, \lambda)$ such that a high search intensity equilibrium only exists if $N_1(c/v, \lambda) \leq N \leq N_2(c/v, \lambda)$. Moreover, the same as in (i) applies with respect to the moderate and the low search intensity equilibria. This implies that for $N_1(c/v, \lambda) \leq N \leq N_2(c/v, \lambda)$ two equilibria exist.

(iii) Fix $c/v \leq \max_{\mu_1} \rho(\mu_1, 2, \lambda)$. Then a high search intensity equilibrium always exists and this equilibrium is not necessarily unique. Indeed, there exists a critical number of firms $N_3(c/v, \lambda)$ such that a moderate search intensity equilibrium also exists if $N_3(c/v, \lambda) \leq N \leq \widetilde{N}(c/v, \lambda)$, and a low search intensity equilibrium exists if $N \geq \widetilde{N}(c/v, \lambda)$.

Situations in which there may be multiple equilibria are illustrated in Figure 7(a) and (b). In the left graph, we have represented a market where $c/v = 0{\cdot}08$ and $\lambda = 0{\cdot}75$. This parameter constellation corresponds to case (iii) above. For $2 \leq N \leq 13$, only a high search intensity equilibrium exists. For $14 \leq N \leq 63$, there are two equilibria, a moderate search intensity one and a high search intensity one. For $N \geq 64$, also two equilibria exist, a low search intensity one and a high search intensity one. The graph thus illustrates how for low search costs and a high fraction of informed consumers, multiple equilibria only arise when there are many firms in the market. It is clearly seen that expected prices in the low and moderate search intensity equilibria are higher than in the high search intensity equilibrium. This observation can be proven in general. First, for given parameters c/v, λ and N the equilibrium expected price when consumers search with low intensity satisfies $v - E[p; N] - c = 0$; by contrast, if buyers search with high intensity, equilibrium expected price satisfies $v - E[p; N] - c > 0$. This shows that if the two equilibria exist for a given set of parameters, the former always yields a higher expected price than the latter. Second, applying the implicit function theorem to equation (7), it is easy to see that $F(p; N)$ decreases monotonically in μ_1 and converges to the equilibrium price distribution

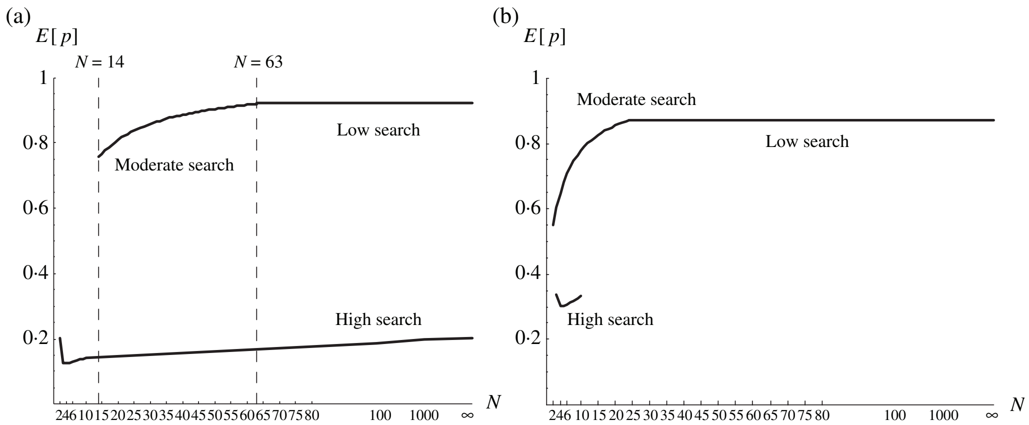

FIGURE 7

(a) ($\lambda = 0{\cdot}75$; $c/v = 0{\cdot}08$); (b) ($\lambda = 0{\cdot}5$; $c/v = 0{\cdot}13$)

under moderate search intensity as $\mu_1 \to 1$. This shows that, for any c/v, λ and N such that both a moderate and a high search intensity equilibrium exist, expected price is always lower under high search.

In Figure 7(b), we have depicted a market with higher search costs and fewer informed consumers, $c/v = 0{\cdot}13$ and $\lambda = 0{\cdot}5$. This corresponds to case (ii) above. In this situation, for $N = 2$, only a moderate search intensity equilibrium exists. This equilibrium continues to exist until $N = 24$, where the economy moves to a low search intensity equilibrium. Interestingly, from $N = 3$ to 10, a high search intensity equilibrium also exists. In contrast to Figure 7(a), we see now that multiple equilibria only exist when the market accommodates a few firms and that the difference between expected prices across equilibria is not as large as before.

7. CONCLUSIONS

This paper has presented an oligopoly model where a certain fraction of consumers engages in costly non-sequential search to discover prices. We have shown that there can be three distinct price-dispersed equilibria. These equilibria can be characterized in terms of how intensively consumers search. There may be low, moderate and high search intensity equilibria. If the economy is in a low search intensity equilibrium, an increase in the number of competitors does not influence expected price and results in greater price dispersion and in a decrease in welfare. In a moderate search intensity equilibrium, the effect of bringing more competitors together is an unambiguous increase in expected price, but this type of equilibrium fails to exist when N becomes large. Finally, when consumers search with high intensity, entry of a new firm leads to a lower expected price when the number of competitors in the market is small to begin with, but to a higher expected price otherwise. Thus, in this case expected price exhibits a non-monotonic relationship with respect to the number of firms.

Three more results are worth emphasizing. First, we have seen how consumers' search behaviour prevents firms from charging monopoly prices, even when the market accommodates a very large number of competitors. Thus, a Diamond (1971) type of result cannot arise in our model. This is explained by the following observations. First, in a high search intensity equilibrium, consumers keep comparing prices with positive probability, for any number of firms, which prevents monopoly pricing. Second, if the economy is initially in a moderate search intensity equilibrium, as the number of firms grows beyond a critical number, less-informed consumers find it profitable to economize on search and start exiting the market with positive probability, which also prevents monopoly pricing. This result, which is at odds with previous research on search and oligopoly, is important because it shows that when all price quotations are costly to obtain price dispersion does not vanish as the number of firms approaches infinity.

The second result worth emphasizing is that in a high search intensity equilibrium, two firms and infinitely many firms yield the same market outcome, while three or more firms yield typically lower expected prices. The reason why the limit economy and the duopoly case are alike is that in the former case an individual firm is in effect also competing with just one other firm; this is because the probability of being undercut by some other firm is close to one when there are infinitely many firms, and thus firms ignore the possibility of selling to the fully-informed consumers altogether. When the market accommodates three or more competitors, a more competitive outcome emerges as firms find themselves effectively competing for three groups of consumers (those comparing all prices, those comparing just two prices and those not comparing prices at all). This result is also at odds with previous research on search and oligopoly, highlighting the strong influence of consumer search on firms' pricing behaviour.

A third result is that there are sets of parameters for which multiple equilibria emerge. A high search intensity equilibrium may coexist with a low or with a moderate search intensity equilibrium, but the latter two types of equilibria will never coexist together. For the same parameter values, expected prices are lower in the high search intensity equilibrium than in the other types of equilibria. This multiplicity of equilibria can potentially explain price differences across seemingly identical markets. Moreover, it suggests that consumers would gain if they did collectively agree to search with high intensity. We note however that consumers acting unilaterally would not have an incentive to engage in more search when firms expect them not to search much and choose their prices accordingly.

APPENDIX

Proof of Lemma 1. (i) Suppose $\mu_0 = 1$. Then, only fully-informed consumers would remain in the market and in any symmetric equilibrium all firms would charge prices equal to marginal cost, *i.e.* $p_i = 0$, $i = 1, 2, \ldots, N$. As a consequence, less-informed consumers would find it beneficial to search at least once. Therefore, $\mu_0 = 1$ cannot be part of an equilibrium.

(ii) Suppose $\mu_n = 1$, for a single $n = 2, 3, 4, \ldots, N$. Then, all consumers would exercise price comparisons at least once, and thus in a symmetric equilibrium all firms would charge prices equal to marginal cost. But if this was so, less-informed consumers would search only once to economize on search costs. Therefore $\mu_n = 1$, for some $n = 2, 3, 4, \ldots, N$ cannot be part of an equilibrium either.

(iii) The same reasoning as before rules out equilibria where $\mu_1 = 0$ and some $\mu_n > 0$, $n = 3, 4, \ldots, N$. It thus remains to show that $\mu_1 > 0$ together with some $\mu_n > 0$, $n = 3, 4, \ldots, N$ cannot be part of an equilibrium either. Suppose, on the contrary, that an equilibrium of this type existed. The first observation is that the equilibrium would be characterized by an atomless price distribution (see Lemma 2 and the arguments thereafter). Moreover, it would be the case that $v - E[p] - c = v - E[\min\{p_1, p_2, \ldots, p_n\}] - nc$, for some $n = 3, 4, \ldots, N$; or, in words, less-informed consumers would be indifferent between searching for one price and searching for *n* prices. This equality can be rewritten as $E[p] - E[\min\{p_1, p_2, \ldots, p_n\}] = (n-1)c$. Note now that the expected value of the minimum of a random sample of *n* observations is a decreasing function of *n*, and, further, that such a minimum falls at a decreasing rate (see, *e.g.* Stigler, 1961). This implies that $E[p] - E[\min\{p_1, p_2, \ldots, p_{n-1}\}] > (n-2)c$ for any $n \geq 3$. But this can be rewritten as $v - E[\min\{p_1, p_2, \ldots, p_{n-1}\}] - (n-1)c > v - E[p] - c$, which implies that the proposed search strategy is not optimal.

(iv) Similar considerations as in (ii) select away equilibria where $\mu_1 = 0$ and $\mu_0 + \mu_2 = 1$. So, let us consider the case where $\mu_1 > 0$. In this case, the following two conditions must hold:

$$v - E[p] - c = 0 = v - E[\min\{p_i, p_j\}] - 2c, \tag{A.1}$$

i.e. less-informed consumers must be indifferent between searching for one price, not searching at all, and searching for two prices. Again in this case the equilibrium would be characterized by an atomless price distribution with upper bound equal to v (see Lemma 2). We now show that both equalities cannot hold together. We note first that

$$E[\min\{p_i, p_j\}] = 2\int_{\underline{p}}^{v} p(1-F(p))f(p)dp = 2E[p] - \int_{\underline{p}}^{v} 2pF(p)f(p)dp.$$

Thus, the R.H.S. of (A.1) can be written as

$$\begin{aligned} v - E[\min\{p_i, p_j\}] - 2c &= v - 2E[p] + \int_{\underline{p}}^{v} 2pF(p)f(p)dp - 2c = \int_{\underline{p}}^{v} 2pF(p)f(p)dp - E[p] - c \\ &= \int_{\underline{p}}^{v} 2pF(p)f(p)dp - v, \end{aligned}$$

where the second equality follows from $v - E[p] - c = 0$. Integrating by parts, we can show that

$$\int_{\underline{p}}^{v} pF(p)f(p)dp = v - \int_{\underline{p}}^{v} [F(p)]^2 dp - \int_{\underline{p}}^{v} pF(p)f(p)dp.$$

Thus,

$$v - E[\min\{p_i, p_j\}] - 2c = -\int_{\underline{p}}^{v} [F(p)]^2 dp < 0,$$

which constitutes a contradiction. The proof is now complete. ‖

Proof of Lemma 2. Suppose firms charged a particular price with positive probability. Then, there would be a positive probability that a firm tied with another firm for having the lowest price in the market. This implies that a small reduction in that price by one of the rivals would be beneficial as it would attract all fully-informed consumers. As a result, the only price that could be proposed as having positive probability equals marginal cost. However, since in any equilibrium $\mu_1 > 0$, there is a positive probability that a firm charging $p = 0$ attracts a less-informed consumer and then such a firm would increase its (expected) profit by raising its price.

Proof of Proposition 1. Expected price can be written as

$$E[p; N] = \int_{\underline{p}(N)}^{v} \frac{(1-\lambda)v}{\lambda p N(N-1)} \left(\frac{(1-\lambda)(v-p)}{\lambda N p}\right)^{\frac{2-N}{N-1}} dp.$$

Consider the following variable change:

$$z = \left(\frac{(1-\lambda)(v-p)}{\lambda N p}\right)^{\frac{1}{N-1}}.$$

Then we have $p = \frac{v}{1+bNz^{N-1}}$ and $dp = \frac{-vbN(N-1)z^{N-2}}{(1+bNz^{N-1})^2}dz$, where $b = \lambda/(1-\lambda) > 0$. This enables us to rewrite the expected price as follows:

$$E[p; N] = \int_0^1 \frac{v}{1+bNz^{N-1}} dz.$$

Then we can compute the difference

$$\begin{aligned} E[p; N+1] - E[p; N] &= v\int_0^1 \left[\frac{1}{1+b(N+1)z^N} - \frac{1}{1+bNz^{N-1}}\right] dz \\ &= v\int_0^1 \left[\frac{bz^{N-1}(N-(N+1)z)}{(1+b(N+1)z^N)(1+bNz^{N-1})}\right] dz. \end{aligned}$$

To complete the argument it is enough to show that this last integral is positive. To show this, note that $1 + b(N+1)z^N$ and $1 + bNz^{N-1}$ are both positive and strictly increasing in z. Then,

$$\begin{aligned} & v\int_0^1 \left[\frac{bz^{N-1}(N-(N+1)z)}{(1+b(N+1)z^N)(1+bNz^{N-1})}\right] dz \\ &\quad = v\int_0^{\frac{N}{N+1}} \left[\frac{bz^{N-1}(N-(N+1)z)}{(1+b(N+1)z^N)(1+bNz^{N-1})}\right] dz - v\int_{\frac{N}{N+1}}^1 \left[\frac{bz^{N-1}((N+1)z-N)}{(1+b(N+1)z^N)(1+bNz^{N-1})}\right] dz \end{aligned}$$

$$\geq v\int_0^{\frac{N}{N+1}}\left[\frac{bz^{N-1}(N-(N+1)z)}{\left(1+b(N+1)\left(\frac{N}{N+1}\right)^N\right)\left(1+bN\left(\frac{N}{N+1}\right)^{N-1}\right)}\right]dz$$
$$-v\int_{\frac{N}{N+1}}^{1}\left[\frac{bz^{N-1}((N+1)z-N)}{\left(1+b(N+1)\left(\frac{N}{N+1}\right)^N\right)\left(1+bN\left(\frac{N}{N+1}\right)^{N-1}\right)}\right]dz$$
$$=\frac{bv}{\left(1+b\frac{N^N}{(N+1)^{N-1}}\right)}\int_0^1 z^{N-1}((N+1)z-N)dz=0.$$

To establish that the expected price approaches v when N converges to infinity, it is enough to note that $F(p,N)$ converges to zero as N goes to infinity. The proof is now complete. ‖

Proof of Proposition 6. We note that $E[p;N,\mu_1]=v-\int_{\underline{p}(N,\mu_1)}^{v}F(p;N,\mu_1)dp$. As argued above, there exists a unique solution to equation (7) that is monotonically increasing in p. Thus, we can invert the function $F(p;N,\mu_1)$ to obtain:

$$p(z;N,\mu_1)=\frac{v}{g(z;N,\mu_1)}$$

where

$$g(z;N,\mu_1)=1+2(a-1)(1-z)+abN(1-z)^{N-1},$$

with $a=1/\mu_1$, $b=\lambda/(1-\lambda)$ and $z\in[0,1]$. We note that $E[p;N,\mu_1]=\int_0^1 p(z;N,\mu_1)dz$; therefore $E[p;N+1,\mu_1]\geq E[p;N,\mu_1]$ if and only if

$$\int_0^1(p(z;N+1,\mu_1)-p(z;N,\mu_1))dz\geq 0. \tag{A.2}$$

The L.H.S. of (A.2) can be written as

$$\int_0^1\frac{abv(N(1-z)^{N-1}-(N+1)(1-z)^N)}{g(z;N+1,\mu_1)g(z;N,\mu_1)}dz$$
$$=\int_{\frac{1}{N+1}}^1\frac{abv(N(1-z)^{N-1}-(N+1)(1-z)^N)}{g(z;N+1,\mu_1)g(z;N,\mu_1)}dz-\int_0^{\frac{1}{N+1}}\frac{abv((N+1)(1-z)^N-N(1-z)^{N-1})}{g(z;N+1,\mu_1)g(z;N,\mu_1)}dz$$
$$\geq\int_{\frac{1}{N+1}}^1\frac{abv(N(1-z)^{N-1}-(N+1)(1-z)^N)}{g\left(\frac{1}{N+1};N+1,\mu_1\right)g\left(\frac{1}{N+1};N,\mu_1\right)}dz-\int_0^{\frac{1}{N+1}}\frac{abv((N+1)(1-z)^N-N(1-z)^{N-1})}{g\left(\frac{1}{N+1};N+1,\mu_1\right)g\left(\frac{1}{N+1};N,\mu_1\right)}dz$$
$$=\frac{abv}{g\left(\frac{1}{N+1};N+1,\mu_1\right)g\left(\frac{1}{N+1};N,\mu_1\right)}\left[\int_{\frac{1}{N+1}}^1(N(1-z)^{N-1}-(N+1)(1-z)^N)dz\right.$$
$$\left.-\int_0^{\frac{1}{N+1}}((N+1)(1-z)^N-N(1-z)^{N-1})dz\right]$$
$$=\frac{abv}{g\left(\frac{1}{N+1};N+1,\mu_1\right)g\left(\frac{1}{N+1};N,\mu_1\right)}\int_0^1(N(1-z)^{N-1}-(N+1)(1-z)^N)dz=0,$$

where the inequality follows from the fact that $g(z;N,\mu_1)$ is a decreasing function of z.

It remains to show that expected price does not converge to the monopoly price as N goes to infinity, when consumers search with high intensity. We note that a solution to (7) can be found for $N\to\infty$. The reason is that, for any given value of the other parameters, as N increases, the term $\lambda(1-F(p))^{N-1}$ in (7) approaches zero much more quickly than the other terms. As a result, when $N\to\infty$ firms ignore the informed consumers and we can find the limit price distribution by setting $\lambda=0$ in (7) and solving for $F(p)$:

$$F(p;\infty,\mu_1)=\frac{2-\mu_1}{2(1-\mu_1)}-\frac{\mu_1}{2(1-\mu_1)}\frac{v}{p};\qquad\text{with } p\in\left[\frac{\mu_1 v}{2-\mu_1},v\right].$$

We note that this solution is similar to the one obtained by Burdett and Judd (1983), for competitive markets without fully-informed consumers. A little algebra yields the expected price $E[p;\infty,\mu_1]=\mu_1 v\ln[(2-\mu_1)/\mu_1]/(2(1-\mu_1))<v$, for all $\mu_1\in(0,1)$. ‖

Proof of Proposition 7. Using the inverse function $p(z; N, \mu_1)$ described above, Condition 5.1 can be rewritten as follows:

$$\int_0^1 \frac{p(z; N, \mu_1)(2z-1)}{v} dz = \frac{c}{v}. \tag{A.3}$$

Note that the L.H.S. of (A.3) is $\rho(\mu_1, N, \lambda)$ as defined above. Differentiating with respect to μ_1 yields

$$\frac{\partial \rho(\mu_1, N, \lambda)}{\partial \mu_1} = \int_0^1 \frac{a^2(2z-1)[2(1-z) + bN(1-z)^{N-1}]}{[1 + 2(a-1)(1-z) + abN(1-z)^{N-1}]^2} dz.$$

Differentiating again with respect to μ_1 yields:

$$\frac{\partial^2 \rho(\mu_1, N, \lambda)}{\partial \mu_1^2} = -\int_0^1 \frac{2a^3(2z-1)^2[2(1-z) + bN(1-z)^{N-1}]}{[1 + 2(a-1)(1-z) + abN(1-z)^{N-1}]^3} dz < 0,$$

where the inequality follows from noting that the integrand is a positive number. Therefore, the function $\rho(\mu_1, N, \lambda)$ is strictly concave in μ_1.

We now note that $\rho(1, N, \lambda) > 0$.

$$\begin{aligned}\rho(1, N, \lambda) &= \int_0^1 \frac{2z-1}{1+abN(1-z)^{N-1}} dz = \int_{\frac{1}{2}}^1 \frac{2z-1}{1+abN(1-z)^{N-1}} dz - \int_0^{\frac{1}{2}} \frac{1-2z}{1+abN(1-z)^{N-1}} dz \\ &> \int_{\frac{1}{2}}^1 \frac{2z-1}{1+ab\frac{N}{2^{N-1}}} dz - \int_0^{\frac{1}{2}} \frac{1-2z}{1+ab\frac{N}{2^{N-1}}} dz = \frac{1}{1+ab\frac{N}{2^{N-1}}} \int_0^1 (2z-1) dz = 0.\end{aligned}$$

On the other hand, it is easily seen that $\rho(0, N, \lambda) = 0$. Since $\rho(\mu_1, N, \lambda)$ is strictly concave in μ_1 and $\rho(1, N, \lambda) > 0$, it follows that, provided $c/v \in (0, \overline{\rho}(\mu_1, N, \lambda))$, equation (A.3) has either a single solution or two solutions. Given that μ_1^* is the solution to $\rho(\mu_1, N, \lambda) - c/v = 0$, we can plug it into $F(p; N, \mu_1)$ to obtain the equilibrium price distribution with endogenous search. ‖

Proof of Lemma 6. We first prove that consumers search more under triopoly than under duopoly. To show this, we compare $\rho(\mu_1, N+1, \lambda)$ with $\rho(\mu_1, N, \lambda)$:

$$\begin{aligned}\rho(\mu_1, N+1, \lambda) - \rho(\mu_1, N, \lambda) &= \frac{1}{v}\int_0^1 (2z-1)[p(z; N+1, \mu_1) - p(z; N, \mu_1)] dz \\ &= \int_0^1 (2z-1)\left[\frac{1}{g(z; N+1, \mu_1)} - \frac{1}{g(z; N, \mu_1)}\right] dz \\ &= \int_0^1 (2z-1)\left[\frac{g(z; N, \mu_1) - g(z; N+1, \mu_1)}{g(z; N+1, \mu_1) g(z; N, \mu_1)}\right] dz \\ &= \int_0^1 \frac{ab(1-z)^{N-1}((N+1)z-1)(2z-1)}{g(z; N+1, \mu_1) g(z; N, \mu_1)} dz = \xi_1 + \xi_2,\end{aligned}$$

where

$$\begin{aligned}\xi_1 &= \int_0^{\frac{1}{N+1}} \frac{ab(1-z)^{N-1}((N+1)z-1)(2z-1)}{g(z; N+1, \mu_1) g(z; N, \mu_1)} dz \\ \xi_2 &= \int_{\frac{1}{2}}^1 \frac{ab(1-z)^{N-1}((N+1)z-1)(2z-1)}{g(z; N+1, \mu_1) g(z; N, \mu_1)} dz - \int_{\frac{1}{N+1}}^{\frac{1}{2}} \frac{ab(1-z)^{N-1}((N+1)z-1)(1-2z)}{g(z; N+1, \mu_1) g(z; N, \mu_1)} dz.\end{aligned}$$

We note that since the integrand is positive, $\xi_1 > 0$. Moreover,

$$\xi_2 > \frac{ab}{g\left(\frac{1}{2}; N+1, \mu_1\right) g\left(\frac{1}{2}; N, \mu_1\right)} \int_{\frac{1}{N+1}}^1 (1-z)^{N-1}((N+1)z-1)(2z-1) dz.$$

Note now that

$$\begin{aligned}\int_{\frac{1}{N+1}}^1 (1-z)^{N-1}((N+1)z-1)(2z-1) dz &= 2(N+1)\int_{\frac{1}{N+1}}^1 z^2(1-z)^{N-1} dz + \int_{\frac{1}{N+1}}^1 (1-z)^{N-1} dz \\ &\quad - (N+3)\int_{\frac{1}{N+1}}^1 z(1-z)^{N-1} dz = \frac{\left(\frac{N}{N+1}\right)^N (2 + N(3-N))}{(N+1)^2(N+2)},\end{aligned}$$

where the second equality follows from integration by parts. Then, $\xi_2 > 0$, and by implication $\rho(\mu_1, N+1, \lambda) > \rho(\mu_1, N, \lambda)$, for all λ and μ_1, and $N \leq 3$. This proves that less-informed consumers search more under triopoly than under duopoly.

To complete the argument, we show that there exists a number of firms $\hat{N}$ sufficiently large such that for all $N \geq \hat{N}$, $\rho(\mu_1, N+1, \lambda) < \rho(\mu_1, N), \lambda$ and therefore less-informed consumers' search incentives decline for all $N \geq \hat{N}$. To show this, we study the derivative of $\rho(\mu_1, N, \lambda)$ with respect to N. Recall that

$$\rho(\mu_1, N, \lambda) = \int_0^1 \frac{2z-1}{1+2(a-1)(1-z)+abN(1-z)^{N-1}} dz$$

where $a = 1/\mu_1$ and $b = \lambda/(1-\lambda)$. Differentiation yields

$$\frac{1}{ab}\frac{\rho(\mu_1, N, \lambda)}{\partial N} = \int_0^1 \frac{(1-2z)(1-z)^{N-1}(1+N\ln(1-z))}{(1+2(a-1)(1-z)+abN(1-z)^{N-1})^2} dz. \tag{A.4}$$

We need to show that the R.H.S. of (A.4) is negative for N large. We note that this is equivalent to showing that $I_1 - I_2 + I_3 < 0$, where

$$I_1 = \int_0^{1-e^{-\frac{1}{N}}} \frac{(1-2z)(1-z)^{N-1}(1+N\ln(1-z))}{(1+2(a-1)(1-z)+abN(1-z)^{N-1})^2} dz > 0,$$
$$I_2 = \int_{1-e^{-\frac{1}{N}}}^{\frac{1}{2}} \frac{(2z-1)(1-z)^{N-1}(1+N\ln(1-z))}{(1+2(a-1)(1-z)+abN(1-z)^{N-1})^2} dz > 0,$$
$$I_3 = \int_{\frac{1}{2}}^{1} \frac{(1-2z)(1-z)^{N-1}(1+N\ln(1-z))}{(1+2(a-1)(1-z)+abN(1-z)^{N-1})^2} dz > 0.$$

We now observe that, for N large, the integral

$$\hat{I}_2 = \int_{\frac{\ln N}{2N}}^{\frac{\ln N}{N}} \frac{(2z-1)(1-z)^{N-1}(1+N\ln(1-z))}{(1+2(a-1)(1-z)+abN(1-z)^{N-1})^2} dz < I_2.$$

Therefore, showing that $\hat{I}_2 > I_1 + I_3$ for large N suffices. Notice that

$$I_1 < \frac{1}{\left(1+2(a-1)e^{\frac{-1}{N}}+abNe^{\frac{-(N-1)}{N}}\right)^2} \int_0^{1-e^{-\frac{1}{N}}} (1-2z)(1-z)^{N-1}(1+N\ln(1-z))dz$$
$$< \frac{1}{\left(1+2(a-1)e^{\frac{-1}{N}}+abNe^{\frac{-(N-1)}{N}}\right)^2} \int_0^{1-e^{-\frac{1}{N}}} (1-z)^{N-1}(1+N\ln(1-z))dz$$
$$= \frac{1}{\left(1+2(a-1)e^{\frac{-1}{N}}+abNe^{\frac{-(N-1)}{N}}\right)^2} - (1-z)^N \ln(1-z)\Bigg|_0^{1-e^{-\frac{1}{N}}}$$
$$= \frac{1}{Ne\left(1+2(a-1)e^{\frac{-1}{N}}+abNe^{-1+\frac{1}{N}}\right)^2} <_{\text{for large } N} \frac{e}{a^2b^2N^3}.$$

Moreover,

$$I_3 < \int_{\frac{1}{2}}^{1} (1-2z)(1-z)^{N-1}(1+N\ln(1-z))dz < \int_{\frac{1}{2}}^{1} -(1-z)^{N-1}(1+N\ln(1-z))dz = \frac{\ln 2}{2^N}.$$

Finally, we note that

$$\hat{I}_2 > \frac{1}{\left(1+2(a-1)\left(1-\frac{\ln N}{2N}\right)+abN\left(1-\frac{\ln N}{2N}\right)^{N-1}\right)^2} \int_{\frac{\ln N}{2N}}^{\frac{\ln N}{N}} (2z-1)(1-z)^{N-1}(1+N\ln(1-z))dz$$
$$> \frac{\left(\frac{\ln N}{N}-1\right)}{\left(1+2(a-1)\left(1-\frac{\ln N}{2N}\right)+abN\left(1-\frac{\ln N}{2N}\right)^{N-1}\right)^2} \int_{\frac{\ln N}{2N}}^{\frac{\ln N}{N}} (1-z)^{N-1}(1+N\ln(1-z))dz$$

$$= \frac{\left(\frac{\ln N}{N} - 1\right)\left[\left(1 - \frac{\ln N}{2N}\right)^N \ln\left(1 - \frac{\ln N}{2N}\right) - \left(1 - \frac{\ln N}{N}\right)^N \ln\left(1 - \frac{\ln N}{N}\right)\right]}{\left(1 + 2(a-1)\left(1 - \frac{\ln N}{2N}\right) + abN\left(1 - \frac{\ln N}{2N}\right)^{N-1}\right)^2}$$

$$>_{\text{for large } N} \frac{\left(1 - \frac{\ln N}{N}\right)\left[-\frac{1}{N}\frac{\ln N}{N} + \frac{1}{\sqrt{N}}\frac{\ln N}{2N}\right]}{\left(1 + 2(a-1) + ab\sqrt{N}\right)^2} >_{\text{for large } N} \frac{\ln N}{2a^2b^2N^2\sqrt{N}}.$$

Now, since

$$\frac{\ln N}{2a^2b^2N^2\sqrt{N}} > \frac{e}{a^2b^2N^3} + \frac{\ln 2}{2^N}$$

for large N, the result follows.

It remains to prove that $\mu_1^*(\infty) = (1-\lambda)\mu_1^*(2)$. Setting $N = 2$ in (A.3), integrating and rearranging yields

$$\frac{(1-\lambda)\mu_1^*(2)}{2(1-(1-\lambda)\mu_1^*(2))}\left[\frac{1}{1-(1-\lambda)\mu_1^*(2)}\ln\left(\frac{2-(1-\lambda)\mu_1^*(2)}{(1-\lambda)\mu_1^*(2)}\right) - 2\right] = \frac{c}{v}. \tag{A.5}$$

Similarly, setting $N = \infty$ in (A.3) yields

$$\frac{\mu_1^*(\infty)}{2(1-\mu_1^*(\infty))}\left[\frac{1}{1-\mu_1^*(\infty)}\ln\left(\frac{2-\mu_1^*(\infty)}{\mu_1^*(\infty)}\right) - 2\right] = \frac{c}{v}. \tag{A.6}$$

Inspection of (A.5) and (A.6) reveals the result. ‖

Proof of Proposition 8. Setting $N = 2$ in (7) and solving for $F(p; 2)$ yields

$$F(p; 2, \mu_1^*(2)) = \frac{2-(1-\lambda)\mu_1^*(2)}{2(1-(1-\lambda)\mu_1^*(2))} - \frac{(1-\lambda)\mu_1^*(2)}{2(1-(1-\lambda)\mu_1^*(2))}\frac{v}{p}.$$

Similarly, setting $N \to \infty$ in (7) and solving for the limiting price distribution yields

$$F(p; \infty, \mu_1^*(\infty)) = 1 - \frac{\mu_1^*(\infty)}{2(1-\mu_1^*(\infty))}\frac{v-p}{p}.$$ [19]

Since $\mu_1^*(\infty) = (1-\lambda)\mu_1^*(2)$ as shown in Lemma 6, it follows that $F(p; 2, \mu_1^*(2)) = F(p; \infty, \mu_1^*(\infty))$. The second part of the result follows from Proposition 6 and Lemma 6. ‖

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REFERENCES

BAYE, M. R., KOVENOCK, D. and DE VRIES, C. (1992), "It Takes Two to Tango: Equilibria in a Model of Sales", *Games and Economic Behavior*, **4**, 493–510.

BAYE, M. R. and MORGAN, J. (2001), "Information Gatekeepers on the Internet and the Competitiveness of Homogeneous Product Markets", *American Economic Review*, **91**, 454–474.

BENABOU, R. (1993), "Search Market Equilibrium, Bilateral Heterogeneity and Repeat Purchases", *Journal of Economic Theory*, **60**, 140–158.

BESTER, H. (1988), "Bargaining, Search Costs and Equilibrium Price Distributions", *Review of Economic Studies*, **55**, 201–214.

BESTER, H. (1994), "Price Commitment in Search Markets", *Journal of Economic Behavior and Organization*, **25**, 109–120.

BRAVERMAN, A. (1980), "Consumer Search and Alternative Market Equilibria", *Review of Economic Studies*, **47**, 487–502.

BURDETT, K. and COLES, M. G. (1997), "Steady State Price Distributions in a Noisy Search Equilibrium", *Journal of Economic Theory*, **72**, 1–32.

19. Note that this is exactly the distribution studied by Burdett and Judd (1983) for the case where marginal costs are equal to 0.

BURDETT, K. and JUDD, K. L. (1983), "Equilibrium Price Dispersion", *Econometrica*, **51**, 955–969.

DIAMOND, P. A. (1971), "A Model of Price Adjustment", *Journal of Economic Theory*, **3**, 156–168.

FERSHTMAN, C. and FISHMAN, A. (1992), "Price Cycles and Booms: Dynamic Search Equilibrium", *American Economic Review*, **82**, 1221–1233.

FERSHTMAN, C. and FISHMAN, A. (1994), "The 'Perverse' Effects of Wage and Price Controls in Search Markets", *European Economic Review*, **38**, 1099–1112.

JANSSEN, M. and MORAGA-GONZÁLEZ, J. L. (2000), "Pricing, Consumer Search and the Maturity of Internet Markets" (Tinbergen Institute Discussion Paper TI 2000-042/1, the Netherlands).

MACMINN, R. D. (1980), "Search and Market Equilibrium", *Journal of Political Economy*, **88**, 308–327.

MCAFEE, R. P. (1995), "Multiproduct Equilibrium Price Dispersion", *Journal of Economic Theory*, **67**, 83–105.

MORGAN, P. and MANNING, R. (1985), "Optimal Search", *Econometrica*, **53**, 923–944.

REINGANUM, J. F. (1979), "A Simple Model of Equilibrium Price Dispersion", *Journal of Political Economy*, **87**, 851–858.

ROB, R. (1985), "Equilibrium Price Distributions", *Review of Economic Studies*, **52**, 452–504.

ROSENTHAL, R. W. (1980), "A Model in Which an Increase in the Number of Sellers Leads to a Higher Price", *Econometrica*, **48**, 1575–1579.

RUFFIN, R. J. (1971), "Cournot Oligopoly and Competitive Behavior", *Review of Economic Studies*, **38**, 493–502.

SALOP, S. C. and STIGLITZ, J. E. (1982), "The Theory of Sales: A Simple Model of Equilibrium Price Dispersion with Identical Agents", *American Economic Review*, **79**, 1121–1130.

SATTERTHWAITE, M. (1979), "Consumer Information, Equilibrium Industry Price, and the Number of Sellers", *Bell Journal of Economics*, **10**, 483–502.

STAHL, D. O. (1989), "Oligopolistic Pricing with Sequential Consumer Search", *American Economic Review*, **79**, 700–712.

STAHL, D. O. (1996), "Oligopolistic Pricing with Heterogeneous Consumer Search", *International Journal of Industrial Organization*, **14**, 243–268.

STIGLER, G. (1961), "The Economics of Information", *Journal of Political Economy*, **69**, 213–225.

STIGLITZ, J. E. (1987), "Competition and the Number of Firms in a Market: Are Duopolies More Competitive than Atomistic Markets?", *Journal of Political Economy*, **95**, 1041–1061.

VARIAN, H. R. (1980), "A Model of Sales", *American Economic Review*, **70**, 651–659.