Coupon-advertising under imperfect price information*

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Abstract

This paper studies sales promotions through coupons in an oligopoly under imperfect price information. Sellers can distribute either ordinary coupons, or coupon-(price) advertising, or both types of coupons, at distant locations to attract consumers from their rivals' markets. A unique symmetric pure strategy equilibrium exists where rebates and couponing intensity are always positive. In the ordinary coupons equilibrium, prices, promotional efforts and sellers' profits are higher than in the coupon-advertising equilibrium. However, if sellers are allowed to distribute both types of coupons, only coupon-advertising is sent out in equilibrium.

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1. Introduction

Coupon-advertising (or in-ad coupons) is being extensively used nowadays by manufacturers and retailers to promote their sales. An estimated 85% of retailers used in-ad coupons in 1991, according to NCH Promotional Services.[1] Coupon-advertising is a hybrid sales promotion tool with the characteristics of both ordinary coupons and advertising. In particular, coupon-advertising often includes brand information such as product characteristics, location, quality, guarantees, etc., as well as information about regular prices. At the same time, it includes a coupon that offers a discount off the regular price. Due to the twofold feature of in-ad coupons, the effects of coupon-advertising on consumers' behavior, firms' strategies and market equilibria cannot be explained by the analysis of ordinary coupons, or of advertising alone. As the literature has shown, there are various interactive effects between the coupon and the print advertisement.[2] This literature has restricted attention to the case where coupon-advertising conveys brand information alone. However, casual observation reveals that coupons often post both the seller's regular price and its rebate.[3] So far, the literature has paid little attention to coupon-advertising that conveys price information.

The purpose of this paper is to analyze sales promotions[4] in an oligopolistic market where consumers have imperfect price information. Consumers are uninformed about the price charged at a distant store, and can only learn this price either by venturing, at some cost, to the distant location, or if they receive price advertisements from the distant seller. On the other hand, consumers can learn the local price by visiting the home store at no cost.[5] Moreover, consumers are assumed to know the features of

[1] See Lawrence and Hume (1992); they also report that the number of manufacturers' coupons distributed by supermarkets through their own advertising inserts (in-ad coupons) increased 700% from 1990 to 1992. Leclerc and Little (1997) report that approximately 80% of the over \$6.5 billion value of coupons distributed in the United States in 1995 were delivered through free-standing inserts (FSIs) in magazines and newspapers, and that a coupon in an FSI often appears within an advertisement. Hahn et al. (1995) estimated that over 8% of advertisements in major Korean newspapers and magazines in 1991 were of the sales promotional advertising type.

[2] Leclerc and Little (1997) investigate whether the content of the print advertisement influences the effectiveness of the coupon. They found experimental evidence that the executional cue on the print advertisement can enhance brand attitude, as suggested by Schultz et al. (1993). Hahn et al. (1995) obtain similar results for the Korean markets.

[3] For instance, by visiting Web sites (such as www.savecoupons.com or www.hotcoupons.com), we observe that cybercoupons posting the regular price are usually offered by pizza stores, car and limousine services, dry-cleaning and laundry services, etc. Moreover, casual observation reveals that the same type of sellers often send, via direct mail, coupon-advertising to consumers living in distant neighborhoods.

[4] For a detailed description of various methods of sales promotion used by firms, see e.g., Kotler (1994).

[5] We thus refer to a locationally segmented market. The same analysis, however, applies to a market that is segmented due to consumers' brand loyalty. Note that the key feature in our analysis is the informational segmentation of the market.

all the brands offered in the market. Sellers can attract consumers from the rival's location by sending out coupons with, or without, advertising their regular price on the coupon. That is, sellers can issue two types of coupons: (i) ordinary coupons, i.e. coupons simply offering a discount, and (ii) coupon-advertising, i.e. coupons including price advertisements. Both types of coupons serve as a price discrimination device.[6] However, coupon-advertising also conveys price information, a role of coupons that has been neglected in the literature. Note that, if the costs of issuing and handling coupons are the same as the costs of advertising, a coupon-advertising offering a zero rebate is equivalent to price advertising.

Our paper is thus related to two strands of the sales promotion literature. The first is the literature on coupons as price discrimination devices under perfect price information. Narasimhan (1984) analyzes the case of a competitive firm, while Bester and Petrakis (BP) (1996) and Shaffer and Zhang (1995) consider oligopolistic markets where consumers are differentiated with respect to their brand loyalty or location. The second is the literature on advertising under imperfect price information. Stegeman (1991) studies price and product advertising in a competitive market. In a monopoly model, Bester (1995) considers price advertising, and Caminal (1996) studies price advertising and couponing in the presence of imperfect cost information. Stahl (1994) and Peters (1984) analyze a homogeneous industry under imperfect brand and price information, while Narashiman (1988) and Bester and Petrakis (BP) (1995) analyze price advertising in locationally (or due to brand loyalty) segmented markets. With the exception of Caminal (1996), the effects of coupons and price advertising have been analyzed in separate streams in the sales promotion literature.

Our model combines these two strands of literature by investigating firms' couponing strategies and market equilibria in a locationally (as in BP, 1996) and informationally (as in BP, 1995) segmented market. In addition, our imperfect price information framework allows us to consider price advertising, coupon-advertising and ordinary coupons simultaneously. We show that a symmetric pure strategy equilibrium exists whenever the sellers are allowed to distribute either coupon-advertising alone, or ordinary coupons alone, or both types of coupons simultaneously. In equilibrium, sellers always send out coupons to the distant location to attract consumers from their rivals' markets, and thus increase their profits by price discriminating between coupon holders and non-holders.

Contrary to the price advertising literature where price reductions are temporary,[7]

[6] See e.g. Narasimhan (1984), Caminal (1996), Bester and Petrakis (1996), and Shaffer and Zhang (1995). In this case, coupons foster competition. Coupons may also motivate retail participation in price promotions (Gerstner and Hess, 1991), or create switching costs under repeated purchases (Banerjee and Summers, 1987; Caminal and Matutes, 1990). Contrary to the first case, competition is now relaxed.

[7] For example, in Shilony (1977), Varian (1980), Narasimhan (1988), and Bester and Petrakis (1995), the only symmetric advertising equilibrium is in mixed strategies. The intuition is that an individual seller can gain by advertising his price only if he offers a lower price than his rivals. Obvi-

sales promotions are permanent when carried out through coupon-advertising or ordinary coupons.[8] As all our equilibria are in pure strategies, sellers always offer price discounts by distributing coupons to the consumers in distant locations. Moreover, in the coupon-advertising equilibrium, the sellers' regular prices are also permanently advertised. Interestingly, as equilibrium rebates are positive, sellers will never send out price advertisements if they are able to promote their sales through coupons. This result holds even if the costs of issuing and handling coupons are higher than the price advertising costs. Therefore, our prediction is that price advertising should not be observed in locationally and informationally segmented markets unless the advertising costs are low enough.

Sellers' profits are higher in the ordinary coupon equilibrium than in the coupon-advertising equilibrium, and the latter are higher than in the equilibrium under perfect price information (as in BP, 1996). As coupon-advertising conveys price information, it decreases informational asymmetries among consumers and thus fosters competition. Equilibrium prices, rebates, and couponing intensity are lower when sellers send out coupon-advertising rather than ordinary coupons. Obviously, if consumers are fully informed about all prices in the market, then equilibrium prices, rebates and couponing intensity are lower than under imperfect price information.

If sellers are allowed to send out both types of coupons, we show that they promote their sales by distributing only coupon-advertising in equilibrium. Indeed, Oskin (1993), in his guidelines for smart coupon marketing, advises retailers to print the regular price on the coupon together with the cents-off discount. The sellers would prefer to distribute ordinary coupons alone, since in this way their profits are higher. However, an individual seller, by advertising his regular price (besides offering a rebate) in the distant location, can steal business from his rival and thus increase his profits. Sellers find themselves into a Prisoner's Dilemma situation, and hence attain lower profits in equilibrium. Therefore, our analysis predicts that ordinary coupons should not be observed in markets presenting locational as well as informational segmentation.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 and Section 4 characterize the symmetric pure strategy equilibria when sellers are allowed to send out coupon-advertising alone and ordinary coupons alone, respectively. Section 4 also compares the above two market equilibria and the equilibrium under perfect price information. Section 5 extends the model by allowing sellers to issue both types of coupons and shows that, in equilibrium, sellers send exclusively coupon-

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ously, this cannot be simultaneously fulfilled for all sellers in a symmetric pure strategy equilibrium. This literature gives a dynamic interpretation to the mixed strategy equilibria arguing that temporal price dispersion occurs (as each store varies its price over time).

[8] According to Kotler (1994), many products such as soft-drinks and cereals are permanently on sale. Litwak (1996) reports that for some product categories, price reductions have become the norm and sales at the everyday price the rarity. World Wide Web coupons is an example of permanent promotions since consumers can print and use them at any moment. Oskin (1993), who provides guidelines for smart coupon marketing, suggests that the sellers should send coupons, via direct mail, over and over again so that the new coupon arrives just before the old one expires.

advertising. Section 6 contains comparative statics and welfare results. Section 7 concludes. All the proofs are relegated to the Appendix.

2. The model

As in Bester and Petrakis (1996), we consider a market with two firms, A and B, located in different regions. Firms produce a homogeneous good at zero cost. The neighborhood of each firm is inhabited by a unit mass of consumers who have unit demands and a common reservation utility $v > 0$. A consumer j can costlessly visit the store at his home location, but incurs a transportation cost $s_j \geq 0$ to visit the distant store, where s_j is uniformly distributed on $[0; \bar{s}]$. To avoid local monopolies, assume that $v > \bar{s}$; i.e. transportation costs are not too high.

Consumers are aware of the existence, characteristics and availability of both goods and can learn at no cost the price charged at their home location. Consumers are uninformed about the price charged at the distant location, and can only learn this price either by venturing to the distant store at a certain cost, or if the distant seller advertises his price. Firms can promote their sales by distributing coupons in their rivals' regions. Under our imperfect price information setting, sellers can issue two types of coupons: (i) coupon-advertising, i.e. coupons that offer a rebate and, at the same time, advertise the regular price, and (ii) ordinary coupons, i.e. coupons that only offer a discount. Both types of coupons serve as a price discrimination device[9] as sellers can charge different prices to consumers from distinct locations. Coupon-advertising also serves as a vehicle to transmit price information as firms inform distant consumers about their regular prices. Note that a coupon-advertising offering a zero rebate is equivalent to pure price advertising.

Formally, seller i sends out coupons to a fraction $\lambda_i \in [0; 1]$ of the consumers at location j, offering a rebate r_i on his regular price p_i; $0 \leq r_i \leq p_i$; $i = A; B$.[10] Without loss of generality, let $0 \leq p_i \leq v$; $i = A; B$. Since individual transportation costs are unobservable, λ_i represents the probability that any consumer in region j receives a coupon. Consumers in region j receiving a coupon from firm i can buy the good at price $p_i - r_i$, while the rest have to pay p_i. To reach a fraction of consumers λ, a seller incurs a cost $k(\lambda)$. Following the literature (see e.g. Butters, 1977; Grossman and Shapiro, 1984; BP, 1995, 1996), we assume that $k(\cdot)$ is increasing, convex and $k(0) = 0$.[11]

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[9] As in Bester and Petrakis (1996), Caminal (1996), Narashiman (1984) and Shaffer and Zhang (1995).

[10] It can be checked that in our model sellers cannot increase their profits by distributing coupons to their home locations.

[11] Note that we implicitly assume that printing the price on a coupon has no extra cost, i.e. coupon-advertising and ordinary coupons are equally costly.

3. Coupon-Advertising Equilibrium

In this section we assume that sellers can only send out coupon-advertising. Firms compete in the market by simultaneously choosing their prices, rebates and intensities of couponing. Let $x_i = (p_i; r_i; \lambda_i)$; $i = A; B$ denote seller i's marketing strategy. Firm i chooses x_i to maximize its profits taking as given its rival's strategy.

We first determine firm i's demand and profits. We distinguish four types of buyers. First, a consumer located at A receiving a coupon-advertising from B learns firm B's regular price and rebate. He then buys at his home store if $p_A \leq p_B - r_B + s$ and $v \geq p_A$: Second, a consumer at A not receiving firm B's coupon purchases at A whenever $p_A \leq p_B^e + s$ and $v \geq p_A$; where p_B^e is the price he expects to be charged at store B. Third, a consumer at B having a coupon from A learns firm A's price and rebate. He switches store and buys at A if $p_A - r_A + s \leq p_B$ and $v \geq p_A - r_A + s$: Finally, a consumer at B not receiving a coupon from A buys at store A if $p_A^e + s \leq p_B$ and $v \geq p_A^e + s$; where p_A^e is his expected price at A. Then firm A's profits are:

$$\begin{aligned}\Pi_A(x_A; x_B) = {} & p_A(1-\lambda_B)\max\left\{0; \min\left\{1-\frac{p_A-p_B^e}{\bar{s}}; 1\right\}\right\} \\ & + p_A\lambda_B \max\left\{0; \min\left\{1-\frac{p_A-(p_B-r_B)}{\bar{s}}; 1\right\}\right\} \\ & + (p_A-r_A)\lambda_A \max\left\{0; \min\left\{\frac{p_B-(p_A-r_A)}{\bar{s}}; \frac{v-(p_A-r_A)}{\bar{s}}; 1\right\}\right\} \\ & + p_A(1-\lambda_A)\max\left\{0; \min\left\{\frac{p_B-p_A^e}{\bar{s}}; \frac{v-p_A^e}{\bar{s}}; 1\right\}\right\} - k(\lambda_A):\end{aligned} \tag{1}$$

By symmetry, firm B's profits are given by an analogous expression. We shall restrict attention to symmetric Nash-equilibria where uninformed consumers anticipate correctly the prices charged by the distant firm.

Definition 1. A pair of marketing strategies $(x_A^*; x_B^*)$ is a symmetric pure strategy equilibrium if (i) $\Pi_i(x_i^*; x_j^*) \geq \Pi_i(x_i; x_j^*)$ for all $x_i \neq x_i^*$; $i \neq j$; $i; j = A; B$; (ii) $x_A^* = x_B^*$; and (iii) $p_i^e = p_i^*$; $i = A; B$:

Condition (i) says that each marketing strategy is a best-reply to the rival's. Condition (ii) imposes symmetry. Finally, condition (iii) requires consumers' price expectations to be fulfilled in equilibrium. Let $x = (p; r; \lambda)$ denote the symmetric equilibrium market strategy. The following Proposition characterizes the interior equilibrium with coupon-advertising and provides conditions for its existence.

Proposition 1. If sellers promote their sales by sending out coupon-advertising, a unique interior symmetric pure strategy equilibrium[12] exists if and only if (a) $k'(1) >$

[12] We are unable to prove, or disprove, the existence of an asymmetric pure strategy equilibrium

$\bar{s}/9$ and (b) $k'(2\bar{s}/3v) < v^2/4\bar{s}$. This equilibrium is given by the unique solution to the following system of equations:

$$p = 2\bar{s}/3\lambda; \quad r = 0.5p; \quad p^2/4\bar{s} = k'(\lambda): \qquad (2a,b,c)$$

In equilibrium, a fraction $\min[1; 1/3\lambda]$ of consumers redeem their coupons at the distant store, while the rest buy from their home store at the regular price.

The equilibrium price and coupon-advertising intensity are shown in Figure 1. The downward-sloping curve PP depicts equation (2a) and the upward-sloping curve KK depicts equation (2c). The intersection of these two curves provides the equilibrium price and coupon-advertising intensity (point E). The role that conditions (a) and (b) play in guaranteeing an interior solution can easily be seen in Figure 1.[13]

< insert figure 1 here>

The fact that a pure strategy equilibrium exists when sellers send out coupon-advertising suggests that sales promotions are permanent under imperfect price information. This contrasts with the existing literature where sales promotions are temporary.[14] In this literature a pure strategy equilibrium fails to exist and the equilibrium in mixed strategies is interpreted as if each store were varying its price over time. In this sense, sales promotions generate temporal price dispersion as sellers advertise their prices only with some probability at each date.

In equilibrium the rebate is strictly positive and, in particular, equals half the regular price.[15] As a coupon offering zero rebate is equivalent to pure price advertising in our model, Proposition 1 predicts that pure price advertising should not be observed in markets with both informational and locational (e.g. due to brand loyalty) segmentation.

One might argue, however, that handling coupon-advertising generates extra costs for the seller, and thus sending out price advertisements is significantly cheaper than distributing coupons (see e.g. Caminal, 1996). It can be shown that if the price

in the general case. In a candidate asymmetric equilibrium, if $p_A < p_B$ then it must also hold that $\lambda_A < \lambda_B$; that is, the seller with the higher price must distribute more coupon-advertising. A number of simulations that we have conducted using the family of couponing costs functions, $k(\lambda) = m\lambda^a$; $a \geq 2$; indicate that the above conditions cannot hold simultaneously, and thus an asymmetric equilibrium fails to exist.

[13] Clearly, if marginal couponing costs are too low, sellers will set $\lambda^* = 1$; while, if they are too high, sellers will charge $p = v$ to appropriate the entire home consumers' surplus (with or without sending out coupons to the distant location).

[14] See Shilony (1977), Butters (1977), Varian (1980), Narasimhan (1988), Stahl (1994) and Bester and Petrakis (1995).

[15] This is due to our assumption that transportation costs are uniformly distributed. It can be checked that if this distribution is biased towards low-transportation-cost consumers (e.g. s is distributed exponentially), the equilibrium rebate is less than half the regular price.

advertising costs are much lower than the coupon-advertising costs, the marketing strategies of Proposition 1 will no longer constitute an equilibrium, provided that the costs of couponing are sufficiently convex. Consider an extended model where sellers can send out both pure price advertising and coupon-advertising. Let the advertising costs be $k_A(\lambda) = \delta k(\lambda)$; $0 \leq \delta < 1$: Note first that price advertising and coupons cannot coexist in a symmetric pure strategy equilibrium. The intuition is simply that by advertising his price, a seller can attract consumers from the distant location only if he offers a lower price than his rival's. This however cannot happen simultaneously for all the sellers in a symmetric pure strategy equilibrium (see BP, 1995). Now if δ is sufficiently small, an individual seller may find it profitable to deviate by lowering his price and advertising the lower price to all the consumers at the distant location. It can be shown that, if the equilibrium couponing intensity is not too high, such a deviation will be profitable for the seller. Then the marketing strategies in Proposition 1 cannot be sustained as an equilibrium in the extended model.

4. The equilibrium with ordinary coupons

Assume now that sellers can only promote their sales by sending out ordinary coupons. We first determine seller A's profits. As previously, four different groups of consumers can be distinguished. A consumer at A receiving firm B's coupon learns only the rebate offered by firm B. He buys at A if $p_A \leq p_B^e - r_B + s$ and $v \geq p_A$: A consumer at A not receiving firm B's coupon purchases at his home store whenever $p_A \leq p_B^e + s$ and $v \geq p_A$: A consumer at B who receives a coupon from A only learns the rebate offered by firm A. He switches store and buys at A if $p_A^e - r_A + s \leq p_B$ and $v \geq p_A^e - r_A + s$. Finally, a consumer at B not receiving firm A's coupon buys at A if $p_A^e + s \leq p_B$ and $v \geq p_A^e + s$: Firm A's profits are then

$$\begin{aligned}\Pi_A(x_A; x_B) = {} & p_A(1-\lambda_B)\max\left\{0; \min\left\{1-\frac{p_A-p_B^e}{\bar{s}}; 1\right\}\right\} \qquad (3)\\ & + p_A\lambda_B \max\left\{0; \min\left\{1-\frac{p_A-(p_B^e-r_B)}{\bar{s}}; 1\right\}\right\}\\ & + (p_A-r_A)\lambda_A \max\left\{0; \min\left\{\frac{p_B-(p_A^e-r_A)}{\bar{s}}; \frac{v-(p_A^e-r_A)}{\bar{s}}; 1\right\}\right\}\\ & + p_A(1-\lambda_A)\max\left\{0; \min\left\{\frac{p_B-p_A^e}{\bar{s}}; \frac{v-p_A^e}{\bar{s}}; 1\right\}\right\} - k(\lambda_A):\end{aligned}$$

Firm B's profits are analogous by symmetry. The following proposition characterizes the equilibrium with ordinary coupons.

Proposition 2. If sellers promote their sales by sending out ordinary coupons, a unique symmetric interior pure strategy equilibrium exists if and only if $k'(1) > \bar{s}/4$; and $k'(\bar{s}/v) < v^2/4\bar{s}$. This equilibrium is given by the solution to the following system of equations:

$$p = \bar{s}/\lambda; \quad r = 0.5p; \quad p^2/4\bar{s} = k'(\lambda): \qquad (4a,b,c)$$

In equilibrium, a fraction $\min[1; 1/2\lambda]$ of consumers redeem coupons at the distant store, while the rest buy from their home store at the regular price.

In Figure 1, the equilibrium price and couponing intensity are given by the intersection of curves $P'P'$ and KK (point E'). Equation (4a) is depicted by $P'P'$, while KK represents equation (4c), which is the same as (2c). Note that $P'P'$ lies entirely above PP.

As in the coupon-advertising equilibrium, sellers always offer positive rebates that are equal to half the regular price in equilibrium. Moreover, sales promotions are not temporary, since coupons offering rebates occur in a pure strategy equilibrium.

It is interesting to compare our results (Propositions 1 and 2) with those under perfect price information (BP, 1996). The equilibrium under full information is characterized by three equations (see their Proposition 1). Their last two equations coincide with the corresponding equations of both of our equilibria. The only difference lies in the first equation: under full information, the equilibrium price is given by, $p = \bar{s}/(1 + 0.5\lambda)$, which is depicted by $P''P''$ in Figure 1. As KK is common to all cases, E'' in Figure 1 represents the equilibrium price and advertising intensity under perfect price information. Note further that $P''P''$ lies entirely below PP.

Interestingly, the equilibrium rebate equals half the regular price ($r = 0.5p$) under both imperfect and perfect price information. In all these cases, r is chosen to maximize the seller's profits obtained from those consumers receiving a coupon. Since coupon-advertising also conveys price information, this part of the maximization problem in Proposition 1 is identical to the corresponding one in Bester and Petrakis (1996). On the other hand, a consumer receiving an ordinary coupon does not learn the regular price of the good. Then the equilibrium discount equals half the expected regular price. However, since consumers anticipate correctly the price charged at the distant location, the discount equals half the regular price.

Further, we observe from Figure 1 that equilibrium prices and couponing intensities are higher whenever price information is imperfect (E'' lies to the southwest of both E and E'). Obviously, imperfect information generates informational segmentation and thus reduces price competition. Firms can then send out coupons to more consumers at the distant store without having to cut their regular prices substantially. Interestingly, the price differential between the ordinary coupons and the perfect price information equilibrium decreases as marginal costs of couponing decrease (as $k'(\lambda)$ decreases, KK shifts to the right). Firms with lower marginal costs can send out coupons to a bigger fraction of consumers, and thus decrease the informational segmentation of the market. In fact, if the marginal costs of couponing are sufficiently low, the coupon-advertising equilibrium converges to that under full information. On the other hand, the ordinary-coupon equilibrium always involves higher prices and advertising intensities than the coupon-advertising equilibrium (E lies to the southwest of E^0). The intuition is that sellers, by using coupon-advertising instead of ordinary coupons, spread price information, and thus reduce informational segmentation and increase competition in the market. Then to avoid a substantial reduction of his regular price, a seller optimally reduces his sales promotion intensity.

Finally, we compare sellers' profits in the three equilibria. Our previous discussion reveals that sales promotion costs are higher in the ordinary-coupon equilibrium than in the coupon-advertising equilibrium; and the latter are higher than those under perfect price information. The same is true for the regular prices and rebates, while the opposite holds for the sales promotion intensities. As a result, sellers' revenues (and hence profits) are not easily comparable across equilibria. The following proposition compares equilibrium profits for a family of cost functions that is common in the literature.

Proposition 3. If $k(\lambda) = m\lambda^{\alpha}$, $\alpha > 3$, then equilibrium profits are higher under ordinary coupons than under coupon-advertising, and the latter are higher than under perfect price information.

The intuition is that as the information available in the market increases, sellers face stronger price competition, and thus typically obtain lower profits. Note however that, as the market becomes more informed, there is also less need for firms to spend on price ads. Finally, equilibrium profits under coupon-advertising approach those under perfect information as the marginal cost becomes small; while the equilibrium profits under ordinary coupons always remain higher.

5. A model with coupon-advertising and ordinary coupons

So far we have assumed that sellers can promote their sales by sending out either coupon-advertising or ordinary coupons. We have seen that sellers' profits are typically higher when they distribute ordinary coupons. A natural question then arises: If sellers can use both coupon-advertising and ordinary coupons, what type(s) of coupons will be distributed in equilibrium? The following proposition answers:

Proposition 4. If sellers can send out both coupon-advertising and ordinary coupons to promote their sales, then ordinary coupons will be never be distributed in equilibrium. The unique symmetric equilibrium in pure strategies is the coupon-advertising equilibrium described in Proposition 1.[16]

Our model predicts that ordinary coupons should not be observed in markets that are both locationally and informationally segmented. As we have seen, both

[16] The proof of Proposition 4 is available from the authors upon request.

sellers would earn higher profits if they could agree to send out ordinary coupons alone. However, an individual seller, by lowering his price and informing consumers at the rival's location about this reduction, is able to attract more customers and thus increase his profits. Thus, whenever the rival distributes ordinary coupons alone, the seller has an incentive to send out coupon-advertising to the distant location. Sellers find themselves in a Prisoners' Dilemma, and thus obtain lower profits in equilibrium.

Interestingly, in the unique symmetric pure strategy equilibrium, sellers send out only coupon-advertising to the distant location. The intuition is as follows. A deviation from the equilibrium marketing strategy described in (2a,b,c) can only be profitable if e.g. seller A, who decides to send-out ordinary coupons as well, raises his regular price. Let p_A (> p) be his regular price, r_A the rebate offered in a coupon-advertising, and r^0_A the rebate of an ordinary coupon. The key to our argument is how consumers, who unexpectedly receive an ordinary coupon, form their (out-of-equilibrium) expectations, p^e_A. In equilibrium, those consumers expect to receive either a coupon with the price printed on it or nothing. They receive, instead, an ordinary coupon which moreover offers a rebate different from the equilibrium rebate. Rational consumers, however, know that the deviator sets its rebate in an optimal manner for any regular price chosen. Hence, those consumers receiving an ordinary coupon form rational out-of-equilibrium beliefs, and can thus infer the regular price from the rebate printed on the coupon. In fact, these expectations are the only "consistent" beliefs based on their information. In particular, a consumer in region B receiving an ordinary coupon observes r^0_A and can infer the equilibrium price p; he also knows that seller A will set optimally his regular price and rebate, taking as given consumers expectations p^e_A: That is, the consumer, knowing seller A's optimal rule, $r^0_A = 0.5(p_A + p^e_A - p)$, can conduct a simple thought experiment and infer the only regular price p_A that is consistent with his own beliefs (see also footnote 19). This, in turn, implies that consumers receiving ordinary coupons behave in the same way as those receiving coupon-advertising. Hence, if seller A did not have an incentive to deviate by raising his price in the equilibrium with coupon-advertising, he does not have an incentive to increase his price when sending out ordinary coupons either. Therefore, this deviation cannot be profitable.[17]

6. Comparative statics

Figure 1 allows us to derive comparative statics results for the coupon-advertising equilibrium of the extended model. We first study the effect of the degree of product differentiation (measured by the size of transportation costs, $\bar{s}$) on the equilibrium

[17] Proposition 4 holds true for other out-of-equilibrium beliefs, too. For instance, it holds if consumers receiving ordinary coupons are "pessimists", i.e. they believe that seller A charges the (higher) equilibrium price under ordinary coupons. It also holds if those consumers are "optimists", i.e. they believe that seller A did not print the price on the coupon by mistake, but only if the couponing cost function is sufficiently convex.

outcome. An increase in $\bar{s}$ shifts the curve PP to the right and the KK to the left. Then, in line with the literature (see e.g. Perloff and Salop, 1985; Shaked and Sutton, 1982), the equilibrium price increases with the degree of product differentiations. Moreover, the equilibrium couponing intensity increases with $\bar{s}$ (since from (4) we see that $\bar{s} = 9\lambda^2 k'(\lambda)$ and $k''(\lambda) > 0$). The equilibrium redemption rate, however, (weakly) decreases with $\bar{s}$. Since only the consumers with transportation costs lower than or equal to the rebate r redeem their coupons, the redemption rate equals $\min[1; r/\bar{s}] = \min[1; 1/3\lambda]$. Therefore, although more consumers receive a coupon, a smaller percentage of them redeem it.

Finally, from (2) we get $p = 6\lambda k'(\lambda)$; and the equilibrium profits become $\Pi = p - \lambda p^2/4\bar{s} - k(\lambda) = 5\lambda k'(\lambda) - k(\lambda)$. Thus, sellers' profits increase with $\bar{s}$ (since λ increases with $\bar{s}$ and $\partial\Pi/\partial\lambda = 4k'(\lambda) + 5\lambda k''(\lambda) > 0$): The intuition is simple. As $\bar{s}$ increases, it becomes more difficult to attract consumers from the distant location. As competition becomes softer, sellers can charge higher prices. On the other hand, sellers increase their couponing effort to motivate more consumers to switch stores. Contrary to the first effect, the latter fosters competition. However, the first effect is dominant and thus sellers' profits increase.

We next analyze the effect of marginal couponing costs on the equilibrium outcome. As $k'(\lambda)$ increases, the curve KK shifts to the left, while the curve PP remains unchanged. As a result, the equilibrium price and rebate rise, while the couponing intensity decreases. In fact, if the marginal cost is prohibitively high, coupons will not be issued, and sellers will extract all the surplus of the local consumers by charging the monopoly price v. Further, the equilibrium redemption rate (weakly) increases with the marginal costs of couponing. Even though fewer coupons are sent out, a higher fraction is redeemed because the rebate offered is higher. Finally, to study the effect of an increase in $k'(\lambda)$ on sellers' profits, we consider the family of costs functions $k(\lambda) = m\lambda^\alpha; \alpha \geq 3$: Our model confirms the well-known result in the informative advertising literature: Sellers' profits increase when the marginal costs of couponing rise (see e.g. Grossman and Shapiro, 1984; Peters, 1984; Bester and Petrakis, 1995). While an increase in the marginal costs, and hence in the total costs of couponing, has a negative direct effect on profits, there is also a positive strategic effect: Sellers reduce their couponing intensity and thus competition is relaxed. The strategic effect dominates the direct effect, and thus sellers' profits increase when couponing becomes more expensive. Thus, in line with the literature, sellers would prefer couponing to be illegal in our model too. Summarizing,

Proposition 5. a) As the degree of product differentiation increases, equilibrium prices, rebates, couponing intensity and sellers' profits increase, while equilibrium redemption rates decrease. b) As the marginal cost of couponing increases, equilibrium prices, rebates and redemption rates increase, while equilibrium couponing intensity decreases. Further, equilibrium profits increase with m if the couponing costs are $k(\lambda) = m\lambda^\alpha; \alpha \geq 3$:

Finally, we turn to the welfare analysis. Since all consumers buy in equilibrium, and production costs are zero, total welfare equals gross consumer surplus minus couponing costs, i.e. $SW = 2(v - k(\lambda))$. The consumer surplus is obtained by subtracting sellers' total profits, $10\lambda k'(\lambda) - 2k(\lambda)$, from total welfare, i.e.

$$CS = 2v - 10\lambda k'(\lambda) = 2v - 10\bar{s}/9\lambda \tag{5}$$

(since (2a,b,c) implies $\lambda k'(\lambda) = \bar{s}/9\lambda$). As λ increases with $\bar{s}$, both consumer surplus and welfare decrease with the degree of product differentiation. Since sales promotion costs are socially wasteful in our model, total welfare is higher when sellers spend less on couponing. Consumer surplus also decreases with $\bar{s}$, as a higher degree of product differentiation implies softer competition, and thus higher equilibrium prices. Moreover, as marginal costs of couponing increase, consumer surplus decreases. Sellers not only charge higher prices, but also send fewer coupons to the distant location. Finally, to study how an increase in the marginal cost of couponing affects total welfare, we consider the family of cost functions $k(\lambda) = m\lambda^{\alpha}; \alpha > 3$. As both marginal and total cost of couponing increase, there is a negative direct effect on total welfare, and a positive indirect effect due to less intensive sales promotion. The direct effect, however, dominates and welfare decreases with m. Summarizing,

Proposition 6. a) Consumer surplus and welfare decrease with the degree of product differentiation. b) Consumer surplus decreases as the marginal cost of couponing increases. Further, total welfare decreases with m when the couponing costs are $k(\lambda) = m\lambda^{\alpha}; \alpha > 3$.

7. Conclusions

This paper analyzes the sellers' sales promotion strategies in markets where consumers are uninformed about the prices charged in distant locations or the prices of other brands. Pizza stores, food delivery, hairdressing, car and limousine services, dry-cleaning and laundry, super- and mini-markets, video stores and electronics are typical examples of markets presenting informational and locational (or due to brand loyalty) segmentation. Sellers in these markets can attract customers from distant locations, or from other brands, by targeting coupons to the customers of their rivals.[18] Consumers with low transportation costs, or low brand loyalty, will switch store only if they

[18] The analysis of the growing panel data on household purchase behavior with the help of new advanced statistical procedures allows firms to target coupons to selected consumers with considerable accuracy. Catalina Marketing Co., Citicorp P.O.S. Information Services, as well as some marketing firms and retail chain stores, have created customer databases which are currently used for a more effective targeting of coupons. Moreover, coupons are targeted to selected customers via direct mail. For instance, Computerized Marketing Technologies, Inc. and Donnelly Marketing (Carol Wright program) mail coupons to millions of selected households each year (for details see Shaffer and Zhang (1995) and the references therein).

receive a coupon and also believe, or know, that the discounted price charged at the distant store is sufficiently lower than the regular price at the home location. By printing their regular price on the coupon (coupon-advertising), sellers have the option to inform consumers at the distant location about their undiscounted price at no additional cost. It is shown that sellers' profits are higher when they distribute ordinary coupons instead of coupon-advertising. However, an individual seller, by lowering his regular price and printing it on the coupons, can attract more customers from his rival's location and thus increase his profits. Sellers find themselves in a Prisoner's Dilemma. In equilibrium, both sellers promote their sales by sending out coupon-advertising alone and, as a result, their profits are lower.

Our model predicts that ordinary coupons should not be observed in locationally and informationally segmented markets where the coupons are issued by the sellers themselves. Clearly, coupons not posting the regular price are often distributed in real-world markets. Of course, one reason is that a large number of coupons (about 70%, according to NCH Promotional Services) are issued by manufacturers who sell their goods through distribution channels. For obvious legal reasons, those coupons cannot have the price printed on them. Of the remaining 30% of the coupons that are issued by retailers or manufacturers selling to consumers directly, to the best of our knowledge, there is no information about the usage rates of coupon-advertising and ordinary coupons. As most of the reports on coupon usage are prepared by coupon redemption agents or clearing houses, retailer-initiated coupon promotions are not included in those databases because sellers hardly ask for the services of these agents. An empirical test of our findings will be hard to conduct until data on the usage rates of various types of coupons are available for those markets. Casual observation, however, reveals that both ordinary coupons and coupon-advertising are distributed in those markets.

One possible explanation for the existence of ordinary coupons could be that sellers are colluding in an infinitely repeated market interaction. This collusion is, however, partial, since sellers could further increase their profits by not issuing coupons at all, and behaving as monopolists in their home location. A better explanation might be that sellers, by printing their regular price on coupons with relatively large duration, lose their flexibility to modify those prices if they face random shocks, such as unexpected demand and cost variations. Sellers then prefer to issue coupons offering discounts as percentages off the regular price. This subject is open to further research.

Appendix

Proof of Proposition 1: First, we show that any symmetric interior equilibrium satisfies (2a,b,c). Assume, for the moment, that consumers at i not receiving coupons believe that $p_j^e \geq p_i$; also, that $p_j \geq p_i - r_i$; $i, j = A, B$. We check below that these assumptions are indeed satisfied in equilibrium. Then firm A solves (by symmetry, B's problem is analogous),

$$\max_{p_A, r_A, \lambda_A} [p_A(1-\lambda_B) + p_A\lambda_B(1-(p_A-(p_B-r_B))/\bar{s} + \lambda_A(p_A-r_A)(p_B-(p_A-r_A))/\bar{s} - k(\lambda_A). \tag{6}$$

The first order conditions (focs) are

$$\begin{aligned} 1-\lambda_B(2p_A-(p_B-r_B))/\bar{s}+\lambda_A(p_B-2(p_A-r_A))/\bar{s} &= 0 \\ -\lambda_A(p_B-2(p_A-r_A))/\bar{s} &= 0 \\ (p_A-r_A)(p_B-(p_A-r_A))/\bar{s}-k'(\lambda_A) &= 0 \end{aligned} \tag{7}$$

Imposing symmetry, i.e. $x_A = x_B = x$ and using the rational expectations hypothesis, i.e. $p_A^e = p_A$; $p_B^e = p_B$; we obtain (2a,b,c). Note that all the assumptions made above are satisfied.

We next show that the system (2a,b,c) has a unique interior solution if and only if $k'(1) > \bar{s}/9$ and $k'(2\bar{s}/3v) < v^2/4\bar{s}$: Equation (2a) defines $p_1(\lambda) = 2\bar{s}/3\lambda$ with $p_1'(\lambda) < 0$; $p_1(0) = \infty$ and $p_1(1) = 2\bar{s}/3$ (curve PP in Figure 1). Equation (2c) defines $p_2(\lambda) = (4\bar{s}k'(\lambda))^{0.5}$, with $p_2'(\lambda) > 0$; $p_2(0) < \infty$ and $p_2(1) = (4\bar{s}k'(1))^{0.5}$ (curve KK). From Figure 1, it can be checked that $p_1(\lambda)$ and $p_2(\lambda)$ intersect at an interior point such that $p \leq v$ if and only if the above conditions are verified.

Finally, we check that no firm has an incentive to deviate. Assume firm B follows the equilibrium strategy $(p; r; \lambda)$: If A deviates, consumers at B not receiving a coupon have no reason to change their expectations, as they remain uninformed of A's price. Nor do consumers at A have any reason to change their expectations about B's price. We first check A's incentives to deviate by lowering his price together with some rebate and couponing intensity. There are two cases. First, profits from a deviation such that $0.5p \leq p_A \leq p$ are given by

$$\begin{aligned} \Pi_A = \; & p_A(1-\lambda)+p_A\lambda(1-(p_A-0.5p)/\bar{s}) \\ & +\lambda_A(p_A-r_A)(p-(p_A-r_A))/\bar{s}-k(\lambda_A). \end{aligned} \tag{8}$$

Since these profits equal those of equation (71), it is obvious that A cannot gain by adopting such a deviation. Second, if A charges $p_A < 0.5p$; then he obtains

$$\Pi_A = p_A + \lambda_A(p_A-r_A)(p-(p_A-r_A))/\bar{s} - k(\lambda_A). \tag{9}$$

Firm A would optimally set $(p_A - r_A) = 0.5p$: Then $\partial\Pi_A/\partial\lambda_A = 0$ reduces to $p^2/4\bar{s} =$

$k'(\lambda_A)$. Thus, A would optimally choose $\lambda_A = \lambda$. To complete the argument, note that $\partial\Pi_A/\partial p_A = 1 > 0$; i.e. profits decrease as the price falls. Consequently, A would optimally set $p_A = 0.5p$; which is not a profitable deviation (see above). Therefore, A cannot gain by lowering his price.

We next check A's incentives to raise its price. Then A's deviation profits are,

$$\Pi_A = p_A(1-\lambda)(1-(p_A-p)/\bar{s}) + p_A\lambda(1-(p_A-0.5p)/\bar{s}) + \lambda_A(p_A-r_A)(p-(p_A-r_A))/\bar{s} - k(\lambda_A). \quad (10)$$

Seller A would optimally set $(p_A - r_A) = 0.5p$. Then $\partial\Pi_A/\partial\lambda_A = 0$ reduces to $p^2/4\bar{s} = k'(\lambda_A)$. Again, A would optimally choose $\lambda_A = \lambda$. Finally, it can be checked that $\partial\Pi_A/\partial p_A = (p(1+\lambda) - 2p_A)/\bar{s} < 0$, i.e. profits decrease as the price increases. As a result, A cannot gain by raising its price. The proof is now complete.■

Proof of Proposition 2: The first two steps are similar to those in the proof of Proposition 1, and we will not repeat them here (for details see Moraga and Petrakis, 1997). It remains to show that no seller has incentive to deviate. Assume that B follows the equilibrium strategy (p, r, λ) in (4a,b,c), while seller A deviates by choosing $p_A \neq p$. At this point, it is crucial to carefully specify the expectations of consumers at B after seller A's deviation. Consumers at B not receiving a coupon do not observe anything new; thus, their expectations remain unchanged. However, those consumers at B getting a coupon from A should change their expectations whenever they observe a rebate different from the equilibrium one, i.e. $r_A \neq 0.5p$. In fact, they know that seller A will set his rebate in an optimal manner, which from the seller foc is, $r_A = 0.5(p_A + p^e_A - p)$. By conducting a simple thought experiment, they can infer A's regular price.[19] This demonstrates that the out-of-equilibrium expectations of those consumers receiving a coupon must be rational, i.e. $p^e_A = p_A$.

First, A deviates by lowering his price such that $0.5p \leq p_A < p$. Then his profits are,

$$\Pi_A = p_A(1-\lambda) + p_A\lambda(1-(p_A-0.5p)/\bar{s}) + \lambda_A(p_A-r_A)(p-(p^e_A-r_A))/\bar{s} - k(\lambda_A). \quad (11)$$

From our previous argument, $p^e_A = p_A$. Hence, the optimal rebate must satisfy $0.5p = (p_A - r_A)$. Further, the couponing intensity has to satisfy $\partial\Pi_A/\partial\lambda_A = p^2/4\bar{s} - k'(\lambda_A) = 0$; which implies $\lambda_A = \lambda$. As a result, $\partial\Pi_A/\partial p_A = 2\lambda(p - p_A)/\bar{s} > 0$; i.e. profits decrease as p_A decreases, thus seller A cannot gain by setting $0.5p \leq p_A < p$. Suppose next

[19] Assume that $p = 10$ and $r = 5$ in the equilibrium with ordinary coupons. Assume, further, that consumers at B receive a coupon from A offering $r_A = 6$. If, for instance, they form expectations $p^e_A = 12$; then those consumers, using the sellers' optimal rebate rule, $r_A = 0.5(p_A + p^e_A - p)$, infer that p_A must equal 10. Thus, their expectations are not correct. The only beliefs consistent with their information are then $p^e_A = 11$. In fact, the optimal rebate rule implies that $p_A = 11$; thus satisfying $p^e_A = p_A$.

that A chooses a price such that $p_A < 0.5p$. Profits are then,

$$\pi_A = p_A + (p_A - r_A)\lambda_A(p_B - (p_A^e - r_A))/\sigma - k(\lambda_A). \tag{12}$$

As above, p_A^e must equal p_A; and thus r_A satisfies $0.5p = (p_A - r_A)$. Also, $\partial\pi_A/\partial\lambda_A = 0$ implies that $\lambda_A = \lambda$. As a result, $\partial\pi_A/\partial p_A = 1.5 > 0$. Thus, A would choose $p_A = 0.5p$; which cannot be a profitable deviation (see above).

Finally, consider that A raises its price, $p_A > p$. Profits are then

$$\begin{aligned}\pi_A &= p_A(1-\lambda_B) + p_A\lambda_B(1-(p_A-(p_B^e-r_B))/\sigma) \\ &\quad + (p_A - r_A)\lambda_A(p_B - (p_A^e - r_A))/\sigma - k(\lambda_A).\end{aligned} \tag{13}$$

As before, $p_A^e = p_A$. Note that these profits equal those where A deviates by choosing $0.5p \geq p_A < p$. Thus, the optimal rebate and couponing intensity satisfy $0.5p = (p_A - r_A)$ and $\lambda_A = \lambda$. Then, $\partial\pi_A/\partial p_A = 2\lambda(p - p_A)/\sigma < 0$. This proves that A will not raise his price. The proof is now complete.■

Proof of Proposition 3: Denote the optimal couponing intensity with coupon advertising as λ, with ordinary coupons as λ', and under perfect information as λ''. The firms' profits are π, π', and π'', respectively. First, we compare π and π'. From (2a,b,c), $\pi = 5\lambda k'(\lambda) - k(\lambda)$, and from (4a,b,c), $\pi' = 3\lambda' k'(\lambda') - k(\lambda')$. For $k(\lambda) = m\lambda^\alpha$, $\alpha \geq 3$, we have $\pi' > \pi$ if and only if $\lambda'^\alpha(3\alpha - 1) > \lambda^\alpha(5\alpha - 1)$, or $\lambda'/\lambda > ((5\alpha - 1)/(3\alpha - 1))^{1/\alpha}$. Further, from (2a,b,c) and (4a,b,c), $\sigma = 9\lambda^2 k'(\lambda) = 4\lambda'^2 k'(\lambda')$. Then for $k(\lambda) = m\lambda^\alpha$, we have $\lambda'/\lambda = (9/4)^{\alpha/(1+\alpha)}$. Therefore, $\pi' > \pi$ if and only if $2.25 > ((5\alpha - 1)/(3\alpha - 1))^{(1+\alpha)/\alpha}$. This inequality is clearly satisfied for the case $\alpha = 3$. Further, since its right-hand side is decreasing in α, it can be checked that $\pi' > \pi$ for all $\alpha \geq 3$.

Second, we compare π and π''. The latter are $\pi'' = (4 + \lambda'')k'(\lambda'') - k(\lambda'')$ (see BP, 1996). Proceeding as before, and given that λ'' satisfies $\sigma = 4(1 + 0.5\lambda'')^2 k'(\lambda'')$, one obtains that $\pi \geq \pi''$ if and only if

$$(5\alpha - 1)(2 + \lambda)^{\frac{2\alpha}{\alpha+1}}\lambda^{\frac{\alpha(\alpha-1)}{\alpha+1}} - 3^{\frac{2\alpha}{\alpha+1}}[\alpha(4+\lambda)\lambda^{\alpha-1} - \lambda^\alpha] \geq 0, \tag{14}$$

for all $\alpha \geq 3$ and $0 < \lambda < 1$. It is clear that in the extreme case $\lambda = 1$, $\pi = \pi''$. By plotting the left-hand side of inequality (14), it is easily seen that it is always positive.■

Proof of Proposition 5: It only remains to show that profits are decreasing in m. For $k(\lambda) = m\lambda^\alpha$, $\alpha \geq 3$, we have $\lambda = (\sigma/9\alpha m)^{1/(1+\alpha)}$. Profits are then, $\pi = m^{1/(\alpha+1)}(5\alpha - 1)(\sigma/9\alpha)^{\alpha/(\alpha+1)}$, which increases with m.■

Proof of Proposition 6: It only remains to show that welfare is decreasing in m. For $k(\lambda) = m\lambda^\alpha$, $\alpha \geq 3$, we have $SW = 2v - 2m(\sigma/9\alpha m)^{(\alpha/1+\alpha)}$ (using the optimal λ), which decreases with m.■

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