What is product differentiation?

• Products differ in their characteristics and attributes. It is useful to distinguish between:
  – 1. HORIZONTAL: Consumers do not agree in the ranking they give to the goods
    • some people prefer blue jeans while others prefer black jeans
    • some people prefer a supermarket in the south of Rotterdam while others prefer one in the north of Rotterdam
  – 2. VERTICAL. Consumers give the same ranking to the goods
    • (almost) everyone prefers a color printer over a b&w printer
    • (almost) everyone prefers a BMW over a SEAT
    • (almost) everyone prefers a Pentium III processor over a Pentium II
A tour on Horizontal product differentiation

Two approaches to horizontal product differentiation

1. The non-address approach: consumer have preferences over the goods and a taste for variety

   Representative consumer
   Two types of models:
   • Oligopoly competition
     • Bertrand paradox
     • Efficiency of Bertrand vs. Cournot
   • Monopolistic competition
     • Does the market provide the optimal # of brands?

A tour on Horizontal product differentiation (cont.)

2. The address (or location) approach: consumers have preferences over the characteristics of products

   Hotelling model
   Main questions are:
   • What products does the market provide? Are they the “right” ones?
The non-address approach:  
Oligopolistic competition

- Consumers have preferences over two products and like variety
- \[ U(q_1,q_2) = a_1 q_1 + a_2 q_2 - \frac{1}{2} (b_1 q_1^2 + 2c q_1 q_2 + b_2 q_2^2) \]
- All parameters positive except possibly c. Assume also \( b_1 b_2 - c^2 > 0 \), to ensure \( U \) strictly concave.
- A duopoly selling the two horizontally differentiated goods

Cournot vs. Bertrand competition

- Assume symmetric situation: \( a_1 = a_2, b_1 = b_2 \)
- The system of (inverse) demand functions is
  \[
  p_1 = a - b q_1 - c q_2 \\
  p_2 = a - b q_2 - c q_1 \\
  b > 0, \text{ and } b > c 
  \]
  __own-price effect is stronger than cross-price effect.__

  Useful to analyze Cournot competition.

- For Bertrand, invert the demand system to obtain:
  \[
  q_1 = d - e p_1 + f p_2 \\
  q_2 = d - e p_2 + f p_1 \\
  \text{with } d = a(b-c)/(b^2-c^2); \ e = b/(b^2-c^2); \ f = c/(b^2-c^2) 
  \]

  If \( b = c \), goods are homogeneous; as \( b \to 0 \) become independent.
Cournot case

- Firm 1’s demand: \( p_1 = a - b q_1 - c q_2 \)
- Firm 1 cost: normalized to zero
- Firm 1 acts in the belief that firm 2 will put some amount \( q_2 \) in the market.
- Then firm 1 maximizes profits obtained from serving the residual demand: \( p_1 = (a - c q_2) - b q_1 \)
- Profits: \( \pi_1 = ((a - c q_2) - b q_1) q_1 \)
- Best-response functions are downward-sloping: quantities are strategic substitutes.

Cournot Equilibrium

- \( q_1^* \) maximizes firm 1’s profits, given that firm 2 produces \( q_2^* \)
- \( q_2^* \) maximizes firm 2’s profits, given firm 1’s output \( q_1^* \)
- No firm wants to change its output, given the rival’s output.
- Beliefs are consistent: each firm “thinks” rivals will stick to their current output, and they do so!
Cournot equilibrium

- Cournot equilibrium with differentiated products:
  \[ q_i = \frac{a}{2b+c}; \]
  \[ p_i = \frac{ab}{2b+c}; \]
  \[ \pi_i = \frac{a^2b}{(2b+c)^2}; \]

- In a Cournot game with differentiated products firms’ profits increase as products become more differentiated.

Bertrand case

- Firm 1’s demand: \( q_1 = d - e p_1 + f p_2 \)
- Firm 1 cost: normalized to zero
- Firm 1 acts in the belief that firm 2 will set a price \( p_2 \) for its product.
- Then firm 1 maximizes profits:
  \[ \pi_1 = (d - e p_1 + f p_2 ) p_1 \]
- Best-response functions are upward-sloping: prices are strategic complements.
Bertrand Equilibrium

- $p_1^*$ maximizes firm 1’s profits, given that firm 2 charges $p_2^*$
- $p_2^*$ maximizes firm 2’s profits, given firm 1’s price is $p_1^*$
- No firm wants to change its price, given the rival’s beliefs are consistent: each firm “thinks” rivals will stick to their current price, and they do so!

Bertrand equilibrium

- Bertrand equilibrium with differentiated products:
  \[ p_i = \frac{d}{2e-f} = \frac{a(b-c)}{(2b-c)} \]
  \[ q_i = \frac{de}{2e-f} \]
  \[ \pi_i = \frac{d^2e}{(2d-f)^2} = \frac{a^2b(b-c)^2}{(2b-c)^2} \]

- In a Bertrand game with differentiated products, firms’ profits decrease when products are less differentiated, and converge to zero as product differentiation vanishes.

Bertrand paradox
Cournot vs. Bertrand

In a differentiated products industry:

- Cournot price $> \text{Bertrand price}$.
  - The more differentiated the products, the lower the difference between the two, and converges to zero when the products become independent.
  - **Intuition:** Firms perceive a higher elasticity of demand under Bertrand competition.
    - Bertrand: $\frac{dq}{dp} = -e$ vs. Cournot $\frac{dq}{dp} = -\frac{1}{b}$
    - But $\frac{e}{b(b^2-c^2)} > \frac{1}{b}$

- CS under Bertrand $> \text{CS under Cournot}$

- Bertrand competition is more efficient (SW is higher!)

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The non-address approach
Monopolistic competition

- Numerous buyers with symmetric preferences and a taste for variety
- Numerous sellers of differentiated products
  - **Implication:** Since products are differentiated, each firm faces a downward sloping demand curve.
    (Firms have limited market power.)
- Easy (free) entry and exit
  - **Implication:** Firms will earn zero profits in the long run.
Taste for variety

Monopolistic Competition

- Market power permits the seller to price above marginal cost, just like a monopolist does.
- The # of units the seller puts in the market depends on his price, just like a monopolist. But …

The presence of other brands in the market makes the demand for a seller’s brand more elastic than if he was a monopolist.

A monopolistically competitive seller has limited market power.
Monopolistic Competition: Profit Maximization

- Maximize profits like a monopolist:
- A seller puts a # of units of its brand such that it makes MR and MC equal.
- The price for the brand is the price on the demand curve that corresponds to that quantity
Easy (free) entry and exit of brands
Long-run equilibrium

If the industry is truly monopolistically competitive, other “greedy capitalists” enter, and their new brands steal market share of others. This reduces the demand for the product of a seller until profits are ultimately zero.

How many varieties?

- # of firms (brands, or varieties) in a free-entry monopolistically competitive equilibrium depends on:
  - (i) demand elasticities,
  - (ii) the importance of economies of scale (fixed costs of entry).
- The more important economies of scale are and the smaller the degree of differentiation, the fewer the number of firms and varieties.
Too many of too few detergents?

• Private incentives to introduce new brands are generally mis-aligned with respect to social incentives:
  – *Business-stealing* effect: this effect leads to an excessive # of firms when $\pi > f > \Delta TS$
  – *Non-appropriability of total surplus*: buyers appropriate some of the surplus, so a social planner would introduce new products in situations in which a firm would not. $\Delta TS > f > \pi$

The address approach
What products does the market provide?

The Hotelling’s linear city model:

2 issues:
• Competition over location
• Competition over location and price
Competition over location
The principle of minimum differentiation

• Can a location choice like this be equilibrium?

• For two firms, equilibrium locations exhibit *minimum differentiation*:

The principle of minimum differentiation with more than two firms

• For four firms, equilibrium locations do not exhibit minimum differentiation strictly speaking but *bunching*
Welfare maximizing product choices

- For any number of firms \( N \): locate the firms equidistantly so that there are \( 1/2N \) half-market lengths of equal distance.
- For two firms:

\[
\begin{align*}
0 & \quad 1/4 & \quad \theta_1 & \quad \theta_2 & \quad 3/4 & \quad 1 \\
\end{align*}
\]

- For four firms:

\[
\begin{align*}
0 & \quad 1/8 & \quad 3/8 & \quad 5/8 & \quad 7/8 & \quad 1 \\
\end{align*}
\]

Competition over location and prices: the principle of maximum differentiation

- Suppose firms locate at the end-points of the city: differentiation is maximal.

\[
\begin{align*}
A & \quad t x^2 & \quad x & \quad t (1-x)^2 & \quad B \\
\end{align*}
\]

- Assume consumer \( x \)’ utility is \( U = V - k d^2(x,i) - p_i \)
- For a pair of prices \((p_A, p_B)\), there is a consumer \( x_0 \) indifferent between the two varieties:

\[
V - k d^2(x_0 A) - p_A = V - k d^2(x_0 B) - p_B
\]
- Consumer $x_0$ is $x_0 = (p_B - p_A)/2k + 1/2$. Consumers to the left of $x_0$ prefer to buy from A while consumers to the right of $x_0$ prefer to buy from B.

- To find the NE in prices, we derive the best-response functions:

Maximize $\pi_A = ((p_B - p_A)/2k + 1/2) p_A$ in $p_A$

$p_A = (p_B + k)/2$

By symmetry $p_B = (p_A + k)/2$

Solving yields $p_A = p_B = k$

$\sigma = k/2$
Consider firms relocate to a distance $x$ from their respective initial end-points locations.

The new indifferent consumer is $x_1$, with valuation:

$$x_1 = \left( \frac{p_B - p_A}{2k} \right) + \frac{1}{2}$$

It is useful to compute demand elasticity:

$$\epsilon = \frac{1}{2k(1-2x)} q_i$$

Following the same steps as above, the new NE is

$$p = k (1 - 2x)$$

and profits $\pi = p / 2$

This shows how location affect equilibrium prices and profits.

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The principle of maximum differentiation

Consider a two-stage game:

- Stage 1: Firms locate their products
- Stage 2: Firms compete in prices

Two effects to take into account:

- **Demand effect**: move towards the center to increase the size of their captive markets
- **Strategic effect**: move towards the end-points of the city to minimize price competition with the rival.

In the quadratic cost example discussed here, the 2nd effect dominates and firms end up locating the furthest possible from their rival’s location.
Caveats

- Maximum differentiation not always:
  - Factors that make the price effect weaker tend to restore the principle of minimum differentiation
    - Differentiation in some other dimensions, e.g., quality
    - Cournot competition rather than Bertrand competition
    - Meeting the competition clauses
    - Some imperfect information regarding prices of competitors
    - Some uncertainty regarding sellers’ products features.
    - Collusion reducing the incentives to undercut rival’s price

Vertical product differentiation

- Products are vertically differentiated if buyers all agree in the way they rank the distinct products. In these models goods differ in their ‘quality.’
- Quality can be ‘observable’ or ‘unobservable.’
  - Goods whose quality is observable are called *search goods*
  - Goods whose quality is unobservable before the purchase are called *experience goods.*
  - There are goods whose quality is (almost) never observed: *credence goods.*
Vertical product differentiation

- Here we discuss questions pertaining to search goods: Does the market offer the “right” quality?
  - Monopoly and oligopoly are considered

- Questions that pertain to experience goods have a larger scope, including the effects of warranties and signaling of quality (advertising).

Monopoly Provision of quality

- Demand is \( P = P(q,s) \)
- Consumers like quality: if \( s_1 > s_2 \), then \( P(q,s_1) > P(q,s_2) \)
- Quality is costly to produce: \( C(s), C'(s) > 0 \)
- Compare quality offered by monopolist and social planner. In general they differ because:
  - Monopolist chooses quality to equalize Mg gains of an increase in quality to Mg cost of quality.
  - Social planner chooses quality to make Mg social gains of an increase in quality equal to the Mg cost of quality.
- Result: monopoly underprovides quality in some cases and oversupplies quality in other cases, compared to what would be socially desirable.
Monopoly (under)Provision of quality

Effect of an increase in quality on the monopolist’s revenue.

Socially desirable provision of quality

Effect of an increase in quality on social welfare.
Monopoly (over)Provision of quality

Effect of an increase in quality on the monopolist’s revenue.

In this case, a monopolist over-provides quality.

SP Provision of quality

Effect of an increase in quality on the monopolist’s revenue.

Willingness to pay for an additional unit of quality of the marginal consumer

P(q,s+ds)

P(q,s)

MC

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Quality and competition

- Consumer preferences:
  - \( U = Y_s - p \) if they buy a product of quality \( s \) at price \( p \).
  - They get no utility if they do not buy.
  - They buy at most a single unit of the product.
- Buyers valuations differ; e.g., \( Y \) is uniformly distributed over the set \([0,1]\).
- Quality is costly to produce: \( C(s), C'(s) > 0 \).
- Competition develops over two stages:
  - 1st stage: firms compete over ‘location’ in the quality ladder
  - 2nd stage: firms compete to sell their products (either Cournot or Bertrand)

Quality and price competition

- Solve the game for a SPE
- It can be shown that:
  - Firms have generally an incentive to differentiate their products (a version of the principle of maximum differentiation holds here).
  - Indeed second stage prices are:
    \[
    p_l = \frac{q_l(q_h - q_l)}{4q_h - q_l}; \quad p_h = \frac{2q_h(q_h - q_l)}{4q_h - q_l}
    \]
Quality and price competition

- First stage reaction functions:

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In equilibrium:
- High-quality producer obtains higher profits than low-quality producer
- Firms differentiate more their products under Bertrand competition than under Cournot competition.
- Bertrand competition is more efficient than Cournot competition
- If there is free entry in this model, it can be seen that the market can only accommodate a finite # of firms (natural oligopoly)

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