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Quality uncertainty and informative advertising

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Abstract

We present a price signalling model with informative advertising. A costly advertisement informs of the good's quality directly and therefore the seller determines the fraction of informed buyers endogenously. We show that informative advertising only occurs in pooling equilibria. For an advertising pooling equilibrium to exist, consumer valuation for high-quality, advertising cost, prior probability that quality is high, and inaccuracy of the buyers' pre-purchase information must be sufficiently high. For some parameters there is a unique undefeated advertising pooling equilibrium. If advertising is used in equilibrium, the adverse selection problem is mitigated.

Keywords: Informative advertising; Quality uncertainty; Signalling

JEL classification: L15; D42; D82; M37

1. Introduction

A substantial amount of advertising is observed in markets where the quality of the goods is imperceptible by consumers before they are purchased (*experience goods*).[1] Nelson (1974) suggests that most of the advertising of experience goods

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[1] The conceptual distinction between experience and search goods was proposed by Nelson (1970), (1974). The quality of experience goods can only be ascertained after consumers purchase them. The quality of search products is learned by buyers on observation.

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cannot be of the *informative* type.[2] He argues that, since experience qualities are unverifiable before purchase, sellers' advertisements claiming that they are offering a higher quality product can be misleading. Consequently, rational consumers will disregard them. Nelson then reasons that, when there exist market mechanisms that positively relate products' qualities and advertising outlays, *uninformative* advertising may be observed since it may be indirectly informative. His ideas have been formalized by Milgrom and Roberts (1986) and Kihlstrom and Riordan (1984). The mechanism Nelson refers to is defined as a signalling mechanism in these articles.[3]

The role of advertising and the reasons for it in experience goods markets is however more subtle. The models mentioned above only apply to markets where, first, advertising expenditures are publicly observable, and, second, publicity cannot convey verifiable information about products' quality. Certainly, environments exist where customers can find out firms' advertising expenditures. For instance, this may happen when sellers hire popular actors or models for their TV-commercials at very high prices. In other markets, however, buyers cannot find out suppliers' advertising efforts accurately. Evidently, if this is so, then advertising cannot be used as a signal.[4] Note that if, in addition, publicity does not contain 'hard' information about the goods' characteristics, then it should never be observed in equilibrium.

In many other real-world markets, advertising conveys useful information. Sellers often provide direct information about the value of their goods by advertising activities such as distributing *free samples*[5] or carrying out *point-of-sale* or *point-of-purchase demonstrations*.[6] Whether advertising functions as a signal, or, instead, conveys direct information depends on the type of context

[2] Even though uninformative advertising may become informative in separating equilibria (e.g. as in Milgrom and Roberts, 1986), we will use the term informative advertising for publicity that conveys direct information about a good's quality.

[3] In Milgrom and Roberts (1986), a monopolist sets both the price and the level of uninformative advertising to introduce an experience good into the market. In equilibrium, both variables may simultaneously be used as *signals* of quality. On the other hand, in Kihlstrom and Riordan (1984), sellers spend on publicity to introduce their goods: advertising functions as an entry fee into the market for high-quality products.

[4] See Hertzendorf (1993) for a model where advertising is noisily observable.

[5] Free samples are small portions of a new good that are made available to consumers with the purpose of proving its value. They are widely used to introduce shampoos, beauty aids, cookies, cleaning products, etc. One also finds free samples of new software in Internet and academic books or journals in Scientific Meetings. Marketing researchers agree that distributing free samples is the most effective manner to introduce a good into the market when sellers are very confident of their products' characteristics. For instance, Lever Brothers successfully introduced its detergent Surf by sending out more than 4 million free samples (Kotler, 1994). The Gillete campaign to introduce its Trac II Razor consisted of sending out 12 million free samples (Assael, 1993).

[6] Suppliers usually sponsor training seminars and demonstrations for distributors, or facilitate hands-on experimentation of their goods.

considered. Here we focus on experience goods markets where advertising is informative and publicity expenses are not observable.[7]

We present a single-period adverse selection model (in the spirit of Milgrom and Roberts, 1986) where a monopolist may employ informative advertising to introduce its product into the market. The (experience) good can be either high or low quality. Before making any price or advertising decision, the producer observes the true quality of the product and all consumers receive an independent market signal that is positively correlated with the true quality. Then the seller sets the price and the informative advertising intensity. Naturally, advertising is costly for the producer. Each consumer has a given probability that he will be reached by an advertisement, and this probability is increasing in the amount the firm spends on advertising. It is assumed that all buyers observe the price charged but, in contrast, the advertising effort is not observable.[8] Hence, only prices may function as signals of quality in our model.

Since a consumer receiving an advertisement fully learns quality, our approach brings about a theoretical innovation. Namely, we deal with an incomplete information model where the seller determines the percentage of informed consumers endogenously. Consumers are fully rational. Therefore, those receiving an advertisement ascertain the true quality of the product and disregard any other (noisy) information received. The others remain uninformed. Since they understand the (high-quality) firm's incentives to advertise, they form beliefs about its advertising effort. Buyers then decide whether or not to buy taking into consideration the price, the external signal observed, and their beliefs. In equilibrium, the fraction of informed consumers depends on the price and the conjectures that uninformed buyers form are correct.

We first show that informative advertising never occurs in any separating equilibrium. Since prices signal quality in a signalling equilibrium, consumers, after observing the price, perfectly learn the true quality of the product.[9] Consequently, advertising expenditures are completely unnecessary. This contrasts with models where uninformative advertising functions as a quality signal.

[7] Recent empirical investigation supports the idea that much of the advertising in experience goods markets is informative. Caves and Greene (1996) compute global rank correlations between quality and prices and advertising expenses for about 200 products evaluated by Consumers Reports. They find the positive relationship between quality and advertising to be very weak and conclude that quality signalling is not the function of much of the advertising of consumption goods. Instead, advertising is found to serve as a source of direct and relevant information about many goods' quality: advertising expenditures tend to increase with quality if higher quality products have better features or capabilities that consumers may learn from verifiable advertised information. Their results are consistent with previous studies (e.g. Rotfeld and Rotzoll, 1976; Archibald et al., 1983; Phillips et al., 1983).

[8] An individual consumer only observes whether or not he has received an informative advertisement.

[9] See, inter alia, Milgrom and Roberts (1986), Bagwell and Riordan (1991) and Ellingsen (1997) for signalling models where quality is exogenously given. In Chan and Leland (1982), Wolinsky (1983), Cooper and Ross (1984), (1985) and Riordan (1986) prices signal quality choices.

Milgrom and Roberts (1986), in a repeat purchase context, show that uninformative advertising may occur in a separating equilibrium. Advertising is used because it contributes to the signalling role of prices to achieve separation at minimal cost. In other words, it helps the price to signal quality.[10] In Kihlstrom and Riordan (1984), in contrast, uninformative advertising is the only manner to signal qualities because firms do not choose their prices.

We then investigate the constellations of parameters for which advertising is used in both full pooling and partial pooling (or semi-separating) equilibria. Informative advertising never occurs in any type of pooling equilibrium if both the difference between consumers' valuations for the high and the low-quality, and the cost of advertising are sufficiently small. Furthermore, for an advertising full pooling equilibrium to exist, the informativeness of the market signal must be low enough and the consumers' prior probability of high-quality must be sufficiently high. Existence of an advertising partial pooling equilibrium requires similar conditions but the consumers' prior probability that quality is high cannot be too large. For some parameter constellations, partial and full pooling equilibria coexist.

The typical multiplicity problem arising in signalling models is tackled by first applying the Intuitive Criterion. It is shown that all equilibria found pass the test. We then apply the recently-introduced notion of *undefeated equilibrium*.[11] If advertising cost is sufficiently high and the degree of informativeness of the external signal is sufficiently small, then advertising arises in the unique undefeated outcome. More precisely, it is shown that under those parametric conditions the most efficient (from the seller's point of view) pooling equilibrium with advertising defeats the rest of pooling equilibria with and without advertising.

In our model some consumers become perfectly informed through advertising in a pooling equilibrium. In contrast, in the papers on uninformative advertising signals, consumers are only perfectly informed in a separating equilibrium (or exogenously). Here consumers may learn quality either from the price (if separation occurs) or from advertising (if pooling with advertising happens).[12] Interestingly, the quantity traded when advertising arises is higher than if advertising were forbidden in any type of pooling equilibrium. As a result, informative advertising mitigates the adverse selection problem.[13]

The remainder of the paper is organized as follows. Section 2 presents the model. Separating equilibria are analyzed in Section 3. In Section 4, we investigate

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[10] In Milgrom and Roberts (1986) advertising in a signalling equilibrium relies on repeat purchases. Otherwise, only prices may signal quality in equilibrium (as in Bagwell and Riordan, 1991).

[11] See Mailath et al. (1993).

[12] Vettas (1997) develops a dynamic model where consumers, after the first period, can also learn the true quality of the product from two alternative sources: from the price (in a separating outcome) and from word-of-mouth communication (if there is separation and/or pooling).

[13] The quantity traded tends to be small in adverse selection models (see e.g. Akerlof, 1970).

(full and semi) pooling equilibria with and without advertising. Section 5 is devoted to refine the set of equilibria. Finally, Section 6 concludes.

2. The model

Consider a single period monopoly market where a new product of uncertain quality q is introduced. For simplicity, quality can only be either high or low. Only the producer observes the true quality of the product.[14] In what follows, the *monopolist when the quality is low (high)* will be referred to as the *low- (high-) quality seller*. The cost of producing one unit of the high-quality good is $c > 0$ while the unit production cost of the low-quality product is normalized to zero.

On the demand side of the market, there is a large number of potential consumers whose mass is normalized to 1. Each buyer will at most purchase one unit of the product. All customers have identical reservation values for the products, namely, q_h for the high-quality good and q_l for the low-quality one, $q_h > q_l \geqslant 0$. Prior to purchase and before the seller sets its marketing strategy (price and advertising intensity), all buyers are fully uninformed of the true quality. Consumers' prior belief that good is of high-quality is denoted β, $0 < \beta < 1$. This probability is common knowledge.

It is further assumed that (a) $q_h - c > q_l$ and (b) $q_l - c > 0$. Assumption (a) means that producing the high-quality product is socially more efficient. On the other hand, if assumption (b) were not satisfied, then the high-quality seller would never mimic its low-quality counterpart.[15]

All consumers receive an *independent* signal s that is informative about the actual quality. Buyers observe this external information before any pricing and/or advertising decision takes place. The signal can be either a high-quality signal (s_h) or a low-quality one (s_l). It is assumed that:[16]

$$\Pr\{s_h|q_h\} = \gamma;\ \Pr\{s_h|q_l\} = 1 - \gamma. \tag{2.1}$$

We assume that $\gamma \in (1/2, 1)$ which implies that the external signals are informative but there is enough noise in the market so that they are imperfect. For example, suppose that buyers read a number of different reports or newspapers that

[14] By assuming this, we are replacing the 'incomplete' information game by a game of 'complete' but 'imperfect' information (see Harsanyi, 1967, 1968). In other words, we are employing an adverse selection model instead of a moral hazard one.

[15] This latter assumption simplifies the presentation. Dropping it would lead to many different subcases depending on parameters, and make the analysis less clear.

[16] We are imposing symmetry on the structure of the independent signals. A more general formulation could assign different probabilities conditional on the product being high-quality or low-quality, e.g. $\Pr\{s_h|q_h\} = \gamma_1$; $\Pr\{s_l|q_l\} = \gamma_2$. To economize on parameters we are assuming $\gamma_1 = \gamma_2$.

announce that the product will be introduced into the market. Suppose that, on average, a fraction γ of them reports the expected quality of the product correctly. Then, a percentage γ of the population would receive a correct signal about the actual quality while the rest of the buyers would receive wrong information.[17] The role of these signals is to smooth demand functions by introducing some heterogeneity into consumers' valuations. This will become clear in Section 4.

The (high-quality) seller can inform buyers of its actual quality through advertising activities. Each consumer is equally likely to receive an advertisement (free sample) of the product. Advertising is costly. For simplicity and computational convenience, we assume that the cost of informing a fraction λ of the consumers is quadratic, i.e. $C(\lambda)=0.5k\lambda^2$, $k>0$, $0\leq\lambda\leq1$.[18] According to this specification, informing a larger fraction of buyers is more costly and, further, the advertising technology exhibits decreasing returns to scale. This latter feature is standard in the literature on informative advertising.[19] Further, we assume that neither the total amount of money spent on publicity nor the advertising intensity is observable by consumers. Finally, to ensure that the optimal fraction of consumers reached by the seller's advertisements is smaller than 1, we assume that $k>q_h-c$.[20]

Before continuing the analysis, let us make clear the sequence of events in our model: first, Nature selects the quality of the product. Then the monopolist observes its choice and, simultaneously, consumers receive the independent signals of quality. After this, the seller sets its marketing strategy (advertising intensity and price). Finally, consumers decide whether or not to purchase and the seller satisfies demand.

If information were complete, none of the sellers would obviously advertise in equilibrium. The low-quality seller would charge $p_l^*=q_l$ and make profits $\Pi_l^*=q_l$, while the high-quality seller would set $p_h^*=q_h$ and obtain profits $\Pi_h^*=q_h-c$. In what follows, we will refer to these prices and profits as the *optimal prices and profits under complete information*. As $q_h-c>q_l$ (see above) the high-quality product is socially more efficient (under complete information, if the producer

[17] Wolinsky (1983) employs similar market signals to model the pre-purchase information about the goods' quality that consumers obtain as a by-product of their shopping processes.

[18] Since advertising perfectly reveals quality, it is obvious that only the high-quality seller will have an incentive to advertise. Thus, we assume that advertising costs do not depend on the quality advertised.

[19] See Butters (1977) and Grossman and Shapiro (1984) inter alia. The underlying idea is that an advertisement may fail to reach an uninformed buyer. For instance, if a seller inserts a number m of free samples in a number of newspapers or magazines, it is reasonable to think that fewer than m consumers will become fully informed (as some consumers buy more than one newspaper).

[20] Assuming that advertising costs are quadratic is a simplification to save on computations. We conjecture that if we had alternatively used a different convex functional form, similar conditions would have emerged and the entire intuition behind our analysis would have remained intact.

were able to choose the quality of the product, it would select the high-quality good).

Under incomplete information, our model defines a signalling game. However, our game is not standard as the seller's marketing strategy is twofold. Moreover, only the price is observable by consumers. When the seller sets the marketing strategy, the population of consumers is endogenously divided into two groups. From now on, those buyers receiving an advertisement will be referred to as *informed consumers*. The rest of customers, who do not obtain direct quality information, will accordingly be called *uninformed consumers*.

We throughout employ the notion of Perfect Bayesian equilibrium. As usual, it requires the monopolist's strategy to be sequentially rational and consumers' beliefs to conform with Bayes' rule whenever it applies. We will analyze separating, pooling and partial pooling (or semi-separating) equilibria. In a separating equilibrium, both types of sellers choose different prices, and the uninformed consumers, after observing the price, infer the true quality of the product. In contrast, in a full pooling equilibrium, both types of firms set the same price and the uninformed buyers cannot ascertain the true quality using only this observation. This feature also appears in a partial pooling equilibrium, where the high-quality seller always sets a pooling price and the low-quality seller randomizes between the pooling price and its optimal price under complete information.[21] Since the focus of our research is on advertising, we will also distinguish between equilibria with and without it.

As is typical in signalling models, many perfect Bayesian equilibria arise here. There may exist many different separating, pooling and partial pooling equilibria, with and without advertising. The source of this multiplicity of equilibria is the indeterminacy of the beliefs that consumers form after observing out-of-equilibrium movements. For simplicity, we will throughout place a restriction on consumers' beliefs off-the-equilibrium path, namely, that they consider any deviating price to be quoted by the low-quality monopolist. These are the beliefs that support the largest set of equilibria. In Section 5 we will argue that some equilibria are less reasonable than others and some refinements will be applied.

The following observation is convenient here. Since the seller employs a twofold strategy, it matters whether advertising is chosen before or after prices, specially when considering deviations. To see this, recall that we have assumed that the beliefs consumers form after observing a deviating price are such that they put probability 1 on the event that the monopolist is of low-quality. These beliefs may not be 'reasonable' if the deviator is actually the high type and makes a non-zero advertising effort (later in the paper we will consider these types of deviations). In fact, some consumers, believing that the type is low with

[21] As explained below, a partial pooling equilibrium where the high-quality seller randomizes does not exist.

probability 1 also receive an advertisement demonstrating that quality is high. This is contradictory. To avoid this trouble, let us assume that consumers receive the advertisements before prices are observed. Therefore, only the uninformed consumers form conjectures after seeing a deviating price.[22,23]

3. Separating equilibrium

In any separating equilibrium both types of sellers charge different prices. Therefore, prices signal the true quality of the product and consumers disregard the (noisy) information that they receive through the independent signals. This characteristic allows us to conclude that:

Lemma 1. *Informative advertising never occurs in any separating equilibrium.*

The reason for this is that the information provided through advertising would simply be redundant. Prices solve the incomplete information problem and therefore advertising the product would only generate additional costs for the high-quality seller.

The following remarks are appropriate here: first, observe that the absence of informative advertising in any separating equilibrium is based on the assumption that the seller's advertising effort is not observable.[24] Second, notice that if one acknowledges that customers can be unaware of firms' advertising expenditures completely, then the result in lemma 1 is a general property (not model specific).[25]

Observe, further, that in any separating equilibrium both sellers' demands would be equal to those under full information, i.e. both suppliers would serve the entire market. This leads us to conclude that:

Proposition 1. *A separating equilibrium does not exist.*

Intuitively, this is because all consumers have identical valuations for both types of goods. Then, if a consumer buys the high-quality product in a proposed

[22] This seems reasonable if consumers receive free samples at their addresses and later on discover prices in the store.

[23] I am indebted to a referee for pointing out this issue.

[24] There is a technical observation to point out here. We are implicitly assuming that the seller cannot make sure that all consumers receive an advertisement, i.e. $\lambda = 1$. If this were possible in a proposed equilibrium, then deviations consisting of changing the advertising effort would be detectable by consumers. We consider that $\lambda = 1$ is a zero-probability event and therefore rule out this possibility.

[25] Matters would be different if the price signalling mechanism were noisy, perhaps because the seller was not perfectly informed about quality or because discovering prices were costly for the buyers. Independently of the reason, if prices do not convey the entire truth, informative advertising might occur in separating equilibria because it would help consumers to estimate quality better.

separating equilibrium, the rest of them will also buy it. Consequently, the low-quality seller will always have an incentive to mimic its high-quality counterpart, as it will not lose any buyer. Technically, the so-called 'single-crossing property' is not verified in our model, i.e. sending higher messages (here prices) is not 'easier' for the high-quality seller.[26]

4. Pooling equilibrium

In any full pooling equilibrium, both types of sellers set the same price p. Thus, uninformed consumers are unable to infer the true quality from the observed price. Consumers are fully rational and therefore understand that the high-quality seller may have an incentive to advertise. In other words, uninformed buyers know that if quality were high, they might have received a free sample with some positive probability. Let $\lambda_e(p)$ be the common consumers' expectation about the high-quality seller's advertising effort.[27] While those customers receiving a free sample learn the true quality of the product and disregard any signal observed, the rest of them will use all the available information to update their beliefs. Thus, conditional upon observing the price p and a high-quality signal s_h, the expected quality for consumers who do not receive an advertisement is (by Bayes' rule)[28]

$$q_{eh}(\lambda_e) = \frac{\gamma\beta(1-\lambda_e)q_h + (1-\gamma)(1-\beta)q_l}{\gamma\beta(1-\lambda_e) + (1-\gamma)(1-\beta)}. \tag{4.1}$$

If, on the other hand, uninformed consumers observe a low-quality signal s_l, they expect the quality to be

$$q_{el}(\lambda_e) = \frac{(1-\gamma)(1-\lambda_e)\beta q_h + \gamma(1-\beta)q_l}{(1-\gamma)(1-\lambda_e)\beta + \gamma(1-\beta)}. \tag{4.2}$$

Thus, for any price p, sellers' demand $D_i(p, \lambda, \lambda_e)$ depends on the price p, the

[26] Unlike Lemma 1, the absence of separating equilibria is specific to our model. This stems from the fact that all consumers' valuations are equal under separation. Here, buyers' valuations will only differ if they take into account the information provided through the independent signal s, but this information will be disregarded in a separating equilibrium. Bagwell and Riordan (1991) obtain separation by considering heterogeneous consumers' willingness to pay for the high-quality. Assuming this in our model would substantially complicate the rest of the analysis, without adding much to it, since our focus is on informative advertising.

[27] From now on, to save space, we will write λ_e to denote this probability, but note that it is conditional on the price observed. As we show below, the higher the price, the higher is the probability that a consumer receives an advertisement.

[28] Here, the probability of receiving a free sample depends on whether the producer is the high or the low type. Thus, consumers not receiving direct information will use this fact to update their beliefs on quality by Bayes' rule. This is similar to Vettas (1997), where the probability of being informed through word-of-mouth communication also depends on whether the firm is the high or the low type.

high-quality seller's advertising intensity λ, and the consumers' conjectures about it λ_e. Sellers maximize profits $\Pi_i(p, \lambda, \lambda_e)$ taking the expectation λ_e as fixed. Of course, in equilibrium we will require λ_e to be consistent with the actual advertising intensity chosen by the high-quality seller (rational expectations hypothesis).

We now derive both sellers' demand functions. First, consider that the product is the low-quality one. Then, a fraction γ of the population observes the right signal s_l. These consumers will buy the product whenever $p \leqslant q_{el}(\lambda_e)$. The rest of consumers, a fraction $1-\gamma$, observes the wrong signal s_h and will then purchase the good whenever $p \leqslant q_{eh}(\lambda_e)$. Obviously, the low-quality seller will not advertise its product since it is not interested in revealing itself as a low-quality firm. Thus, in a pooling situation, demand for the low-quality product is

$$D_l(p, 0, \lambda_e) = \begin{cases} 1 & \text{if } p \leqslant q_{el}(\lambda_e) \\ 1-\gamma & \text{if } q_{el}(\lambda_e) < p \leqslant q_{eh}(\lambda_e). \\ 0 & \text{otherwise} \end{cases} \tag{4.3}$$

Consider second that the actual quality is high. Then, a fraction γ of the population receives the correct signal s_h while a fraction $1-\gamma$ observes the wrong one s_l. Disregarding, for the moment, the possibility of advertising its product, the high-quality seller would serve the entire market for those prices such that $p \leqslant q_{el}(\lambda_e)$. If the price lies on the interval $q_{el}(\lambda_e) < p \leqslant q_{eh}(\lambda_e)$, it would obtain a demand of γ. However, if $p \leqslant q_h$, the high-quality seller can increase its demand by advertising the product (since consumers getting an advertisement will learn the true quality and will be willing to pay as much as q_h). Hence, the high-quality seller faces the following demand function:

$$D_h(p, \lambda, \lambda_e) = \begin{cases} 1 & \text{if } p \leqslant q_{el}(\lambda_e) \\ \gamma + \lambda(1-\gamma) & \text{if } q_{el}(\lambda_e) < p \leqslant q_{eh}(\lambda_e) \\ \lambda & \text{if } q_{eh}(\lambda_e) < p \leqslant q_h \\ 0 & \text{otherwise} \end{cases}. \tag{4.4}$$

We are now ready to compute the high-quality seller's optimal advertising effort. As we have seen, if $p \leqslant q_{el}(\lambda_e)$, the high-quality seller serves the entire market. Therefore, advertising the product would only generate extra costs for the firm. If $q_{el}(\lambda_e) < p \leqslant q_{eh}(\lambda_e)$, consumers observing the wrong signal will not purchase unless they receive a proof of quality. For this interval of prices, the high-quality seller chooses λ to maximize profits $\Pi_h(p, \lambda, \lambda_e) = (\gamma + \lambda(1-\gamma))(p-c) - 0.5k\lambda^2$. The first and second order conditions give $\lambda^* = (1-\gamma)(p-c)/k$. Finally, if $q_{eh}(\lambda_e) < p \leqslant q_h$, consumers will not purchase the good unless they know the actual quality. In this case, the monopolist selects λ to maximize $\Pi_h(p, \lambda, \lambda_e) = \lambda(p-c) - 0.5k\lambda^2$, that is $\lambda^* = (p-c)/k$. Note that the assumption $k > q_h - c$ (see above) ensures that the optimal advertising effort is always an interior solution. We summarize these findings next:

Lemma 2. *In any pooling equilibrium, given the price p and the expected advertising effort λ_e, the high-quality seller's optimal advertising strategy is given by*:

$$\lambda^*(p, \lambda_e) = \begin{cases} \frac{(1-\gamma)(p-c)}{k} & \text{if } q_{el}(\lambda_e) < p \leq q_{eh}(\lambda_e) \\ \frac{(p-c)}{k} & \text{if } q_{eh}(\lambda_e) < p \leq q_h \\ 0 & \text{otherwise} \end{cases} \quad (4.5)$$

Fig. 1 depicts both sellers' demands. The step function represented by the solid lines depicts the low-quality seller's demand while the one represented by the dashed lines shows the demand of the high-quality type. Observe that this latter demand exhibits two flat steps corresponding to those prices for which the optimal advertising effort is zero (price intervals $0 < p < q_l$ and $q_h < p < \infty$). There are also two upward-sloping intervals ($q_{el} < p < q_{eh}$ and $q_{eh} < p < q_h$). The positive slope stems from the fact that, within each interval, the optimal advertising effort is an increasing function of the price (see Lemma 2). The higher the price, the higher is the surplus the monopolist gets from each unit of good sold and, therefore, the higher is its incentive to advertise.

Let us clarify the role of the external signals now. Note that a necessary condition for advertising to occur is that the high-quality firm can increase its demand. This is exactly what the signals do: they introduce the necessary smoothness into the demand function for advertising to have scope. Suppose that consumers did not receive signals or, equivalently, they were not informative (i.e. $\gamma = 0.5$). Then, all consumers' decisions would be identical, independently of the

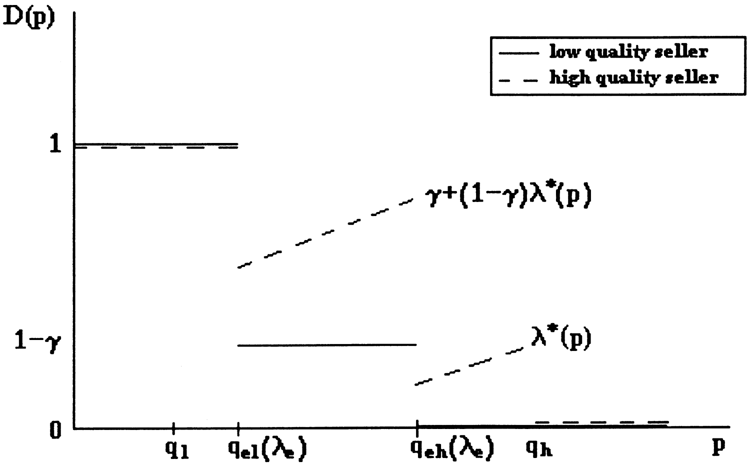

Fig. 1. Sellers' demands when they are believed to produce the high quality with probability β.

type of signal received (low or high). As a consequence, if a consumer purchased in a pooling equilibrium, the rest of them would also buy. Then, advertising would never occur and the interest of the analysis would vanish.[29]

Observe that, since marginal cost of informing a small fraction of consumers is arbitrarily low, the high-quality seller will always increase its demand by sending out some advertisements.[30] From this observation, it turns out that two types of equilibria may emerge: those where advertising does not occur, which we term *no-advertising pooling equilibria* and those with advertising, which we call *advertising pooling equilibria*. From our discussion above, it naturally follows that both firms will serve the entire market in any no-advertising equilibrium.

To ensure that a proposed equilibrium is indeed an equilibrium, we must check that agents cannot profitably deviate from it. Note first that consumers can only observe price-deviations.[31] Since Bayes' rule does not pin down determinate beliefs off-the-equilibrium path, when a firm deviates from a proposed equilibrium by charging a different price (i.e. sending a disequilibrium 'message'), consumers may in general infer any possible expected quality. As explained above, to simplify the presentation, we will throughout assume that the beliefs that consumers form after observing a deviating price are the worst possible from the sellers' point of view, i.e. any disequilibrium price will always be assumed to be quoted by the low-quality seller.[32] Note that employing more general posterior belief functions to enunciate the Propositions 2 and 4 that follow would only restrict the set of potential equilibria.

To analyze the equilibria, we then need to characterize both sellers' best deviating strategies and their profits thereafter. Let p^* be a proposed equilibrium. When a seller deviates from p^* by charging $\tilde{p}$, it will be believed to produce low-quality with probability 1. Consider first that the deviator is the low-quality firm. When it deviates by charging $\tilde{p} \leqslant q_l$, it serves the entire market. Otherwise, i.e. $\tilde{p} > q_l$, it obtains zero demand. Suppose now that the deviator is the high-quality monopolist. Note that it can advertise its product and, to some extent, diminish the negative effect derived from being considered the low-quality type with certainty. Of course, it advertises at a level determined by Lemma 2. Therefore, it serves the entire market whenever $\tilde{p} \leqslant q_l$, and faces demand of $(\tilde{p} - c)/k$ if $q_l < \tilde{p} \leqslant q_h$. Otherwise, its demand is zero.

[29] An alternative (and perhaps more natural) modeling choice is to allow for two groups of consumers with different valuations for the high-quality good. However, it can be easily shown that this is equivalent for our purposes, i.e. it generates the same types of demands as those in Fig. 1. Our model saves on parameters.

[30] Of course, this is so whenever it does not serve the entire market and advertising costs are not prohibitive ($k < \infty$).

[31] Again, note that for this statement to be true, it is crucial that $\lambda < 1$ in any proposed equilibrium. The assumption $k > q_h - c$ guarantees this here.

[32] We postpone the use of refinements to Section 5.

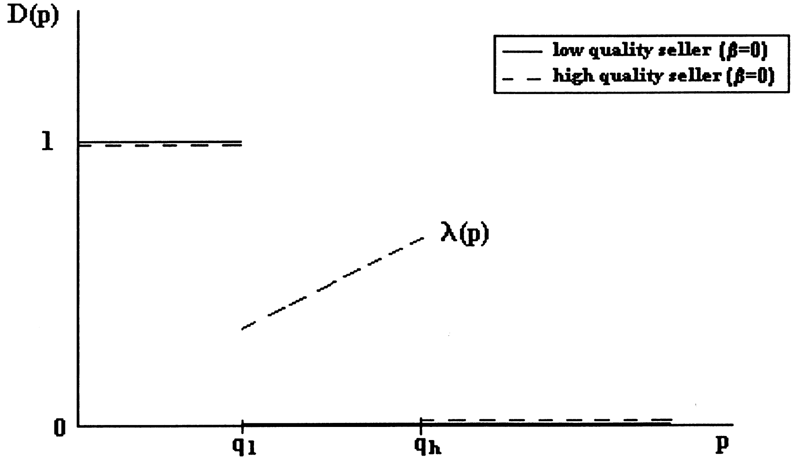

Fig. 2. Sellers' demands when they are believed to produce the low quality with probability 1.

We have depicted these demands in Fig. 2. Again, the demand for the low-quality good is represented by the solid lines and the dashed lines show the high-quality seller's one. Note that there is an upward-sloping interval in the latter. As before, this appears because for prices such that $q_l < \tilde{p} \leqslant q_h$, the supplier's optimal advertising effort is an increasing function of the price.

As a result, when consumers believe that quality is low with probability 1 after observing a disequilibrium message, the best deviating price for the low-quality seller is $\tilde{p} = q_l$, which gives benefits $\tilde{\Pi}_l = q_l$. On the other hand, the high-quality seller has two alternative best deviations. Namely, deviating by either charging $\tilde{p} = q_l$ and not advertising at all, which yields profits $\tilde{\Pi}_h = q_l - c$; or charging $\hat{p} = q_h$ and advertising $\hat{\lambda} = (q_h - c)/k$, which gives benefits $\hat{\Pi}_h = (q_h - c)^2/2k$. Of course, it will choose the one giving the highest profits. In what follows, we will use these best deviating strategies and profits to characterize the pooling equilibria with and without advertising.

4.1. No-advertising pooling equilibria

In any no-advertising pooling equilibrium both sellers must serve the entire market (see above). Then, upon observing Fig. 1, we can state that:

Lemma 3. *In any no-advertising pooling equilibrium $q_l \leqslant p \leqslant q_{el}(\lambda_e)$.*

Proof. Suppose not, then there are three possibilities. First, if $p < q_l$, the low-quality seller would deviate by setting $p = q_l$. Second, if $q_{el}(\lambda_e) < p \leqslant q_{eh}(\lambda_e)$, the high-quality seller would deviate by advertising $\lambda = \arg\max_{\lambda}\{(\gamma + (1-\gamma)\lambda)(p-c) - 0.5k\lambda^2\}$. Finally, if $p > q_{eh}(\lambda_e)$, the low-quality seller would face zero demand and then deviate to q_l. ■

For (p^*, λ^*) being a no-advertising equilibrium, in addition, it must be the case that neither the high nor the low-quality seller has an incentive to deviate and, importantly, that consumers' conjectures are confirmed, i.e. $\lambda^* = \lambda_e = 0$. A firm has no incentive to deviate from the proposed equilibrium (p^*, λ^*) whenever it makes higher profits from adopting this strategy than from using its best deviating one. Thus, for the low-quality seller one must have that $p^* \geqslant q_l$. For the high-quality seller it must be the case that $(p^* - c) \geqslant \max\{q_l - c, (q_h - c)^2/2k\}$. Therefore:

Proposition 2. (p^*, λ^*) *is a no-advertising pooling equilibrium if and only if:*

(a) $p^* \geqslant q_l$

(b) $p^* \leqslant q_{el}(\lambda_e)$

(c) $p^* \geqslant \dfrac{(q_h - c)^2}{2k} + c$

(d) $\lambda^* = \lambda_e = 0.$

We next turn to study the existence of no-advertising pooling equilibria. For this purpose, we define

$$\Psi_1(\beta,\gamma) = \frac{(q_h - c)^2((1-\gamma)\beta + \gamma(1-\beta))}{2((1-\gamma)\beta(q_h - c) + \gamma(1-\beta)(q_l - c))}. \quad (4.6)$$

Proposition 3. *A no-advertising pooling equilibrium exists if and only if* $k \geqslant \Psi_1(\beta, \gamma)$.

Proof. (⇒) Assume $(p^*, 0)$ is a no-advertising pooling equilibrium. Then, from (b) and (c) in Proposition 2, $p^* - c \geqslant (q_h - c)^2/2k$ and $q_{el}(0) - p^* \geqslant 0$. By adding these two inequalities, it is obtained that $q_{el}(0) - c \geqslant (q_h - c)^2/2k$. Then, by substituting Eq. (4.2) into $q_{el}(0)$ and isolating k, we obtain $k \geqslant \Psi_1(\beta, \gamma)$.

(⇐) We show that $(p, \lambda) = (q_{el}(0), 0)$ is a no-advertising equilibrium. First, Lemma 2 ensures that the optimal advertising effort is zero for any λ_e when $p = q_{el}(\lambda_e)$. Second, the low-quality seller does not deviate since $q_{el}(0) \geqslant q_l$. Finally, the high-quality seller does not deviate whenever $q_{el} - c \geqslant (q_h - c)^2/2k$, which is ensured by the condition that $k \geqslant \Psi_1(\beta, \gamma)$. ■

The intuition behind the condition in Proposition 3 is as follows. For a full pooling equilibrium without advertising to exist, it is necessary that k is sufficiently high. If, on the contrary, k were very small, the high-quality firm would profitably deviate by charging its optimal price under full information (q_h). It would simply disregard the fact that its product would be considered of low-quality with probability 1, because it would be able to profitably inform most of the consumers about the true quality at a low cost. If k is high enough, which is ensured by the condition that $k \geq \Psi_1(\beta, \gamma)$, such a deviation is no longer profitable. This is the only condition needed to guarantee the existence of a no-advertising pooling equilibrium since neither the low nor the high-quality seller would deviate by lowering the price (note that they face full demand).

To gain further intuition consider the function $\varphi(k, \beta, \gamma) = k - \Psi_1(\beta, \gamma)$. A pooling equilibrium without advertising exists whenever $\varphi(\cdot) \geq 0$. It is easily checked that the set of parameters for which a no-advertising pooling equilibrium exists is non-empty. Moreover, note that $\partial \Psi_1 / \partial \beta < 0$. Therefore, as the consumers' prior probability that quality is high (β) decreases, to sustain an equilibrium, it is necessary that the cost of advertising increases. The reason is that as β decreases, the expected quality $q_{el}(0)$ approaches q_l, and, as a result, the price charged in equilibrium is lower. The incentive of the high-quality seller to deviate to the strategy (q_h, $\lambda^*(q_h)$) is then higher because its equilibrium profits decrease. On the contrary, notice that $\partial \Psi_1 / \partial \gamma > 0$. Therefore, when the accuracy of the consumers' pre-purchase information (γ) increases, to sustain an equilibrium, it is needed that k increases. The intuition is the same as before.

4.2. Advertising pooling equilibria

In any advertising pooling equilibrium, the price must be high enough so that consumers receiving wrong signals do not purchase the product unless they are informed of the true quality. As a consequence, the high-quality seller will have an incentive to advertise. This indeed happens for those prices such that $q_{el}(\lambda_e) < p^* \leq q_{eh}(\lambda_e)$ (see Fig. 1). Note also that a price higher than $q_{eh}(\lambda_e)$ cannot be an equilibrium because the low-quality seller faces zero demand. Therefore:

Lemma 4. *In any advertising pooling equilibrium* $q_{el}(\lambda_e) < p^* \leq q_{eh}(\lambda_e)$.

Additionally, the following is needed for (p^*, λ^*) to be an advertising pooling equilibrium. First, the advertising effort has to be optimal, that is, λ^* must equal $\arg\max_\lambda \Pi_h(p^*, \lambda, \lambda_e) = (1-\gamma)(p^* - c)/k$ (see Lemma 2). Second, buyers' expectation about the high-quality seller's advertising intensity must coincide with the actual one. Finally, both the high and the low-quality seller cannot have incentives to deviate: the high-quality firm does not deviate from the proposed equilibrium whenever its equilibrium profits $(p^* - c)(\gamma + \lambda^*(1-\gamma)) - 0.5k\lambda^{*2}$ exceed benefits from its best deviating strategy $\max\{q_l - c, (q_h - c)^2/2k\}$. Analogously, the low-quality seller does not deviate if $(1-\gamma)p^* \geq q_l$. We summarize this next:

Proposition 4. (p^*, λ^*) *is an advertising pooling equilibrium if and only if:*

(a) $\lambda^* = \lambda_e = \dfrac{(1-\gamma)(p^* - c)}{k}$

(b) $q_{el}(\lambda_e) < p^* \leqslant q_{eh}(\lambda_e)$

(c) $(1-\gamma)p^* \geqslant q_l$

(d) $\gamma(p^* - c) + \dfrac{(1-\gamma)^2(p^*-c)^2}{2k} \geqslant \max\left\{ q_l - c, \dfrac{(q_h - c)^2}{2k} \right\}.$

We now turn to study the conditions under which an advertising pooling equilibrium exists. For this purpose we define

$$X^- = \frac{B - (B^2 - 4AC)^{\frac{1}{2}}}{2A}, \tag{4.7}$$

where $A = \gamma\beta(1-\gamma)$, $B = A(q_h + c) + k(\gamma\beta + (1-\beta)(1-\gamma))$ and $C = Aq_h c + k(\gamma\beta q_h + (1-\beta)(1-\gamma)q_l)$.

The following proposition, whose proof is relegated to Appendix A, exhibits necessary and sufficient conditions for the existence of a pooling equilibrium with advertising.

Proposition 5. *An advertising full pooling equilibrium exists if and only if:*

(i) $q_h > 2q_l$

(ii) $\gamma < \dfrac{q_h - q_l}{q_h}$

(iii) $\beta \geqslant \dfrac{k(1-\gamma)q_l}{(q_h(1-\gamma) - q_l)(k - (q_l - (1-\gamma)c)) + k(1-\gamma)q_l}$

(iv) $2k\gamma(X^- - c) + (1-\gamma)^2(X^- - c)^2 - (q_h - c)^2 \geqslant 0.$

The intuition behind these conditions is as follows. Consider first the low-quality firm. In comparison with its best deviating strategy $(q_l, 0)$, in an advertising pooling equilibrium, it charges a higher price $(p^* > q_l)$ but sells a lower quantity $(1-\gamma < 1)$. To ensure that the low-quality firm does not deviate, neither its sales nor the price can be too low. This is guaranteed by conditions (i)–(iii). On the one hand, (i) and (iii) ensure that the price charged is not too low. Condition (i) requires the consumers' reservation value for the high-quality good to be sufficiently large compared with the one for the low-quality product. This guarantees that the buyers' expected qualities (and consequently the price charged in equilibrium) are sufficiently high. However, this is not enough. Condition (iii) is also needed, i.e. the consumers' prior probability that quality is high must be sufficiently large. Otherwise, the price charged would be too low. Note also that the higher the prior for the high-quality good, the higher are consumers' expected qualities and, as a result, the higher is consumers' willingness to pay. When either β or q_h is too low, the pooling price is very close to q_l and, as a consequence, the price cannot be sustained in equilibrium any more (see Eqs. (4.1) and (4.2)). On the other hand, condition (ii) guarantees that equilibrium sales are not too low. It requires the market to be noisy enough (γ small). In fact, in equilibrium the low-quality seller only sells to those consumers receiving the wrong signal (i.e. the high-quality one), namely, a fraction $1-\gamma$. Therefore, this percentage of consumers has to be large enough for an equilibrium of this type to exist. Note that condition (i) also ensures that the set of γs for which an equilibrium exists is non-empty.

Consider now the high-quality seller. The above arguments also allow us to rule out a deviation where the high-quality seller lowers the price. Notice that when the low-quality firm has no incentive to deviate to the price q_l, then the high-quality firm has no incentive to deviate to the strategy $(q_l, 0)$ either. The reason is simply that even if it disregards the possibility of advertising its product, the high-quality seller is better off by charging the pooling price. Finally, condition (iv) must be satisfied to ensure that the high-quality seller does not deviate by raising the price. This condition requires the cost of advertising to be sufficiently high. If it were small, deviating by charging the consumers' reservation price for the high-quality (q_h) and extensively advertising the product would always be profitable for the high-quality seller.[33]

We summarize the previous arguments next: for an advertising pooling equilibrium to exist, it is necessary that (a) the consumers' reservation price for the high-quality product is large enough in comparison with that for the low-quality one, (b) the informativeness of the independent signal is sufficiently low, (c) the consumers' prior probability that quality is high is large enough and, finally, (d) the cost of advertising is sufficiently high.

The set of parameters for which an advertising pooling equilibrium exists is non-empty. In Fig. 3 we have depicted conditions for its existence. Schedule C-C represents condition (iii) while D-D depicts condition (iv). The lower bound $\underline{\beta}$ has been obtained from condition (iii). Of course, the rest of the parameters have been chosen satisfying conditions (i) and (ii). The shaded area then represents the constellation of parameters $k-\beta$ for which informative advertising occurs in a full pooling equilibrium. Interestingly, as the consumers' prior probability that quality is high (β) decreases, a higher advertising cost is required to sustain the equilibrium. The intuition behind this observation is again found in the fact that the price charged decreases as β diminishes (since consumers' willingness to pay

[33] Note that this basically requires the same as what is required by the condition in Proposition 3.

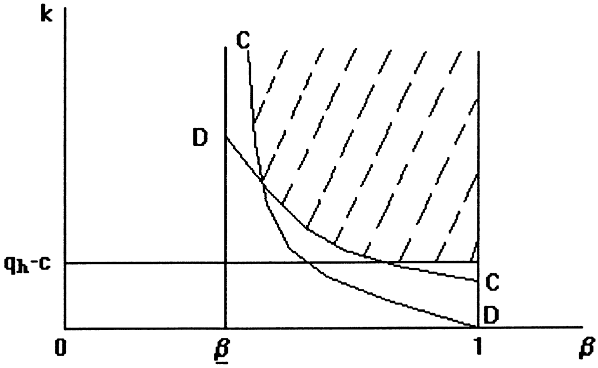

Fig. 3. Existence of an advertising pooling equilibrium.

decreases). So, from both sellers' point of view, a higher parameter k is required to sustain the equilibrium (both C-C and D-D decrease with k).

We have also analyzed partial pooling (or semi-separating) equilibria. Here we will only describe our findings very briefly (see Moraga-González (1997) for details). In any partial pooling equilibrium, the low-quality monopolist randomizes between its optimal price under complete information q_l, and a pooling price $p*$, while the high-quality monopolist always sets the pooling price.[34] Let ρ be the probability that the low-quality seller charges the pooling price and ρ_e the consumers' expectation about it. In a partial pooling equilibrium, first, consumers observing the price q_l ascertain that quality is low; second, buyers receiving a free sample learn that the true quality is high; and finally, those uninformed consumers observing the price $p*$ and a high-quality signal s_h, and not receiving an advertisement expect quality to be (by Bayes' rule)

$$\tilde{q}_{eh}(\lambda_e,\rho_e) = \frac{\gamma\beta(1-\lambda_e)q_h + (1-\gamma)(1-\beta)\rho_e q_l}{\gamma\beta(1-\lambda_e) + (1-\gamma)(1-\beta)\rho_e}, \tag{4.8}$$

while those buyers receiving a low-quality signal s_l form beliefs

$$\tilde{q}_{el}(\lambda_e,\rho_e) = \frac{(1-\gamma)(1-\lambda_e)\beta q_h + \gamma(1-\beta)\rho_e q_l}{(1-\gamma)(1-\lambda_e)\beta + \gamma(1-\beta)\rho_e}. \tag{4.9}$$

The same types of arguments as in the previous section allow us to state that advertising always occurs in a partial pooling equilibrium. The conditions for which an advertising partial pooling equilibrium exists are very similar to those in Proposition 5. Indeed, the only difference is that for a semi-separating equilibrium

[34] A semi-separating equilibrium where the high-quality seller randomizes does not exist.

to exist, it is needed that the consumers' prior probability that quality is high is not too large. If, on the contrary, β were very high, then the low-quality monopolist would charge the pooling price with probability one.

5. Refinements

This section is devoted to refine the set of equilibria. We first apply the well-known and widely-employed Intuitive Criterion of Cho and Kreps (1987) and show that all the equilibria previously found survive. In words, a proposed equilibrium p^* is 'intuitive' if there does not exist another price $\tilde{p}$ to deviate to for which the high type is better off while the low one is worse off when consumers believe that the deviator sells the high-quality product. If this price existed, consumers should correctly infer that only the high-quality firm would charge such a price. This indeed makes the deviation to $\tilde{p}$ to be profitable for the high type, and, as a result, the proposed equilibrium to fail.[35]

We first show that any no-advertising pooling equilibrium is intuitive. Let p^* be such an equilibrium. It satisfies the Intuitive Criterion if there does not exist $\tilde{p}$ such that, when buyers believe it to be quoted by the high type, this firm is actually better off, i.e. $\tilde{p}-c>p^*-c$, and the low-quality one is worse off, i.e. $\tilde{p}<p^*$. Clearly, such a price can never exist.

We next show that any advertising pooling equilibrium also satisfies the Intuitive Criterion. Let (p^*, λ^*) be an equilibrium with advertising. It is intuitive if there does not exist another price $\tilde{p}$ for which both conditions $\tilde{p}-c>\gamma(p^*-c)+(1-\gamma)^2(p^*-c)^2/2k$ and $\tilde{p}<(1-\gamma)p^*$ hold. That is, whenever $\gamma(p^*-c)+(1-\gamma)^2(p^*-c)^2/2k+c\geqslant(1-\gamma)p^*$. Rewriting, it must be the case that $(1-\gamma)^2(p^*-c)^2/2k\geqslant p^*(1-2\gamma)-c(1-\gamma)$. But this inequality is always satisfied since $\gamma>0.5$ (its right-hand side is negative).[36]

As we have seen, the Intuitive Criterion has no bite in our setting. Following the discussion by Mailath et al. (1993) on belief-based refinements, we turn to consider their alternative notion, namely *undefeated equilibrium*. In our setting, this concept refines away all those equilibria that seem unreasonable. We refer to equilibria that are inefficient from the seller's point of view. For instance, note that in any equilibrium without advertising (i.e. $p^*\leqslant q_{el}(0)$) all consumers buy. An equilibrium where the firm charges $p^*<q_{el}(0)$ does not seem reasonable since it does not charge the highest possible price for which all buyers purchase. We show that these types of equilibria are not undefeated.

Intuitively, the refinement works as follows. Consider a proposed pooling

[35] For a formal definition see Cho and Kreps (1987).

[36] All the advertising partial pooling equilibria also survive the Intuitive Criterion (see Moraga-González, 1997).

equilibrium (p^*, λ^*) and an out-of-equilibrium price $\tilde{p}$. Suppose that there is an alternative equilibrium where one or both types of monopolists charge $\tilde{p}$. Suppose further that it is precisely such a type or both who obtain higher profits in the alternative equilibrium than in the proposed one. Then, after observing the disequilibrium message $\tilde{p}$, the test requires consumers to form the same beliefs as those that they would have formed in the alternative equilibrium. If the beliefs sustaining the original equilibrium are not consistent in this manner, it is said that the second equilibrium defeats the proposed one.[37]

In our model, this refinement works adequately and allows us to state conditions under which there is a unique outcome. Note first that the unique no-advertising pooling equilibrium that may be undefeated is $(p^*, \lambda^*)=(q_{el}(0), 0)$, i.e. the highest price sustainable as a pooling equilibrium without advertising (see Fig. 1). In fact, any no-advertising pooling equilibrium such that $p^*<q_{el}(0)$ is defeated by the alternative equilibrium $(q_{el}(0), 0)$. To see this, fix a no-advertising pooling equilibrium where $p^*<q_{el}(0)$ and take $\tilde{p}=q_{el}(0)$ as the disequilibrium message. Note that $q_{el}(0)$ is charged by both types of monopolists in the alternative equilibrium $(q_{el}(0), 0)$. Observe further that, precisely, both types of sellers obtain higher profits in this equilibrium than in the original one. Then, consumer beliefs must be consistent with the fact that either seller may have sent the disequilibrium message. If this is so, a deviation to $q_{el}(0)$ is profitable for either of the types. Consequently, $(q_{el}(0), 0)$ defeats the previous equilibrium.

Analogously, it can be seen that the only pooling equilibrium with advertising that may be undefeated is $(p^*, \lambda^*)=(q_{eh}(\lambda^*), \lambda^*)$, i.e. the highest price sustainable as an advertising pooling equilibrium accompanied by its corresponding optimal publicity effort (see Fig. 1). To conclude the analysis, we need to find the conditions under which the advertising equilibrium defeats the no-advertising one. Given the spirit of the refinement, this is simply done by comparing profits in both situations. Therefore:

Proposition 6. *Suppose that the conditions in Propositions 3 and 5 hold. Then,*

$$(p^*, \lambda^*)=\left(X^-, \frac{(1-\gamma)(X^- - c)}{k}\right)$$

is the unique undefeated pooling equilibrium if and only if $(1-\gamma)X^- \geqslant q_{el}(0)$. *In this equilibrium, the high-quality seller introduces the good employing informative advertising.*

Proof. The unique pooling equilibrium price that may be undefeated is given by the equation $p^*=q_{eh}(\lambda^*(p^*))$ (see above). Using Lemma 2 and isolating p^*, it is

[37] The refinement proposed by Grossman and Perry (1986) called 'perfect sequential equilibrium' has a similar spirit and functions identically in our context (it selects away the same sets of equilibria). However, it does not require the disequilibrium message to be sent in another equilibrium.

obtained that either $p^* = X^-$ or $p^* = X^+$. However, proof of Proposition 5 shows that $p^* = X^+$ cannot be an equilibrium price (see Appendix A). Therefore, the unique pooling equilibrium that may be undefeated is precisely $(p^*, \lambda^*) = (X^-, (1-\gamma)(X^- - c)/k)$. By adopting this strategy, the low-quality monopolist gets profits $(1-\gamma)X^-$ while the high-quality type obtains $\gamma(X^- - c) + (1-\gamma)^2(X^- - c)^2/2k$. Now, fix the unique equilibrium without advertising that may be undefeated $(q_{el}(0), 0)$ and take the disequilibrium price $\tilde{p} = X^-$. Note that both types of monopolist set X^- in the alternative equilibrium with advertising. Observe further that if condition $(1-\gamma)X^- \geqslant q_{el}(0)$ holds, both types of firms obtain higher profits in the equilibrium with advertising, as $\gamma X^- + (1-\gamma)c + (1-\gamma)^2(X^- - c)^2/2k > (1-\gamma)X^- \geqslant q_{el}(0)$ (since $\gamma > 0.5$). Then, since consumers' beliefs must be consistent with this fact, a deviation to $\tilde{p} = X^-$ will be profitable for either of the types. Consequently, the equilibrium with advertising defeats the no-advertising one. ■

The condition in Proposition 6 guarantees that both types of sellers obtain higher profits in the equilibrium with advertising. To be sure that a unique undefeated equilibrium with advertising actually exists, we have to check that the set of parameters satisfying conditions in Propositions 5 and 6 is not empty. To demonstrate this, consider that k is sufficiently high. Then, condition (iii) in Proposition 5 will hold (see Eq. (6.1) in Appendix A). Since λ tends to zero and X^- to $q_{eh}(0)$, condition (iv) will also hold. Finally, it is easy to show that $(1-\gamma)q_{eh}(0) \geqslant q_{el}(0)$ can be rewritten as $q_h\beta(1-\gamma)((1-\beta)(\gamma^2+\gamma-1) - \gamma^2\beta) + q_l(1-\beta)(\beta((1-\gamma)^3 - \gamma^2) - \gamma^2(1-\gamma)(1-\beta)) \geqslant 0$, which holds for q_h sufficiently high. By continuity, for $\lambda > 0$ but small, the condition in Proposition 6 will hold too.

6. Conclusions

In this paper we have studied a supplier's decision to use informative advertising to introduce an experience good into a market where prices are observable and advertising efforts are not. Informative advertising never occurs in a separating equilibrium since prices convey full information about quality, so advertising is redundant. We have shown that a full pooling equilibrium with informative advertising exists if and only if (a) the consumers' valuation for the high-quality is sufficiently large, (b) the informativeness of the market signal is low enough, (c) the consumers' prior probability of high-quality is sufficiently high and (d) the cost of advertising is high enough. Existence of an advertising semi-separating equilibrium requires similar conditions. Under some parameters, there is a unique undefeated equilibrium with advertising. When informative advertising occurs in equilibrium, the adverse selection problem is mitigated. Moreover, the lower the advertising cost, the further is the alleviation of that problem.

Several questions remain open. In many cases, sellers can choose the type of advertising campaign to employ when launching a new good. Therefore, extending the analysis to a context where a seller can employ advertising either as a signal or as information would be interesting. The difference between the cost of producing each quality will be important since it determines the extent to which price signalling may occur and, consequently, the potential for uninformative advertising signals.

Some of our assumptions could be relaxed to test the robustness of our predictions. Throughout, it has been assumed that advertising conveys perfect information about a good's quality. This is reasonable for products such as shampoos or cosmetics since a free sample of the good conveys the entire truth. For sophisticated products, for instance software, encyclopedias etc., it is more likely that advertising conveys information that is noisy. The main implication is that no consumer would be perfectly informed in equilibrium. Instead, there would be four groups of consumers with different sets of information, depending on the signal observed and whether or not they had received an advertisement. It seems likely that there would exist advertising equilibria if the precision of the information conveyed through the noisy publicity was sufficiently high. As in the present model, the seller would determine the level of information in the market endogenously.[38]

The observation that prices do not reveal quality in equilibrium is specific to our model. This strongly depends on the assumption that consumers' valuations for the products are identical. Considering heterogeneous consumers' willingness to pay for the goods, as in Bagwell and Riordan (1991), would be appropriate to check whether our results are robust. In a model in that spirit, a unique intuitive separating equilibrium emerges if advertising is impossible (the Intuitive Criterion selects away all pooling equilibria). However, proving that separation may be ruled out when advertising is possible and inexpensive is easy. Intuitively, if advertising cost is sufficiently low, the high quality seller may find it profitable to deviate from a proposed separating equilibrium by charging a lower price and informing to many consumers through advertising. Whether advertising pooling equilibria would survive the Intuitive Criterion is dubious and should be carefully investigated.

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[38] Note that if the informative content of advertisements were confusing (e.g. if sellers could lie), they would simply be disregarded by consumers. As a result, advertising would never occur.

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Appendix A

Proof of Proposition 5

We use the following definitions and lemmas. Define:

$$A = \gamma\beta(1-\gamma)$$

$$B = A(q_h + c) + k(\gamma\beta + (1-\gamma)(1-\beta))$$

$$C = Aq_hc + k(\gamma\beta q_h + (1-\gamma)(1-\beta)q_l)$$

$$X^+ = \frac{B + (B^2 - 4AC)^{\frac{1}{2}}}{2A}$$

$$X^- = \frac{B - (B^2 - 4AC)^{\frac{1}{2}}}{2A}$$

Lemma 5. $B^2 - 4AC > 0$.

Proof. Equation $B^2 - 4AC > 0$ can be rewritten as $a_1k^2 + a_2k + c > 0$, where $a_1 = (\gamma\beta + (1-\gamma)(1-\beta))^2$, $a_2 = -2A(\gamma\beta(q_h - c) - (1-\beta)(1-\gamma)(q_h - 2q_l + c))$ and $a_3 = A^2(q_h - c)^2$. Consider the quadratic equation $a_1k^2 + a_2k + c = 0$, which is convex since $a_1 > 0$. Its solutions are given by $k = (-a_2 \pm (a_2^2 - 4a_1a_3)^{0.5})/2a_1$. Note that $a_2^2 - 4a_1a_3 = -(q_h - q_l)\gamma^2(1-\gamma)^3\beta^2(1-\beta)(\gamma\beta(q_h - c) + (1-\gamma)(1-\beta)(q_l - c)) < 0$. Therefore, equation $a_1k^2 + a_2k + c = 0$ has no real solution. This implies that $a_1k^2 + a_2k + c = B^2 - 4AC > 0$. ■

Lemma 6. $X^+ > q_h$.

Proof. $B/2A > q_h$ as long as $A(q_h + c) + k(\gamma\beta + (1-\gamma)(1-\beta)) > 2Aq_h$. This inequality can be rewritten as $k(\gamma\beta + (1-\gamma)(1-\beta)) > A(q_h - c)$. Since $A < \gamma\beta + (1-\gamma)(1-\beta)$ and, by assumption, $k > q_h - c$ it follows that $B/2A > q_h$. Then, by using Lemma 5, the result directly follows. ■

Lemma 7. *If* $(q_l-(1-\gamma)q_h)>0$, *then* $\dfrac{\beta(q_l-(1-\gamma)q_h)(q_l-(1-\gamma)c)}{\beta(q_l-(1-\gamma)q_h)+(1-\beta)(1-\gamma)q_l}<q_h-c$.

Proof. Note that the left hand side of the inequality increases with β. Then, it suffices to show that the inequality holds in the worst of the cases, i.e. $\beta=1$. In such a case, the inequality reduces to $(q_l-(1-\gamma)q_h)(q_l-(1-\gamma)c)-(q_h-c)(q_l-(1-\gamma)q_h)<0$. Rearranging terms, it can be rewritten as $(q_l-(1-\gamma)q_h)(q_l-q_h+\gamma c)<0$. By using the assumption $q_h-c>q_l$ and the hypothesis $q_l-(1-\gamma)q_h>0$, it is easily checked that this inequality is satisfied. ■

The proof of Proposition 5 now follows:

Proof. (⇒) Assume that (p^*, λ^*) is an advertising pooling equilibrium. Then, from Proposition 4, it must satisfy equation $p^*\leqslant q_{eh}(\lambda^*(p^*))$. Solving this inequality for p^*, it is obtained that $p^*\leqslant X^-$ or $p^*\geqslant X^+$. Lemma 5 ensures that X^- and X^+ are well defined. In addition, Lemma 6 allows us to ignore those prices $p^*\geqslant X^+$. From (b) and (c) in Proposition 4 one has that $X^- - p^*\geqslant 0$ and $p^*\geqslant q_l/(1-\gamma)$. By adding these two inequalities, it follows that $(1-\gamma)X^-\geqslant q_l$. This inequality can be rewritten as:

$$k(\beta(q_h(1-\gamma)-q_l)-(1-\beta)(1-\gamma)q_l)\geqslant\beta(q_h(1-\gamma)-q_l)(q_l-(1-\gamma)c). \tag{6.1}$$

Assume that (i) does not hold, that is, $q_h-2q_l\leqslant 0$. Then, since $\gamma>0.5$, one must have $q_h(1-\gamma)-q_l\leqslant 0$. Otherwise, there would not exist any feasible γ. Then, both sides of the inequality (6.1) are negative. Rewriting this inequality, it requires that

$$k\leqslant\frac{\beta(q_l-q_h(1-\gamma))(q_l-(1-\gamma)c)}{\beta(q_l-q_h(1-\gamma))+(1-\beta)(1-\gamma)q_l}. \tag{6.2}$$

However, Lemma 7 shows that

$$\frac{\beta(q_l-q_h(1-\gamma))(q_l-(1-\gamma)c)}{\beta(q_l-q_h(1-\gamma))+(1-\beta)(1-\gamma)q_l}<q_h-c, \tag{6.3}$$

which, since $k>q_h-c$, constitutes a contradiction. As a result, (i) must be satisfied and, since $\gamma>0.5$, (ii) must also hold. Condition (iii) is nothing else than Eq. (6.1) properly rearranged.

Finally, by condition (d) in Proposition 4, one obtains that $\gamma(p^*-c)+(1-\gamma)^2(p^*-c)^2/2k\geqslant(q_h-c)^2/2k$. Since the left hand side of this inequality is strictly increasing in p^* and $p^*\leqslant X^-$, (iv) follows.

(⇐) We show that, if (i)–(iv) are satisfied, then $(p, \lambda)=(X^-, (1-\gamma)(X^- - c)/k)$ is an advertising pooling equilibrium. First, the optimal advertising intensity follows from substituting X^- into the optimal advertising function (see Lemma 2).

Condition (ii) ensures that the low-quality seller does not deviate. On the other hand, condition (iv) guarantees that the high-quality seller does not deviate by using the strategy $(\hat{p}, \hat{\lambda}) = (q_h, (q_h - c)/k)$. To complete the proof, we have to show that the high-quality seller does not deviate by using the alternative strategy $(\tilde{p}, \tilde{\lambda}) = (q_l, 0)$. Profits from using such a strategy equal $q_l - c$. From condition (ii), one has that $\gamma(X^- - c) \geqslant \gamma(q_l/(1-\gamma) - c)$. Since $\gamma > 0.5$, it follows that $q_l - c < \gamma(q_l/(1-\gamma) - c) \leqslant \gamma(X^- - c)$. Therefore, $\gamma(X^- - c) - (q_l - c) + (1-\gamma)^2(X^- - c)^2/2k \geqslant 0$; thus, the proposition follows.

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