Advertising, Consumer Search and Product Differentiation

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Disclaimer: preliminary and incomplete!
1. Introduction
Introduction

In markets for (horizontally) differentiated goods, the prices firms charge depend on the amount of (price and product) information consumers have.

(Price and product) Information can be

- acquired by consumers themselves (consumer search)
- disclosed by the firms themselves (advertising)

Main contribution of this paper is to present a tractable oligopoly framework for the study of firm incentives to disclose price and/or product information to consumers who otherwise can engage in (costly) search.

Starting point is Wolinsky’s (1986) work-horse model for markets for differentiated goods with costly consumer search (further studied by Anderson and Renault, 1999).
In Wolinsky’s model consumers only know that the firms retail products they like; however, they do not know

- how much they really like the product of a particular firm
- deviation prices.

To discover such critical information, they have to (costly) search the firms. In such a setting, one can demonstrate that

- prices and profits increase in search costs

Most of the literature building on Wolinsky, however, neglects the possibility that firms advertise their prices, and perhaps even product information.
We start by first considering the role of price advertising in consumer search markets for differentiated products.

It turns out that modelling price advertising into Wolinsky’s framework is intractable.

- the reason is that a pure-strategy eq. fails to exist and the mixed-strategy eq. proves to be very difficult to characterize

As a fix to this problem, we present a new model where consumers’ utility depends on (before-search) observed and unobserved product characteristics.

Because some product characteristics are observed before search, in this new model search is not any more random but directed:

- consumers search first the firms that are more promising ex-ante (Weitzman, 1979)
By allowing for price advertising, our paper contributes to a better understanding of the relation between market power, advertising and search costs.

We show that the price-advertising game has the characteristics of a prisoner’s dilemma when advertising costs are negligible:

- If a firm does not advertise its price, then the rival firm has an incentive to cut its price and disclose it to consumers.
- Moreover, if a firm does advertise its price, then the rival firm also wishes to advertise its price.*

As a result, a symmetric price-advertising equilibrium exists. And firms obtain lower profits than in the Wolinsky’s benchmark.

*For this we need a “restriction” on beliefs off-the-equilibrium path. More on this later.
One way to read this result, is that the much studied framework of Wolinsky (1986) can only be rationalised on the basis of significant advertising frictions.

Interestingly, when firms advertise their prices, prices are a decreasing function of search costs.

- that is, in contrast to conventional wisdom, firms do not benefit from search costs becoming higher
- intuition: a visit to a firm is more valuable the higher the search cost
We then consider **match-value advertising**, or product advertising.

Whether a firm will use it depends on whether prices can be advertised or not.

If firms cannot advertise prices, we do not expect firms to disclose match-value information.

However, when match-value advertising is used along with price advertising, then we expect firms to reveal their product information.

Interestingly, in a price-and-product-advertising equilibrium, firms obtain higher profits than in a price-advertising equilibrium.

- in this sense product advertising does not have the features of a prisoner’s-dilemma.

Product advertising has social value: it serves to direct consumers to better deals, thereby reducing (useless) traffic from shop to shop to discover satisfactory products.
Related literature

**Advertising literature:** Though the literature is large, little work has studied the interaction between search costs and (price and/or product) advertising in oligopolistic settings. Exceptions are:

- Bagwell and Ramey (AER, 1994), advertising as a coordination device
- Robert and Stahl (Etrica, 1993) with price advertising for homogeneous products.
- Haan and Moraga-González (EJ, 2011), persuasive advertising

For monopoly, Anderson and Renault (AER, 2006) show that a monopolist does not have incentives to provide precise product information only.

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†see also Janssen and Non (IJIO, 2008)
**Consumer search with differentiated products:** Most literature considers models of random search, e.g. Wolinsky (QJE, 1986), Anderson and Renault (RJE, 1999).

- Paper adds to the recent efforts towards modelling ordered-search, e.g. Arbatskaya (RJE, 2007), Armstrong et al. (RJE, 2009).
- By advertising price and/or product information, firms turn out to be visited in order, so advertising functions as a source of market prominence. Differently from Haan and Moraga-González (EJ, 2011), here ads are informative.
Comparative advertising: The literature deals with models in which search costs are equal to zero, e.g. Meurer and Stahl (IJIO, 1994) and Anderson and Renault (AER, 2009).

- With zero search costs comparative advertising is inconsequential because firms will reveal their match value information anyway.
- With positive search costs, we show this cannot occur in equilibrium and market product revelation is imperfect.
- Comparative ads simply do not occur in eq.
2. A Framework to Study Price and Product Advertising in Markets for Differentiated Products with Costly Consumer Search
The model

Firms:

- Duopoly market, firms labelled 1 and 2
- Price competition
- Firms’ products are horizontally differentiated in two dimensions
  - One dimension, denoted $\eta$, is observable before search‡
  - The other dimension, denoted $\varepsilon$ can be:
    - a "search characteristic," that is, only observable after search
    - or a characteristic that can also be advertised by the firms
- We adopt random utility framework:
  - $\eta$ follows CDF $G(\eta)$, defined on $[0, \bar{\eta}]$
  - $\varepsilon$ follows CDF $F(\varepsilon)$ defined over $[0, \bar{\varepsilon}]$, with $1 - F(\varepsilon)$ log-concave
  - densities are denoted $g$ and $f$, respectively
- constant returns to scale, mg. cost normalized to zero

‡Think of firm location for example. This dimension of product diff. is introduced for technical reasons: it is necessary for the existence of pure-strategy equilibria.
Consumers:

- Unit mass of consumers
- \textit{perfectly informed} about the $\eta$’s they get at the two firms
- \textit{imperfectly informed} about the $\varepsilon$’s and (deviation) prices (unless advertised)
- Search sequentially, no replacement, costless recall, hold equilibrium beliefs about prices
- Search cost, denoted $s$, is common across consumers, and small enough so that a first search is always worth
- The utility of a consumer $i$ who buys product $j$ equals

$$u^i_j = \eta^i_j + \varepsilon^i_j - p_j$$

(will drop super index $i$ in what follows)

- match values are consumer’s private information (no price discrimination)
- the market is fully covered (outside option gives utility $-\infty$)
a. Useful notation

Define $\Delta_\eta \equiv \eta_2 - \eta_1$.

The support of $\Delta_\eta$ is $[-\eta, \eta]$ and the distribution is

\[
\Gamma(z) \equiv \Pr[\Delta_\eta < z] = \begin{cases} 
1 - \int_{-\eta}^{\eta} G(\eta - z) \, dG(\eta) & z \geq 0 \\
\int_{-\eta}^{\eta} \int_{0}^{\eta_1 + z} dG(\eta_2) \, dG(\eta_1) = \int_{-\eta}^{\eta} G(\eta + z) \, dG(\eta) & z < 0
\end{cases}
\]

with density function

\[
\gamma(z) = \begin{cases} 
\int_{0}^{\eta - z} g(\eta + z) \, dG(\eta) & z \geq 0 \\
\int_{-\eta}^{\eta} g(\eta + z) \, dG(\eta) & z < 0
\end{cases}
\]

(Note that the density function is symmetric with respect to zero.)
b. Assumption

We will make the assumption that search costs are not very large, or in other words that the “search characteristic” is sufficiently important for consumers.

This boils down to assuming that

$$\int_{\bar{\eta}}^{\bar{\varepsilon}} (\varepsilon - \bar{\eta}) f(\varepsilon)d\varepsilon > s$$

The LHS represents the gains from search for a consumer who has drawn the highest possible $\eta$ but the worst possible $\varepsilon$. The RHS is the search cost.

For the uniform distribution the assumption is $\bar{\varepsilon} > \bar{\eta} + \sqrt{2s\bar{\varepsilon}}$. (Or equivalently $\bar{\eta} < \hat{x}$.)
No advertising benchmark

In what follows we characterise the benchmark equilibrium where firms do not advertise.

- This should be seen as an extension of Wolinsky (1986) to a situation where some products attributes are observed before search and therefore search is directed

Let $p^*$ be the symmetric equilibrium price. Since match values $\eta$ are observed:

- consumers for whom $\Delta \eta \geq 0$ will start searching at firm 2,
- while consumers for whom $\Delta \eta < 0$ will start at firm 1.

In order to compute the price equilibrium, we consider the gains a firm obtains by deviating from the equilibrium price.

- Let us assume that a firm, say 1, deviates by charging $p$.
- Notice that since consumers do not see the deviation price, they will start their search for satisfactory goods in the usual (equilibrium) way.
Take a consumer who visits firm 1 in her first search, observes the deviation price \( p \) and contemplates going to firm 2. Let \( \varepsilon_1 + \eta_1 - p \) the observed utility at firm 1.

Since a buyer observes \( \eta_2 \) and expects the equilibrium price \( p^* \) to prevail at firm 2, her gains from search are

\[
\int_{\varepsilon_1+\Delta}^{\varepsilon} [\varepsilon_2 - (\varepsilon_1 + \Delta)] f(\varepsilon_2) d\varepsilon_2
\]

where

\[
\Delta = \Delta_p - \Delta_\eta; \text{ and } \Delta_p = p^* - p; \Delta_\eta = \eta_2 - \eta_1
\]

Denoting by \( x = \varepsilon_1 + \Delta \), the buyer will conduct a search provided that \( x < \hat{x} \) where \( \hat{x} \) is the solution to

\[
\int_{\hat{x}}^{\varepsilon} (\varepsilon - \hat{x}) dF(\varepsilon) = s
\]

Otherwise, the consumer will buy product 1 right away.

For consumers who start searching at firm 2, the story is similar.
The probability that a consumer that starts searching at firm 1 ($\Delta \eta \leq 0$) buys from firm 1 is given by:

- the probability she stops right away, denoted $q^1_1$, which is given by
  
  $$q^1_1(\Delta \eta) = Pr[\varepsilon_i > \hat{x} - \Delta p + \Delta \eta] = 1 - F(\hat{x} - \Delta p + \Delta \eta)$$

- the probability she walks away from 1, visits 2 and returns to 1 because 1 offers her a better deal after all, denoted $q^1_{12}$, which is given by
  
  $$q^1_{12}(\Delta \eta) = Pr[\varepsilon_2 - \Delta p + \Delta \eta < \varepsilon_1 < \hat{x} + \Delta \eta - \Delta p]$$
  
  $$= \int_{0}^{\hat{x} - \Delta p + \Delta \eta} F(\varepsilon + \Delta p - \Delta \eta) \, dF(\varepsilon)$$

Since $\Delta \eta$ varies across the consumers who start searching at firm 1 (with $\Delta \eta \leq 0$), we have a total demand from these consumers equal to:

$$D^1_1 = \int_{-\infty}^{0} [q^1_1(\Delta \eta) + q^1_{12}(\Delta \eta)] \gamma(\Delta \eta)) \, d\Delta \eta$$
Firm 1 also sells to consumers who start searching at firm 2 \((\Delta_\eta > 0)\), walk away from it, inspect product 1 and decide to buy it. This occurs with a probability we denote by \(q_{21}^1\), which is given by:

\[
q_{21}^1 = Pr[\Delta_\eta > 0, \varepsilon_2 < \hat{x} - \Delta_\eta, \varepsilon_1 > \varepsilon_2 - \Delta_p + \Delta_\eta] = \\
F(\hat{x} - \Delta_\eta)[1 - F(\hat{x} - \Delta_p)] + \int_{\max\{0,\Delta_\eta - \Delta_p\}}^{\hat{x} - \Delta_p} F(\varepsilon + \Delta_p - \Delta_\eta) f(\varepsilon) d\varepsilon
\]

Since \(\Delta_\eta\) varies across the consumers who start searching at firm 2 (with \(\Delta_\eta > 0\)), we have a total demand from these consumers equal to:

\[
D_{21}^1 = \int_0^{\Delta_p} \left( F(c - \Delta_\eta)(1 - F(\hat{x} - \Delta_p)) + \int_0^{\hat{x} - \Delta_p} F(\varepsilon - \Delta_p - \Delta_\eta) dF(\varepsilon) \right) d\Gamma(\Delta_\eta) + \\
\int_0^{\infty} \left( F(\hat{x} - \Delta_\eta)(1 - F(\hat{x} - \Delta_p)) + \int_{\Delta_\eta - \Delta_p}^{\hat{x} - \Delta_p} F(\varepsilon - \Delta_p - \Delta_\eta) dF(\varepsilon) \right) d\Gamma(\Delta_\eta)
\]
The payoff of firm 1 is then given by

$$\pi_1 = p[D_1^1 + D_2^1]$$

Taking the first order condition (FOC) and applying symmetry we get:

$$\frac{1}{2} - p^* \int_{-\infty}^{0} \left[ f(\hat{x} + \Delta_\eta)(1 - F(\hat{x})) + 2 \int_{0}^{\hat{x} + \Delta_\eta} f(\epsilon - \Delta_\eta) dF(\epsilon) \right] d\Gamma(\Delta_\eta) = 0$$
Proposition 1: If an equilibrium exists where firms do not advertise their prices, then the price is given by:

\[ p^* = \frac{1/2}{\int_{-\infty}^{0} \left[ f(\hat{x} + \Delta \eta)(1 - F(\hat{x})) + 2 \int_{0}^{\hat{x} + \Delta \eta} f(\varepsilon - \Delta \eta) dF(\varepsilon) \right] d\Gamma(\Delta \eta)} \]

and firms obtain profits equal to

\[ \pi^* = \frac{1}{2} p^* \]

Moreover, if \( 1 - F \) is log-concave, then prices are an increasing function of search costs.\\[^{\S}]\\

[Note that when \( \overline{\eta} \to 0 \), this price gives the price in Anderson and Renault (1999).]

[^{\S}]: Proofs are at the end of the presentation slides.
Example:

The uniform distribution: For the case in which $\varepsilon$ and $\eta$ are uniformly distributed, the equilibrium price is

$$p^* = \frac{1}{2} \left[ \int_{-\infty}^{0} \left( \frac{\varepsilon - \hat{x} + 2(x + z)}{\varepsilon^2} \right) \frac{\eta + z}{\eta^2} dz \right]^{-1} = \frac{3\varepsilon^2}{3(\varepsilon + \hat{x}) - 2\eta}$$

Remark: The equilibrium price decreases in $\hat{x}$ (so increases in search cost), and increases in the product differentiation parameters $\bar{\eta}$ and $\varepsilon$. When $\bar{\eta} \to 0$, price is $\varepsilon^2/(\varepsilon + \hat{x})$, which is the AR price.
3. Price advertising
Incentives to reveal the price

In order to check the robustness of the equilibrium in Proposition 1, we allow firm 1 to deviate by charging a price \( p \neq p^* \) and making it public. Let \( \Delta_p = p^* - p \).

The probability a consumer who starts searching at firm 1 buys from firm 1 is given by:

- the probability she stops right away, denoted \( q^1_1 \), which is given by
  \[
  q^1_1 = Pr[\varepsilon_1 > \hat{x} - \Delta_p + \Delta_\eta] = 1 - F(\hat{x} - \Delta_p + \Delta_\eta)
  \]
- and the probability she walks away from 1, visits 2 and returns to 1 because 1 offers her a better deal after all, denoted \( q^1_{12} \), which is given by
  \[
  q^1_{12} = Pr[\varepsilon_2 - \Delta_p + \Delta_\eta < \varepsilon_1 < \hat{x} + \Delta_\eta - \Delta_p]
  = \int_{\hat{x} - \Delta_p + \Delta_\eta}^{\hat{x}} F(\varepsilon + \Delta_p - \Delta_\eta) dF(\varepsilon)
  \]
Since $\Delta \eta$ varies across the consumers who start searching at firm 1 (with $\Delta \eta \leq \Delta p$), we have a total demand from these consumers equal to:

$$D_1^1 = \int_{-\infty}^{\Delta p} [q_1^1(\Delta \eta) + q_{21}^1(\Delta \eta)] \gamma(\Delta \eta) d\Delta \eta$$

Firm 1 also sells to consumers who start searching at firm 2 ($\Delta \eta > \Delta p$), walk away from it, inspect product 1 and decide to buy it. This occurs with a probability we denote by $q_{21}^1$, which is given by:

$$q_{21}^1 = Pr[\varepsilon_2 + \Delta \eta - \Delta p < \hat{x} \& \varepsilon_1 + \Delta p - \Delta \eta < \varepsilon_2]$$

$$= F(\hat{x} - \Delta \eta + \Delta p)(1 - F(\hat{x})) + \int_{\Delta \eta - \Delta p}^{\hat{x}} F(\varepsilon + \Delta p - \Delta \eta) f(\varepsilon) d\varepsilon$$

Since $\Delta \eta$ varies across the consumers who start searching at firm 2 (with $\Delta \eta > \Delta p$), we have a total demand from these consumers equal to:

$$D_2^1 = \int_{\Delta p}^{\infty} q_{21}^1 d\Gamma(\Delta \eta)$$
The payoff of the deviating firm is then

\[ \pi_1 = p \left( D_1^1 + D_2^1 \right) - k \]

Taking the FOC gives:

\[
\int_{-\infty}^{\Delta_p} \left[ 1 - F (\hat{x} + \Delta_\eta - \Delta_p) + \int_0^{\hat{x}+\Delta_\eta-\Delta_p} F (\varepsilon - \Delta_\eta + \Delta_p) \, dF (\varepsilon) - p^* f (\hat{x} + \Delta_\eta - \Delta_p) (1 - F (\hat{x})) - p^* \int_0^{\hat{x}+\Delta_\eta-\Delta_p} f (\varepsilon - \Delta_\eta - \Delta_p) \, dF (\varepsilon) \right] \, d\Gamma (\Delta_\eta) + \\
\int_{\Delta_p}^{\infty} \left[ F (\hat{x} - \Delta_\eta + \Delta_p) (1 - F (\hat{x})) + \int_{\Delta_\eta-\Delta_p}^{\hat{x}} F (\varepsilon - \Delta_\eta + \Delta_p) \, dF (\varepsilon) - p^* f (\hat{x} - \Delta_\eta + \Delta_p) (1 - F (\hat{x})) - p^* \int_{\Delta_\eta-\Delta_p}^{\hat{x}} f (\varepsilon - \Delta_\eta + \Delta_p) \, dF (\varepsilon) \right] \, d\Gamma (\Delta_\eta) - p^* [1 - F (\hat{x})]^2 \gamma (\Delta_p) = 0
\]
If we evaluate this FOC at $p = p^*$ gives:

$$-p^* (1 - F (\hat{x})) \left[ (1 - F (\hat{x})) \gamma (0) + \int_0^\infty f (\hat{x} - \Delta \eta) d\Gamma (\Delta \eta) \right] < 0$$

This implies that the payoff obtained from advertising a price is decreasing at $p^*$. As a result, advertising a lower price is a profitable deviation.

We conclude that:

**Proposition 2:** *If advertising costs are negligible, a no-advertising equilibrium does not exist.*

The relevance of this Proposition is that the framework proposed by Wolinsky (1986) can only be rationalised when advertising frictions exist (high advertising costs).

¶

¶More on this later (page XX).
Price-advertising equilibrium

Suppose firms advertise their prices in equilibrium. Again let $p^*$ be the equilibrium price.

In order to derive a tentative equilibrium, we compute the payoff of a firm that deviates by advertising a different price, say $p \neq p^*$.

Note that now both match values $\varepsilon$’s and prices are observed from home. Therefore

- consumers for whom $\Delta_\eta - \Delta_p \geq 0$ will start searching at firm 2,
- while consumers for whom $\Delta_\eta - \Delta_p < 0$ will start searching at firm 1.
Take a consumer who visits firm 1 in her first search, observes the match value $\varepsilon_1$ and contemplates going to firm 2. Let $\varepsilon_1 + \eta_1 - \rho$ be the observed utility at firm 1.

The consumer search rule is the same as the one discussed above, that is, the consumers will walk away from firm 1 whenever

$$\varepsilon_1 + \Delta \rho - \Delta \eta < \hat{x}$$

and will buy product 1 directly otherwise.

Given this we can compute the deviant’s demand as follows.
The probability a consumer who starts searching at firm 1 buys from firm 1 is given by:

- the probability she stops right away, denoted \(q_1^1\), which is given by

\[
q_1^1 = Pr[\varepsilon_1 > \hat{x} - \Delta_p + \Delta_\eta] = 1 - F(\hat{x} - \Delta_p + \Delta_\eta)
\]

- the probability she walks away from 1, visits 2 and returns to 1 because 1 offers her a better deal after all, denoted \(q_{12}^1\), which is given by

\[
q_{12}^1 = Pr[\varepsilon_2 - \Delta_p + \Delta_\eta < \varepsilon_1 < \hat{x} + \Delta_\eta - \Delta_p]
= \int_{0}^{\hat{x} - \Delta_p + \Delta_\eta} F(\varepsilon + \Delta_p - \Delta_\eta) dF(\varepsilon)
\]

Since \(\Delta_\eta\) varies across the consumers who start searching at firm 1 (with \(\Delta_\eta \leq \Delta_p\)), we have a total demand from these consumers equal to:

\[
D_1^1 = \int_{-\infty}^{\Delta_p} [q_1^1(\Delta_\eta) + q_{12}^1(\Delta_\eta)] \gamma(\Delta_\eta) d\Delta_\eta
\]
Firm 1 also sells to consumers who start searching at firm 2 ($\Delta_\eta > \Delta_p$), walk away from it, inspect product 1 and decide to buy it. This occurs with a probability we denote by $q_{21}^1$, which is given by:

$$q_{21}^1 = Pr[\varepsilon_2 + \Delta_\eta - \Delta_p < \hat{x} \& \varepsilon_1 + \Delta_p - \Delta_\eta < \varepsilon_2]$$

$$= F(\hat{x} - \Delta_\eta + \Delta_p)(1 - F(\hat{x})) + \int_{\Delta_\eta - \Delta_p}^{\hat{x}} F(\varepsilon + \Delta_p - \Delta_\eta)f(\varepsilon)d\varepsilon$$

Since $\Delta_\eta$ varies across the consumers who start searching at firm 2 (with $\Delta_\eta > \Delta_p$), we have a total demand from these consumers equal to:

$$D_{21}^1 = \int_{\Delta_p}^{\infty} q_{21}^1 d\Gamma(\Delta_\eta)$$

The payoff of the firm is then

$$\pi_1 = p(D_{11}^1 + D_{21}^1) - k$$
Proposition 3: If a price-advertising equilibrium exists, then prices are given by

\[ p^* = \frac{1}{2} \sqrt{2 \int_{-\infty}^{0} \left[ f(\hat{x} + \Delta \eta)(1 - F(\hat{x})) + \int_{\hat{x} + \Delta \eta}^{\hat{x} + \Delta \eta} f(\varepsilon - \Delta \eta) dF(\varepsilon) \right] d\Gamma(\Delta \eta) + (1 - F(\hat{x}))^2 \gamma(0)} \]

and the equilibrium profits are then:

\[ \pi^* = \frac{1}{2} p^* - k. \]

Moreover, the equilibrium price (and so profits) is a decreasing function of search costs.

When prices can be advertised, they will be advertised, in which case search costs are pro-competitive!

• (different from the standard search cost literature.)
Example:

The uniform distribution: For the case in which $\varepsilon$ and $\eta$ are uniformly distributed, the equilibrium price is

$$p^* = \frac{1}{2} \left[ 2 \int_{-\infty}^{0} \left( \frac{\bar{\varepsilon} + z}{\varepsilon^2} \right) \frac{\bar{\eta} + z}{\eta^2} dz + \left( 1 - \frac{\hat{x}}{\bar{\varepsilon}} \right)^2 \frac{1}{\bar{\eta}} \right]^{-1}$$

$$= \frac{3\bar{\eta}\varepsilon^2}{6(\bar{\varepsilon} - \hat{x})^2 + 6\bar{\eta}\varepsilon - 2\eta^2},$$

Remark: This price increases in $\hat{x}$ (so decreases as search costs go up) and increases in the product differentiation parameters $\bar{\eta}$ and $\bar{\varepsilon}$. 
Deviations by concealing the price

In order to check the robustness of the equilibrium in Proposition 3, we allow a firm, say 1, to deviate by concealing its price, and possibly changing it.

Let $p \neq p^*$ be the concealed price and again let $\Delta p = p^* - p$.

To study this deviation we need to address the following question:

• What is reasonable for consumers to believe after noticing that a firm has deviated and concealed its price?

It is clear that if consumers hold “passive” beliefs –that is, if they expect the deviant to continue to charge the equilibrium price– the deviation will be profitable

• with such beliefs the deviant firm has an incentive to “hold-up” consumers: given that they visit as usual, because search costs are positive, the firm has an incentive to raise the price –similar to the reasoning behind the Diamond (1971) paradox.
But are “passive beliefs” reasonable? In a sense, they are not consistent with the information consumers have: they know that a deviation has occurred since ads have not been sent.

They also “know” that if they had passive beliefs they would be held-up. Therefore they should expect a higher price.

- We will make the assumption that consumers will expect $p^e \neq p \neq p^*$. 
- Let $\Delta^e_p = p^* - p^e$ be the difference in prices expected by the consumers off-the-equilibrium.
As before, part of consumers start searching at firm 1 and part searches starting at firm 2.

- Consumers, for whom $\Delta \eta < \Delta ^e_p$ start searching at firm 1.
- Otherwise, they start searching at firm 2.

The probability a consumer who starts searching at firm 1 buys from firm 1 is given by:

- the probability she stops right away, denoted $q^1_1$, which is given by

$$q^1_1 = Pr[\varepsilon_1 > \hat{\varepsilon} - \Delta p + \Delta \eta] = 1 - F(\hat{\varepsilon} - \Delta p + \Delta \eta)$$

- and the probability she walks away from 1, visits 2 and returns to 1 because 1 offers her a better deal after all, denoted $q^1_{12}$, which is given by

$$q^1_{12} = Pr[\varepsilon_2 - \Delta p + \Delta \eta < \varepsilon_1 < \hat{\varepsilon} + \Delta \eta - \Delta p] = \int_{\hat{\varepsilon} - \Delta p + \Delta \eta}^{\infty} F(\varepsilon + \Delta p - \Delta \eta) \ dF(\varepsilon)$$
We need to integrate $\Delta_{\eta}$ out, which gives

$$D_1^1 = \int_{-\infty}^{\Delta_p^e} [q_1^1(\Delta_{\eta}) + q_{21}^1(\Delta_{\eta})] \gamma(\Delta_{\eta}))d\Delta_{\eta}$$

The deviant firm also obtains demand from the consumers who start searching at firm 2 ($\Delta_{\eta} > \Delta_p^e$), walk away from it, inspect product 1 and decide to buy it. This occurs with a probability we denote by $q_{21}^1$, which is given by:

$$q_{21}^1 = Pr[\varepsilon_2 + \Delta_{\eta} - \Delta_p^e < \hat{x} \& \varepsilon_1 + \Delta_p - \Delta_{\eta} > \varepsilon_2]$$

$$= F(\hat{x} - \Delta_{\eta} + \Delta_p^e)(1 - F(\hat{x} + \Delta_p^e - \Delta_p))$$

$$+ \int_{\Delta_{\eta} - \Delta_p}^{\hat{x} + \Delta_p^e - \Delta_p} F(\varepsilon + \Delta_p - \Delta_{\eta})f(\varepsilon)d\varepsilon$$

Since $\Delta_{\eta}$ varies across the consumers who start searching at firm 2 (with $\Delta_{\eta} > \Delta_p^e$) and $\Delta_p^e > \Delta_p$, we have a total demand from these consumers equal to:

$$D_2^1 = \int_{\Delta_p^e}^{\infty} q_{21}^1 d\Gamma(\Delta_{\eta})$$
Then the total payoff of the deviant firm 1 is

$$\pi_1 = p(D_1^1 + D_2^1)$$

Taking the FOC w.r.t. $p$ and setting $\Delta^e_p = \Delta_p$ gives:

$$\int_{-\infty}^{\Delta_p} \left[ 1 - F(\hat{x} + \Delta_\eta - \Delta_p) + \int_{0}^{\hat{x} + \Delta_\eta - \Delta_p} F(\varepsilon - \Delta_\eta + \Delta_p) dF(\varepsilon) - pf(\hat{x} + \Delta_\eta - \Delta_p)(1 - F(\hat{x})) - p\int_{0}^{\hat{x} + \Delta_\eta - \Delta_p} f(\varepsilon - \Delta_\eta + \Delta_p) dF(\varepsilon) \right] d\Gamma(\Delta_\eta) +$$

$$\int_{\Delta_p}^{\infty} \left[ F(\hat{x} - \Delta_\eta + \Delta_p)(1 - F(\hat{x})) + \int_{\Delta_\eta - \Delta_p}^{\hat{x}} F(\varepsilon - \Delta_\eta + \Delta_p) dF(\varepsilon) - p\int_{\Delta_\eta - \Delta_p}^{\hat{x}} f(\varepsilon - \Delta_\eta + \Delta_p) dF(\varepsilon) \right] d\Gamma(\Delta_\eta) = 0$$

We are still working on a proof that this deviation is not profitable. For the moment, let us look at the case in which the match values are uniformly distributed.
With uniformly distributed math values, the equilibrium payoff and the deviation payoff are shown in the following graph:

As can be seen, the deviant does not gain by concealing its price.
Therefore:

**Proposition 4:** If advertising costs are negligible, a price-advertising equilibrium exists.
When advertising costs matter

Overview of the equilibria for positive ad costs

Adv. costs

0.05

0.04

0.03

0.02

0.01

No price advertising

Price advertising

Both cases possible

s
4. Match value advertising
Suppose the match value $\varepsilon$ can be advertised, that is, the necessary information for consumers to learn their $\varepsilon$’s can be digitised and shown on-line, or printed on ads.

Consider the benchmark equilibrium of Proposition 1 where (deviation) prices are only discovered upon search. We ask:

- Does a firm have an incentive to deviate and make its match value information public?

**Proposition 5:** An equilibrium where both firms advertise their match values does not exist. ■

Argument of proof: Following Diamond (1971), if both firms advertise their match values, firms have an incentive to raise prices (till infinity).
Corollary of Proposition 5: No firm has an incentive to engage in comparative advertising for otherwise the market would collapse.

Proposition 6 (still a conjecture!): An equilibrium where one firm imparts match value information while the other firm abstains from doing it does not exist.

Argument of proof: Suppose firm 1 imparts its match-value information so all consumers know the eq. utility they get at firm 1.

A consumer knows that if she happened to be at firm 1 it would be worthwhile to inspect product 2 whenever \( \varepsilon_1 < \hat{x} - \Delta_p + \Delta_\eta \); otherwise not.
Anticipating this, consumers for whom

- \( \varepsilon_1 > \hat{x} - \Delta p + \Delta \eta \) will go to firm 1 and buy there.
- The rest will go first to firm 2, inspect product 2 and decide whether to buy product 1 or 2.

And here the Diamond’s argument enters again: firms 2 can hold-up the consumers who pay it a visit. After visiting, those consumers know their match values at the two firms. Firm 2 can raise its price to \( p_2 + c \) without demand consequences.

Anticipating this, no consumer will visit firm 2. Given this, firm 1 can also hold-up consumers and the market collapses.
Match-value and price advertising

Suppose the match value $\varepsilon$ can be advertised in the price-advertising equilibrium of Proposition 3. We ask:

- Does a firm have an incentive to deviate and make its match value information public?

Situation is similar to the analysis before. Main difference is that firms commit to prices, so consumers cannot be held-up after paying a visit.

**Proposition 7 (still a conjecture!):** If match values and prices can be advertised, firms will advertise both. ■

While price-advertising has the features of a prisoner's dilemma, match-values advertising does not. Collectively the firms would not be better off if they agreed not to disclose their match value information.
Corollary of Proposition 7: With positive advertising costs, asymmetric equilibria may exist where one firm uses comparative advertising along with price advertising and the other just price-advertising. ■
Some implications for platform design

The ranking of prices (and so profits) is shown in this graph.

A platform (that obtains the bulk of its profits from firms) would prefer to obfuscate price and match value information.

- However, if consumers search on price then it is better for the platform to disclose all possible information.
Concluding remarks

The paper presents a new directed-search model that enables us to study price and product advertising into the work-horse framework of consumer search model with differentiated products.

We first consider situations where match values cannot be advertised. In this case, the unique equilibrium (with zero-adv.-costs) involves firms disclosing their prices

- in such a case, prices and profits decrease as search costs increase
- the reason is that persuading (via low prices) consumers to visit is the more attractive the higher the search costs
- so firms end up killing the “chicken of the golden eggs”.

When advertising is relatively costly, a no-price-advertising equilibrium can be sustained. The incentives to deviate are increasing in search costs so higher adv. costs are needed to sustain it when search costs go up.
If a firm can advertise its match value information, whether it will do it depends on whether prices can be advertised or not.

If firms cannot advertise prices, then

- an equilibrium where both firms impart match-value information does not exist.
- We conjecture that an eq. where one firm discloses product match information does not exist either

If match-value advertising is used along with price advertising, then

- an individual firm has an incentive to communicate its match-value information
- incentives also exist for comparative advertising so if advertising costs are non-trivial we may expect asymmetric equilibria where one firm reveals all match values and the other free-rides
• While price-advertising has the features of a prisoners’ dilemma, match-value advertising has not.

• match value advertising is a public good for the firms: it weakens competition because information on yet another source of product differentiation is made readily available for consumers to compare products

• though prices increase, match value advertising need not be bad for consumers because they are directed to the best deals and (costly) traffic from shop to shop is eliminated
Thanks much for your attention!
Proof of Proposition 1

The equilibrium price (and so firm’s profits) decreases with search cost \( s \) if the expression in the denominator increases in \( \hat{x} \). Taking the derivative gives

\[
\frac{\partial}{\partial \hat{x}} \left( f \left( \hat{x} + \Delta \eta \right) (1 - F (\hat{x})) + 2 \int_0^{\hat{x} + \Delta \eta} f (\varepsilon - \Delta \eta) \, dF (\varepsilon) \right) =
\]

\[
f' \left( \hat{x} + \Delta \eta \right) (1 - F (\hat{x})) + f (\hat{x} + \Delta \eta) f (\hat{x}) =
\]

\[
[1 - F (\hat{x})] f (\hat{x} + \Delta \eta) \left[ \frac{f' (\hat{x} + \Delta \eta)}{f (\hat{x} + \Delta \eta)} + \frac{f (\hat{x})}{1 - F (\hat{x})} \right] >
\]

\[
[1 - F (\hat{x})] f (\hat{x} + \Delta \eta) \left[ \frac{f' (\hat{x} + \Delta \eta)}{f (\hat{x} + \Delta \eta)} + \frac{f (\hat{x} + \Delta \eta)}{1 - F (\hat{x} + \Delta \eta)} \right] > 0.
\]

Both inequalities follow from the log-concavity of \( 1 - F \):

- the first inequality follows from the fact that \( \Delta \eta < 0 \) and the hazard rate \( f/(1 - F) \) is increasing
- the second follows from the fact that \( f'(1 - F) + f^2 > 0 \) under log-concavity
Observation of the price formula immediately leads to the conclusion that an increase in $\bar{\eta}$ leads to a higher price.

Taking the derivative with respect to $\bar{\varepsilon}$ gives

$$\frac{dp^*}{d\bar{\varepsilon}} = \frac{3\bar{\varepsilon}(-4\bar{\eta} + 6\hat{x} + 3\bar{\varepsilon})}{(3(\hat{x} + \bar{\varepsilon}) - 2\bar{\eta})^2}$$

The sign of this expression depends on the sign of $6\hat{x} + 3\bar{\varepsilon} - 4\bar{\eta}$. Since our assumption (in page XX) requires that search costs are small, $\bar{\eta} < \hat{x}$ and therefore $6\hat{x} + 3\bar{\varepsilon} - 4\bar{\eta} > 0$. ■
Proof of Proposition 3

We need to prove that the denominator of the expression of $p^*$ in page 32 decreases in $\hat{x}$. If we take the derivative we get

$$2 [1 - F(\hat{x})] \left[ \int_{-\infty}^{0} [f'(\hat{x} + \Delta\eta)] d\Gamma(\Delta\eta) - f(\hat{x}) \gamma(0) \right]$$

Integrating by parts gives

$$2 [1 - F(\hat{x})] \left[ - \int_{-\infty}^{0} [f(\hat{x} + \Delta\eta)] d\gamma(\Delta\eta) - f(\hat{x} - \overline{\eta}) \gamma(-\overline{\eta}) \right] =$$

$$-2 [1 - F(\hat{x})] \left[ \int_{-\infty}^{0} [f(\hat{x} + \Delta\eta)] d\gamma(\Delta\eta) \right]$$

where the last equation follows form $\gamma(-\overline{\eta}) = 0$. 
The expression inside the squared brackets is positive. To see this, we take the derivative with respect to $\bar{\eta}$:

$$f (\hat{x} - \bar{\eta}) \gamma' (-\bar{\eta}) = f (\hat{x} - \bar{\eta}) g(\bar{\eta}) g(0) > 0$$

Since it increases in $\bar{\eta}$, we can write

$$\int_{-\bar{\eta}}^{0} [f (\hat{x} + \Delta_{\eta})] d\gamma (\Delta_{\eta}) > \lim_{\bar{\eta} \to 0} \left( \int_{-\bar{\eta}}^{0} [f (\hat{x} + \Delta_{\eta})] d\gamma (\Delta_{\eta}) \right) = 0$$

Therefore,

$$\int_{\eta - \bar{\eta}}^{0} [f (\hat{x} + \Delta_{\eta})] d\gamma (\Delta_{\eta}) > 0$$

and

$$-2 [1 - F(\hat{x})] \left[ \int_{\eta - \bar{\eta}}^{0} [f (\hat{x} + \Delta_{\eta})] d\gamma (\Delta_{\eta}) \right] < 0$$

Hence, $p^{*}_{A}$ increases in $\hat{x}$. ■
Proof of Proposition 5

By contradiction. Suppose firms charge a price $p^*$. Consumers who pay a visit to firm 1 in equilibrium have $\eta_1 + \varepsilon_1 \geq \eta_2 + \varepsilon_2$.

Given this, if firm 1 deviates to a price $p^* + c$, no consumer will walk away to exercise the option to buy product 2. As a result, the deviation is profitable and no consumer should expect a price less than infinity, which makes the market to collapse. □
Therefore, the demand of firm 1 is equal to the
• demand from consumers who decide not to check the product
  of firm 2, which is given by:
  \[ q_1^1 = Pr[\varepsilon_1 > \hat{x} - \Delta_p + \Delta_\eta] = 1 - F(\hat{x} - \Delta_p + \Delta_\eta) \]
• plus the demand from consumers who check it and return to
  1, which is given by
  \[ q_{12}^1 = Pr[\varepsilon_2 - \Delta_p + \Delta_\eta < \varepsilon_1 < \hat{x} + \Delta_\eta - \Delta_p] = \int_{\max\{\Delta_\eta - \Delta_p, 0\}}^{\min\{\varepsilon, \hat{x} - \Delta_p + \Delta_\eta\}} F(\varepsilon + \Delta_p - \Delta_\eta) \, dF(\varepsilon) \]

Integrating across all \( \Delta_\eta \) we have a total demand for the deviant equal to
\[ D_1^1 = \int_{-\infty}^{\bar{\varepsilon} - \hat{x} + \Delta_p} [q_1^1(\Delta_\eta) + q_{12}^1(\Delta_\eta)] \gamma(\Delta_\eta) \, d\Delta_\eta + \int_{\bar{\varepsilon} - \hat{x} + \Delta_p}^{\infty} q_{12}^1(\Delta_\eta) \gamma(\Delta_\eta) \, d\Delta_\eta \]
We now argue that the sales of firm 1 go up after advertising the match values, even keeping price constant.

The intuition is as follows:

- demand for firm 1 from the consumers who visited firm 1 first remains the same (this is clear because they used to visit firm 1 first and after the deviation it is as if they visit firm 1 first too
- Therefore, all the action comes from consumers with $\Delta_\eta > \Delta p$, which used to visit firm 2 first.
  - From these consumers, those for whom $\Delta_\eta > \hat{x} + \Delta p$ just stayed at firm 2; this continues to be true because $\varepsilon_1 - \Delta_\eta + \Delta p < -\hat{x}$.
  - So we need to look at the demand from consumers for whom $\Delta_\eta < \hat{x} + \Delta p$
Therefore, we need to compare the following:

- **Sales before advertising the match value for consumers with** $\Delta_\eta < \hat{x} + \Delta_p$:

$$\int_{\Delta_p}^{\hat{x} + \Delta_p} \left[ F(\hat{x} + \Delta_p - \Delta_\eta)(1 - F(\hat{x})) + \int_{\Delta_\eta - \Delta_p}^{\hat{x}} F(\varepsilon + \Delta_p - \Delta_\eta)dF(\varepsilon) \right] d\Gamma(\Delta_\eta)$$

(1)

- **Sales after advertising the match value for consumers with** $\Delta_\eta < \hat{x} + \Delta_p$:

$$\int_{\Delta_p}^{\hat{x} + \Delta_p} \left[ 1 - F(\hat{x} - \Delta_p + \Delta_\eta) + \int_{\Delta_\eta - \Delta_p}^{\min\{\varepsilon, \hat{x} - \Delta_p + \Delta_\eta\}} F(\varepsilon + \Delta_p - \Delta_\eta)dF(\varepsilon) \right] d\Gamma(\Delta_\eta)$$

(2)

Suppose the deviant just advertises the match value and does not change price. Then $\Delta_p = 0$.

In order to compare (1) and (2), take first the set $0 < \Delta_\eta < \bar{\varepsilon} - \hat{x}$. In this case we note that

$$1 - F(\hat{x} + \Delta_\eta) > (1 - F(\hat{x}))F(\hat{x} - \Delta_\eta)$$

and

$$\int_{\Delta_\eta}^{\hat{x} + \Delta_\eta} F(\varepsilon + \Delta_p - \Delta_\eta)dF(\varepsilon) > \int_{\Delta_\eta}^{\hat{x}} F(\varepsilon + \Delta_p - \Delta_\eta)dF(\varepsilon)$$
For the other $\Delta_\eta$’s, that is, for $\bar{\varepsilon} - \hat{x} < \Delta_\eta < \hat{x}$ we argue as follows. Take the derivative of (1) with respect to $\hat{x}$:

$$f (\hat{x} - \Delta_\eta) (1 - F (\hat{x})) > 0$$

Since (1) increases, we can write

$$F (\hat{x} - \Delta_\eta) (1 - F (\hat{x})) + \int_{\Delta_\eta}^{\hat{x}} F (\varepsilon - \Delta_\eta) dF(\varepsilon) <$$

$$F (\bar{\varepsilon} - \Delta_\eta) (1 - F (\bar{\varepsilon})) + \int_{\Delta_\eta}^{\bar{\varepsilon}} F (\varepsilon - \Delta_\eta) dF(\varepsilon) =$$

$$\int_{\Delta_\eta}^{\bar{\varepsilon}} F (\varepsilon - \Delta_\eta) dF(\varepsilon),$$

which is equal to (2). This concludes the argument.