Hybrid R&D*

Sanjeev Goyal†
Alexander Konovalov‡
José Luis Moraga-González§

This version: May 2007

Abstract

We develop a model of R&D collaboration in which individual firms carry out in-house research on core activities and undertake bilateral joint projects on non-core activities with other firms. We develop conditions on the profit functions of the firm under which R&D investments in different projects of a firm are complementary. We show that this condition is met by standard price and quantity setting oligopoly models. We then examine the relation between number of joint projects and investments and profits. In this context, we identify a second aspect of complementarity: equilibrium investments in the in-house project as well as in each joint project are increasing in the number of projects. However, we find that an increase in number of joint projects for all firms lowers collective profits, suggesting the presence of excessive incentives for conducting research.

JEL classification: L13, L14, L22, O31, O32

*We thank the editor, Xavier Vives, and two anonymous referees for very useful comments. We also thank Rabah Amir, René Belderbos, Francis Bloch, Bruno Cassiman, Sergei Guriev, Rachel Kranton, Pierre Regibeau, Oz Shy, Ernst Verwaal, Reinhilde Veugelers and Helen Weeds, for their suggestions. This paper has also benefited from the comments of seminar participants at Amsterdam, CEFIR (Moscow), Cergy-Pontoise, Erasmus, Essex, Groningen, Lausanne, Leuven, Utrecht, WZB Berlin, the VI Summer School on Economics and Philosophy in San Sebastián, ESEM 2003 and EARIE 2003. Financial support from the Netherlands Organization for Scientific Research (NWO) is gratefully acknowledged.

†University of Essex. E-mail: sgoyal@essex.ac.uk
‡University of Innsbruck. E-mail: Alexander.Konovalov@uibk.ac.at
§University of Groningen. E-mail: j.l.moraga.gonzalez@rug.nl

1 Introduction

In the last three decades there has been a significant increase in the number of R&D collaboration agreements between firms. This number rose sharply in the 1990's and has remained high in recent years.[1] As a result of this trend, a typical firm nowadays carries out independent in-house R&D and in addition is engaged in several joint research projects with distinct partners. For instance, in the pharmaceutical sector, leading firms such as Bayer, Bristol-Myers-Squibb, and Glaxo-Smith-Kline now explicitly state that their research strategy relies upon a combination of in-house research and collaborations with other firms to deliver innovation.[2] We refer to this organization of research as *hybrid R&D*.[3]

This hybrid form of organizing research, which combines in-house R&D and collaborative R&D with distinct partners, raises the following questions:

1. What are the relative merits of in-house and joint research projects and how do firms allocate resources between different projects?
2. What are the circumstances under which such hybrid forms of organization are optimal for firms?

Existing work on research collaboration among firms proceeds by assuming that firms either work all together or carry out independent research.[4] This formulation rules out the possibility of hybrid forms of research organization which have assumed center-stage in the last two decades. The principal contribution of our paper is a model of R&D which accommodates such forms of

[1] See Hagedoorn (2002) for an analysis of general trends and patterns in inter-firm R&D partnering since the early 1960s.

[2] For example, GSK has on-going collaborations with Bristol-Myers, Merck and Hoffmann-La Roche, while Bayer currently collaborates with Millenium and CuraGen and ComGenex. In the aeronautic defense sector Boeing has been engaged in joint research with Alenia Spazio and EADS in the area of missile defense, while EADS (Eurocopter) collaborates with Onera on developing the technology for high-speed helicopter flight. Finally, research collaborations are very popular among firms in the IT sector. For instance, IBM and Intel work on blade servers, IBM and Apple work together on a new 64-bit Power PC processor, CSC conducts joint research with Oracle, IBM and SAS.

[3] The hybrid nature of R&D is also reflected in patent grants. For example, Daimler-Chrysler AG has independently obtained a series of inventions in the area of gear change transmissions for motor vehicles and, at the same time, has been engaged in collaborative research with BMW Group and Volkswagen AG on processes and systems aiming at reducing emissions of internal combustion engines. Similarly, Toyota Motor Co. has patented a series of inventions developed in-house in the area of carburetors while it has been collaborating with Daihatsu Motor Co. Ltd. in intake manifolds for multi-cylinder internal combustion engines, and with Aisin-Warner in the area of four-wheel drive transmissions.

[4] There is a long and distinguished history of work in this area; influential contributions include Amir, Evstigneev and Wooders (2003), d'Aspremont and Jacquemin (1988), Kamien, Muller and Zang (1992), Kamien and Zang (2000), Katz (1986), Leahy and Neary (1997), and Suzumura (1992).

organization. Specifically, we introduce a model in which an individual firm chooses levels of investments in its in-house research as well as in each of its different joint research projects. This innovation yields a framework in which the above questions can be addressed.

When firms produce complex products involving a range of technologies and skills that are difficult to master individually, hybrid R&D arises naturally. A firm prefers to undertake research related to its *core activities* independently, that is, projects related the set of technologies, activities or research areas mastered by the firm are more effectively done in-house. By contrast, the potential for efficiency gains in non-core activities is relatively large so firms prefer to work in these areas with collaborators.[5]

Our model of hybrid R&D incorporates these ideas. Each firm has a set of distinctive core research competences which are carried out in-house. In addition, pairs of firms can combine skills and open new avenues of research in non-core areas. A firm's investments in its in-house project and in its joint projects taken along with the investments of its collaborators determine its operating costs.[6] Given these costs, firms compete in the market by choosing prices or quantities.

We first explore the relationship between investments in in-house research and in joint research projects. Our main result derives a necessary and sufficient condition, on the profit function of a firm, for *all the projects in the research portfolio of an individual firm to be complementary.* We emphasize that this complementarity is driven entirely by the market advantages generated by being a lower cost firm and thus obtains in the *absence* of any technological spillovers across projects.[7] The intuition behind this result is as follows: on the one hand, as a firm i invests more in a joint project with firm j, it lowers its own costs and expands its quantity sold. This has a direct effect on the marginal returns from investments in the in-house project. On the other hand, as a firm invests more in a joint project, the costs of the partner firm go down, and this raises the relative output of the collaborating firm. This has a negative effect on the marginal returns from lowering its own costs. The prospects of the complementarity result thus depend on the relative magnitude

[5]The notion of core activity or core competence was developed by Prahalad and Hamel (1990) in the business literature. 'Miniaturization' at Sony, 'small engine design and manufacture' at Honda, 'total vehicle architecture' at Ford Motor Co., 'four wheel drive' at Subaru and 'measurement technology' at Hewlett Packard are examples of core competences often mentioned in the management literature (Leonard-Barton, 1992; Petts, 1997).

[6]We assume that research lowers marginal costs of production; in this formulation we are following the standard models in the literature see e.g., d'Aspremont and Jacquemin (1988), Kamien, Muller and Zhang (1992), Leahy and Neary (1997) and Suzumura (1992).

[7]For a general examination of incentives for innovative activity under different forms of market competition, see Vives (2005). That paper also makes the point that the incentives for innovation are similar across a range of models with price and quantity competition.

of these two effects. We show that in standard models of oligopoly when firms compete in prices or in quantities the positive effect dominates and investments in in-house and joint research projects are complements. We also note that if firms operate in independent markets, the second negative effect is absent, and so complementarity between in-house and joint research always obtains.

We then turn to an examination of equilibrium investments and profits under different forms of market competition. Here we uncover a second general aspect of the complementarity across projects: *a firm's investment in in-house project as well as in each of the joint projects increases as the number of joint projects every firm is engaged in rises.* This result has the following natural interpretation in the context of the research on core competence: it says that as a firm diversifies its portfolio of research projects, the incentive to invest in core in-house research increases.

This result also yields an insight into the level of efforts undertaken by firms with different number of research collaborations: in a collaboration between a highly linked firm and a poorly linked firm, the former will exert higher research efforts. Related to this is the observation that total research efforts will be higher in a joint project between two highly linked firms as compared to the research efforts in a joint project between a highly and a poorly linked firm.[8]

We then turn to the profit implications of collaborative research. In our setting, efficiency gains from joint projects are significant and so collaboration is individually incentive compatible provided that firms face symmetric research opportunities. The above results on effort levels suggest that denser networks of collaboration lead to more efficient industries and thus to larger aggregate quantity. However, we find that in large industries the *profits of the firms decrease as the number of partners of a typical firm increases.* This points to excessive individual incentives to invest in R&D and also suggests that collaboration may exacerbate this problem. To the best of our knowledge this finding of a tension between individual incentives and collective interests in the context of collaborative R&D is novel.

Our paper contributes to two strands of research. The first strand is the literature in business strategy on core competences and the role of alliances and networks. The second strand of research is the recent work on network effects.

The notion of core competence has been central to theorizing in business strategy and management science since the work of Prahalad and Hamel (1990). The model of hybrid R&D presented in

[8]This result on unequal investment levels arising from differences in the number of joint projects of the partner firms is developed with the help of a 4 firm example in section 5.

this paper offers a natural formulation of core and non-core competences. One of the recurring themes in the strategy literature is whether investments in different projects are substitutes or complements and whether collaboration in non-core areas divert investments from core areas (see e.g., Doz and Hamel (1998)). Our analysis shows that investments in core and non-core areas are complements and that an increase in the joint research opportunities of a firm increases investments in independent as well as in each of the existing joint projects. Existing empirical work suggests that collaborative alliances and in-house projects reinforce each other and are complementary, see e.g., Arora and Gamberdella (1994), Cassiman and Veugelers (2002a, 2002b), Mowery and Rosenberg (1989). While these papers emphasize the role of technological spillovers in explaining the observed complementarity, our paper shows how in-house and joint projects can be complementary purely due to market related considerations and in the *absence* of any technological spillovers. Thus our paper provides an alternative foundation for the observed complementarity.[9]

In recent years, a number of authors have explored the effects of network architecture on individual incentives and aggregate outcomes (see e.g., Ballester, Calvó-Armengol and Zenou (2006), Bramoulle and Kranton (2007), Deroian and Gannon (2006), Galeotti (2004), and Goyal and Moraga-González (2001)). In these models, individual players are embedded in a network and each player chooses a *single action* which affects the payoff of those who are connected to the player in question. The novelty of the present paper is that actions (investments) are *specific* to a partner. This model of partner specific investments constitutes a natural first step in the study of strength of links. It opens up the possibility of studying questions like free riding and exploitation in bilateral relations, which are of general interest in economics. This potential is illustrated by our finding that partners in a joint research project will put unequal efforts into their common project if they have a different number of joint projects, with the more linked partner putting in a higher level of effort. We also find that relationship specific investments have strong implications for economic performance. For example, Goyal and Moraga-González (2001) study firms which choose a single R&D level and share it with all their collaborators. They find that R&D investments are decreasing and profits are non-monotonic in the number of partners while in the present paper we find that R&D investments are increasing and profits are decreasing in the number of collaborators.

The rest of the paper is organized as follows. The general model is presented in section 2. In section

[9]This raises the issue about the relative magnitude of the two sources of complementarity and as an anonymous referee said, this raises the question also of how the two sources of complementarity can be distinguished. These are important empirical questions but we believe that they lie outside the scope of the present paper. We thank an anonymous referee for bringing these issues to our attention.

3 we present a general result on the complementarity of investments in different projects. Section 4 examines equilibrium investments and profits under quantity and price competition, when all firms have the same number of collaboration partners. Section 5 discusses issues relating to asymmetric collaboration structures, endogenous levels of collaboration and the social welfare implications of hybrid R&D. Section 6 concludes.

2 The model

We consider a two-stage game. In the first stage, each firm allocates resources to its private R&D project and to every joint R&D project available to the firm. These decisions determine the *effective* costs of production of every firm. In stage two, firms compete in the market by setting prices or quantities. The details of the game and the notation follow.

Collaboration networks and research strategies: Let $N = \{1, 2, \ldots, n\}$ be the set of ex-ante identical firms. We shall assume that $n \geq 2$. Each firm is endowed with a set of core research capabilities, i.e., distinctive competence and skills within the firm that enable it to undertake in-house R&D. Let us normalize the number of in-house projects per firm to one. In addition, a firm can potentially engage in bilateral collaborative work with other firms when research opportunities between the parties are available. We normalize the number of projects two firms can undertake together to one.[10]

The set of potential joint research projects with other firms is represented by an (undirected) network g with n nodes. Whenever $ij \in g$, this means that there exists a potential *collaborative research project* between firms i and j; this project is activated if firms i and j pool their differentiated expertise in a new research trajectory.[11]

Let $N_i(g) = \{i \in N | ij \in g\}$ be the set of firms with which firm i can potentially initiate a joint project in network g. Part of our work focuses on symmetric networks, i.e., where all firms have the same joint research opportunities, $|N_i(g)| = |N_j(g)| = k$. In this case, we shall refer to k as the *degree of collaborative activity* in the industry. Clearly, k can take on values 0,1,2,...,$n-1$.

[10]In actual practice collaboration projects often involve 3 or more firms and two firms may have more than one joint project going on at the same time. The assumption of bilateral collaboration ties and a maximum of one project per pair of firms is made for analytical simplicity.

[11]In the basic model we are assuming that the research projects available to a firm are exogenously given; however projects are only activated if firms allocate resources to them. For a discussion of endogenous determination of the level of collaboration see section 5.

Given the set of potential collaboration projects g, a firm i chooses the amount of money $x_{ij} \in I\!\!R_+$ to be spent on the joint project with firm j, $j \in N_i(g)$, as well as the level of the R&D expenditure in its own core (in-house) project $x_{ii} \in I\!\!R_+$. Let $x_i = (x_{ii}, (x_{ij})_{j \in N_i(g)})$ be the research strategy of firm i and $x = (x_i)_{i \in N}$ denote a strategy profile for the n firms.

The nature of research and cost-reducing R&D: We assume that the returns to R&D investment in in-house research are given by the function $f : I\!\!R_+ \to I\!\!R_+$, while every collaborative project yields returns given by the function $h : I\!\!R_+^2 \to I\!\!R_+$. We normalize the number of in-house projects to one, and also suppose that any pair of firms can only undertake one joint project. We assume that f and h are strictly increasing, strictly concave, and twice differentiable on $I\!\!R_+$ and $I\!\!R_+^2$, respectively. Denote by $h_d(y)$ the constriction of h to the diagonal: $h_d(y) = h(y, y)$. We impose some boundary conditions on f and h to guarantee the existence of symmetric equilibrium: $f(0) = 0$, $h(0, 0) = 0$, the derivative of f and the partial derivatives of h at zero are sufficiently large and, finally, functions f and h_d are bounded for large investment levels.

Given the research strategy x_i of a firm i, this firm obtains a research output $f(x_{ii})$ from its in-house project, while it obtains a research outcome $h(x_{ij}, x_{ji})$ from the research project with firm j, $j \in N_i(g)$. Given the outcomes of all research projects of a firm i, its unit cost of production is given as follows:

$$c_i(g, x) = \bar{c} - f(x_{ii}) - \sum_{j \in N_i(g)} h(x_{ij}, x_{ji}). \tag{1}$$

This cost-reduction formulation is important in our analysis and we discuss the ideas underlying it. First, we note that research activity yields cost reduction in a deterministic manner. This formulation naturally suggests itself as a model of process innovation; while this is quite specific, it is worth emphasizing that a similar formulation has been used in many of the seminal papers in this literature, see e.g., d'Apremont and Jacquemin (1988), Leahy and Neary (1998), Kamien, Mueller and Zhang (1992) and Suzumura (1992).[12]

Second, we note that the additive structure reflects the fact that distinct in-house and joint projects go along different research trajectories and there are no technological spillovers across them. The absence of technological spillovers is motivated by the growing complexity of many products and

[12]For an empirical assessment of the relationship between product vs process innovation and the size of firms, see Cohen and Klepper (1996). They observe that larger firms allocate relatively a larger share of resources to process innovations as compared to small firms.

the distinct technologies that are involved in different aspects of the product. As an example of this consider the automobile industry: investments in improving lamp illumination or in reducing lamp costs have no influence on the cost of producing engine starting procedures, nor on the cost of producing collision safety devices like airbags.[13]

Third, we note that, since f and h are concave, the additive structure also provides an incentive for spreading efforts across different projects and therefore an increase in the number of projects embodies efficiency gains. To clarify the role of this additive structure, we have also considered the case of a single research trajectory. In this case all efforts of a firm enter the same R&D production function: we assume that $c_i(g, x) = \bar{c} - F(x_{ii} + \sum_{j \in N_i(g)}(x_{ij} + x_{ji}))$, where F is a positive, symmetric, increasing, and concave function. Clearly, here every firm has an incentive to engage in independent research only (see the Appendix for a proof of this result).

Finally, two firms can combine their skills and find new avenues of research that lie outside their core competences. This is motivated by the idea that core capabilities are quite specific to the firm and so different pairs of firms can potentially collaborate on distinct research projects. This however will only be possible when a research opportunity between the two firms exists, which is exogenously given to the firms. In case of symmetric networks, the number of joint research opportunities is given by k; more generally, the set of opportunities for collaboration by g. In all cases, the cost specification in (1) assumes that projects with different partner firms go along different research trajectories for otherwise only one of the joint projects would be financed.

The market stage: Given the costs $c_i(g, x)$, firms operate in the market by choosing quantities or prices. Assume that there exists a unique equilibrium in pure strategies in the market stage. This allows us to write, for any collaboration pattern g and investment profile x (and a specified mode of market competition), the payoff function of an individual firm. Let $\pi_i(g, x)$, for $i = 1, 2, ..., n$, denote firm i's payoff.

3 Complementarity of research projects

The aim of this section is to show that there exists a natural complementarity in investments across different projects of the same firm. The interest of this result derives from the fact that the

[13] In an earlier version of our paper, we developed an extension of the model in which technological spillovers across projects were present as well; the presence of such spllovers does not alter our basic results, see Goyal, Moraga-González and Konovalov (2006).

complementarity arises purely due to market related forces and in the *absence* of any technological spillover across projects.

Given a collaboration and investment pair, (g, x), let $c_i(g, x)$ be the marginal cost of firm i, $i = 1, 2, ..., n$. Suppose firms choose quantities or prices in the market stage. Denote the market stage strategy of firm i by $(s_i(c(g, x)))_{i \in N}$. The reduced form profits for firm i are then given by $\pi_i(g, x)$. The effects of investments in in-house project on the incentives to invest in a joint project are reflected in the following equation:

$$\frac{d^2\pi_i(g,x)}{dx_{ii}dx_{ij}} = \frac{df(x)}{dx_{ii}}\frac{\partial h(x)}{\partial x_{ij}}\Big[\frac{d^2\pi_i(s(c(g,x)))}{d^2c_i} + \frac{d^2\pi_i(s(c(g,x)))}{dc_i dc_j}\Big] \tag{2}$$

Under our assumptions, the first two terms are positive, and so we are led to the observation that the investments in in-house projects and in joint projects are complementary if and only if the expression within the square bracket is positive. A similar condition can be formulated for the complementarity of distinct joint research projects. These observations are summarized in our first result.

Proposition 3.1 *A firm i's investments in in-house research x_{ii} and in joint research x_{ij}, $j \neq i$ are complementary if and only if*

$$\frac{d^2\pi_i(s(c))}{d^2c_i} + \frac{d^2\pi_i(s(c))}{dc_i dc_j} > 0. \tag{3}$$

A firm i's investments in joint research projects with different partners j and k, $j \neq k \neq i$ are complementary if and only if

$$\frac{d^2\pi_i(s(c))}{d^2c_i} + \frac{d^2\pi_i(s(c))}{dc_i dc_j} + \frac{d^2\pi_i(s(c))}{dc_i dc_k} + \frac{d^2\pi_i(s(c))}{dc_j dc_k} > 0. \tag{4}$$

This result provides a foundation for the well known argument in the business strategy literature that investments in bilateral relations reinforce the incentives for investing in core competences (see e.g., Doz and Hamel (1998), Prahalad and Hamel (1990)). Athey and Schmutzler (1995) find a related complementarity result between the process and the product innovation strategies of a firm.

On the one hand, if a firm i increases investment in a joint project it lowers its own costs, which has a direct effect on the marginal returns from investments in the in-house project. On the other hand, if a firm invests more in a joint project, the costs of the partner firm go down, and this has an indirect effect on the marginal returns from lowering its own costs. The two terms in (3) capture these two effects, respectively. Condition (3) characterizes the circumstances in which investments in own in-house project and joint projects with other firms will be complements. Condition (4) refers to complementarity of investments in distinct joint research projects and has a similar interpretation. We now explore the economic circumstances in which these conditions are satisfied. Perhaps the simplest way to understand the economic forces involved is to locate the above research collaboration problem in the context of standard price and quantity setting models and ask if the profit functions in these models satisfy the conditions.

As a first step then it is useful to start with the case where firms are located in independent markets, and therefore act as monopolists in the market stage. To fix ideas, suppose that firms choose quantities and that the inverse demand is given by $P(Q_i)$, where Q_i is the quantity produced by firm i. Assume $P'(Q_i) < 0$ and that $P''(Q_i)$ is well defined. It then follows that a firm i's profits are given by

$$\pi_i(g,x) = [P(Q_i) - c_i(g,x)]Q_i - x_{ii} - \sum_{j \in N_i(g)} x_{ij}. \tag{5}$$

First observe that the independent markets assumption implies that the costs of other firms $j \neq i$ have no bearing on profits of firm i. Next observe that $d\pi_i(.)/dc_i = -Q_i$ and that $d^2\pi_i(.)/d^2c_i = -dQ_i/dc_i$. Straightforward computations – which exploit the second order conditions of profit maximization – now tell us that the sign of this last derivative is positive. We have thus shown that a firm operating in an independent market satisfies conditions (3) and (4). The intuition underlying this result is really quite simple. Let us compare the marginal returns from investment in an in-house project for a firm given two levels of investment x_{ij} and x'_{ij}, where $x'_{ij} > x_{ij}$. For fixed x_{ii}, the costs of the firm are lower at x'_{ij} than at x_{ij}, and so its optimal quantity is higher in the first case. Therefore, when the firm considers raising its investments in the in-house project, the effects of the lower cost will act on a larger output base with x'_{ij} than with x_{ij}, which translates into a higher marginal return from the in-house project. The same arguments apply to investments in different joint projects.

We now examine how the complementarity result is affected by market competition. The key point to note about competition is that now there are additional terms which reflect the effects of other firms' costs on firm i's profits. Consider in particular the complementarity of in-house and joint project research. As the investment in joint project x_{ij} increases it lowers costs of the partner firm as well, which alters the competitive position of the firms. The marginal returns to in-house project must take this indirect strategic effect into account. Indeed, as the costs of the firm j fall, its quantity increases and the quantity of firm i falls, so we should expect this indirect effect to be negative. The prospects of the complementarity result thus hinge on the relative magnitude of the two terms in condition (3). We now show that this condition is indeed satisfied in the classical models of price and quantity competition. The same applies to the condition for complementarity of distinct joint research projects.

We start with an examination of the classical quantity competition model with linear demand. The inverse demand function is given by $p = A - \sum_{i \in N} q_i(g,x)$. From standard arguments, it follows that the equilibrium quantity of firm i is

$$q_i(g,x) = \frac{A - nc_i(g,x) + \sum_{j \neq i} c_j(g,x)}{n+1}, \tag{6}$$

and the profits of firm i are given by

$$\pi_i(g,x) = (q_i(g,x))^2 - \sum_{j \in N_i(g)} x_{ij} - x_{ii}. \tag{7}$$

Simple computations reveal that $d^2\pi_i(.)/d^2c_i = 2n^2/(n+1)^2$ and that $d^2\pi_i(.)/dc_i dc_j = -2n/(n+1)^2$; summing the two expressions, we find that condition (3) is indeed satisfied so in-house and joint work are complementary. Using the fact that $d^2\pi_i(.)/dc_j dc_k = 2/(n+1)^2$, it is readily seen that condition (4) also holds.

We next examine whether the complementarity results depend on the homogeneity of the goods and the assumption of quantity competition. To this end we study a model of price competition with differentiated goods. Suppose $n \geq 2$ firms operate in a market for differentiated products. Let the (inverse) demand of firm i be given by:

$$p_i(q) = A - q_i - \delta \sum_{j \neq i} q_j;\ 0 < \delta < 1;\ i = 1, 2, ..., n. \tag{8}$$

where $q = (q_1, q_2, ..., q_n)$ denotes the vector of quantities firms put in the market. Parameter δ measures the degree of product differentiation. When $\delta \to 0$ products are independent and when $\delta \to 1$ products become homogeneous.

Given the vector of investments x and the marginal costs $c_i(g, x)$, $i = 1, 2, ..., n$, firms operate in the market by setting prices. The direct demands are given by:

$$q_i(p) = a - bp_i + d \sum_{j \neq i} p_j;\ i = 1, 2, ..., n. \tag{9}$$

where

$$a = \frac{A}{1 + \delta(n-1)};\ b = \frac{1 + \delta(n-2)}{(1-\delta)(1 + \delta(n-1))};\ d = \frac{\delta}{(1-\delta)(1 + \delta(n-1))} \tag{10}$$

and $p = (p_1, p_2, ..., p_n)$ denotes firm prices. A firm i chooses its price p_i to maximize its profits $\pi_i(p; g, x) = (p_i - c_i(g, x))q_i(p)$. The system of first order conditions can be solved for a Nash equilibrium in prices:

$$\begin{aligned} p_i(g, x) &= \frac{A(1-\delta)}{(2 + \delta(n-3))} + \frac{(1 + \delta(n-2))\delta}{(2 + \delta(2n-3))(2 + \delta(n-3))} \sum_{j \neq i} c_j(g, x) \\ &\quad + \frac{(1 + \delta(n-2))(2 + \delta(n-2))}{(2 + \delta(2n-3))(2 + \delta(n-3))} c_i(g, x) \end{aligned} \tag{11}$$

Note that the price of a firm is increasing in own-cost and in the cost of the other firms. Substituting these prices in the profits expression yields the following reduced-form equilibrium profits:

$$\pi_i(g, x) = \frac{1 + \delta(n-2)}{(1-\delta)(1 + \delta(n-1))} (p_i(g, x) - c_i(g, x))^2 - x_{ii} - \sum_{j \in N_i(g)} x_{ij}. \tag{12}$$

Now simple computations reveal that $d^2\pi_i(.)/d^2c_i = 2b(\partial p_i/\partial c_i - 1)^2$ and that $d^2\pi_i(.)/dc_i dc_j = 2b(\partial p_i/\partial c_i - 1)(\partial p_i/\partial c_j)$. Putting these two terms together, we obtain $d^2\pi_i(.)/d^2c_i + d^2\pi_i(.)/dc_i dc_j = 2b(\partial p_i/\partial c_i - 1)(\partial p_i/\partial c_i + \partial p_i/\partial c_j - 1)$. Using (11) it is easy to verify that this expression is positive so condition (3) is satisfied. To check condition (3) we also need $d^2\pi_i(.)/dc_j dc_k = 2b(\partial p_i/\partial c_j)(\partial p_i/\partial c_k)$, and the same conclusion obtains. Therefore, the complementarity of investments across projects result also holds with price competition and differentiated goods. [14]

The above discussion shows that condition (3) holds in textbook models of price and quantity competition if demand is linear. We now ask if there are circumstances in which condition (3) is violated? To study this issue we consider a market with non-linear demand and quantity competition. In contrast to the linear demand case discussed above, when demand is non-linear a firm's price-cost margin is no longer equal to its quantity. As a result, the marginal gains to investment in a project depend not only on the responsiveness of its equilibrium quantity to cost decreases but also on how price-cost margin changes with cost. For the family of demand functions $p = A - Q^{\alpha}$, $\alpha \geq 1$, we have found that the complementarity result holds whenever α is low; for large α, in-house and joint research may fail to be complementary: in a setting where the demand curve is 'very' concave, increases in quantity can lower price sharply. If firms have very different costs then their incentives to increase quantity are quite different since the impact of lower prices is much larger for the lower cost (and higher quantity) firm. This divergence in incentives results in overproduction and the fall in prices leads to a violation of the complementarity property. Our calculations for this family of demand functions are placed in the Appendix.

4 Equilibrium Investments and Profits

In this section our interest is in understanding whether firms have an incentive in activating potential joint projects and the implications of having different numbers of potential projects. This requires us to analyze equilibrium behavior of firms. We carry out this equilibrium analysis for two classical oligopoly models, quantity competition with homogenous goods and price competition with differentiated goods. We obtain similar results for the two models. We show that firms will activate all potential projects. We then show that for each firm investments in in-house research as well as in each of the joint projects are increasing in the level of collaborative activity, i.e., in number of potential projects. However, firm profits do not necessarily increase as the level of collaborative activity rises.

We shall focus on the case where all firms have the same number of potential partners $k = 0, 1, 2, ..., n-1$. The case of unequal number of research opportunities is taken up in Section 5

[14] We also note that the conditions in Proposition 3.1 are satisfied if products are differentiated and firms compete in quantities; the derivations are available from the authors upon request.

below. Recall, that the number k is referred to as the *degree of collaborative activity* in the industry. In the context where every firm has the same number of partners, it is reasonable to focus on symmetric equilibrium in investments. An equilibrium x is called *symmetric* if $x_{ij} = x_{\ell m}$ whenever $i \neq j$ and $\ell \neq m$, and $x_{ii} = x_{jj}$ for every pair of firms $i, j \in N$.

Given the network g and other firms' R&D investments, firm i solves the following problem:

$$\begin{array}{rl} \max & \pi_i(g,x) \\ \text{s.t.} & x_{ii} \geq 0, \\ & x_{ij} \geq 0, \quad j \in N_i(g). \end{array} \tag{13}$$

4.1 Quantity competition

Under Cournot competition, the first-order conditions for the interior solution of the problem in (13) are

$$\frac{\partial \pi_i(g,x)}{\partial x_{ii}} = \frac{2q_i(g,x)n}{(n+1)} \frac{\partial f(x_{ii})}{\partial x_{ii}} - 1 = 0, \tag{14}$$

and

$$\frac{\partial \pi_i(g,x)}{\partial x_{ij}} = \frac{2q_i(g,x)(n-1)}{(n+1)} \frac{\partial h(x_{ij},x_{ji})}{\partial x_{ij}} - 1 = 0, \quad j \in N_i(g). \tag{15}$$

These first order conditions say that a firm should continue to invest in a research project up to the point where the value of the marginal cost reduction obtained from the project equals the marginal cost of R&D investment.

A symmetric equilibrium must satisfy the first order conditions (14) and (15). These can be simplified now and written as follows:

$$\frac{2n}{(n+1)^2}(A - \bar{c} + f(x_{ii}) + kh_d(x_{ij}))f'(x_{ii}) - 1 = 0, \tag{16}$$

$$\frac{(n-1)}{(n+1)^2}(A - \bar{c} + f(x_{ii}) + kh_d(x_{ij}))h'_d(x_{ij}) - 1 = 0, \tag{17}$$

Our next result proves existence and uniqueness of symmetric equilibrium in investment levels. This result requires the costs of research to be sufficiently large relative to its returns. For convenience, we parameterize functions f and h as follows: $f(x_{ii}) = \frac{1}{\gamma}\tilde{f}(x_{ii})$, $h(x_{ij}, x_{ji}) = \frac{1}{\gamma}\tilde{h}(x_{ij}, x_{ji})$. Parameter $\gamma \in (0, +\infty)$ is a shift-scalar that reduces the returns from R&D.

Proposition 4.1 *Let g^k be a symmetric network of degree k. Let $f(x) = \frac{1}{\gamma}\tilde{f}(x_{ii})$, $h(x_{ij}, x_{ji}) = \frac{1}{\gamma}\tilde{h}(x_{ij}, x_{ji})$. Then if*

$$\min\{f'(0), \frac{\partial h(0,0)}{\partial x_{ij}}\} > \frac{(n+1)^2}{2(n-1)(A-\bar{c})}$$

there exists γ_0 such that for all $\gamma \geq \gamma_0$ there exists a unique symmetric equilibrium in investment levels $x^* \in I\!R_{++}^{(k+1)n}$.

The proof of this result is given in the appendix.

We now examine the sensitiveness of equilibrium to the degree of collaborative activity k. Equation (16) implicitly defines a function $x_{ii}^*(x_{ij})$. Due to complementarity of in-house and joint research, this function is increasing. Moreover, the assumptions above on the function f imply that the intercept of $x_{ii}^*(x_{ij})$ is positive and independent of k. Similarly equation (17) implicitly defines a relationship $x_{ij}^*(x_{ii})$; this equation is also increasing and has a positive intercept, which increases in k. The symmetric equilibrium in Proposition 4.1 is given by the intersection of these two functions. Using (16) and (17) it is possible to check that an increase in k increases the slopes of $x_{ii}^*(x_{ij})$ and $x_{ij}^*(x_{ii})$. As a result, research expenditures in own project as well as in joint projects increase in the degree of collaborative activity k. This is illustrated in Figure 1, where the intersection of the solid lines $E^*(k)$ determines the investment levels for the degree of collaborative activity k and the crossing point of the dashed lines $E^*(k+1)$ shows R&D levels for a larger degree of collaboration $k+1$.

Figure 1: Cournot R&D investments and the degree of collaborative activity.

Proposition 4.2 *An increase in the degree of collaborative activity k leads to an increase in firm investment in in-house research as well as in each of its joint projects. As a result, cost reduction, aggregate quantity and consumer surplus increase in the degree of collaboration k.*

Proof: The equilibrium investment levels x_{ii}^* and x_{ij}^* are the solutions of the system (16) − (17). The left-hand-side of (16) is increasing in x_{ij} and k. The left-hand-side of (17) is increasing in x_{ii} and k. By virtue of Theorem 4 in Milgrom and Roberts (1994), the (unique) solution of this system of equations is non-decreasing in k.

We note further that, for all k, the equilibrium investment levels x_{ii}^* and x_{ij}^* satisfy the equation

$$2nf'(x_{ii}^*) = (n-1)h_d'(x_{ij}^*). \tag{18}$$

From this it follows that investment levels must increase in k. To see this suppose, on the contrary, that $x_{ii}^*(k+1) = x_{ii}^*(k)$. Then by equation (18), it follows that $x_{ij}^*(k+1) = x_{ij}^*(k)$ (and vice versa), which contradicts the fact that the left-hand side of the first order conditions is strictly increasing in k. ■

This finding that more research partners leads to higher investment in R&D per project contrasts with the results of earlier work studying the effects of bilateral collaboration in oligopoly. In Goyal and Moraga-González (2001) firms invest in a *single* research project and its outcome is in turn shared with the collaborators. They find that individual firm R&D investment *falls* as the degree of collaborative activity increases. The main force operating there is the positive externality that a firm investment confers upon the collaborators, which reduces the incentives to do research as the degree of collaboration rises. Why does our model yield opposite predictions? There are two reasons. First, in our model individual efforts in joint projects are project-specific and therefore only shared with individual partners. This implies that the spillover externalities associated to collaboration are *independent* of the degree of collaboration. Second, in our model an increase in the degree of collaboration is materialized in a new avenue of research, which, by complementarity, increases firms incentives to invest.

Another paper where more collaboration leads to lower investment is Kamien *et al.* (1992). In their paper, they compare the case of no partners, called *R&D competition*, with the case in which all firms work together, called *RJV competition*.[15] They find that firms invest less when they

[15] Under R&D competition firms carry out only private research and spillovers are absent, while under RJV com-

collaborate than when they work independently. This is analogous to comparing $k = 0$ in our case with $k = n - 1$. Again, the difference in results arises due to the fact that in Kamien *et al.* the spillover effects associated to collaborative work are much larger than in the present paper.

We now turn to the effects of increased partnering on firm profits. The following result summarizes our findings.

Proposition 4.3 *If n is large enough, then individual firm profits decrease in the level of collaborative activity k.*

Proof: For a symmetric network g^k, let $x^*(k)$ be a symmetric equilibrium in investment levels. We investigate the total derivative of the profit function with respect to k :

$$\frac{d\pi_i(x^*(k))}{dk} = \frac{\partial \pi_i}{\partial k} + \frac{\partial \pi_i}{\partial x^*_{ii}}\frac{\partial x^*_{ii}}{\partial k} + \frac{\partial \pi_i}{\partial x^*_{ij}}\frac{\partial x^*_{ij}}{\partial k}.$$

This expression can be rewritten as follows:

$$\frac{d\pi_i(x^*(k))}{dk} = \frac{\partial x^*_{ii}}{\partial k}\left(\frac{2q_i}{n+1}f'(x^*_{ii}) - 1\right) + k\frac{\partial x^*_{ij}}{\partial k}\left(\frac{2q_i}{n+1}h'_d(x^*_{ij}) - 1\right) + \left(\frac{2q_i}{n+1}h_d(x^*_{ij}) - x^*_{ij}\right).$$

Using the first order conditions, we obtain

$$\frac{d\pi_i(x^*(k))}{dk} = \frac{\partial x^*_{ii}}{\partial k}\left(\frac{1}{n} - 1\right) + k\frac{\partial x^*_{ij}}{\partial k}\left(\frac{2}{n-1} - 1\right) + \left(\frac{2q_i}{n+1}h_d(x^*_{ij}) - x^*_{ij}\right).$$

The first two terms of the RHS of this equation are negative. The last term is negative if and only if $\frac{2q_i h_d(x^*_{ij})}{(n+1)x^*_{ij}} - 1$ is negative. A majorant of this function $\frac{2q_i}{(n+1)}h'_d(0) - 1$ becomes negative if n grows sufficiently large. However, if n becomes too large, the boundary condition $h'_d(0) > \frac{(n+1)^2}{2(n-1)(A-\bar{c})}$ may be violated, in which case firms do not allocate resources to joint research projects. Since $\lim_{n\to+\infty}\left(\frac{n+1}{2q_i} - \frac{(n+1)^2}{2(n-1)(A-\bar{c})}\right) = +\infty$, there exists a sufficiently large number of firms n for which profits are strictly decreasing in k, provided $h'_d(0)$ is large enough. ■

This result should be seen together with Proposition 3.1. Our analysis provides support to the view expressed in the business strategy literature that, at an individual level, firms benefit from undertaking collaborative projects (see e.g., Powell, Koput and Smith-Doerr, 1996). However, the aggregate effects of collective firm partnering turn out to be negative for the firms since individual

petition firms collaborate with all other firms and so spillovers are perfect. In both cases, firms make R&D decisions non-cooperatively.

profits fall in the number of partners that every firm has; to the best of our knowledge this point has not been made in this literature before.

We now elaborate on the arguments underlying Proposition 4.2. The effects of an increase in the degree of collaboration are gathered in the following equation:

$$\frac{d\pi_i(x^*(k))}{dk} = \frac{\partial \pi_i}{\partial x^*_{ii}}\frac{\partial x^*_{ii}}{\partial k} + \sum_{j \in N_i(g)} \frac{\partial \pi_i}{\partial x^*_{ij}}\frac{\partial x^*_{ij}}{\partial k} + \sum_{\ell \neq i} \frac{\partial \pi_i}{\partial x^*_{\ell\ell}}\frac{\partial x^*_{\ell\ell}}{\partial k} + \sum_{\ell \neq i}\sum_{m \in N_\ell(g)} \frac{\partial \pi_i}{\partial x^*_{\ell m}}\frac{\partial x^*_{\ell m}}{\partial k} + \frac{\partial \pi_i}{\partial k} \tag{19}$$

We note that as more research opportunities become available to the firms (k rises) all R&D investments by firms increase. Since a firm chooses its R&D efforts to maximize its own profits, the profit effect of an increase in its own research variables is zero (by the envelope theorem). As a result, the first and second terms in the RHS of (19) can be ignored. The influence of an increase in the in-house R&D of other firms is clearly negative because these investments reduce the costs of production of these firms and not the unit cost of firm i. The effect of changes in the joint investments of other firms is also negative because only k of those variables reduce the cost of production of firm i while $(n-2)k$ other variables reduce each the costs of the competitors. As a result, the effect of an increase in collaborative activity via R&D investments is negative. The last term in the RHS of (19) represents the effect of opening a new avenue of research for every firm. This term is also negative for large n.**Alex, Jose, The referee wants more discussion of this result; can you make an effort?**

The above result obtains for large n; what is the relation between collaborative activity and firm profits in market with a few firms? We have been unable to provide a general answer to this question. Analysis of an example with specific functional forms for the return functions f and h, however, suggests that if the number of firms is small then there is a positive relationship between the level of collaborative activity and firm profits.[16]

4.2 Price competition

When $n \geq 2$ firms operate in a market for differentiated products, the first order conditions of the problem stated in (13) are:

[16] In the example, we assume that $f(x_{ii}) = \frac{1}{\gamma}\sqrt{x_{ii}}$ and $h(x_{ij}, x_{ji}) = \frac{1}{\gamma}(\sqrt{x_{ij}} + \sqrt{x_{ji}})$. In this setting, we find that firm profits are increasing in k for $n < 5$, while they are declining in k for $n \geq 5$.

$$2b\frac{(1-\delta)(A-\bar{c}+f(x_{ii}^*)+kh_d(x_{ij}^*))}{2+\delta(n-3)}\Psi_1\frac{df(x_{ii}^*)}{dx_{ii}}-1 \;=\; 0 \tag{20}$$

$$2b\frac{(1-\delta)(A-\bar{c}+f(x_{ii}^*)+kh_d(x_{ij}^*))}{2+\delta(n-3)}\Psi_2\frac{\partial h(x_{ij}^*,x_{ji}^*)}{\partial x_{ij}}-1 \;=\; 0 \tag{21}$$

We first show that there exists an interior equilibrium in investments, for any level of collaborative activity k. Again, we parameterize functions f and h by γ.

Proposition 4.4 *Let g^k be a symmetric network of degree k. Then if*

$$f'(0) > \frac{(2+\delta(n-3))(1+\delta(n-1))}{2(1+\delta(n-2))(A-\bar{c})\Psi_1} \tag{22}$$

and

$$\frac{\partial h(0,0)}{\partial x_{ij}} = \frac{\partial h(0,0)}{\partial x_{ji}} > \frac{(2+\delta(n-3))(1+\delta(n-1))}{2(1+\delta(n-2))(A-\bar{c})\Psi_2}, \tag{23}$$

there exists γ_0 such that for all $\gamma \geq \gamma_0$ there exists a unique symmetric equilibrium in investment levels $x^* \in I\!R_{++}^{(k+1)n}$.

The proof of this existence result is given in the appendix.

We now examine the effects of k on the equilibrium investments and profits. We will establish the following result.

Proposition 4.5 *An increase in the degree of collaborative activity k leads to an increase in firm investment in in-house research as well as in each of its joint projects. If n is large enough, then individual firm profits decrease in the level of collaborative activity k.*

Proof: Note that the LHS of (20) is strictly increasing in x_{ij}^* and in k; likewise, the LHS of (21) is strictly increasing in x_{ii}^* and in k. These two facts can be used as in the proof of Proposition 4.2 to show that equilibrium investments are increasing in k.

Finally, consider equilibrium profits with respect to the degree of collaborative activity. Taking the total derivative of the profit function with respect to k:

$$\frac{d\pi_i(x^*(k))}{dk} = \frac{\partial \pi_i}{\partial x_{ii}^*}\frac{\partial x_{ii}^*}{\partial k} + \frac{\partial \pi_i}{\partial x_{ij}^*}\frac{\partial x_{ij}^*}{\partial k} + \frac{\partial \pi_i}{\partial k}. \tag{24}$$

This expression can be rewritten as follows:

$$\begin{aligned}\frac{d\pi_i(x^*(k))}{dk} &= \left(2b(p_i - c_i)\frac{1-\delta}{2+\delta(n-3)}\frac{df(x_{ii}^*)}{dx_{ii}} - 1\right)\frac{\partial x_{ii}^*}{\partial k} \\ &+k\left(2b(p_i - c_i)\frac{1-\delta}{2+\delta(n-3)}\frac{\partial h(x_{ij}^*, x_{ji}^*)}{\partial x_{ij}} - 1\right)\frac{\partial x_{ij}^*}{\partial k} \\ &+2b(p_i - c_i)\frac{1-\delta}{2+\delta(n-3)}h_d(x_{ij}^*) - x_{ij}^* \end{aligned} \tag{25}$$

Using the first order conditions, we obtain

$$\begin{aligned}\frac{d\pi_i(x^*(k))}{dk} &= \frac{\partial x_{ii}^*}{\partial k}\left(\frac{-\delta(n-1)\,(1+\delta(n-2))}{2+3\delta(n-2)+\delta^2(5+n(n-5))}\right) \\ &+\frac{\partial x_{ij}^*}{\partial k}\left(\frac{-\delta(n-2)(1+\delta(n-2))}{2+\delta(3n-7)+\delta^2(7+n(n-6))}\right) \\ &+\left(2b(p_i - c_i)\frac{1-\delta}{2+\delta(n-3)}h_d(x_{ij}^*) - x_{ij}^*\right). \end{aligned} \tag{26}$$

The first two terms of the right-hand side of this equation are clearly negative. The last term is also negative if and only if $\frac{2b(p_i-c_i)(1-\delta)h_d(x_{ij}^*)}{(2+\delta(n-3))x_{ij}^*} - 1$ is negative. Since h is concave, a majorant of this function is $\frac{2b(p_i-c_i)(1-\delta)}{2+\delta(n-3)}h_d'(0) - 1$; using b and the expression for $(p_i - c_i)$ in a symmetric equilibrium this function can be written as $\frac{2(A-c_i)(1-\delta)(1+\delta(n-2))}{(2+\delta(n-3))^2(1+\delta(n-1))}h_d'(0) - 1$, which becomes negative if n grows sufficiently large. The negativity of this expression is compatible with the boundary conditions that guarantee the existence of an interior equilibrium. Therefore, there exists a sufficiently large number of firms n for which profits are strictly decreasing in k. The critical minimal value of n, for which the last term is negative, is decreasing as δ grows.[17] ■

5 Discussion

The analysis in the previous section assumed that every firm has the same number of collaboration projects and that this number is exogenously given. This section first explores the effects of asymmetries in number of collaboration projects and then examines the incentives of firms to start new collaboration projects. In sections 3 and 4 we focused on firm investments and profits; to complete the analysis in the last part of this section, we briefly discuss the implications of hybrid

[17] Note that this analysis excludes the case of local monopolies ($\delta = 0$).

R&D for social welfare. For expositional simplicity, we present our analysis within the quantity setting oligopoly framework.

5.1 Core and periphery firms

It is frequently argued that firms forming research alliances enhance their market dominance. This suggests that inequality in the set of joint research opportunities available to various firms may be the very source of market power in an industry. Partner firms involved in a collaborative agreement may allocate unequal amounts of resources to their joint project; some firms may 'exploit' their partners by having them make greater investments in joint projects.

An examination of these arguments requires an analysis of networks in which the number of research opportunities of a firm differs across firms, i.e., *asymmetric collaborative structures*. It is also worth noting that recent empirical work shows that inter-firm collaboration networks in biotechnology exhibit considerable inequality with some key firms having a very large number of joint projects while most firms have a few partners only.[18]

We have been unable to solve for equilibrium in investments for general patterns of collaboration. The technical problem here is that when the number of firms in an asymmetric network is large, equilibrium firm investments are given by the solution of a system of first order conditions which are non-linear equations. To get an idea of the economic issues involved we proceed as follows: first, we provide a complete analysis of a four firm example and second, we study equilibrium for specific networks for general n.

Four firm example: In this example the cost reduction function from in-house research is given by $f(x_{ii}) = 1/\gamma\sqrt{x_{ii}}$ while the cost-reduction obtained from a joint project between firms i and j is $h(x_{ij}, x_{ji}) = 1/\gamma(\sqrt{x_{ij}} + \sqrt{x_{ji}})$.

In total there are 11 different collaborative structures. The computations for investments and profits are presented in the appendix. The analysis yields the following findings:

1. A firm with a larger number of joint projects invests more in independent and in every joint project and earns a higher profit as compared to a firm with less joint projects.

2. Opening a new joint project increases the profits of the two participating firms while it

[18]For a discussion of empirical work on inter-firm network structures, see e.g., Baker, Gibbons and Murphy (2004).

decreases the profits of the rest of the firms.

The first finding implies that partners in a joint research project will put unequal efforts into their common project if they have a different number of joint projects, with the more linked partner putting in a higher level of effort. Moreover, this greater investment is worthwhile since it yields a higher profit for the more connected firm. Our second observation substantiates nicely the claim that strategic partnering enhances market dominance. It also suggests that in this four firm example, the complete network is the unique incentive-compatible network. How general are these findings? The issue of equilibrium collaboration structures is studied in the next section; the nature of investments in asymmetric investments for general n is briefly discussed here.

We find that in a star network for general n,[19] the central firm puts in greater investment in in-house projects as well as in each of the joint projects as compared to the peripheral firms. The central firm also makes a higher profit as compared to the peripheral firms. Similarly, in a network with two inter-linked stars,[20] we find that the central firms put in higher investment in in-house projects and in each of the joint projects as compared to the peripheral firms. This also has the interesting implication that the total investment in the joint project between the two central firms is larger than the total investment in a joint project between a peripheral firm and a central firm. The central firms also make a higher profit than the peripheral firms.[21]

5.2 Equilibrium collaboration structures

Given any distribution of joint research opportunities g, consider the set of joint projects that are activated in equilibrium:

$$g^* = \{ij \in g \mid x^*_{ij} + x^*_{ji} > 0\}. \tag{27}$$

Suppose that each firm is given the opportunity to initiate at most k joint projects with other firms (that is, $g = g^k$). In this case, Proposition 4.1 tells us that equilibrium investments are positive,

[19] A star network is one in which a single firm has a joint project with each of the other firms who in turn have no other joint projects.

[20] Suppose that n is even. A inter-linked star network is one where there are two central firms, (say) $(n-1)$ and n; $(n-1)$ has joint projects with (say) each of $1, ..., (n-2)/2$, while firm n has joint projects with each of $1 + (n-2)/2, ..., (n-2)$ firms and $(n-1)$ and n have a joint project. There are no other joint projects. The inter-linked star is a stylized version of the collaboration network observed in the biotechnology industry, where a few inter-linked firms have a large number of joint projects, while most firms have few partners.

[21] The computations are available from the authors upon request.

and this means that every firm has an incentive to activate all research opportunities provided the rest of the firms do so, that is, $g^* = g^k$.

This result that firms will work on all projects available to them can be made stronger. Assume that $h'_d(0)$ is significantly high. This along with some additional technical requirements on the functions $f(\cdot)$ and $h(\cdot)$, guarantees that $\frac{\partial \pi_i(g,x)}{\partial x_{ij}}|_{x_{ij}=0,\, x_{ji}=0} > 0$ for any i, j, g, and x. In this case, for any given set of joint research opportunities the only equilibrium is one in which all possible projects are funded.

If these conditions do not hold firms may not open all projects in equilibrium. Consider the following example. Let the outcome of a joint project ij be $h(x_{ij}, x_{ji}) = 1 - e^{-\alpha(x_{ij}+x_{ji})}$ and set the returns to the in-house project $f(x_{ii}) \equiv 0$ for simplicity. Set $\alpha = 0.501$, $n = 5$, and $A - \bar{c} = 9$ and suppose that every joint research project is available: $g = g^{n-1}$. It can be seen that there is an asymmetric equilibrium where a firm i invests $x_{ij} \approx 1.16$ in each of its projects and the rest of the firms invest nothing at all; this equilibrium corresponds to a star network where firm i is the hub firm. The reason for this is that firms which are marginalized in the market may have no incentives to conduct R&D.

5.3 Social welfare

Our analysis in section4 so far has clarified that in markets with a large number of firms individual research efforts – both in independent research and in each of the joint projects – are increasing in the number of partners, while individual profits are decreasing in the number of partners. Increasing research efforts imply lower costs and therefore higher quantities and consumers surplus. However, this increase in consumers surplus takes place along with a fall in firm profits. So the effects of increasing collaboration on social welfare are unclear.

To shed some light on this issue, we assume the following quadratic specification of the return to R&D functions: $f(x_{ii}) = 1/\gamma\sqrt{x_{ii}}$ and $h(x_{ij}, x_{ji}) = 1/\gamma(\sqrt{x_{ij}} + \sqrt{x_{ji}})$. These functions satisfy all the conditions mentioned above except for differentiability at zero. We note, however, that all results presented above are also true for this family of functions. To guarantee existence of equilibrium in investment levels, we simply require that γ is sufficiently large ($\gamma > 2\sqrt{n}$ suffices).

Equilibrium investment levels are given by the solution to (14)-(15). In this case we obtain:

$$x_{ii}^* = \left(\frac{(A-\bar{c})n\gamma}{\gamma^2(n+1)^2 - n - 2k(n-1)}\right)^2, \tag{28}$$

$$x_{ij}^* = \left(\frac{(A-\bar{c})(n-1)\gamma}{\gamma^2(n+1)^2 - n - 2k(n-1)}\right)^2, \tag{29}$$

We will compare social welfare under different levels of collaborative activity. For his comparison we shall employ the usual measure of social welfare:

$$W(g,x) = \sum_{i\in N} \pi_i(g,x) + \frac{1}{2}Q(g,x)^2, \tag{30}$$

where $Q(g,x) = \sum_{i\in N} q_i(g,x)$.

Plugging these R&D efforts in (30), we can calculate the level of welfare:

$$W(x^*, g^k) = \frac{(A-\bar{c})^2\gamma^2[n\gamma^2(n+2)(n+1)^2 - 2kn(n-1)^2 - 2n^3]}{2\left(\gamma^2(n+1)^2 - n - 2k(n-1)\right)^2} \tag{31}$$

We can first use this expression to examine the welfare implications of an increase in the degree of collaborative activity k. For this we compute:

$$\frac{\partial W(g^k)}{\partial k} = \frac{(A-\bar{c})^2 n(n-1)\gamma^2(\gamma^2(n+1)^2(n+5) - 2k(n-1)^2 - n(3n+1))}{\left(\gamma^2(n+1)^2 - n - 2k(n-1)\right)^3} \tag{32}$$

It can now be verified that social welfare is increasing in the level of collaborative activity (for all parameters γ for which equilibrium exists). This implies that larger degrees of collaboration are more attractive than partnerships or just in-house research from the social point of view. This result shows that an increase in collaboration leads to gains for the consumers (via larger R&D investments, lower cost of production, and larger aggregate quantity) that offset the fall in firms profits that occurs when n is large (see Proposition 4.3).

6 Concluding remarks

When firms produce complex products involving a range of technologies and skills that are difficult to master individually, hybrid R&D arises naturally. A firm prefers to undertake research in its core activity independently while non-core activities are more effectively researched with partners. This leads us to model the idea that research activity in firms takes the form of different projects, some of which are carried out in-house, while others are carried out jointly with other firms. Our interest in in understanding the incentives of firms to invest in different types of projects and the implications of these investments for the collective profits of the firms.

We first explore the relationship between investments in in-house research and in collaborative R&D. Our first result derives a necessary and sufficient condition, on the profit function of a firm, for *all the projects in the research portfolio of an individual firm to be complementary.* We show that a number of standard models including a price and quantity competition with differentiated goods as well as firms in independent markets satisfy this condition. The interest behind this result is that complementarity across projects of a firm can arise purely due to market advantages generated by a being a lower cost firm even in the *absence* of any technological spillovers across projects.

We then turn to an examination of equilibrium investments and profits under different forms of market competition. Here we uncover a second general aspect of the complementarity across projects: *a firm's investment in in-house project as well as in each of the joint projects increases as the number of joint projects every firm is engaged in rises.* This result has the following natural interpretation in the context of the research on core competence: it says that as a firm diversifies its portfolio of research projects, the incentive to invest in core in-house research increases. Thus research diversification reinforces incentives to invest and build core competence of a firm.

Finally, we examine the effects of increasing levels of collaborative activity on firm profits. Our final result is that that inter-firm collaborations have serious and potentially negative aggregate effects: an increase in overall level of partnering in an industry lowers the profit for each firm (if the number of firms is large). In spite of this negative effect, we find that firms have an incentive to undertake joint projects with every other firm.

In carrying out our analysis we made two assumptions which appear to be worth discussing. We assumed that firms know the skills and core competence of other firms; in actual practice, we believe that asymmetric information about skills and competence of individual firms is an important factor. We also assumed that all ties are bilateral and only one project is possible between any pair of firms. In future work, we hope to explore the implications of relaxing these assumptions.

7 Appendix

7.1 Single research trajectory

Consider a collaboration structure g and an investment profile x. Let $c(g,x)$ be the vector of firms' marginal costs. Denote the market stage strategy of firm i by $(s_i(c(g,x)))_{i\in N}$. The reduced form profits for firm i are then given by $\pi_i(g,x)$. In the case of a single research trajectory, the first order conditions with respect to R&D variables are

$$\frac{\partial \pi_i(g,x)}{\partial x_{ii}} = -\frac{\partial \pi_i(s(g,x))}{\partial c_i}\frac{\partial F(x)}{\partial x_{ii}} - 1 = 0, \tag{33}$$

and

$$\frac{\partial \pi_i(g,x)}{\partial x_{ij}} = -\left(\frac{\partial \pi_i(s(g,x))}{\partial c_i} + \frac{\partial \pi_i(s(g,x))}{\partial c_j}\right)\frac{\partial F(x)}{\partial x_{ij}} - 1 = 0, \quad j \in N_i(g). \tag{34}$$

Since there is a single research trajectory, it follows that $\partial F(x)/\partial x_{ii} = \partial F(x)/\partial x_{ij}$. Therefore, since $\partial \pi_i(s(g,x))/\partial c_j > 0$, from the first order conditions it follows that marginal returns to in-house research are always greater than the marginal returns to collaborative research. As a result, firms invest only in own research. The idea here is also related to the fact that collaborative research brings about detrimental business-stealing effects. When all research activities are substitutes because they all go along the same research trajectory, a firm always gains by shifting money from any joint research project to own research, since in this way it eliminates business-stealing effects.

7.2 Complementarity of research projects and concave demand

Consider the quantity competition model with homogeneous products and assume now that demand is given by the family of functions $p = a - Q^\alpha$, with $Q = \sum q_i$, and $\alpha > 0$. The profit of a firm i is $\pi_i = (a - (\sum q_i)^\alpha - c_i)q_i$. The solution of the quantity setting game gives $q_i = (a - Q^\alpha - c_i)/\alpha Q^{\alpha-1}$, and the expression for equilibrium reduced-form profits is $\pi_i = (a - Q^\alpha - c_i)^2/\alpha Q^{\alpha-1}$, $i = 1, 2, ..., n$.

We now note that

$$\frac{d\pi_i}{dc_i} = q_i \frac{(a-c_i)(\alpha-1) + Q^\alpha(\alpha+1-2\alpha(n+\alpha))}{\alpha(n+\alpha)Q^\alpha}.$$

Using this expression, some calculations reveal that in-house and joint project research are complementary whenever

$$(\alpha-1)(2\alpha-1)(a-c_i)^2 - (\alpha-1)(a-c_i)(3\alpha(n+\alpha)-2)Q^\alpha + (1+\alpha(\alpha((n+\alpha)^2-3)-3n+1))Q^{2\alpha} > 0. \tag{35}$$

The expression in the left-hand side of (35) is quadratic in Q^α. Its sign is equal to the sign of $(\alpha-1)(2\alpha-1)$ whenever its discriminant is negative. The latter condition is satisfied if $\alpha > 1$ and

$$-8+12n-5n^2+12\alpha-10n\alpha+n^2\alpha-5\alpha^2+2n\alpha^2+\alpha^3<0. \tag{36}$$

The left-hand side of (36) is negative as long as α is between zero and some positive number $\bar{\alpha}(n)$, which constitutes its unique positive real root. Therefore, a sufficient condition for complementarity is that α belongs to the interval between 1 and $\bar{\alpha}(n)$, the approximate values of $\bar{\alpha}(n)$ are given in the following table:

n	$\bar{\alpha}(n)$
2	2.87
3	3.29
4	3.55
5	3.73
...	...
10	4.19
...	...

For large values of n $\bar{\alpha}(n)$ converges to 5. Once α is between 1 and 2.87, the complementarity holds for any number of firms.

It is illustrative to set $n = 2$ in (35). Suppose the costs of the two competitors are similar, i.e., $c_i = c_j = c$. Then the expression in (35) simplifies to $(a-c)^2\alpha^2(5+3\alpha)/(2+\alpha)^2$, which implies that the complementarity result holds no matter the degree of concavity of the demand function provided firms have symmetric costs.

Now assume that, say, firm i is more efficient than the competitor, i.e., $c_i < c_j$; assuming that α is relatively large, the sign of the expression in (35) is equal to the sign of $(a-c_j)(c_i-c_j)\alpha^4$, which is negative and therefore the complementarity result is violated (for the efficient firm).

7.3 Proofs of propositions

Proof of Proposition 4.1: We first show that the assumptions imposed on functions $f(\cdot)$ and $h(\cdot,\cdot)$ guarantee the existence of a symmetric solution to the system of the first order conditions (16)-(17). Consider a function $R(y) = (h'_d)^{-1}(\frac{2n}{n-1}f'(y))$, and put $R(y)$ equal to 0 if $(h'_d)^{-1}(\frac{2n}{n-1}f'(y))$ is not determined. By the concavity of the functions f and h (and, therefore, $h_d(y)$), the function $R(y)$

is continuous, non-negative and monotonically non-decreasing for all $y \geq 0$. Moreover, if (x_{ii}, x_{ij}) is an interior solution of (16)-(17), then $x_{ij} = R(x_{ii})$. A function $\frac{2n}{(n+1)^2}(A-\bar{c}+f(y)+kf(R(y)))f'(y)-1$ is continuous and strictly positive at zero by the boundary conditions. It is strictly negative if y is sufficiently large. Therefore, there is a point $y = x^*_{ii} > 0$ such that

$$\frac{2n}{(n+1)^2}(A - \bar{c} + f(x^*_{ii}) + kf(R(x^*_{ii})))f'(x^*_{ii}) - 1 = 0.$$

Let $x^*_{ij} = R(x^*_{ii})$. By construction, (x^*_{ii}, x^*_{ij}) is a solution of the system (16)-(17). Since $h'_d(0) > \frac{(n+1)^2}{(n-1)(A-\bar{c})}$, x^*_{ij} is strictly positive. Show now that every function $\pi_i(x_i, x^*_{-i})$, $i \in N$ is strictly concave if γ sufficiently large. Without loss of generality, assume that x_i belongs to a compact convex set $K_i \subset I\!R^{k+1}_+$. It is sufficient to verify that $D^2_{x_i}\pi_i(x_i, x^*_{-i})$ is negative definite for every $x_i \in K_i$. The matrix $D^2_{x_i}\pi_i(x_i, x^*_{-i})$ can be represented as a linear combination

$$D^2_{x_i}\pi_i(x_i, x^*_{-i}) = \frac{1}{\gamma}B_i(x_i, x^*_{-i}) + \frac{1}{\gamma^2}C_i(x_i, x^*_{-i}),$$

where matrices $B(.)$ and $C(.)$ do not depend on γ, and $B(.)$ is diagonal and strictly negative definite for all $x_i \in K_i$. The functions $u \cdot B_i(x_i, x^*_{-i})u$ and $u \cdot C_i(x_i, x^*_{-i})u$ are continuous and bounded if $x_i \in K_i$ and $u \in \{v \in I\!R^{k+1} \mid ||v|| = 1\}$. Therefore it is possible to find a sufficiently large γ such that for all such x_i and u the inequality $\frac{1}{\gamma}|u \cdot B_i(x_i, x^*_{-i})u| > \frac{1}{\gamma^2}u \cdot C_i(x_i, x^*_{-i})u$ is satisfied. This implies that $D^2_{x_i}\pi_i(x_i, x^*_{-i})$ is also negative definite for every x_i in K_i and all sufficiently large γ. Hence, a profile of investment levels $x^* = (x^*_{ii}, x^*_{ij})_{i \in N, ij \in g}$ is an interior symmetric Nash equilibrium.

To prove the uniqueness of a symmetric equilibrium, it is sufficient to show that the function $\tilde{F} : I\!R^2 \to I\!R^2$ defined by the right-hand side of the system (16)-(17) is strictly monotonic on some compact set containing all (u^*, v^*) such that $x_{ii} = u^*$, $i \in N$, $x_{ij} = v^*$, $ij \in g^k$, is a symmetric equilibrium. This can be done by analogy with the first part of the proof.

The condition that γ should be sufficiently large is essential. As numerical examples show, if returns from R&D activity are too high, no equilibrium exists in the model. ■

Proof of Proposition 4.4: We first show that the assumptions imposed on functions $f(\cdot)$ and $h(\cdot, \cdot)$ guarantee the existence of a symmetric solution to the system of the first order conditions (20)-(21). Consider a function $R(y) = (h'_d)^{-1}(\frac{2\Psi_1}{\Psi_2}f'(y))$, and put $R(y)$ equal to 0 if $(h'_d)^{-1}(\frac{2\Psi_1}{\Psi_2}f'(y))$ is not determined. Essentially, the function $R(\cdot)$ provides the corresponding optimal level of x_{ij} for the given optimal level of x_{ii}. By the concavity of the functions f and h (and, therefore, $h_d(y)$), the function $R(y)$ is continuous, non-negative and monotonically non-decreasing for all $y \geq 0$. If (x_{ii}, x_{ij}) is an interior solution of (20)-(21), then $x_{ij} = R(x_{ii})$. The function

$$2b\frac{(1-\delta)(A-\bar{c}+f(y)+kh_d(R(y)))}{2+\delta(n-3)}\Psi_1 f'(y) - 1$$

is continuous and strictly positive at zero due to the boundary conditions we imposed. It is strictly negative if y is sufficiently large. Therefore, there is $x^*_{ii} > 0$ such that

$$2b\frac{(1-\delta)(A-\bar{c}+f(x^*_{ii})+kh_d(R(x^*_{ii})))}{2+\delta(n-3)}\Psi_1 f'(x^*_{ii}) - 1 = 0.$$

Let $x^*_{ij} = R(x^*_{ii})$. By construction, (x^*_{ii}, x^*_{ij}) is a solution of the system (20)-(21). By condition (23) x^*_{ij} is strictly positive. Show now that every function $\pi_i(x_i, x^*_{-i})$, $i \in N$ is strictly concave if γ sufficiently large. Without loss of generality, assume that x_i belongs to a compact convex set $K_i \subset I\!R^{k+1}_+$. It is sufficient to verify that $D^2_{x_i}\pi_i(x_i, x^*_{-i})$ is negative definite for every $x_i \in K_i$. The matrix $D^2_{x_i}\pi_i(x_i, x^*_{-i})$ can be represented as a linear combination

$$D^2_{x_i}\pi_i(x_i, x^*_{-i}) = \frac{1}{\gamma}B_i(x_i, x^*_{-i}) + \frac{1}{\gamma^2}C_i(x_i, x^*_{-i}),$$

where matrices $B(.)$ and $C(.)$ do not depend on γ, and $B(.)$ is diagonal. Moreover, since

$$p_i(g,0) - c_i(g,0) = \frac{(A-\bar{c})(1-\delta)}{2+\delta(n-3)} > 0,$$

$\Psi_1 > 0$, $\Psi_2 > 0$, and the functions $f(\cdot)$ and $h(\cdot,\cdot)$ are strictly concave, all diagonal elements of the matrix $B(\cdot)$ are strictly negative. The functions $u \cdot B_i(x_i, x^*_{-i})u$ and $u \cdot C_i(x_i, x^*_{-i})u$ are continuous and bounded if $x_i \in K_i$ and $u \in \{v \in I\!R^{n+1} \mid ||v|| = 1\}$. Therefore it is possible to find a sufficiently large γ such that for all such x_i and u the inequality $\frac{1}{\gamma}|uB_i(x_i, x^*_{-i})u| > \frac{1}{\gamma^2}uC_i(x_i, x^*_{-i})u$ is satisfied. This implies that $D^2_{x_i}\pi_i(x_i, x^*_{-i})$ is negative definite for every x_i in K_i and all sufficiently large γ. Hence, a profile of investment levels $x^* = (x^*_{ii}, x^*_{ij})_{i\in N, ij\in g}$ is an interior symmetric Nash equilibrium.

To prove the uniqueness of a symmetric equilibrium, it is sufficient to show that the function $\tilde{F} : I\!R^2 \to I\!R^2$ defined by the right-hand side of the system (20)-(21) is strictly monotone on some compact set containing all (u^*, v^*) such that $x_{ii} = u^*$, $i \in N$, $x_{ij} = v^*$, $ij \in g^k$, is a symmetric equilibrium. This can be done by analogy with the first part of the proof.

As before, the condition that γ should be sufficiently large is essential. If returns from R&D activity are too high, no equilibrium exists in the model. ■

7.4 Four firm example

Suppose that $n = 4$. Since the factor $A - \bar{c}$ has no influence on the comparison of profit and investment levels, we can safely put it equal to 1. Then for a symmetric network g^k of degree k, the equilibrium R&D expenditure levels are

$$x_{ii} = \left(\frac{n\gamma}{\gamma^2(n+1)^2 - n - 2k(n-1)} \right)^2,$$

$$x^*_{ij} = \left(\frac{(n-1)\gamma}{\gamma^2(n+1)^2 - n - 2k(n-1)} \right)^2,$$

for $i \in N$, $ij \in g^k$, and the profit of firm i is

$$\pi_i = \frac{\gamma^2(4(25\gamma^2 - 16) - 36k)}{(50\gamma^2 - 12k - 8)^2}.$$

Furthermore, there are 7 asymmetric network architectures. Solving the corresponding systems of the first order conditions gives the following equilibrium in-house investment levels and profits. For the network $g = \{(12)\}$ the equilibrium R&D investment levels are

$$x_{11} = x_{22} = \left(\frac{4\gamma(5\gamma^2 - 4)}{5(25\gamma^4 - 42\gamma^2 + 8)} \right)^2,$$

$$x_{33} = x_{44} = \left(\frac{4\gamma(\gamma^2 - 2)}{25\gamma^4 - 42\gamma^2 + 8} \right)^2,$$

and the profits are

$$\pi_1 = \pi_2 = \frac{\gamma^2(4 - 5\gamma^2)^2(-1 + \gamma^2)}{(8 - 42\gamma^2 + 25\gamma^4)^2},$$

$$\pi_3 = \pi_4 = \frac{\gamma^2(2 - \gamma^2)^2(-16 + 25\gamma^2)}{(8 - 42\gamma^2 + 25\gamma^4)^2}.$$

For the network $g = \{(12), (13)\}$ the equilibrium investment levels are

$$x_{11} = \left(\frac{4\gamma(-1 + 5\gamma^2)}{52 + 25\gamma^2(-9 + 5\gamma^2)} \right)^2,$$

$$x_{22} = x_{33} = \left(\frac{4\gamma(-7 + 5\gamma^2)}{52 + 25\gamma^2(-9 + 5\gamma^2)} \right)^2,$$

$$x_{44} = \left(\frac{4\gamma(-13 + 5\gamma^2)}{52 + 25\gamma^2(-9 + 5\gamma^2)} \right)^2,$$

and the profits are

$$\pi_1 = \frac{(-34 + 25\gamma^2)(\gamma - 5\gamma^3)^2}{(52 + 25\gamma^2(-9 + 5\gamma^2))^2},$$

$$\pi_2 = \pi_3 = \frac{25\gamma^2(7 - 5\gamma^2)^2(-1 + \gamma^2)}{(52 + 25\gamma^2(-9 + 5\gamma^2))^2},$$

$$\pi_4 = \frac{\gamma^2(13-5\gamma^2)^2(-16+25\gamma^2)}{(52+25\gamma^2(-9+5\gamma^2))^2}.$$

For the "chain" network $g = \{(12), (23), (34)\}$ the equilibrium investment levels are

$$x_{11} = x_{44} = \left(\frac{20\gamma(-2+\gamma^2)}{82-240\gamma^2+125\gamma^4}\right)^2,$$

$$x_{22} = x_{33} = \left(\frac{4\gamma(-4+5\gamma^2)}{82-240\gamma^2+125\gamma^4}\right)^2,$$

and the profits are

$$\pi_1 = \pi_4 = \frac{625\gamma^2(-2+\gamma^2)^2(-1+\gamma^2)}{(82-240\gamma^2+125\gamma^4)^2},$$

$$\pi_2 = \pi_3 = \frac{\gamma^2(4-5\gamma^2)^2(-34+25\gamma^2)}{(82-240\gamma^2+125\gamma^4)^2}.$$

For the "star" network $g = \{(12), (13), (14)\}$ the equilibrium investment levels are

$$x_{11} = \left(\frac{4\gamma(2+5\gamma^2)}{64-240\gamma^2+125\gamma^4}\right)^2,$$

$$x_{22} = x_{33} = x_{44} = \left(\frac{20\gamma(-2+\gamma^2)}{64-240\gamma^2+125\gamma^4}\right)^2,$$

and the profits are

$$\pi_1 = \frac{\gamma^2(2+5\gamma^2)^2(-43+25\gamma^2)}{(64-240\gamma^2+125\gamma^4)^2},$$

$$\pi_2 = \pi_3 = \pi_4 = \frac{625\gamma^2(-2+\gamma^2)^2(-1+\gamma^2)}{(64-240\gamma^2+125\gamma^4)^2}.$$

For the "triangle" network $g = \{(12), (13), (23))\}$ the equilibrium R&D levels are

$$x_{11} = x_{22} = x_{33} = \left(\frac{4\gamma(-4+5\gamma^2)}{64-240\gamma^2+125\gamma^4}\right)^2,$$

$$x_{44} = \left(\gamma\left(\frac{1}{8-5\gamma^2} + \frac{9}{-8+25\gamma^2}\right)\right)^2,$$

and the profits are

$$\pi_1 = \pi_2 = \pi_3 = \frac{\gamma^2(-4+5\gamma^2)^2(-34+25\gamma^2)}{(64-240\gamma^2+125\gamma^4)^2},$$

$$\pi_4 = \frac{\gamma^2(-16+5\gamma^2)^2(-16+25\gamma^2)}{(64-240\gamma^2+125\gamma^4)^2}.$$

For the "flower" network $g = \{(12), (13), (14), (23)\}$ the equilibrium R&D levels are

$$x_{11} = \left(\frac{4\gamma(5\gamma^2-1)}{125\gamma^4-255\gamma^2+94}\right)^2,$$

$$x_{22} = x_{33} = \left(\frac{4\gamma(5\gamma^2-7)}{125\gamma^4-255\gamma^2+94}\right)^2,$$

$$x_{44} = \left(\frac{4\gamma(5\gamma^2 - 13)}{125\gamma^4 - 255\gamma^2 + 94} \right)^2,$$

and the profits are

$$\pi_1 = \frac{\gamma^2(5\gamma^2 - 1)^2(25\gamma^2 - 43)}{(125\gamma^4 - 255\gamma^2 + 94)^2},$$

$$\pi_2 = \pi_3 = \frac{\gamma^2(5\gamma^2 - 7)^2(25\gamma^2 - 34)}{(125\gamma^4 - 255\gamma^2 + 94)^2},$$

$$\pi_4 = \frac{25\gamma^2(5\gamma^2 - 13)^2(\gamma^2 - 1)}{(125\gamma^4 - 255\gamma^2 + 94)^2},$$

Finally, for the network $g = \{(12), (13), (14), (23), (24)\}$ the equilibrium R&D levels are

$$x_{11} = x_{22} = \left(\frac{4\gamma(-4 + 5\gamma^2)}{124 - 270\gamma^2 + 125\gamma^4} \right)^2,$$

$$x_{33} = x_{44} = \left(\frac{20\gamma(-2 + \gamma^2)}{124 - 270\gamma^2 + 125\gamma^4} \right)^2.$$

and the profits are

$$\pi_1 = \pi_2 = \frac{\gamma^2(5\gamma^2 - 4)^2(25\gamma^2 - 43)}{(125\gamma^4 - 270\gamma^2 + 124)^2},$$

$$\pi_3 = \pi_4 = \frac{25\gamma^2(\gamma^2 - 2)^2(25\gamma^2 - 34)}{(125\gamma^4 - 270\gamma^2 + 124)^2}.$$

For the equilibrium joint project research investments it is always the case that $x_{ij} = \frac{9}{16} x_{ii}$ for every $i \in N, ij \in g$. Assume $\gamma > 4$, a condition sufficient for the equilibrium existence. It is easy to check then that, if $|N_i(g)| > |N_j(g)|$, then $x_{ii}(g) > x_{jj}(g)$ and $\pi_i(g) > \pi_j(g)$. If additionally $|N_j(g)| > 0$, then $x_{is}(g) > x_{jt}(g)$ for every $s \in N_i(g)$ and $t \in N_j(g)$. For instance, for the "flower" network it holds that $x_{11} > x_{22} = x_{33} > x_{44}$. Consequently,

$$x_{12} = x_{13} = x_{14} > x_{21} = x_{31} = x_{32} = x_{23} > x_{41}.$$

For the profit levels it holds that $\pi_1(g) > \pi_2(g) = \pi_3(g) > \pi_4(g)$.

One can also observe that establishing a link always increases the profits of participating players while it decreases the profits of the rest of the firms. For example, let g be the "flower" network and consider the network $g' = \{(12), (13), (14), (23), (24)\}$, $g' = g + (24)$. It is easy to check that for $\gamma > 4$ $\pi_1(g') < \pi_1(g)$, $\pi_2(g') > \pi_2(g)$, $\pi_3(g') < \pi_3(g)$, $\pi_4(g') > \pi_4(g)$.

References

[1] R. Amir, I. Evstigneev and J. Wooders (2003), Non-cooperative R& D versus Cooperative R& D with Endogenous Spillover Rates, *Games and Economic Behavior*, 42, 183-207.

[2] A. Arora and A. Gambardella (1990), Complementarity and external linkages: the strategies of the large firms in biotechnology, *Journal of Industrial Economics*, 38, 361-379.

[3] A. Arora and A. Gamberdella (1994), Evaluating technological information and utilizing it: scientific knowledge, technological capability, and external linkages in biotechnology, *Journal of Economic Behavior and Organization*, 24, 91-114.

[4] S. Athey and A. Schmutzler (1995), Product and process flexibility in an innovative environment, *Rand Journal of Economics*, 26, 557-574.

[5] G. Baker, R. Gibbons and K. Murphy (2004), Strategic Alliances: Bridges between 'Islands of Conscious power', *mimeo*, MIT Sloan School.

[6] C. Ballester, A. Calvó-Armengol and Y. Zenou (2006), Who's who in networks. Wanted: the key player, *Econometrica*, 74, 5, 1403-1417.

[7] R. De Bondt and R. Veugelers (1991), Strategic investment with spillovers, *European Journal of Political Economy* 7, 345-66.

[8] Y. Bramoulle and R. Kranton (2007), Public goods in networks, *Journal of Economic Theory*, forthcoming.

[9] J. Brander and B. Spencer (1983), Strategic commitment with R&D: the symmetric case, *Bell Journal of Economics*, 14, 225-35.

[10] B. Cassiman and R. Veugelers (2002a), R&D cooperation an spillovers: some empirical evidence from Belgium, *American Economic Review*, 92, 4, 1169-1184.

[11] B. Cassiman and R. Veugelers (2002b), Complementarity in the innovation strategy: interal R&D, external technology acquisition and cooperation in R&D, mimeo, DTEW, KULeuven.

[12] W. M. Cohen and S. Klepper (1996), Firm size and the nature of innovation within industries: the case of process and product R&D, *Review of Economics and Statistics*, 78, 2, 232-243.

[13] I. Cockburn and R. Henderson (1998), Absorptive capacity, coauthoring behavior and the organization of research in drug discovery, *Journal of Industrial Economics*, 46, 157-182.

[14] C. d'Aspremont and A. Jacquemin (1988), Cooperative and non-cooperative R& D in duopoly with spillovers, *American Economic Review*, 78, 1133-1137.

[15] M. Delapierre and L. Mytelka (1998), Blurring boundaries: new inter-firm relationships and the emergence of networked, knowledge-based oligopolies, in M. G. Colombo (ed.), *The changing boundaries of the firm*, Routledge Press, London.

[16] F. Deroian and F. Gannon (2006), Quality-improving alliances in differentiated oligopoly, *International Journal of Industrial Organization* 24-3, 629-637.

[17] Y. L. Doz and G. Hamel (1998), *Alliance advantage. The art of creating value through partnering*, Harvard Business School Press: Boston, MA.

[18] A. Galeotti (2004), Consumer networks and search equilibria, *Tinbergen Institute Discussion Paper 2004-75.*

[19] S. Goyal and J. L. Moraga-González (2001), R&D networks, *Rand Journal of Economics*, Vol. 32, No. 4, 686-707.

[20] S. Goyal, J. L. Moraga-González and A, Konovalov (2006), Hybrid R&D, *mimeo*, Essex University and Erasmus University.

[21] J. Hagedoorn (2002), Inter-firm R&D partnerships: an overview of major trends and patterns since 1960, *Research-Policy.* May 2002; 31(4): 477-92.

[22] M. I. Kamien, E. Muller and I. Zang (1992), Research joint ventures and R&D cartels, *American Economic Review*, 82(5), 1293-1306.

[23] KZ) Kamien, M. and I. Zang (2000), Meet me half-way: Research Joint Ventures and Absorptive Capacity, *International Journal of Industrial Organization*, 18, 7, 995-1015.

[24] M. Katz (1986), An analysis of cooperative R&D, *Rand Journal of Economics*, 17, 527-543.

[25] T. Leahy and J. P. Neary (1997), Public policy towards R&D in oligopolistic industries, *American Economic Review*, 87(4), 642-662.

[26] D. Leonard-Barton (1992), Core capabilities and core rigidities: a paradox in managing new product development, *Strategic Management Journal* 13, 111-125.

[27] P. Milgrom and J. Roberts (1994), Comparing equilibria, *American Economic Review*, 84(3), 441-459.

[28] D. Mowery and N. Rosenberg (1989), *Technology and the pursuit of economic growth.* Cambridge University Press, Cambridge.

[29] N. Petts (1997), Building growth on core competences– A practical approach, *Lon Range Planning* 30-4, 551-61.

[30] J. M. Podolny and K. P. Page (1998), Network forms of organization, *Annual Review of Sociology*, 24, 57-76.

[31] W. Powell, K. Koput and L. Smith-Doerr (1996), Inter-organizational collaboration and the locus of innovation: networks of learning in biotechnology, *Administrative Science Quarterly*, 41, 116-145.

[32] C. K. Prahalad and G. Hamel (1990), The core competence of the corporation, *Harvard Business Review*, 68, 79-91.

[33] K. Suzumura (1992), Cooperative and non-cooperative R&D in oligopoly with spillovers, *American Economic Review*, 1307-1320.

[34] X. Vives (2005), Innovation and competitive pressure, *mimeo*, ICREA-UPF and INSEAD.