R&D NETWORKS

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Supplementary Appendix

Proof of Proposition 6 (the general γ case): We first consider the instability of the empty network. A comparison between equilibrium (A2) and deviating profits for general values of γ requires computing equilibrium profits for an interior Nash equilibrium. Solving the set of best-replies given in the main body of the text yields

$$e_i(g^e + g_{ij}) = \frac{(a-\overline{c})(n-1)(\gamma(n+1)-n)}{\gamma^2(n+1)^3 - \gamma(2n^3+n^2+n+2)+2n(n-1)}$$

$$e_l(g^e + g_{ij}) = \frac{(a-\overline{c})\, n\, [\gamma(n+1)-2(n-1)]}{\gamma^2(n+1)^3 - \gamma(2n^3+n^2+n+2)+2n(n-1)}$$

Note that $e_l(g^e + g_{ij}) > 0$ if and only if the condition $\gamma > 2(n-1)/(n+1)$ holds. Plugging these efforts into (A3) yields:

$$\pi_i(g^e + g_{ij}) = \frac{(a-\overline{c})^2\, \gamma\, [\gamma(n+1)-n]^2\, [\gamma(n+1)^2-(n-1)^2]}{[\gamma^2(n+1)^3 - \gamma(2n^3+n^2+n+2)+2n(n-1)]^2} \qquad (1)$$

With the help of software Mathematica 3.0 we can compare (1) and (A2) for general values of γ. Figure 1 depicts $\pi(g^e) - \pi_i(g^e + g_{ij})$ for $n \in [4, 30]$ and $\gamma \in [2, 100]$.[1] Upon the observation of this graph one sees that the empty network is generally not stable.

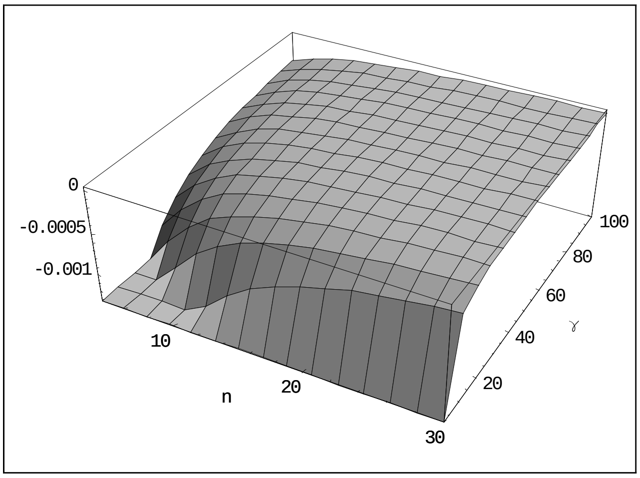

Figure 1: $\pi(g^e) - \pi_i(g^e + g_{ij})$

[1] Plotting $\pi(g^e) - \pi_i(g^e + g_{ij})$ rather than $\pi_i(g^e + g_{ij}) - \pi(g^e)$ allows for a better visualization.

We now consider the stability of the complete network.A comparison between (A5) and (A6) for general values of γ can be done with the help of Mathematica 3.0. Figure 2 exhibits the difference $\pi_i(g^{n-1} - g_{ij}) - \pi(g^{n-1})$ for $\gamma \in [4/25, 100]$ and $n \in [4, 30]$.[2] The graph suggests that the complete network is stable in general.

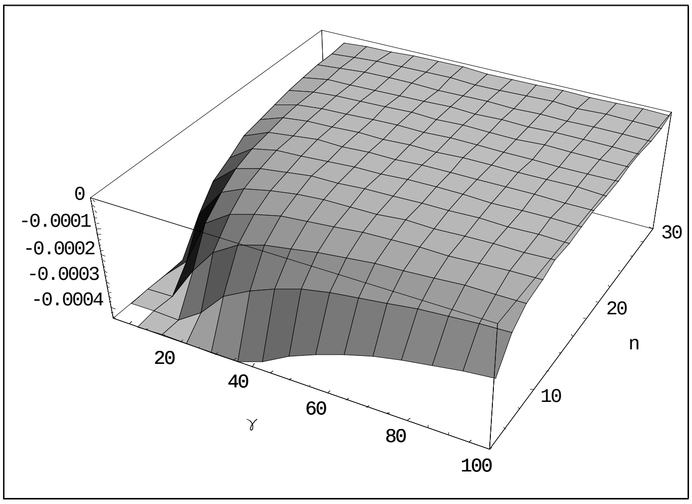

Figure 2: $\pi_i(g^{n-1} - g_{ij}) - \pi(g^{n-1})$

△

Proof of Proposition 9: Consider first the complete network g^c. The stability condition (i) is trivially satisfied because there are no links to be formed. Thus, g^c is stable if and only if no firm has a unilateral interest in breaking one of its ties (stability condition (ii)). To prove this, consider a firm in g^c that deviates by severing a link. Note that the resulting collaboration network is the star network g^s, and also that the deviating firm becomes a spoke firm in g^s. Then, the profits such firm would attain from this deviation would be given by $\pi_s(g^s)$ in (A10). We need to establish a comparison between equilibrium profits $\pi(g^c)$ in (A7), and deviating benefits $\pi_s(g^s)$. Since the mathematical formulae for this comparison are lengthy, we have chosen to compare them by means of Figure 5 (given in the paper). In this graph we have represented the different profits obtained by the distinct firms in the different networks of collaboration (we have normalized $a - \overline{c} = 1$, without loss of generality). It can be seen that the curve $\pi_s(g^s)$ lies below $\pi(g^c)$ for all spillover parameters β, which shows that no firm in the complete network would like to severe a link. Observe that this also proves that the star network is not stable.

Consider now the partially connected network g^p. To see that the stability condition (i) is satisfied, let us check that no linked firm in g^p can gain by breaking its tie. Notice that if a linked firm severs its link, the resulting network is the empty network g^e. The deviator would obtain profits $\pi(g^e)$ in (A16). We need to compare the profits $\pi_l(g^p)$ in (A15) with $\pi(g^e)$. Let us again resort to Figure 5 in the paper for this profits

[2] We have plotted $\pi_i(g^{n-1} - g_{ij}) - \pi(g^{n-1})$ rather than $\pi(g^{n-1}) - \pi_i(g^{n-1} - g_{ij})$ to improve visualization.

comparison. It is clearly seen that $\pi(g^e)$ lies below $\pi_l(g^p)$ for all β. Therefore, no firm in g^p has an unilateral incentive to break its collaborative link. Notice that this argument also proves that the empty network g^e is not stable because any pair of firms would deviate by forming a collaborative arrangement.

Let us now examine when stability condition (ii) is satisfied too. Notice first that the isolated firm always wishes to form a collaboration link with one of the linked firms in g^p. Indeed, if such a link is formed, then the isolated firm becomes a spoke firm in g^s, with profits $\pi_s(g^s)$ in (A10). In g^p the isolated firm obtains equilibrium benefits $\pi_i(g^p)$. To compare these two profits, we use Figure 5 again. The graph shows that the curve $\pi_i(g^p)$ lies below $\pi_s(g^s)$ for all β. This shows that the isolated firm in g^p always wishes to form a collaboration.

However, since links are pair-wise in our model of networks, we have to check whether or not a linked firm in g^p desires to form a link with the isolated firm. Notice that if the linked is formed, the resulting network is the star network g^s, and that the deviating firm would be at the hub of the network. Thus, we have to compare equilibrium profits $\pi_l(g^p)$ in (A14) with deviating profits $\pi_h(g^s)$ in (A11). Figure 5 shows that these two curves intersect approximately at the point $\beta_1 = 0.03649165$, and that deviating profits curve $\pi_h(g^s)$ lies thereafter below the equilibrium profits curve $\pi_l(g^p)$. This completes the proof.

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