Pricing, Consumer Search and Maturity of Internet Markets*

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Abstract

Despite the mixed empirical evidence, many economists still hold to the view that Internet will promote competition, thereby lowering prices and price dispersion and increasing welfare. This paper presents a search model that provides a different view. We analyze the market for a homogeneous good where some consumers are fully informed while others are not. Depending on the size of the purchase and the maturity of the market, the economy can be in one of three qualitatively different types of equilibria. Interestingly, comparative statics results are different for each of these equilibria. For example, a reduction in search cost may raise expected prices and price dispersion when consumers' search intensity is low, while the opposite occurs when consumers search intensity is high. The impact of the proliferation of search engines has also diverse consequences. The different comparative statics results may explain the controversial empirical evidence found so far.

Keywords: Internet, price dispersion, search, search agents

JEL Classification: D40, D83, L13

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1 Introduction

Throughout economic history, changes in technology have had a substantial impact on consumers' search and transportation costs and, consequently, on the size of the relevant market. One example is the progressive decline in transportation costs that historically has taken place through the use of faster means of transportation (sailing ships, machine ships, trains, cars, airplanes, etc.). This reduction in transportation costs has made it possible for consumers to search for products in markets that were previously beyond their horizon. In our present times, the increased use of Internet can be viewed in a similar way. Due to a reduction in search costs, Internet allows consumers to become active in markets where they were not active before. In this paper we take the Internet as our leading example, but the mechanisms we discover may be applied more broadly.

The general consensus among academics and leading businessmen seems to be that increased use of Internet will lower consumers' search costs and consequently intensify price competition. Internet is thus regarded as reducing commodity prices and promoting economic efficiency. Bakos (1997), for example, argues that[1]

> "electronic marketplaces are likely to move commodity markets closer to the classical ideal of a Walrasian auctioneer where buyers are costlessly and fully informed about seller prices. ...we expect that electronic marketplaces typically will sway equilibria in commodity markets to favor the buyers, will promote price competition among sellers, and will reduce sellers' market power."

Moreover, Jeffrey P. Bezos of Amazon.com has recently argued that:

> "We on the Internet should be terrified of customers because they are loyal to us right up to the point that someone else offers a better service. The power shifts to the consumer on-line."[2]

Recent empirical studies investigating Internet market efficiency do not unambiguously support this view. Concerning *price levels* in electronic markets, some studies, notably Lee (1997) and Lee *et al.* (1999) on cars, and Bailey (1998) on books and CDs, find that they are higher than corresponding prices in conventional markets. Other analyses, Friberg *et al.* (2000) and Clay *et al.* (2000a) on books, report that prices in on-line and physical stores are similar. Finally, Brynjolfsson and Smith (1999) in a study on books and CDs

[1] See also Bailey and Bakos (1997), and Vulkan (1999: F69-70).

[2] Quoted in Victoria Griffith (1999).

find prices to be lower in electronic markets.[3] On *price dispersion,* the effect of moving markets on-line also seems to be ambiguous empirically. Bailey (1998) and Brynjolfsson and Smith, (1999) find that books and CDs price dispersion is not lower on-line than in traditional outlets, whereas Bailey (1998) notes that the opposite holds true for software. Other studies also emphasize that on-line prices exhibit substantial dispersion (Baye and Morgan, 2000; Brown and Goolsbee, 2000; Clay *et al.*, 2000a; Clemons *et al.,* 1998).[4]

In this paper we present an endogenous search model that identifies basic market conditions that may help explain and organize the mixed empirical evidence. By doing so, we develop a more cautious view on the economic implications of electronic marketplaces than the consensus view expressed in the above quotes. We study an economy where firms produce a homogeneous product and compete to sell their good to a number of consumers. There are two types of consumers: k *informed* buyers who incur no search cost, and m *less-informed* consumers who have positive search cost c. We shall refer to the percentage of informed consumers in the economy as *market maturity,* the more informed consumers the more mature the market. For simplicity, in the main part of the paper all consumers have identical willingness-to-pay $v > c$. We will refer to v/c as the *relative size of the purchase.* Firms simultaneously choose prices and announce them on the web. Less-informed consumers decide how many searches to make before they know the prices firms set. These buyers can also abstain from searching when they expect search not to be worthwhile. In equilibrium, consumer expectations are fulfilled. Hence, the interaction between firms and buyers is modeled as a simultaneous move game, where (in equilibrium) consumers' search activity impinges on the prices quoted by firms, and the price setting behavior of firms influences buyers' search activity.

There may be three types of price dispersed equilibria in our economy. (i) An equilibrium with *low* search intensity, i.e., where less-informed consumers randomize between one search and no search. (ii) An equilibrium with *moderate* search intensity, i.e., where less-informed buyers search for one price. And (iii) a *high* search intensity equilibrium, i.e., where consumers randomize between one search and two searches.[5] Relative size of the purchase and maturity of the Internet market determine buyers' search incentives. A first lesson our analysis yields is the existence of a correlation between buyers' search propensity and product's value. That is, a high search intensity equilibrium arises when the relative size of the purchase is large, ceteris paribus. In contrast, a low or a moderate search intensity equilibrium results when search cost is relatively important compared to

[3] Recent reports by consultant companies Ernst & Young, Forrester Research and Goldman Sachs have also reached opposite conclusions (OECD, 1999: 73).

[4] See Smith *et al.* (2000) for a recent overview of some of these empirical findings and a discussion of the different methodologies employed.

[5] The low intensity search equilibrium has been disregarded by the search literature (see below).

the size of the purchase. This observation is in line with the empirical findings of Eric Johnson *et al.* (2000), who report that travel shoppers search substantially more than CD shoppers and book purchasers.[6] Since higher search activity is naturally associated with lower price levels one would expect lower price-cost margins for products whose value is greater. This may explain an empirical result found by Clay *et al.* (2000b), namely that bestseller books in on-line markets are generally more discounted than books at random. From the viewpoint of our analysis, search is more intense in the market for bestsellers as buyers value them more.[7]

Our second main observation pertains to comparative statics results. We find that the impact of moving markets on-line on market transparency critically depends on product characteristics and market maturity. We pinpoint the nature of this dependence in more detail below. Our major concern is to ascertain the impact of a change in search technology on expected prices and price dispersion. It turns out that this influence depends on the manner improved search technology is modelled. One may argue that moving markets on-line reduces the cost that less-informed buyers must incur to obtain price quotations. Alternatively, one may reason that moving markets on-line augments the number of informed consumers. Consider first a decline in search cost c, ceteris paribus. When the size of the purchase is relatively small and the market is immature, expected prices rise as c falls because more consumers who do not compare prices enter the market. Price dispersion, in contrast, increases when search cost is high to begin with, and falls otherwise. On the other hand, when the relative size of the purchase is great, expected prices and price dispersion unambiguously decline with c because more buyers exercise price comparisons in this case. In between, when the relative size of the purchase is not very low and not very high, prices and price dispersion are unaffected by a change in c.

The comparative statics impact of an increase in market maturity is also sensitive to product's value. When the size of the purchase is low and the market is immature to begin with an increase in the number of informed buyers leaves expected prices and price dispersion unchanged. When the market is quite mature to begin with, in contrast, an

[6]Incipient research accounting for consumers search behaviour on the Web finds that 70% of CD shoppers, 70% of book purchasers and 36 % of travel buyers were observed to visit just one site. The main issue to observe is that it is easy to visit an Internet site, but it still takes some time to find a particular book or CD and to order it. For a few dollars purchase, it does not seem to be worth to search over and over again.

[7]Sorensen (2000) presents similar evidence in a study of price dispersion in *physical* markets for prescription drugs. He finds that mean prices and price dispersion are sensitive to the characteristics of the drug therapy. In particular, long (multi-month) prescriptions mean prices and price dispersion are lower as compared to regular drug therapies. Seen from our model viewpoint, the market for long treatment drugs would be in an equilibrium with relatively lots of search compared to the market for regular prescriptions.

increase in the number of informed buyers reduces average prices and price dispersion. Finally, when the size of the purchase is large, expected prices and price dispersion are (again) insensitive to changes in the number of informed buyers.

We believe that these results may help explain why, e.g., Bailey (1998) find substantially higher on-line mean prices and price dispersion for books and CDs compared to off-line, while this is not the case for software. For books and CDs the size of the purchase is small relative to software. As outlined above, the larger the relative product's value the more prices decrease as a response to a decline in c. Moreover, the on-line software and hardware market seems to be quite mature compared to books and CDs (Ellison and Ellison, 2001) and this also contributes to explain the observed differences. Furthermore, market maturity may explain why Bailey (1998) found higher on-line prices for books and CDs sold in 1996 and 1997 while Brynjolfsson and Smith (1999) found the opposite for a sample of books sold in 1998 and 1999. The same applies to the study of Brown and Goolsbee (2000) who argue that there is no evidence that Internet usage reduced prices of life insurance policies before comparison websites emerged and proliferated. When market maturity is low, we note that the pro-competitive effects of marginal increases of informed buyers are entirely offset by economizing behaviour of less-informed buyers. As market matures search propensity rises and further increases in λ result in greater transparency.

There is a vast literature on consumer search.[8] The papers that come closest to ours are by Burdett and Judd (1983) and Stahl (1989).[9] Burdett and Judd show that equilibrium price dispersion may occur in *competitive* markets when consumers randomize between searching for two prices and searching for one price, in a non-sequential fashion. Our paper also studies non-sequential search and the price dispersed equilibria we obtain are similar in nature. Our model is, however, more suited for studying the implications of the growth of Internet use for the following two reasons. First, we allow for the presence of fully informed consumers (without search cost). This implies that, in contrast to Burdett and Judd, our model does not have a monopoly price equilibrium and that all the equilibria of our model exhibit price dispersion. More importantly, the proliferation of electronic search agents on the Internet makes it important to study the impact of a growing number of fully informed consumers. Second, Burdett and Judd assume that each consumer makes

[8] See Stiglitz (1989) for a survey of the early literature. Stahl (1996) also surveys critically the most relevant models of consumer search emphasizing whether they fall under the Stackelberg paradigm (buyers know the market distribution of prices and then search) or under the Nash paradigm (buyers ignore the market distribution of prices before search). The present model falls into the second category of work. Other recent contributions include Anderson and Renault (1999) with product heterogeneity, and McAfee (1995) with multiproduct sellers.

[9] An early paper with a model similar to ours is Varian (1980); however, he did not consider endogenous non-sequential consumers search. Fershtman and Fishman (1992) study a dynamic version of Burdett and Judd (1983).

at least one search. As argued above, consumers are steadily entering electronic markets and thus we think that in the context of the Internet discussion, it is important to allow for the possibility that consumers (previously) were not searching, or searching with low intensity.

Stahl (1989) studies a search model where firms price strategically and less-informed buyers search sequentially, i.e., they first observe one price and then decide whether or not to observe a second price, and so on. The first price quotation is observed for free, which implies that every buyer makes at least one search. Moreover, under the optimal sequential search rule a consumer continues searching if, and only if, the observed prices are above a certain reservation price. This implies that in equilibrium no firm charges prices above the reservation price so that, in fact, in every equilibrium buyers search only once. The unique equilibrium of Stahl's model displays properties that are qualitatively similar to properties of our moderate search intensity equilibrium. The sequential search model cannot, however, explain why there is more search in some markets than in others, a feature that is captured in our model by the presence of low and high search intensity equilibria. Using the qualitative properties of these two equilibria allows us to explain some of the empirical findings concerning electronic marketplaces.[10]

Recently, a few papers have appeared that reflect in a more formal way on the changes in the nature of price competition due to the introduction of Internet. These papers, Bakos (1997) and Lal and Sarvary (1999) among others, study heterogeneous goods markets, as they argue that in homogeneous markets it is evident that a reduction in search cost will intensify price competition. As already indicated, our paper argues that this is not necessarily the case. Among other things, Bakos (1997) shows that a reduction in search cost may actually lead to an increase in prices and profits when the market is not fully serviced and when search cost is relatively high. This is similar to one of the results we obtain. Lal and Sarvary (1999) consider a model where commodities have both digital and non-digital attributes and firms compete along multiple channels, Internet and conventional stores. Their main result is that firms may increase their prices by using Internet to sell to satisfied clients who do not want to search for other firms.

The remainder of the paper is organized as follows. Section 2 describes the simple search model we use in the main body of the paper. We characterize the equilibria of the model in Section 3 and give the comparative statics analysis in Section 4. Section 4 also describes in more detail how our results can be used to explain the empirical phenomena

[10] Morgan and Manning (1985) have derived optimal search strategies which combine features of the fixed-sample-size search strategy and the sequential search strategy. Our search method seems more adequate when price observations come after some delay. One of the authors has been recently looking for an apartment in Rotterdam. In this market one must first register at a number of Real State agents electronically to be able to receive offers.

described above. Section 5 discusses the robustness of our results with respect to changes in the main modelling assumptions. We conclude in Section 6. Proofs of some formal statements are given in the appendix.

2 The Model

Consider a market for a homogeneous good. There are $m+k$ buyers who wish to purchase at most a single unit of the good. A number k of the consumers search for prices costlessly. We will refer to these buyers as *informed.* The other m buyers must pay search cost $c > 0$ to observe a price quotation. These buyers, referred to as *less-informed,* may decide to obtain several price quotations, say $n \geq 0$, in which case they incur search cost equal to nc. Let $\lambda = k/(m+k)$ denote the proportion of informed buyers in the market, $0 < \lambda < 1$. We shall refer to λ as the *maturity* of the market. All consumers are fully rational, i.e., informed ones buy the good from the lowest priced store, while less-informed ones acquire it from the store with the sampled lowest price. The maximum price any buyer is willing to pay for the good is $v > c$. We shall refer to v/c as the *relative size of the purchase.*

On the supply side of the market there are $N \geq 2$ firms. Firms produce the good at constant returns to scale and their identical unit production cost is normalized to zero, without loss of generality.

Firms and buyers play a simultaneous move game. An individual firm chooses its price taking price choices of the rivals as well as consumers' search behavior as given. Buyers form conjectures about the distribution of prices in the market and decide how many prices to observe before purchasing from the store with the lowest observed price. Let $F(p)$ denote the distribution of prices charged by a firm. Let μ_n denote the probability with which a less-informed buyer searches for n price quotations. Let $\pi(\cdot)$ be the profits attained by a firm. We only consider symmetric equilibria. An equilibrium is a pair $\{F(p), \{\mu_n\}_{n=0}^{N}\}$ such that (a) $\pi(p) = \overline{\pi}$ for all p in the support of $F(p)$, (b) $\pi(p) \leq \overline{\pi}$ for all p, and (c) $\{\mu_n\}_{n=0}^{N}$ describes the optimal search behavior of less-informed buyers given that their conjectures about the price distribution are correct.

3 Equilibria

In this section and the next we shall concentrate on the case where $N = 2$. Section 5 briefly discusses the N firm case. Taking as given firms' choices of prices, a less-informed consumer must decide whether to visit no store, one store, or two stores. Then, a less-informed buyer's strategy is a probability distribution over these three events. Informed consumers, in contrast, observe all prices at no cost. Our first observation is that there are no equilibria in which less-informed buyers search for both stores' prices, or do not search at all. Moreover, we note that there are no equilibria where less-informed consumers mix between no search and two searches.

Lemma 1 *Assume* $\lambda > 0$ *and* $c > 0$. *Then, equilibria where either (i)* $\mu_2 = 1$, *or (ii)* $\mu_0 = 1$, *or (iii)* $\mu_0 > 0$ *and* $\mu_0 + \mu_2 = 1$ *do not exist.*

Proof. (i) Suppose $\mu_2 = 1$. Then firms would charge Bertrand prices, i.e., $p_i = 0$, $i = 1, 2$. But if this is so, since $c > 0$, less-informed buyers would search only once. Thus, $\mu_2 = 1$ cannot be part of an equilibrium. (ii) Suppose $\mu_0 = 1$. Again, firms would charge zero prices and therefore less-informed buyers would find it beneficial to search once. (iii) Suppose $\mu_0 > 0$ and $\mu_0 + \mu_2 = 1$. Also in this case firms would charge Bertrand prices and therefore less-informed buyers would find it profitable to search once. ■

Lemma 1 implies that the following alternatives exhaust the equilibrium possibilities of less-informed buyers' search behavior: (a) they search with *low intensity*, i.e., $0 < \mu_1 < 1$, $\mu_0 + \mu_1 = 1$, (b) they search with *moderate intensity*, i.e., $\mu_1 = 1$, and finally (c) they search with *high intensity*, i.e., $0 < \mu_1 \leq 1$, $\mu_1 + \mu_2 = 1$.[11]

It is also straightforward to see that there does not exist a symmetric equilibrium where both stores charge a particular price with positive probability.

Lemma 2 *Given the search behavior of less-informed consumers, if* $F(p)$ *is an equilibrium price distribution, then it is atomless. Hence, there is no pure strategy equilibrium.*

Proof. By way of contradiction, suppose in equilibrium a price is charged with positive probability. Then, the chance that such a price is the minimum price charged in the market would be positive. Consequently, a small reduction in that price by one of the firms would be beneficial as it would attract all informed consumers with positive probability. It then follows that the only price that could be proposed as having positive probability is $p = 0$. However, Lemma 1 states that $\mu_1 \leq 1$ in any equilibrium, which implies that there is a positive probability that less-informed buyers obtain only one price quotation. As a result, a single firm would find it beneficial to increase such price. ■

Lemma 2 shows that equilibria exhibit price dispersion. In what follows, we study firms' equilibrium behavior under the various less-informed buyers' behavioral hypotheses.

Case a: Low search intensity

[11] It may also happen that there exists an equilibrium in which less-informed buyers mix between no search, one search and two searches, i.e., $\mu_0 > 0$, $\mu_1 > 0$, $\mu_2 > 0$ and $\mu_0 + \mu_1 + \mu_2 = 1$. This equilibrium is, however, non-generic in the sense that it imposes restrictions on the set of exogenous parameters that are only satisfied in a null set (see footnote 18 below for details).

First consider that less-informed buyers randomize between searching for one price quotation and not searching at all, i.e., $\mu_0 > 0$, $\mu_0 + \mu_1 = 1$. Let $F(p_i)$ be the probability that a firm charges a price that is smaller than p_i. The expected payoff to firm i of charging price p_i when the rival chooses a random pricing strategy according to the cumulative distribution $F(\cdot)$ is in this case

$$\pi_i(p_i, F(\cdot)) = p_i \left[\frac{m\mu_1}{2} + k(1 - F(p_i))\right]. \tag{1}$$

This profit expression is easily understood. Firm i obtains a per consumer profit of p_i. The expected demand faced by a firm stems from the two different groups of consumers. Firm i attracts the k fully informed consumers when it charges a lower price than the rival, which happens with probability $1 - F(p_i)$. The firm also serves the m less-informed consumers whenever they actively search for one price, with happens with probability μ_1, and, particularly, when they visit its store, which occurs with probability one half.

In equilibrium, a firm must be indifferent between charging any price in the support of F. The maximum price a firm will ever charge is v since no buyer who observed a price above his/her reservation price would acquire the good. Further, the upper bound of the price distribution cannot be lower than v because a firm charging the upper bound would gain by slightly raising its price. Thus, it must be the case that $F(v) = 1$, and $F(p) < 1$, for all $p < v$. Any price in the support of F must then satisfy $\pi_i(p_i, F(\cdot)) = \pi_i(v)$, i.e.,

$$p_i \left[\frac{(1-\lambda)\mu_1}{2} + \lambda(1 - F(p_i))\right] = \frac{(1-\lambda)\mu_1 v}{2}, \tag{2}$$

Solving this equation yields

$$F(p) = \frac{2\lambda + (1-\lambda)\mu_1}{2\lambda} - \frac{(1-\lambda)\mu_1}{2\lambda}\frac{v}{p}. \tag{3}$$

Since F is a distribution function there must be some $\underline{p}$ for which $F(\underline{p}) = 0$. Solving for $\underline{p}$ one obtains the lower bound of the price distribution $\underline{p} = (1-\lambda)\mu_1 v / \left(2\lambda + (1-\lambda)\mu_1\right)$.

A mixed strategy over the support $\underline{p} \leq p \leq v$ according to the cumulative distribution function F specified above is an equilibrium if and only if consumers are indeed indifferent between searching for one price and not searching at all. Therefore, it must be the case that $v - E[p] - c = 0$, where E denotes the expectation operator.[12] In other words, the

[12] It must further be checked that it is not profitable for consumers to search more than once, i.e., that $v - E[\min\{p_1, p_2\}] - 2c < 0$. We prove this in the Appendix (see Fact 0).

following condition must be satisfied:[13]

$$1-\frac{(1-\lambda)\mu_1}{2\lambda}\ln\left(\frac{2\lambda+(1-\lambda)\mu_1}{(1-\lambda)\mu_1}\right)=\frac{c}{v} \tag{4}$$

Let us denote the left-hand-side of equation (4) as $\Phi(\mu_1;\lambda)$. The following facts about the function Φ are proved in the Appendix:

Fact 1: $\frac{d\Phi}{d\mu_1}<0$

Fact 2: $\frac{d^2\Phi}{d\mu_1^2}>0$

Fact 3: $\lim_{\mu_1\to 0}\Phi(\mu_1)=1$

Fact 4: $\Phi(1)=1-\frac{(1-\lambda)\ln\left(\frac{1+\lambda}{1-\lambda}\right)}{2\lambda}>0.$

Facts 1 to 4 allow us to represent condition (4) as shown in Figure 1. The decreasing and convex curve represents Φ as a function of μ_1. The flat line is just the right-hand side of (4). An equilibrium consumer's randomization probability is thus given by the intersection of curve Φ and c/v. Facts 1 to 4 also enable us to state that these two curves intersect at most once.

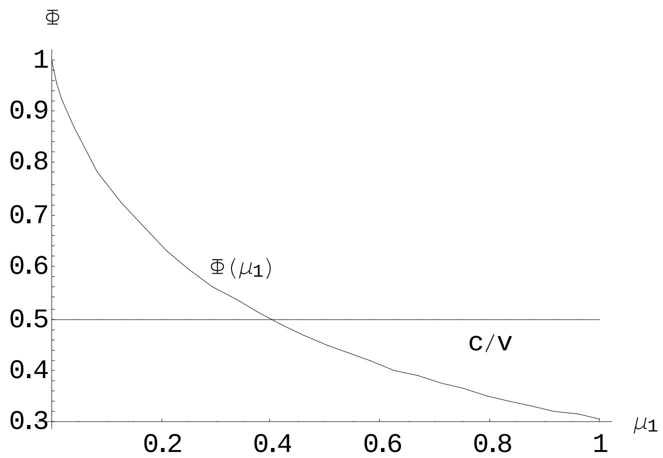

Figure 1: Buyers randomize between one search and no search (λ=1/3)

The following proposition summarizes these findings:

Proposition 1 *[Low search intensity equilibrium]*

Let $1\geq c/v>\Phi(1;\lambda)$*. Then an equilibrium exists where less-informed consumers randomize between searching for one price with probability* μ_1^**, and not searching at all*

[13] For current and future reference, let $H(p)=a-bv/p$ be a distribution function in the support $bv/a\leq p\leq v$, with $a-b=1$. Then $E[p]=bv\ln[a/b]$, $E[\min\{p_1,p_2\}]=2bv(1-b\ln[a/b])$ and $E[\max\{p_1,p_2\}]=2bv[(1+b)\ln[a/b]-1]$.

with probability $1 - \mu_1^*$, *where* $\mu_1^* \in (0,1)$ *solves (4) and firms randomly select prices from the set* $\left[(1-\lambda)\mu_1^* v / \left(2\lambda + (1-\lambda)\mu_1^*\right), v\right]$ *according to the cumulative distribution function (3). There is at most one such equilibrium.*

Case b: High search intensity

We now turn to the case where less-informed consumers randomize between searching for one price quotation with probability μ_1, and searching for two price quotations, with the remaining probability $1 - \mu_1$.[14] The expected payoff to firm i of charging price p_i when the rival chooses a random pricing strategy according to the cumulative distribution function $G(\cdot)$ and less-informed consumers search as specified above is

$$\pi_i(p_i, G(p)) = p_i \left[\frac{m\mu_1}{2} + (k + m(1-\mu_1))(1 - G(p_i))\right]. \quad (5)$$

This profit function can be easily interpreted. Firm i makes a per consumer profit of p_i. The firm's expected number of consumers is $m\mu_1/2 + (k + m(1-\mu_1))(1 - G(p_i))$. The first summand of (5) stems from the less-informed buyers when they search for only one price, which happens with probability μ_1. A firm attracts these $m\mu_1$ consumers with probability one half. The second summand of (5) comes from the fully informed consumers as well as from the less-informed consumers when they search for two prices, which happens with probability $1 - \mu_1$. A firm attracts these consumers, a total of $k + m(1-\mu_1)$, when its price is lower than the rival's one, which occurs with probability $1 - G(p_i)$.

In equilibrium, a firm must be indifferent between charging any price in the support of G. The same arguments employed above allow us to argue that $G(v) = 1$, and $G(p) < 1$, for all $p < v$. Any price in the support of G must satisfy $\pi_i(p_i, G(\cdot)) = \pi_i(v)$, i.e.,

$$p_i \left[\frac{(1-\lambda)\mu_1}{2} + (1 - (1-\lambda)\mu_1)(1 - G(p_i))\right] = \frac{(1-\lambda)\mu_1 v}{2}. \quad (6)$$

We can solve equation (6) for $G(p)$ to obtain

$$G(p) = \frac{2 - (1-\lambda)\mu_1}{2(1 - (1-\lambda)\mu_1)} - \frac{(1-\lambda)\mu_1}{2(1 - (1-\lambda)\mu_1)} \frac{v}{p}. \quad (7)$$

A mixed strategy over the support $\underline{\underline{p}} \leq p \leq v$ according to the cumulative distribution function G specified above is an equilibrium if and only if less-informed buyers are indeed

[14] For ease of exposition, we maintain the notation used so far in the sense that μ_1 denotes the probability with which less-informed buyers search for one price. However, unlike in case a above, $1 - \mu_1$ denotes now the probability with which these consumers search for two prices, i.e., μ_2.

indifferent between searching for only one price and searching for two prices.[15] Therefore it must be the case that $v - E[p] - c = v - E[\min\{p_1, p_2\}] - 2c$. In other words, the following must hold (see footnote 13):

$$\frac{(1-\lambda)\mu_1}{2(1-(1-\lambda)\mu_1)}\left[\frac{1}{1-(1-\lambda)\mu_1}\ln\left(\frac{2-(1-\lambda)\mu_1}{(1-\lambda)\mu_1}\right)-2\right]=\frac{c}{v}. \tag{8}$$

To analyze less-informed consumers' equilibrium condition (8), let us denote the left-hand-side of this equation as $\Gamma(\mu_1;\lambda)$. The following facts, proved in the Appendix, are useful in what follows:

Fact 5: $\Gamma(1) = \frac{(1-\lambda)}{2\lambda}\left(\frac{1}{\lambda}\ln\left[\frac{1+\lambda}{1-\lambda}\right]-2\right) > 0$.

Fact 6: $\lim_{\mu_1 \to 0}\Gamma(\mu_1) = 0$

Fact 7: $\left.\frac{d\Gamma(\cdot)}{d\mu_1}\right|_{\mu_1=1} > 0$ if and only if $\lambda > \overline{\lambda}$, where $\overline{\lambda}$ is defined as the solution to the equation $\ln\left[\frac{1+\lambda}{1-\lambda}\right] - \frac{2\lambda(2+\lambda)}{2+\lambda-\lambda^2} = 0$.[16]

Fact 8: $\frac{d^2\Gamma(\cdot)}{d\mu_1^2} < 0$.

Facts 5 to 8 illustrate that the shape of function $\Gamma(\mu_1;\lambda)$ depends on parameters. When the percentage of informed consumers is large enough, Fact 7 together with Fact 8 indicate that Γ is an increasing and concave function of μ_1, as represented in Figure 2a ($\lambda = 0.83$). A high search intensity equilibrium is given by the intersection of curve Γ with the line c/v. It is readily seen that there is at most one such equilibrium.

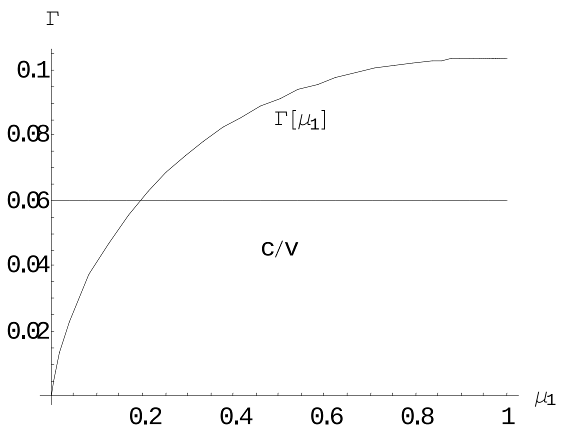

Figure 2a

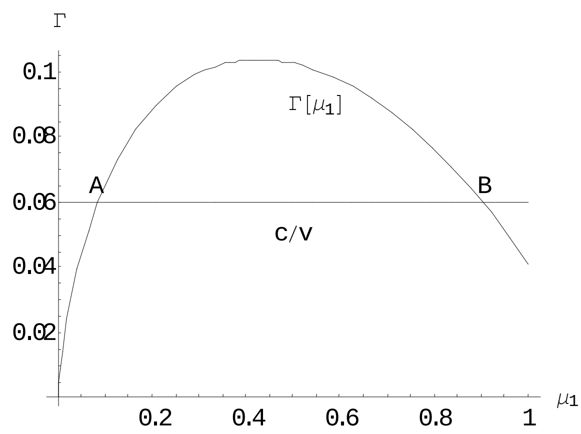

Figure 2b

[15] The lower bound of this distribution is readily derived by setting $G(\underline{p}) = 0$ and solving for $\underline{p}$. We must additionally be sure that no consumer gains by making no search, i.e., it must be the case that $v - E[p] - c > 0$. This is trivially satisfied.

[16] It can be shown numerically that there is a unique solution to this equation, which is approximately $\overline{\lambda} = 0.634816$.

When, in contrast, the percentage of informed consumers is small, Facts 5 to 8 imply that the curve $\Gamma(\mu_1)$ is first increasing and then decreasing. The strict concavity of Γ ensures that there is some unique μ_1 for which Γ reaches a maximum. The shape of Γ when λ is relatively small is illustrated in Figure 2b ($\lambda = 0.16$). This graph shows that there may be either one equilibrium or two equilibria, depending on the relative size of the search cost. Points A and B in the figure depict two equilibria. However, only the equilibrium denoted by point A is stable. In a neighborhood to the left of point B, the expected gains to buyers from searching for two prices instead of searching for one price are larger than the cost of an extra search. Therefore, a small perturbation around point B so that $\mu_1 < \mu_1^*$ would lead consumers to search more intensively, a movement away from B. Similarly, a small perturbation so that $\mu_1 > \mu_1^*$ would lead consumers to search less intensively. These observations show the instability of the equilibrium represented by the point B.[17] A similar argument shows that the equilibrium depicted by point A is a stable equilibrium. In what follows, for our comparative statics results, we will concentrate on this stable equilibrium.

Let $\overline{\Gamma}$ denote the maximum value of $\Gamma(\mu_1)$, i.e., $\overline{\Gamma} = \max_{\mu_1 \in (0,1]} \Gamma(\mu_1)$. From Fact 5 it follows that $\overline{\Gamma} > 0$.

Proposition 2 *[High search intensity equilibrium]*

Let $\overline{\Gamma} \geq \frac{c}{v} \geq 0$. Then either one equilibrium or two equilibria exist where less-informed buyers randomize between searching for one price with probability μ_1^* and searching for two prices with probability $1 - \mu_1^*$, where $\mu_1^* \in (0,1)$ is the solution to (8) and firms randomly select prices from the set $\left[(1-\lambda)\,\mu_1^* / \left(2 - (1-\lambda)\,\mu_1^*\right), v\right]$ according to the cumulative distribution function (7). There is at most one such equilibrium that is stable.

Case c: Moderate search intensity

We finally turn to the case where less-informed consumers search for one price with probability one. Derivations for this case are similar to the computations above and therefore omitted. The equilibrium distribution function for this case can be obtained by plugging $\mu_1 = 1$ in either of the cases a and b above discussed. A mixed strategy distribution function $H(p) = (1+\lambda)/2\lambda - (1-\lambda)v/2\lambda p$ is part of an equilibrium if less-informed consumers indeed find it optimal to search only once. Therefore, the following conditions must hold: (i) $v - E[p] - c \geq 0$ and (ii) $v - E[p] - c \geq v - E[\min\{p_1, p_2\}] - 2c$. Conditions (i) and (ii) relate to the functions $\Phi(1;\lambda)$ and $\Gamma(1;\lambda)$, respectively. In the Appendix we prove that:

Fact 9: $\Phi(1;\lambda) > \Gamma(1;\lambda)$.

[17] See Fershtman and Fishman (1992).

Then, using footnote 13 we can state the following result:

Proposition 3 *[Moderate search intensity equilibrium]*
Let $\Phi(1;\lambda) \geq \frac{c}{v} \geq \Gamma(1;\lambda)$. *Then an equilibrium exists where less-informed buyers search for one price surely and firms randomly select prices from* $[(1-\lambda)v/(1+\lambda), v]$ *according to the cumulative distribution function obtained by substituting* $\mu_1 = 1$ *in (3). There is at most one such equilibrium.*

We are now ready to provide the complete characterization of stable equilibria in our model.[18] The next result states that, depending on market maturity and relative value of the purchase, there may be either a single equilibrium or two equilibria. This statement follows from Facts 9 and 10, proved in the Appendix, and Propositions 1, 2 and 3:

Fact 10: There exists $\widehat{\lambda} < \overline{\lambda}$ such that $\Phi(1) < \overline{\Gamma}$ if and only if $\lambda < \widehat{\lambda}$.

Theorem 1 *[A] Suppose* $1 > \lambda > \overline{\lambda}$. *Then:*
(A.1) For $1 > \frac{c}{v} \geq \Phi(1)$, *there is a single equilibrium in which less-informed buyers search with low intensity (Proposition 1).*
(A.2) For $\Phi(1) \geq \frac{c}{v} \geq \Gamma(1)$, *there is a unique equilibrium in which less-informed consumers search with moderate intensity (Proposition 3).*
(A.3) Finally, for $\Gamma(1) \geq \frac{c}{v} > 0$, *there is a unique equilibrium where consumers search with high intensity (Proposition 2).*
[B] Suppose $\overline{\lambda} > \lambda > \widehat{\lambda}$. *Then:*
(B.1) For $1 > \frac{c}{v} \geq \Phi(1)$, *there is a single low search intensity equilibrium.*
(B.2) For $\Phi(1) \geq \frac{c}{v} \geq \overline{\Gamma}$, *there is a unique moderate search intensity equilibrium.*
(B.3) For $\overline{\Gamma} \geq \frac{c}{v} \geq \Gamma(1)$, *there are two equilibria: (i) a moderate search intensity equilibrium, and (ii) a stable high search intensity equilibrium.*
(B.4) Finally for $\Gamma(1) \geq \frac{c}{v} > 0$, *there is a single stable high search intensity equilibrium.*
[C] Suppose $\widehat{\lambda} > \lambda > 0$. *Then:*
(C.1) For $1 > \frac{c}{v} \geq \overline{\Gamma}$, *there is a single low search intensity equilibrium.*
(C.2) For $\overline{\Gamma} \geq \frac{c}{v} \geq \Phi(1)$ *there are two equilibria: (i) a low search intensity equilibrium, and (ii) a stable high search intensity equilibrium*
(C.3) For $\Phi(1) \geq \frac{c}{v} \geq \Gamma(1)$, *there are two equilibria: (i) a moderate search intensity*

[18] It is now appropriate to argue that an equilibrium where less-informed buyers mix between no search, one search and two searches does not exist generically. To see this notice that such an equilibrium must satisfy (i) $E(p) = v - c$ and (ii) $E(p) - E[\min\{p_1, p_2\}] = c$. Using footnote 13, these conditions can be rewritten as (i) $b = (v-2c)/2c$ and (ii) $b\ln((1+b)/b) = (v-c)/v$, where $b = m\mu_1/2(k+m\mu_2)$. These two equalities hold only if $(v-2c)\ln[v/(v-2c)]/2c = (v-c)/v$,i.e., a restriction that is not satisfied generically by the exogenous parameters c and v.

equilibrium, and (ii) a stable high search intensity equilibrium.
(C.4) Finally for $\Gamma(1) \geq \frac{c}{v} > 0$*, there is a single stable high search intensity equilibrium.*

The three possibilities A, B and C outlined in this theorem are illustrated in Figures 3a, 3b and 3c, respectively. These graphs bring Figures 1, 2 and 3 together for each case. Figure 3a exhibits a mature market where $\lambda = 0.8$. In this case, for large search cost parameters, for instance c_1/v, there is a unique equilibrium where less-informed buyers search with low intensity. As search cost falls, these consumers find it beneficial to search more intensively. Indeed, for intermediate search cost levels, for example c_2/v, the only equilibrium is such that less-informed buyers search for one price with probability one. Finally, when search cost is sufficiently low, for instance c_3/v, buyers search with high intensity in equilibrium. The bold lines depict the loci of equilibrium points.[19]

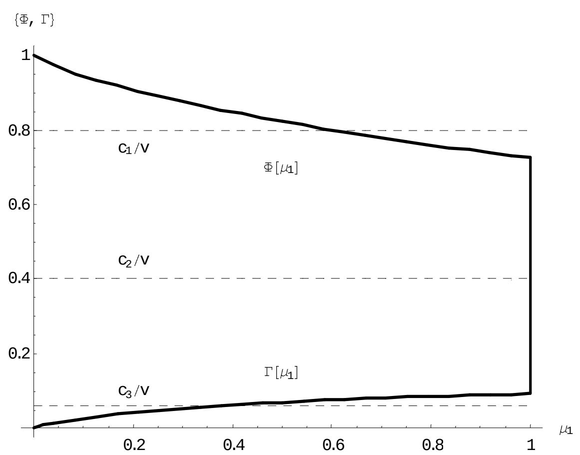

Figure 3a

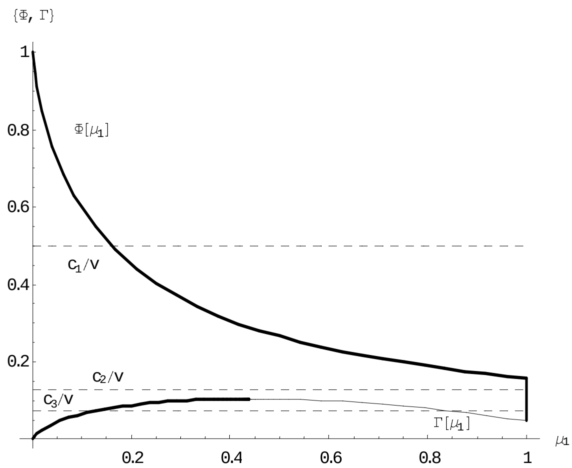

Figure 3b

Figure 3b illustrates a relatively immature market ($\lambda = 0.16$). In this case, for high and intermediate search cost, for example c_1/v, there is a single equilibrium where consumers search with low intensity. However, for a relatively low search cost c_2/v, there is a single moderate intensity equilibrium. For c_3/v, there are two equilibria: one in which buyers search with moderate intensity and another in which they search with high intensity. Finally, for extremely low search costs, there is just one equilibrium with high search intensity. As before, the bold lines depict the loci of equilibrium points.

Figure 3c illustrates the last case mentioned in Theorem 1. For a very immature market ($\lambda = 0.035$) the main distinctive feature arising is that for quite low search costs,

[19] To help to interpret these graphs recall that $1 - \mu_1 = \mu_2$ in a high search intensity equilibrium, while $1 - \mu_1 = \mu_0$ in a low search intensity equilibrium.

for instance c_2/v, there may be an equilibrium with low search intensity and an equilibrium with high search intensity.

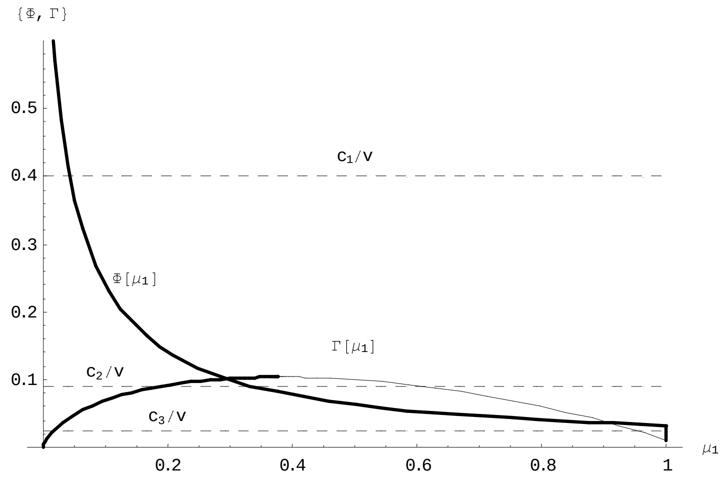

Figure 3c

Theorem 1 yields the important insight that whether the economy is in a low, moderate or high search intensity equilibrium depends on two critical model parameters: (i) the size of the purchase compared to the search cost c/v, and (ii) the maturity of the market λ. As explained in the Introduction this result explains why buyers' propensity to search is higher for some products than for others (Johnson *et al.,* 2000). Although the nature of expected prices and price dispersion is analyzed below in detail, on the basis of Theorem 1, one may already argue that it is natural to find price-cost margins and price dispersion differing across product categories (Clay *et al.*, 2000b), and also that earlier empirical studies on a particular market when market maturity is still low may yield very different results as compared to more recent analyses (Bailey, 1998 versus Brynjolfsson and Smith (1999); see also Brown and Goolsbee, 2000).

4 Comparative statics

In this section we study the impact of changes in the parameters of the model. Our primary interest lies in ascertaining the effect of a reduction in search cost on (i) expected prices, (ii) price dispersion, (iii) firm profits, and (iv) social welfare. As a measure of price dispersion we take the expected difference between the maximum price and the minimum price. There is a mathematical property that makes this measure attractive in our setting:

$$E[\max\{p_1, p_2\}] - E[\min\{p_1, p_2\}] = 2(E[p] - E[\min\{p_1, p_2\}]) \tag{9}$$

Moreover, many of the aforementioned empirical papers have used this measure of dispersion. Considering price variance instead complicates the analysis without bringing about additional insights.

As argued above, a change in the search technology can be captured in our model either by a decline in c, or by an increase in the number of informed consumers k, maintaining the total number of buyers $m + k$ constant. Our analysis yields the important insight that the impact of a change in search technology depends on whether it is modelled as a decline in c or as an increase in λ. This suggests that the long-run influence of Internet on market efficiency depends on whether search engines proliferate and become central places of information exchange for the general public, or, instead, Internet just reduces unit search costs. In addition we shall see that the influence of lower search costs also depends on the intensity with which less-informed buyers search in equilibrium, and hence on the primitives of the market (Theorem 1).

The comparative statics results of a change in search technology are summarized in Table 1. In addition, the table also presents the impact of an increase in the size of the market, modelled either by an increase in the number of less-informed buyers or by an increase in the number of informed buyers. In the table expected prices, price dispersion, profits and welfare are represented by p^e, p^d, π, and W respectively. An upwards (downwards) arrow means that the variable under consideration increases (falls); two arrows together means that the variable may increase or decrease and this depends on the value of the parameter taken to begin with; the symbol "–" means that the variable remains constant. In the discussion below explaining the results summarized in the table, we focus on explaining the empirical studies. Therefore, we concentrate on the implications of a change in search technology on expected prices and price dispersion. The implications for firms' profits and social welfare are discussed in footnotes. Details about the influence of k and m can be found in Janssen and Moraga (2000).

	Low				Moderate				High			
	p^e	p^d	π	W	p^e	p^d	π	W	p^e	p^d	π	W
$\downarrow c$	$\uparrow$	$\uparrow\downarrow$	$\uparrow$	$\uparrow$	–	–	–	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow\downarrow$
$\uparrow \lambda$	–	–	$\uparrow$	$\uparrow$	$\downarrow$	$\uparrow\downarrow$	$\downarrow$	$\uparrow$	–	–	–	$\uparrow$
$\uparrow k$	–	–	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	–	$\uparrow$	–	–	$\uparrow$	$\uparrow$
$\uparrow m$	–	–	–	–	$\uparrow$	$\uparrow\downarrow$	$\uparrow$	$\uparrow$	–	–	$\uparrow$	$\uparrow$

Table 1: Summary of Comparative Statics Results

The following facts prove useful in the discussion that follows. Let p be distributed according to $H(p) = a - bv/p$ in the support $bv/a \leq p \leq v$, with $a - b = 1$. Then:

Fact 11: $\frac{dE[p]}{db} > 0$

Fact 12: $\frac{dE[\min\{p_1,p_2\}]}{db} > 0.$

Fact 13: $\frac{d(E[\max\{p_1,p_2\}]-E[\min\{p_1,p_2\}])}{db} > 0$ if and only if $b < \overline{b} \simeq 0.28763$.

From equations (3) and (7) it follows that b is a measure of the equilibrium fraction of consumers who search only once relative to the number of consumers who search twice. In other words, it measures the fraction of active buyers over which firms have monopoly power.

4.1 The effects of a reduction in search cost c

Expected prices:

Consider first that less-informed consumers search with low intensity in equilibrium (Proposition 1). To ascertain the impact of a decline in c on expected price it is enough to observe that $v - E[p] - c = 0$ in a low search intensity equilibrium. Consequently, the price that less-informed buyers expect increases as c falls! To understand the intuition behind this surprising result let us point out that, as Figure 1 shows, the intensity with which less-informed consumers search in this type of equilibrium rises as c falls. Note further that these consumers are precisely those who do not exercise "price comparisons," and thus they are prepared to accept higher prices. Consequently, a fall in c gives sellers incentives to charge higher prices with higher probabilities, which in turn raises expected prices. Our next observation has to do with the expected price faced by the informed consumers, i.e., $E[\min\{p_1, p_2\}]$. It turns out that increased activity of the less-informed buyers exerts a negative externality on the informed ones. Indeed, a decline in c also increases the expected minimum price. This is readily seen by employing Fact 12 and noting that $b = (1-\lambda)\,\mu_1/2\lambda$ in this equilibrium.[20]

Suppose now that less-informed consumers search with moderate intensity in equilibrium (Proposition 3). Observe that a small change in c leaves less-informed buyers' behavior unchanged. Consequently expected prices remain constant.[21]

[20] It is straightforward to see now how firms' profits and social welfare are affected by a decline in c in a low search intensity equilibrium. Note that profits are proportional to the intensity with which less-informed buyers search: $\pi = v\mu_1/2$ (see (1)). Then it is obvious that firms' profits rise with c. Social welfare, $W = kv + (v-c)m\mu_1$ in this case, also increases. Note, however, that the additional surplus generated by a fall in c is fully captured by the firms!

[21] It is also obvious that firms' profits remain constant with c in a moderate search intensity equilibrium. Social welfare, $W = kv + m(v-c)$ in this case, however rises because less-informed consumers incur lower costs to discover prices.

Finally, consider that less-informed consumers search with high intensity in equilibrium (Proposition 2). Figures 2 and 3 show that a decline in c raises the probability with which less-informed buyers search for two prices, i.e., $d\mu_1/dc > 0$. Since "price comparisons" are more frequent as c falls, price competition between firms is fostered. Thus, one would expect expected prices to fall with c. Indeed $b = (1-\lambda)\,\mu_1/[2(1-(1-\lambda)\,\mu_1)]$ in this type of equilibrium and it is readily seen that $db/d\mu_1 = (1-\lambda)\,/[2(1-(1-\lambda)\,\mu_1)^2)] > 0$. This together with Facts 10 and 11 prove that mean prices for both types of consumers decrease with c.[22]

Price dispersion:

Consider first that consumers search with low intensity in equilibrium. As previously noted, a decline in c increases the shopping activity of the less-informed buyers μ_1. Since $b = (1-\lambda)\,\mu_1/2\lambda$, this implies that b increases as c decreases. Using Fact 13, it is easy to see that price dispersion rises as c decreases if and only if b is small. The fraction of consumers b who search only once can be small for two reasons: first, when the relative number of informed buyers is large or, second, when many less-informed consumers do not find it worthwhile to search (which occurs when c is relatively large). Recall that price dispersion arises due to the existence of two groups of consumers who are asymmetrically informed, and that firms make substantial profits basically by expropriating the less-informed consumers. A search cost reduction brings more price-insensitive buyers to the market, which gives incentives to raise prices. When there are many informed consumers an individual firm will seriously try to keep them by and thus will randomize prices to balance these two conflicting interests. As a consequence, price dispersion increases. In contrast, when there are few informed consumers firms do not need to care so much about them and thus price dispersion does not necessarily increase with a fall in c.

Suppose now that less-informed buyers search with moderate intensity in equilibrium. As observed previously, a small change in c leaves less-informed consumers' behavior

[22]Note that equilibrium profits in a high search intensity equilibrium are $\pi = vm\mu_1/2$ (see (5)). As expected, profits decline with c because μ_1 does so. Interestingly, a fall in c does not increase social welfare necessarily. The reason is that the increased search activity of the less-informed consumers may be excessive from a social welfare viewpoint. This can be seen by noting that welfare is $W = kv + m(v - 2c + \mu_1 c)$ in this case. Thus $dW/dc = m(-2 + \mu_1 + cd\mu_1/dc)$. The sign of this derivative depends on the parameters of the model. To provide an instance in which welfare declines after a decrease in c consider a market setting where $k = 280$, $m = 20$ and $v = 1$. Consider that search costs are initially $c = 0.055$. In the equilibrium with high search intensity, less-informed buyers would search for two prices with probability 0.075646, and social welfare would be 298.817, approximately. A reduction in search cost from 0.055 to 0.054 would increase the incentives to search of the less-informed consumers, who would thus search for two prices with higher probability, 0.105677 approximately. From a social perspective, however, the increased search intensity brought about by the search cost reduction is excessive and welfare attains a lower level, 298.806 approximately.

unchanged. Consequently price dispersion remains constant.

Finally, consider that less-informed consumers search with high intensity in equilibrium. To ascertain the impact of c on price dispersion notice that $E[p]-E[\min\{p_1, p_2\}] = c$ in this case. From (9) it follows that $E[\max\{p_1, p_2\}] - E[\min\{p_1, p_2\}] = 2c$. Consequently, price dispersion unambiguously decreases as the search cost falls.

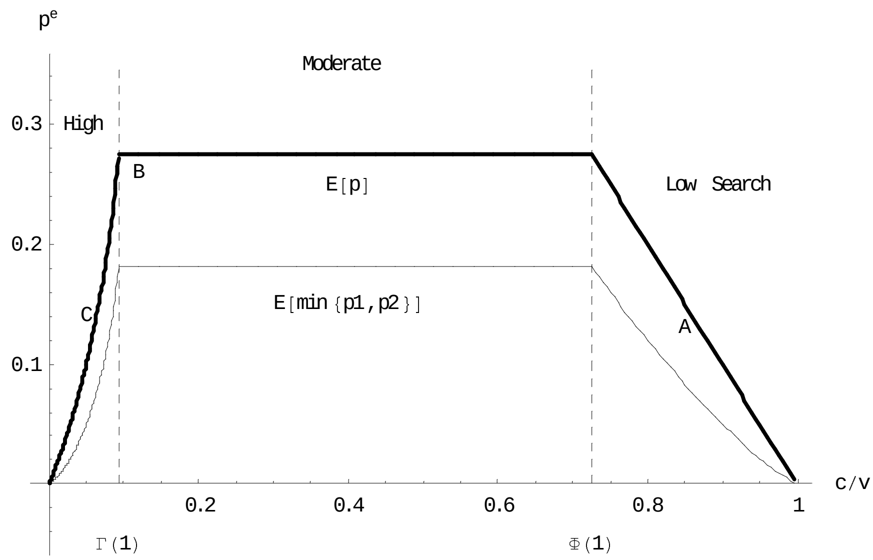

Figure 4: The impact of lower search cost c on a mature market.

Overview of partial results:

Previous observations regarding the impact of a search cost reduction on expected prices and price dispersion are nicely gathered in Figures 4 and 5. In Figure 4 we have simulated an economy where the number of informed consumers is large ($\lambda = 0.8$). This graph depicts expected price (thicker curve) and expected minimum price (thinner curve) as a function of relative search cost. As noted above, for any given value of c/v, the vertical distance between the two lines constitutes a proportional measure of actual price dispersion. When search cost lies in the interval $(\Phi(1), 1)$, less-informed consumers search with low intensity in equilibrium. In this parametrical region, expected prices and price dispersion increase as the search cost falls, as indicated above. For search cost values in between $\Gamma(1)$ and $\Phi(1)$ it pays less-informed consumers to search for one price with probability one. In this parametrical region a decline in search cost has no impact on expected prices and price dispersion. Finally, when search cost lies in the interval $(0, \Gamma(1))$, it pays less-informed consumers to search for two prices with positive probability. As the search cost fall in this parametrical region the frequency with which price comparisons occur in the market increases, which fosters competition between the firms. Consequently, expected prices and price dispersion decline. Eventually, as search cost approaches zero mean prices converge to marginal cost (zero). Figure 4 can also be interpreted bearing in mind that expected prices are highest when b, the ratio of the number of consumers who search once to the number of consumers who search twice, is highest. It readily follows that (for a given maturity of the market) expected prices are highest in the moderate search intensity equilibrium.

It might be argued that moving markets on-line does not marginally decrease search costs but does so dramatically. Upon observing Figure 4 one sees that even a dramatic change in search cost does not necessarily decrease price dispersion and mean prices. To see this, consider for instance a search cost reduction from level A to level B. Even though this is an enormous decline of about 80%, mean prices and price dispersion increase. In contrast consider a smaller cost fall from level B to level C of about 60%. This cost reduction decreases mean prices and price dispersion. These observations illustrate that one cannot conclude that a large cost reduction will enhance market efficiency without taking into account the market status quo.

Figure 5 complements our previous exposition by presenting equilibrium expected prices for a case where the market is more immature ($\lambda = 0.2$). Again expected price is depicted by the thicker curve while expected minimum price is represented by the thinner curve. As before, when search cost lies in the interval $(\Phi(1), 1)$, less-informed buyers search with low intensity in equilibrium. It is easy to see that as c falls mean prices rise while price dispersion first increases and then decreases. When the economy is in a moderate search intensity equilibrium, mean prices and price dispersion remain constant with c. Finally, when less-informed consumers search with high intensity in equilibrium, mean prices and price dispersion decrease as c declines. Figure 5 has the distinctive feature that for some parametrical values there are multiple equilibria. In particular, when search cost lies in the interval $(\Gamma(1), \overline{\Gamma})$ one may have a moderate search equilibrium with high expected prices, or a high search intensity equilibrium with lower mean prices. (This the reason why Figure 5 exhibits a discontinuity in expected prices; see also Figure 2b.) Thus, less-informed consumers face a coordination problem. It pays to search intensively if the others do so, otherwise it is worth to search only once. As argued above, observing price dispersion and expected prices at the search cost levels A, B and C one can state that a dramatic decline in search cost need not increase market transparency.

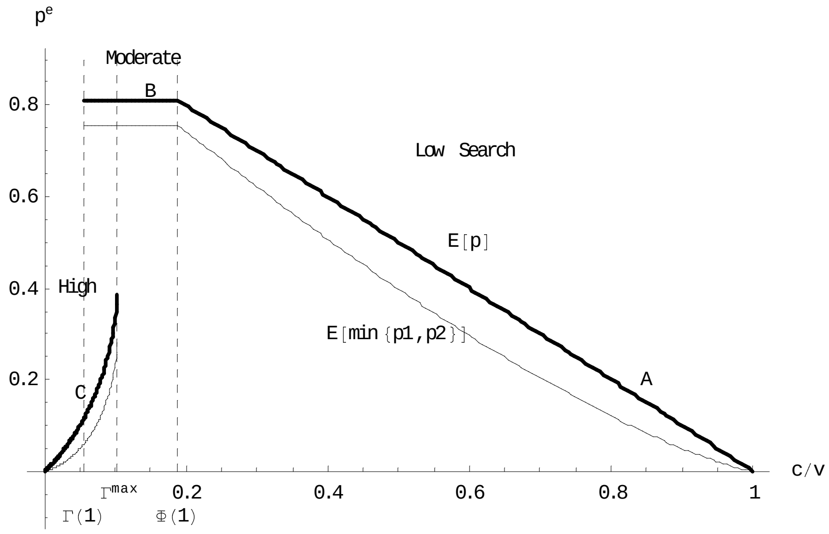

Figure 5: The impact of lower search c on an immature market.

As explained in the Introduction, we see these results as a possible explanation why Bailey (1998) find higher on-line prices for books and CDs compared to off-line, while almost equal on-line and off-line prices for software. For books and CDs the relative size of the purchase is small compared to the search cost and in 1996, 1997 these electronic markets were probably immature. Thus, one would expect a low or a moderate intensity equilibrium to be in place. In such a case, the comparison between the on-line channel and the off-line one would be captured by a decline in c and thus would produce higher or equal expected prices. In contrast, for software one may reasonably expect a moderate or a high search intensity equilibrium, since product value and market maturity seem to be greater for this product. This may explain why software on-line prices were not found to be much higher compared to their off-line counterparts and, further, why price dispersion was found to be lower. Brynjolfsson and Smith (1999) found books and CDs prices to be lower on-line than off-line. Since they took a sample of prices much later than Bailey did, it may be the case that they were looking at a much more mature market. This market may be in a high search intensity equilibrium and thus a decline in search cost would lower expected prices and price dispersion.

4.2 An increase in the relative number of informed buyers λ

Another manner to capture changes in search technology is to consider that the number of informed consumers in the economy increases. This comparative statics exercise gathers the effects of the proliferation of electronic search agents that automatically search for prices in Internet markets. Bearing in mind the definition of λ, we next study how our economy is affected by an increase in λ.

Expected Prices:

Consider first that less-informed consumers search with low intensity in equilibrium. To ascertain the influence of λ on expected prices it is sufficient to observe that $v - E[p] - c = 0$ in equilibrium, and to note that since neither v nor c varies expected price must remain constant. It is worth to disentangle the incidence of changes in λ on consumers search incentives and firms pricing decisions. Notice first that an increase in the number of informed consumers has in principle a pro-competitive effect. Ceteris paribus, firms would tend to charge lower prices with higher probability. One can apply the implicit function theorem to equation (4) to obtain

$$\frac{d\mu_1}{d\lambda} = \frac{\mu_1}{\lambda(1-\lambda)} > 0, \tag{10}$$

which means that an increase in λ results in an increase in the search intensity of the less-informed consumers μ_1. This is obviously due to the fact that more informed consumers in the market makes searching more attractive for the less-informed consumers, as the former buyers put pressure on firms to reduce prices. This in turn implies that the number of price insensitive buyers rises, which gives firms incentives to augment prices. Interestingly, these two opposite forces cancel out so that expected prices remain constant! We also observe that the price informed consumers expect, i.e., $E[\min\{p_1, p_2\}]$, does not change with λ either. To see this, note that in this case $b = (1-\lambda)\mu_1/2\lambda$. Using (10), just a little algebra shows that $db/d\lambda = 0$, which implies that the expected minimum price remains constant too![23]

Suppose now that less-informed buyers search with moderate intensity. Notice that an increase in λ does not alter the behaviour of less-informed buyers. Consequently, since more consumers exercise price comparisons, the economy definitely becomes more competitive. The price expected by the less-informed consumers obviously falls. This is readily seen using Fact 11 and noting that $b = (1-\lambda)/2\lambda$ in this case, and therefore $db/d\lambda = -1/2\lambda^2 < 0$. Analogously, using Fact 12, one proves that the price expected by the informed buyers also declines.[24]

[23] Since expected prices remain constant and more less-informed consumers are active when λ rises, one would expect firms' profits to increase. Indeed, $\pi = v(m+k)(1-\lambda)\mu_1/2$ in this case. The impact of an increase in λ is given by $d\pi/d\lambda = v(m+k)\left[(1-\lambda)d\mu_1/d\lambda - \mu_1\right]/2$. Using (10) we obtain $d\pi/d\lambda = v(m+k)\mu_1(1-\lambda)/2\lambda > 0$, i.e., an increase in λ increases firms' profits!

We finally observe that social welfare increases with λ due to the increased activity of the less-informed consumers (note that $m\mu_1$ rises). To see this, observe that $W = (m+k)\left[\lambda v + (v-c)(1-\lambda)\mu_1\right]$ in this case. Using (10) we can compute $dW/d\lambda = (m+k)[v + (v-c)\mu_1(1-\lambda)/\lambda] > 0$. Note, however, that a great deal of the increase in social welfare is captured by the firms.

[24] From the previous remarks it is readily understood that firm profits, $\pi = v(m+k)(1-\lambda)/2$ in this

Finally, consider that less-informed buyers search with high intensity in equilibrium. As noticed above, when there are more informed consumers in the market firms tend to charge lower prices with higher probability. Applying the implicit function theorem to equation (8), one obtains

$$\frac{d\mu_1}{d\lambda} = \frac{\mu_1}{1-\lambda} > 0, \tag{11}$$

which means that less-informed buyers search less intensively as λ rises. The decreased search activity of these consumers implies that the frequency with which price comparisons occur diminishes. Surprisingly, it turns out that the behavior of less-informed buyers compensates away the pressure that the presence of relatively more informed consumers puts on the firms to cut prices. To see this, note that $b = (1-\lambda)\mu_1/(2(1-(1-\lambda)\mu_1))$ in this case. Using (11), it is easily seen that $db/d\lambda = 0$. This implies that neither the price expected by less-informed consumers who search only once, nor the price expected by buyers who search twice changes![25]

Price Dispersion:

Consider first the economy is in a low search intensity equilibrium. As shown above the pro-competitive effects of an increase in λ are offset by the increased activity of less-informed consumers. This in turn implies that changes in λ have no incidence on price dispersion!

Suppose now that less-informed buyers search with moderate intensity in equilibrium. As shown above $db/d\lambda < 0$, which implies that the influence of λ on price dispersion is ambiguous (Fact 13). When the number of informed consumers is large initially then an increase in λ decreases price dispersion. In contrast, when λ is small initially then an increase in λ augments price dispersion. The intuitive argument has been explained above.

Finally consider the economy is in a high search intensity equilibrium. In this case the pro-competitive effects of an increase in λ are entirely offset by a decrease in the search

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case, decline with λ. In contrast, social welfare, $W = (m+k)[v - c(1-\lambda)]$ here, rises with λ since fewer consumers incur search costs.

[25] An increase in λ also turns out not to have influence on firms' profits. Indeed $\pi = (m+k)v(1-\lambda)\mu_1/2$ in a high search intensity equilibrium and it is readily checked that $d\pi/d\lambda = 0$. In regard to social welfare, note that $W = (m+k)[\lambda v + (1-\lambda)(v - 2c + \mu_1 c)]$ in this case. A little algebra shows that $dW/d\lambda = 2(m+k)c > 0$, which implies that welfare rises with λ. Two factors explain this. First observe that fewer consumers pay search costs when λ rises. Second, less-informed consumers search less intensively and the economy further saves in search costs. Remarkably, the additional surplus is captured by the consumers.

activity of the less-informed buyers. Consequently changes in λ have no impact on price dispersion.

Overview of partial results:

Previous discussions regarding the influence of changes in the number of informed buyers on expected prices and price dispersion are nicely gathered in Figures 6 and 7. Figure 6 simulates an economy where product's valuation is relatively low compared to search cost ($c/v = 0.5$). The figure depicts expected price (thicker curve) and expected minimum price (thinner curve) as a function of λ. Recall that for any given λ, the vertical distance between the two curves provides a measure of price dispersion. Observe that when λ lies in the interval $(0, \Phi(1))$ market equilibrium exhibits low search intensity. In this case, the pro-competitive effects of an increase in λ are entirely offset by the search behaviour of less-informed buyers and so expected prices and price dispersion remain constant. When market maturity is large enough, $\Phi(1) < \lambda < 1$, less-informed consumers search with moderate intensity in equilibrium and prices decline smoothly to marginal cost as λ rises.

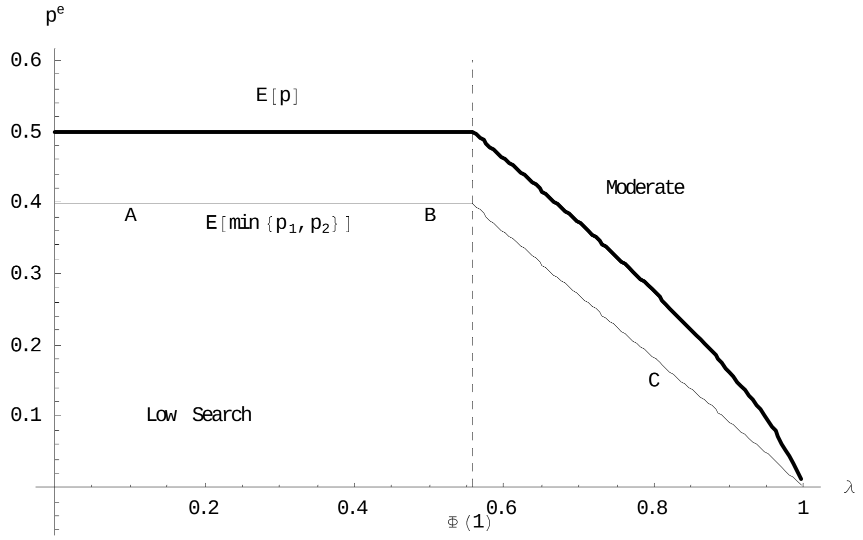

Figure 6: The impact of λ when the relative size of the purchase is small.

In Figure 7 we have simulated an economy where search cost is low ($c/v = 0.05$). We first observe that there may be multiple equilibria in this case. For instance, for a λ level depicted by point A there are two equilibria: one with low search intensity and one with high search intensity. For λ levels given by the points B and C there are also two equilibria: one with moderate search intensity and one with high search intensity. Finally, for a λ level given by point D there is a single moderate search equilibrium. Observe that when there is multiplicity of equilibria a high search intensity equilibrium leads to lower expected prices. Thus, less-informed consumers face a coordination problem. If a high search intensity equilibrium prevails in the economy expected prices and price dispersion are insensitive to changes in λ, as noted above. It may also happen that the nature of buyers' coordination is such that they do not search so much. Then, if λ lies in the interval $(0, \Phi(1))$, the economy would be in a low search intensity equilibrium and changes in λ have no influence on expected prices and price dispersion. If, instead, λ lies in between $\Phi(1)$ and $\Gamma(1)$, consumers would search with moderate intensity and an increase in λ would unambiguously reduce mean prices. However, it would decrease price dispersion only if λ is sufficiently great to begin with. In summary Figures 6 and 7 illustrate that while price levels do not increase with respect to λ, it may very well be the case that price dispersion rises.

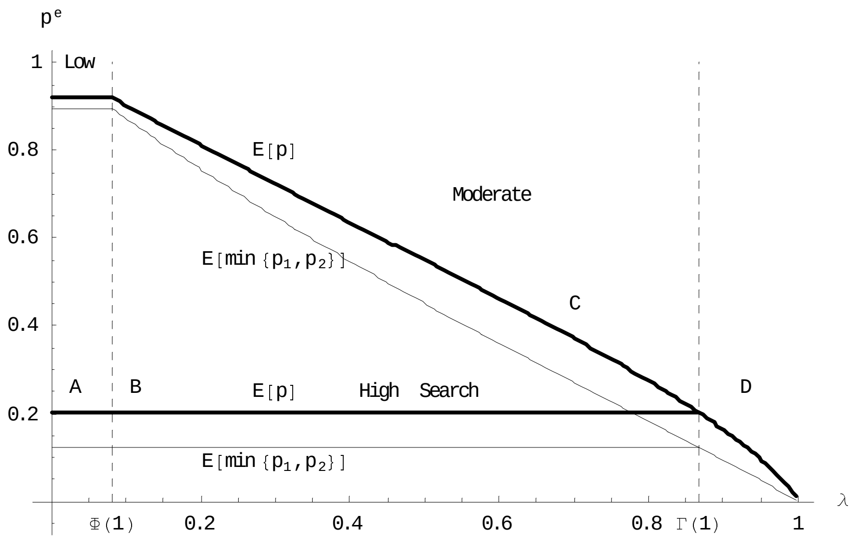

Figure 7: The impact of λ when the relative size of the purchase is large.

A final remark regarding Figure 7 is that a high search intensity equilibrium may exhibit greater price dispersion than a low or moderate search intensity equilibrium. Therefore mean prices and price dispersion need not be perfectly correlated.

5 Discussion

In the previous two sections, we have studied the possible equilibrium configurations of a relatively simple model and their comparative statics properties. We have also argued how the main insights of this model can be interpreted in the light of the debate concerning the implications of Internet usage for market efficiency. In this section we discuss whether these insights hold true in different settings, namely where there are more than two firms operating in the market, where there is consumer heterogeneity, and where less-informed buyers' search rule is sequential.

Large Markets

We first study how the above results would change if we allowed for N firms to be active in the market. To this end, consider first a low search intensity equilibrium. With N firms, the expected payoff to firm i of charging price p_i when competitors choose a random pricing strategy according to the cumulative distribution function $F(p)$ is given by

$$\pi_i(p_i, F(p)) = p_i \left[\frac{m\mu_1}{N} + k(1 - F(p_i))^{N-1}\right]. \tag{12}$$

Equation (12) is interpreted in a similar manner as equation (2). When there are N firms in the market, since informed consumers buy from the lowest priced store, a firm has a chance $(1 - F(p_i))^{N-1}$ of selling to them. Less-informed consumers show up at a firm with probability μ_1/N.

Using the same procedure as above, we can easily solve for the mixed strategy distribution $F(p)$. One obtains

$$F(p) = 1 - \sqrt[N-1]{\frac{\mu_1 (1-\lambda)(v-p)}{N\lambda p}}.$$

The less-informed consumers' stability condition remains $v - E[p] - c = 0$. In a companion paper (Janssen and Moraga, 2001), we show that for a given μ_1, $E[p]$ is increasing in N. This implies that as far as the concern is existence of a low search intensity equilibrium, the range of parameter values c, v and λ for which such an equilibrium exists becomes greater as N rises. In fact, when N grows very large there exists a low search intensity equilibrium even for very small values of the search cost and the moderate search intensity equilibrium disappears.

Apart from the wider range of parameter values for which a low search intensity equilibrium exists, there are no substantial differences between the duopoly case and the more general oligopoly case. In particular, the striking comparative statics properties of the equilibrium, namely that expected prices and price dispersion increase when search costs fall and remain constant as market maturity rises, continue to hold. This is easily seen from the fact that the same indifference equation for the less-informed buyers must hold.

Consider now the case where less-informed consumers search with high intensity in equilibrium. The expected payoff to firm i of charging p_i when rivals choose a random pricing strategy according to $G(p)$ is in this case:

$$\pi_i(p_i, G(p)) = p_i \left[k(1 - G(p_i))^{N-1} + \frac{2m}{N}(1-\mu_1)(1 - G(p_i)) + \frac{m\mu_1}{N} \right].$$

As before, in equilibrium, a firm must be indifferent between charging any price in the support of F and charging a price equal to v. Hence, we must have that

$$p\left[\frac{(1-\lambda)\,\mu_1}{N} + \frac{2\,(1-\lambda)}{N}(1-\mu_1)(1-G(p)) + \lambda(1-G(p))^{N-1}\right] = \frac{(1-\lambda)\,\mu_1 v}{N}. \quad (13)$$

An equilibrium in this case is given by a pair $\{\mu_1, G(p)\}$ which simultaneously solves (13) and the less-informed consumers' stability condition $v - E[p] - c = v - E[\min\{p_i, p_j\}] - 2c$.

Unfortunately, an explicit solution of equation (13) does not exist for general values of N and thus equilibrium analysis becomes intractable for this case. However, we can give an approximate solution for $G(p)$ for large N. For any given value of the other parameters and for large N, the term $k(1 - G(p))^{N-1}$ approaches zero, and the price distribution solution to (13) is approximately equal to

$$G(p) = 1 - \frac{\mu_1(v-p)}{2(1-\mu_1)p} = \frac{2-\mu_1}{2(1-\mu_1)} - \frac{\mu_1}{2(1-\mu_1)}\frac{v}{p},$$

with support $(\underline{p}, v)$ where $\underline{p} = \mu_1 v/(2-\mu_1)$. Note that this solution is similar to the one obtained by Burdett and Judd (1983) for competitive markets without informed consumers. Thus, when there are many firms, the influence of the informed consumers becomes negligible and therefore marginal increases in λ have again no impact on mean prices and price dispersion. Moreover, the comparative statics properties of this equilibrium with respect to c are qualitatively identical to the ones obtained in the duopoly case.

Consumer heterogeneity

So far, we have assumed that all consumers have the same willingness-to-pay for the good, and that all less-informed buyers have identical search costs. In what follows, we allow for consumer heterogeneity and illustrate to what extent our analysis must be modified. A complete analysis of a model where consumers differ with respect to their willingness-to-pay and their search cost turns out to be rather difficult.[26] Since what matters in our model is the relative size of the search cost relative to the willingness-to-pay, we restrict our discussion to the case where buyers have the same search cost, but differ in their willingness-to-pay. In particular, we shall assume that willingness-to-pay is randomly distributed over an interval $[\underline{v}, \overline{v}]$.

[26] Stahl (1996) presents a sequential search model where consumers are identical except for search costs.

A first issue to note is that a high search intensity equilibrium is unaffected by this change in modelling assumption, i.e., if for a less-informed buyer j it holds that $v_j - E[p] - c = v_j - E[\min\{p_i, p_j\}] - 2c$, then this must hold for all less-informed consumers.

Let us now consider a low search intensity equilibrium. One fundamental difference is that not all less-informed buyers will be indifferent between searching for one price and not searching at all. In particular, if for a consumer j it holds that $v_j - E[p] - c = 0$, then all consumers with higher valuations strictly prefer to search once, while those with smaller valuations strictly prefer not to search at all.

Let us assume a low search intensity equilibrium exists and examine the comparative statics effects of a decline in c. Note that some consumers who previously did not search may now find it optimal to search once. This implies that the valuation of the marginal consumer who is indifferent between searching and not searching would fall. However, like in the basic model, this implies that there would be relatively more consumers over which firms have monopoly power. In the new equilibrium, the decrease in c would be partially offset by the fact that the new marginal consumer has a lower willingness-to-pay, but expected prices would still increase as in the results above. The comparative static effects of an increase in λ will be different, however.

Consumers' search behavior

The basic model assumes less-informed consumers adopt a simultaneous search rule, i.e., they decide how many searches they want to make before actual search indeed starts (cf., Burdett and Judd, 1983). It may be argued that in some real markets consumers will search sequentially rather than simultaneously. In the sequential search literature, consumers each time after observing a price decide whether or not to search again. Often, the first price quotation is assumed to be for free (cf., Bester, 1994; Stahl, 1989; Stiglitz, 1989). If we introduce sequential, instead of simultaneous, search into the model, but stick to the assumption that *every* search is costly, then the results would change in the following way. First, the high search intensity equilibrium would no longer exist. This is due to the fact that in sequential search homogeneous good models consumers' search behavior can be characterized by a "stopping rule", i.e., a buyer stops searching and buys when he observes a price below a certain reservation price; otherwise he continues searching. The nature of sequential search implies that no firm would set a price above buyers' reservation price (as it would not result in any sale). Hence, no consumer searches more than once in equilibrium.

A second observation is that a low search intensity equilibrium would display qualitatively similar properties. The major change would be that the upper bound of the price distribution would no longer be equal to v, but instead equal to some reservation price, the value of which would depend on the search cost and the other exogenous parameters. Apart from this change, the main comparative static properties of the basic model would carry over. In particular, if search costs fall, expected prices must rise in order to keep consumers indifferent between searching and not searching. Also, if the number of informed consumers increases, expected prices will remain the same as more less-informed consumers will start searching for one price.

The main reason we have chosen to model buyers search behavior as we did in the basic model, lies in the fact that it is an easy way to analyze the differences between markets where consumers' search activity is important as compared to markets where this is not the case. In real markets, there is a marked difference between buyers' search intensity for different products (cf., the reference to Johnson et al. (1999) in the Introduction). Our basic model is able to explain this difference in terms of the relative size of the search cost compared to the willingness-to-pay, and the maturity of the market. Moreover, our model points out that the impact of improvements in search technology is quite sensitive to the intensity with which buyers search in markets.

6 Conclusion

In a model which may very well resemble an electronic market for a commodity, we have investigated whether improved search technology fosters competition and consequently lowers commodity prices and raises social welfare, as commonly argued. We have found that the impact of Internet usage on commodity markets efficiency may be subtler than previously thought.

Our first primary finding relates to the intensity with which buyers search in equilibrium. We have found that product's value relative to search cost and market maturity are the determinants of consumers' search incentives. More precisely, for a given search cost, buyers' search incentives are (weakly) monotonic in product's value and non-monotonic in market maturity. The direct implication of this observation is that price-cost margins and price dispersion need not be low in all commodity markets. Whether they are low or high depends on how much buyers search, which in turn depends on market characteristics.

Our second major finding is that the comparative statics effects of improved search technology on commodity markets depend on the manner it is modelled, i.e., on whether improved search technology is regarded as lowering unit search costs, or as increasing market maturity. Moreover, these effects are influenced by the intensity with which consumers search in the status quo equilibrium, and hence by initial market characteristics. A unit search cost reduction may result in higher, equal or lower mean prices and price dispersion depending on whether consumers initially search with low, moderate or high intensity, respectively. In contrast, expected prices and price dispersion (weakly) decrease as market maturity rises, irrespective of the buyers' search activity. The latter two remarks suggest that the long run impact of Internet usage on commodity markets will be sensitive to the extent to which search engines proliferate and become central places of information exchange.

As argued in the main body of this paper, our results may help to understand the controversial empirical findings reported so far. Perhaps more interesting is the fact that this research suggests that future empirical research assessing the impact of Internet usage on market efficiency should take into consideration more explicitly market characteristics such as product's value and market maturity.

7 Appendix

Proof of Fact 0 in Footnote 12:

Consider the low search intensity equilibrium (Proposition 1). We prove that less-informed buyers do not find profitable to search for two prices, i.e., that $v - E[\min\{p_1, p_2\}] - 2c < 0$. We can use the observation in Footnote 13 to write this inequality as

$$v - 2bv(1 - b\ln[a/b]) - 2c < 0. \tag{14}$$

In equilibrium, $v - E[p] - c = 0$, i.e., $v - c = bv\ln[a/b]$. Substituting this relationship into (14) we obtain $v - 2c(1+b) < 0$, or $c/v > 1/(2(1+b))$. Since $v - E[p] - c = 0$ in equilibrium, $c/v = 1 - b\ln[a/b] = 1 - b\ln[(1+b)/b]$. From (3), notice that $b = (1-\lambda)\,\mu_1/2\lambda > 0$. So, we need to check that

$$\frac{1+2b}{2b(1+b)} > \ln\left(\frac{1+b}{b}\right) \tag{15}$$

for all $b > 0$. Three observations prove that (15) is indeed satisfied:
(i) $\lim_{b\to 0} \frac{\ln\left(\frac{1+b}{b}\right)}{\frac{1+2b}{2b(1+b)}} > 0$,
(ii) $\lim_{b\to\infty} \frac{\ln\left(\frac{1+b}{b}\right)}{\frac{1+2b}{2b(1+b)}} = 1$
(iii) the LHS of (15) decreases at the rate $\frac{2b^2+2b+1}{2b^2(1+b)^2}$, which is larger than the rate at which the RHS diminishes, namely, $1/(b(1+b))$. The proof is now complete.

Proof of Fact 1: $\frac{d\Phi}{d\mu_1} < 0$.

From (4) it follows that

$$\frac{d\Phi}{d\mu_1} = \frac{(1-\lambda)\left[2\lambda - (2\lambda + (1-\lambda)\,\mu_1)\ln\left(\frac{2\lambda+(1-\lambda)\mu_1}{(1-\lambda)\mu_1}\right)\right]}{2\lambda(2\lambda + (1-\lambda)\,\mu_1)}.$$

The sign of $d\Phi/d\mu_1$ is negative if and only if

$$\frac{2\lambda}{2\lambda + (1-\lambda)\,\mu_1} < \ln\left(\frac{2\lambda + (1-\lambda)\,\mu_1}{(1-\lambda)\,\mu_1}\right) \tag{16}$$

Denote the LHS of (16) as $h_1(\mu_1)$ and its RHS as $h_2(\mu_1)$. Note that $h_1'(\mu_1) = \frac{-2\lambda(1-\lambda)}{(2\lambda+(1-\lambda)\mu_1)^2}$ and that $h_2'(\mu_1) = \frac{-2\lambda(1-\lambda)}{(1-\lambda)\mu_1(2\lambda+(1-\lambda)\mu_1)}$. It is evident that $h_1'(\mu_1) > h_2'(\mu_1)$, which implies that the LHS of (16) decreases at a lower speed than its RHS. Therefore, if (16) holds for

$\mu_1 = 1$, it will also hold for any $\mu_1 \in (0,1)$. Substituting $\mu_1 = 1$ in equation (16) yields

$$\ln\left(\frac{1+\lambda}{1-\lambda}\right) > \frac{2\lambda}{1+\lambda}. \tag{17}$$

To see that (17) holds for all $\lambda > 0$, notice that:
(i) $\lim_{\lambda\to 0}\ln\left(\frac{1+\lambda}{1-\lambda}\right) = \lim_{\lambda\to 0}\frac{2\lambda}{1+\lambda} = 0$
(ii) the LHS of (17) increases at the rate $\frac{2}{1-\lambda^2}$, which is larger than the rate $\frac{2}{(1+\lambda)^2}$ at which its RHS increases. This completes the proof of Fact 1.

Proof of Fact 2: $\frac{d^2\Phi}{d\mu_1^2} > 0$.

It is enough to observe that $d^2\Phi/d\mu_1^2 = \frac{2\lambda(1-\lambda)}{\mu_1(2\lambda+(1-\lambda)\mu_1)^2} > 0$.

Proof of Fact 3: $\lim_{\mu_1\to 0}\Phi(\mu_1) = 1$

Note that $\lim_{\mu_1\to 0}\Phi(\mu_1) = 1 - \lim_{\mu_1\to 0}\frac{(1-\lambda)\mu_1\ln\left(\frac{2\lambda+(1-\lambda)\mu_1}{(1-\lambda)\mu_1}\right)}{2\lambda}$. Applying L'Hopital rule we have $\lim_{\mu_1\to 0}\Phi(\mu_1) = 1 - \lim_{\mu_1\to 0}\frac{(1-\lambda)\mu_1}{2\lambda+(1-\lambda)\mu_1} = 1$.

Proof of Fact 4: $\Phi(1) = 1 - \frac{(1-\lambda)\ln\left(\frac{1+\lambda}{1-\lambda}\right)}{2\lambda} > 0$.

It is enough to prove that

$$\frac{2\lambda}{1-\lambda} > \ln\left(\frac{1+\lambda}{1-\lambda}\right) \tag{18}$$

for all $\lambda \in (0,1)$. The following two observations show that (18) holds:
(i) the LHS of (18) increases at the rate $\frac{2}{(1-\lambda)^2}$, which is higher than the rate at which the RHS rises, namely $\frac{2}{1-\lambda^2}$.
(ii) $\lim_{\lambda\to 0}\frac{2\lambda}{1-\lambda} = \lim_{\lambda\to 0}\ln\left(\frac{1+\lambda}{1-\lambda}\right) = 0$.

Proof of Fact 5: $\Gamma(1) = \frac{(1-\lambda)\left(\ln\left(\frac{1+\lambda}{1-\lambda}\right)-2\lambda\right)}{2\lambda^2} > 0$.

Note that $\Gamma(1) > 0$ whenever $\ln\left(\frac{1+\lambda}{1-\lambda}\right) > 2\lambda$. Proceeding similarly as in Fact 4, a little more algebra shows that this inequality holds for all $\lambda \in (0,1)$.

Proof of Fact 6: $\lim_{\mu_1\to 0}\Gamma(\mu_1) = 0$.

Note that $\lim_{\mu_1\to 0}\Gamma(\mu_1) = \lim_{\mu_1\to 0}\frac{-2+2(1-\lambda)\mu_1+\ln\left(\frac{2-(1-\lambda)\mu_1}{(1-\lambda)\mu_1}\right)}{2\left(\frac{1=(1-\lambda)\mu_1}{(1-\lambda)\mu_1}\right)}$. Applying the L'Hopital rule we obtain $\lim_{\mu_1\to 0}\Gamma(\mu_1) = \lim_{\mu_1\to 0}\frac{-(1-\lambda)^2\mu_1^2+\frac{(1-\lambda)\mu_1}{2-(1-\lambda)\mu_1}}{1-(1-\lambda)^2\mu_1^2} = 0$.

Proof of Fact 7: It can be seen that

$$\frac{d\Gamma}{d\mu_1} = \frac{\mu_1}{2(1-(1-\lambda)\mu_1)^3\,(2-(1-\lambda)\mu_1)} \left(2(3-4(1-\lambda)\mu_1+(1-\lambda)^2\mu_1^2) - (2+(1-\lambda)\mu_1-(1-\lambda)^2\mu_1^2)\ln\left(\frac{2-(1-\lambda)\mu_1}{(1-\lambda)\mu_1}\right)\right.$$

At the point $\mu_1 = 1$, we obtain

$$\left.\frac{d\Gamma}{d\mu_1}\right|_{\mu_1=1} = \frac{2\lambda(2+\lambda) + (\lambda^2 - \lambda - 2)\ln[\left(\frac{1+\lambda}{1-\lambda}\right)}{2\lambda^3(1+\lambda)}.$$

$\left.\frac{d\Gamma}{d\mu_1}\right|_{\mu_1=0} > 0$ if and only if $\frac{2\lambda(2+\lambda)}{2+\lambda(1-\lambda)} > \ln\left(\frac{1+\lambda}{1-\lambda}\right)$. Using, for instance, the software Mathematica 3.0 one can solve this inequality numerically. The critical value $\overline{\lambda} \approx 0.634816$.

Proof of Fact 8: $\frac{d^2\Gamma}{d\mu_1^2} < 0$.

The second derivative of $\Gamma(\mu_1)$ with respect to μ_1 can be written as

$$\frac{d^2\Gamma}{d\mu_1^2} = \frac{\mu_1[2A + (1-\lambda)\mu_1(2 + (1-\lambda)\mu_1)(2 - (1-\lambda)\mu_1)^2 B]}{(1-\lambda)(1-(1-\lambda)\mu_1)^4(2-(1-\lambda)\mu_1)^2},$$

where $A = -1 - 6(1-\lambda)\mu_1 + 13(1-\lambda)^2\mu_1^2 - 7(1-\lambda)^3\mu_1^3 + (1-\lambda)^4\mu_1^4$ and $B = \ln\left(\frac{2-(1-\lambda)\mu_1}{(1-\lambda)\mu_1}\right)$. Figure 8 shows this second derivative in the space $(\mu_1, \lambda) \in (0,1] \times (0,1)$, as plotted by Mathematica 3.0. Upon observation of this graph, it is clear that $d^2\Gamma/d\mu_1^2 < 0$.

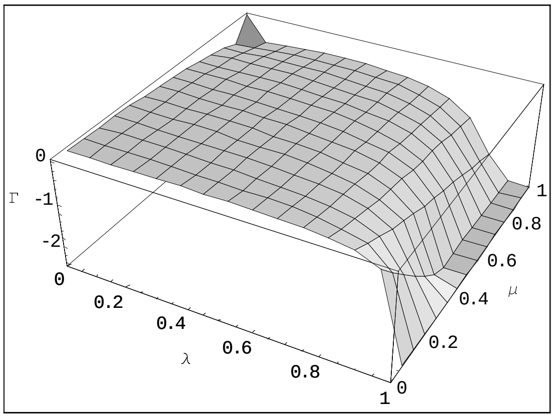

Figure 8

Proof of Fact 9: $\Phi(1) - \Gamma(1) > 0$.

$$\begin{aligned}\Phi(1) - \Gamma(1) &= 1 - \frac{(1-\lambda)\ln\left(\frac{1+\lambda}{1-\lambda}\right)}{2\lambda} - \frac{(1-\lambda)\left(\ln\left(\frac{1+\lambda}{1-\lambda}\right) - 2\lambda\right)}{2\lambda^2} \\ &= \left(2\lambda - \ln\left(\frac{1+\lambda}{1-\lambda}\right)(1-\lambda^2)\right)/2\lambda^2 > 0 \text{ for all } \lambda.\end{aligned}$$

Proof of Fact 10: There exists $\widehat{\lambda} < \overline{\lambda}$ such that $\Phi(1) < \overline{\Gamma}$ if and only if $\lambda < \widehat{\lambda}$.

We first show that $\overline{\Gamma}$ is constant with λ. Define $\widehat{\mu}_1 = \arg\max \Gamma(\mu_1, \lambda)$. By the envelope theorem $\frac{d\overline{\Gamma}}{d\lambda} = \frac{\partial\Gamma(\widehat{\mu}_1;\lambda)}{\partial\lambda}$. But from condition (8) it follows that $\frac{\partial\Gamma(\widehat{\mu}_1;\lambda)}{\partial\lambda} = -\frac{\partial\Gamma(\widehat{\mu}_1;\lambda)}{\partial\mu_1}\frac{\mu_1}{1-\lambda} =$

0. Therefore $\frac{d\overline{\Gamma}}{d\lambda} = 0$. Observe now that $\Phi(1)$ decreases monotonically with λ and that $\lim_{\lambda \to 0} \Phi(1) = 0$. Since $\overline{\Gamma} > 0$ as noticed above, it follows that there exists $\widehat{\lambda}$ such that $\Phi(1) < \overline{\Gamma}$ if and only if $\lambda < \widehat{\lambda}$. It remains to prove that $\widehat{\lambda} < \overline{\lambda}$ but this is trivial because for any $\lambda > \overline{\lambda}$, $\overline{\Gamma} = \Gamma(1)$ and Fact 9 implies that $\Phi(1) > \Gamma(1)$.

Proof of Fact 11: $\frac{dE[p]}{db} > 0$

Using Footnote 13, we can compute

$$\frac{dE[p]}{db} = v\left(\ln\left(\frac{1+b}{b}\right) - \frac{1}{1+b}\right). \tag{19}$$

Note that $\lim_{b\to 0} \ln\left(\frac{1+b}{b}\right) > \lim_{b\to 0} \frac{1}{1+b}$. Observe also that the first summand of (19) decreases at the rate $1/(b(1+b))$, while the second summand falls at the lower rate $1/(1+b)^2$. Then the result follows from the following fact:

$$\lim_{b\to\infty} \frac{\ln\left(\frac{1+b}{b}\right)}{\frac{1}{1+b}} = \lim_{b\to\infty} \frac{1+b}{b} = 1$$

Proof of Fact 12: $\frac{dE[\min\{p_1,p_2\}]}{db} > 0$.

To see this note that $E[\min\{p_1, p_2\}] = 2b(v - E[p])$ (see footnote 13). Using (19), it is easily seen that

$$\frac{dE[\min\{p_1,p_2\}]}{db} = 2bv\left(\frac{1+2b}{b(1+b)} - 2\ln\left(\frac{1+b}{b}\right)\right). \tag{20}$$

The proof of Fact 0 can now be used to demonstrate that this term is positive.

Proof of Fact 13: $\frac{d(E[\max\{p_1,p_2\}] - E[\min\{p_1,p_2\}])}{db} = \frac{d(2(E[p] - E[\min\{p_1,p_2\}]))}{db} > 0$ if and only if $b < \overline{b} \simeq 0.28763$.

Using footnote 13, one readily computes

$$E[\max\{p_1, p_2\}] - E[\min\{p_1, p_2\}] = 2bv((1+2b)\ln[(1+b)/b] - 2).$$

Taking derivatives with respect to b yields

$$\frac{d(E[\max\{p_1,p_2\} - E[\min\{p_1,p_2\}])}{db} = 2v\left((1+4b)\ln\left(\frac{1+b}{b}\right) - \frac{3+4b}{1+b}\right). \tag{21}$$

Thus $E[\max\{p_1, p_2\} - E[\min\{p_1, p_2\}]$ increases with b if and only if $\ln((1+b)/b) > (3+4b)/[(1+b)(1+4b)]$. Solving this inequality numerically, one obtains that price dispersion increases with b if and only if $b < \overline{b} \simeq 0.28763$.

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