# Segmentation, Advertising and Prices*

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**Abstract**

This paper explores the implications of market segmentation on firm competitiveness. In contrast to earlier work, here market segmentation is minimal in the sense that it is based on consumer attributes which are completely unrelated to tastes. We show that when the market is comprised by two consumer segments and when there is sufficient variation in the per-consumer costs firms need to incur to access the different segment populations, then firms obtain positive profits in symmetric equilibrium. Otherwise, the equilibrium is characterized by zero profits. As a result, a minimal form of market segmentation combined with advertising cost asymmetries across consumer segments give firms an opportunity to generate positive rents in an otherwise Bertrand-like environment.

**JEL Classification:** D43, D83

**Keywords:** segmentation, advertising, oligopoly, price dispersion, price discrimination.

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# 1 Introduction

Individuals have unifying characteristics, such as age, gender, mother tongue, profession, sexual orientation, life-style, etc. Often, these characteristics influence individuals' media affinity and, as a result, firms can reach the various groups of buyers by placing advertisements in selected media. Moreover, by choosing advertising strategies appropriately, an individual firm can decide to address its advertisements to just one or, alternatively, more consumer segments at a time.

Consider for example consumer segmentation based on mother tongue, like in Belgium. A firm operating in Belgium may want to address only the French-speaking community by inserting commercials in TV channels that broadcast only in French, or by inserting ads in newspapers and magazines written in French. Alternatively, the firm may decide to address only the Dutch-speaking community, or else all the consumers in the market, and proceed accordingly. Since different advertising strategies have distinct costs, the question for a firm is how to market its product optimally in these circumstances. In this paper we explore the implications of consumer segmentation on firms' profits, the distribution of prices and advertising strategies.[1]

For this purpose, we study a pricing and advertising simultaneous moves game where homogeneous product sellers operate in a market consisting of two consumer segments. Consumers are initially uninformed about the firms' offerings and prices. The key feature of the model is that firms' advertising strategies can be designed to reach the distinct consumer segments or, alternatively, the entire market. Specifically, there are two groups of consumers, groups $A$ and $B$, of possibly different size. Firms may decide to send their ads to segment $A$'s consumers at a cost $\phi_A$ and charge a price $p_A$, or to segment $B$'s ones at a cost $\phi_B$ and charge a price $p_B$, or to all consumers at a cost $\phi_A + \phi_B$ and charge a price $p$. In this setting, advertising costs should be seen as the cost of providing product and price information to the different consumer groups. Ceteris paribus, a segment is relatively more profitable than another if it has a lower per-consumer advertising cost.

We assume that the advertising technology is perfect in the sense that if a firm decides to address its advertisements to one consumer segment, then all individuals in that particular segment observe the firm's ads, while no consumer in the other segment does observe the ads.[2] Since firms sell

[1] In recent days, consumers visit different on-line chat rooms and social networking websites like MySpace and FaceBook. As in the case of mother tongue, the participation of an individual in one of those groups is generally not related to his/her preferences over a particular product. Yet, the mere existence of different websites opens up the possibility for firms to communicate with consumers in a particular on-line group, without affecting the information of consumers in other on-line groups. For a study of firms' use of promotional chat on the internet, see Mayzlin (2006).

[2] We are assuming here that the advertising technology is arbitrarily precise, i.e., a firm can 'communicate' with the consumers in one of the groups without affecting the information set of the consumers in the other group. The

homogeneous goods, this assumption leads to a Bertrand-like environment where, in the absence of segmentation, firms obtain zero profits in equilibrium. This highly competitive environment provides us with a useful benchmark to isolate the effects of segmentation on competition. The main result of the paper is that a minimal amount of market segmentation allows firms to obtain positive profits as long as there is sufficient variation in per-consumer advertising costs across consumer segments. We now explain the economic forces behind this result.[3]

There are two important properties which describe the nature of price-advertising equilibria in segmented homogeneous product markets. The first property is that equilibrium prescribes firms to randomize between sending their ads only to segment $A$'s consumers, sending their ads only to segment $B$'s buyers, and sending their ads to all the consumers in the market. As a result, with strictly positive probability, consumers in different segments observe offerings of distinct firms. We refer to this situation as one where the market outcome is partially segmented because, from time to time, consumers in a given segment are only aware of the offering of one of the firms.

The second property of equilibrium is that pricing behavior, advertising frequencies and firm profits depend on the relative profitability of the different segments. Suppose that segment $A$ is more profitable than segment $B$. This implies that, *ex-ante*, firms find it more attractive to address their ads to segment $A$'s buyers than to segment $B$'s. Since in equilibrium all segments must be equally attractive, segment $A$ must attract more advertising than segment $B$, which creates different intensities of price competition across the two consumer groups. As a result, firms offer better deals to segment $A$'s consumers than to $B$'s. This, in turn, discourages firms to send their ads to all the consumers at a time (because in that case they have to charge a uniform price), which makes partial segmentation likely. We find that, for segmentation to be a source of economic profits, there must be large variation in per-consumer advertising cost across segments. In that case, partial segmentation arises very frequently and firms are able to extract positive rents.

An interesting feature of our model is that it explains firm size dispersion as an equilibrium

other extreme is when the advertising technology is totally imprecise and a firm intending to send its ads to a given group of consumers ends up also reaching the consumers in the other group. In such a case segmentation would be irrelevant and the model equivalent to one where the market is not segmented. The intermediate case where some of a firm's ads intended for one of the two groups of consumers spill over the other group of consumers is equivalent to a model where consumer segments partially overlap. In such a case, if a firm sends its ads to, say, group A, then also a fraction of group B's consumers receive the ads. In our working paper Galeotti and Moraga-González (2003), we show that the main insights we present here carry on if we consider imperfect targeting technologies.

[3]Unlike in recent work by Iyer *et al.* (2005), we obtain this result in a setting where consumers always buy from the lowest-price supplier, i.e., where market segmentation has nothing to do with product differentiation. Despite this, segmentation enables firms to randomize advertising strategies across markets, which weakens price competition and thus opens up the possibility to obtain positive profits.

phenomenon. Moreover, since consumers located in larger consumer segments receive better deals, equilibrium prices reinforce the initial size differences between the segments. An increase in advertising costs or an increase in the asymmetries across consumer groups increases the probability that the market is segmented in equilibrium and therefore the likelihood of unequal firm sizes. This fact increases prices and therefore firms obtain higher revenues. When the advertising cost is low initially, this translates into higher profits, while profits decrease when advertising costs are high.

This paper is a contribution to the study of competitiveness in oligopolistic markets. Since Hotelling (1929), the role of product differentiation in mitigating price competition has been central to microeconomics and industrial organization (see e.g. Shaked and Sutton, 1982 and d'Aspremont, Gabszewicz and Thisse, 1979). In this literature, product differentiation results in imperfect substitutability between the different firms' offerings and this naturally relaxes price competition between firms. The novelty of our contribution is to show that a much weaker form of market segmentation –i.e. based on utility-irrelevant attributes– alongside with asymmetries in per-consumer advertising costs across consumer segments suffice to generate economic rents for the firms, even if firms sell homogenous products and are engaged in a Bertrand-like competitive environment.[4]

Our paper is also related to the work on targeted promotional marketing. Most of this work has focused on targeted promotions and thus the issues of product differentiation and price discrimination have been central.[5] Iyer *et al.* (2005) study targeted advertising and pricing in a market where each firm has a share of loyal consumers. They also obtain the result that segmentation increases profits but their analysis relies on the assumption that a firm is able to send its ads to the consumers who have a high preference for its brand. By contrast, our setting with homogeneous product sellers focuses on the purely strategic effects of targeted advertising and abstracts from the effects of product differentiation.

The paper most closely related to our work is Roy (2000).[6] Roy (2000) studies targeted advertising in a two-stage model where homogeneous product sellers first send product-advertisements to consumers and then choose their prices. The modelling of advertising in Roy's model implies that

[4]Real-world media markets exhibit significant asymmetries in per-reader advertising costs. For example, in the Netherlands, the *Telegraaf*, an Amsterdam-based newspaper with 846000 readers in 2001 charged about 813 Euros per millimeter of advertising space; the Rotterdam-based daily *Algemene Dagblad* charged 430 Euros and had 366574 readers; finally, the *Leeuwarder Courant*, a newspaper based in the capital city of Friesland, Leeuwarden, had 114650 readers and charged 173 Euros (see Handbook of the Dutch Press and Publicity, 2001).

[5]See e.g. Shaffer and Zhang (1995), Bester and Petrakis (1996) and Moraga-González and Petrakis (1999) for coupon targeting; Thisse and Vives (1988) and Chen and Iyer (2002) for personalized pricing; and Chen *et al.* (2001) for imperfect price targeting.

[6]To the best of our knowledge, the other existing papers on targeted advertising have focused on monopoly, namely, Esteban *et al.* (2001, 2006) and Gal-Or and Gal-Or (2005).

advertising has a long-run nature –as in Fudenberg and Tirole (1984)– and applies to markets where advertising provides product information, perhaps intended to create brand image and consumer awareness, and not price information. Roy characterizes an equilibrium where the market is permanently segmented in equilibrium, with one firm serving a group of customers and the rival firm serving a completely different group. Targeted advertising thus enables a firm to commit not to invade the consumers' segment addressed by the rival firm, which generates rents for the competing firms. In our model, advertising conveys price information and thus has a short-run nature instead.[7] Under these circumstances, we find that segmentation alone does not suffice to generate rents for the firms; it is also necessary that there is sufficient variation in ex-ante profitability across consumer segments.

The remainder of the paper is organized as follows. Section 2 describes the basic model. Section 3 presents the equilibria and the comparative statics results. Section 4 concludes.

## 2 The model

We examine an advertising and pricing game between homogeneous product sellers. The key feature of the setting we study is that the consumer market is segmented. By focusing on market for homogeneous products, we consider a minimal form of consumer segmentation, which is based on utility-irrelevant attributes.

On the demand side of the market, there is a unit mass of consumers who hold downward sloping demand functions $D(p)$. For later reference, we define the revenue per consumer as $R(p)$ and assume that $R'(p) > 0$, for all $p \in [0, p^m]$, where $p^m = \arg\max_p R(p)$. Let $R^{-1}(\cdot)$ be the inverse of the revenue function. For our purposes, it will be enough to assume that consumers can be grouped into two market segments $A$ and $B$, with sizes $\mu_A$ and $\mu_B$ respectively, $\mu_j > 0$, $j = A, B$. For simplicity we assume that the market is perfectly segmented, i.e., $\mu_A + \mu_B = 1$, but our main result also holds if some consumers belong to the two segments.[8]

Two firms operate in the industry.[9] Firms produce the good at constant returns to scale and we normalize the marginal cost of production to zero without loss of generality. Consumers ignore,

[7] See e.g. Butters (1977), Grossman and Shapiro (1984), Stegeman (1991) and Stahl (1994).

[8] In our working paper, Galeotti and Moraga-González (2003), we show that our results also hold when consumer segments overlap to some extent. Such case can also be interpreted as one where the targeting technology is imperfect in the sense that if a firm directs its ads to a particular segment of consumers then some of the advertising efforts spill over the other segment of consumers. The only difference with the current setting is that the region of parameters for which firms obtain positive profits becomes smaller as the extent of overlap increases.

[9] The case of $N$ firms is similar and does not bring additional insights (see our working paper).

*a priori*, the existence and the price of the products so that firms must inform them to be able to sell. A firm $i$ may decide to address either the consumers in segment $A$, or in segment $B$, or in both segments $A$ and $B$ (i.e., the entire market), or, finally, stay out of the market altogether. We denote the set of pure advertising-strategies of firm $i$ by $E_i = \{O, A, B, M\}$, where $O$ denotes the decision to stay out of the market and $M$ indicates the decision to send ads to all the consumers in the market. A firm $i$'s mixed advertising-strategy is then a probability function over the set $E_i$. We refer to $\lambda_j^i$ as the probability with which a firm $i$ sends its ads to market $j \in E_i$. We assume that firms face an advertising cost $\phi_j > 0$ to address consumer segment $j$, $j = A, B$. Clearly, a firm sending its ads to the entire market bears a total cost of $\phi_A + \phi_B$.[10] To make the problem interesting, we assume that $\phi_j < \mu_j R(p^m)$, $j = A, B$, i.e., each segment is worth to be served at the monopoly price. The per-consumer advertising cost $\phi_j/\mu_j$ is referred to as the profitability of segment $j$, $j = A, B$. We assume, without loss of generality, that $\phi_A/\mu_A \leq \phi_B/\mu_B$; this implies that, *ex-ante*, firms find segment $A$ more attractive than segment $B$. This will have interesting implications on the nature of the equilibrium advertising and pricing decisions.

For advertising decision $j \in \{A, B, M\}$, a firm $i$'s pricing-strategy is denoted by a distribution of prices $F_j^i(p)$.[11] Let $\sigma_j^i$ denote the support of $F_j^i(p)$ and let $\overline{p}_j^i$ and $\underline{p}_j^i$ denote the maximum and the minimum price in $\sigma_j^i$, respectively. A firm $i$'s strategy is thus denoted by a collection of pairs $s^i = \{(\lambda_j^i, F_j^i(p))\}_{j \in E_i}$. We shall study the existence and characterization of symmetric Nash equilibria.[12]

# 3 Equilibria

Our objective is to examine how consumer segmentation influences market outcomes. To do this we first examine a benchmark case where consumers are all in one group. We then characterize equilibria in the model with two consumer segments and compare the outcomes.

[10] The nature of our results does not change if there are economies or diseconomies of scale in advertising; this will become clear later (cf. footnote 15).

[11] Note that we are assuming that firms cannot price discriminate, i.e., when a firm decides to go for the entire market then it advertises a *uniform* price to both segments of consumers. We discuss the implications of price discrimination in the conclusion. A formal analysis of price discrimination can be found in our working paper.

[12] We also examine all pure asymmetric advertising-strategy profiles (see Lemma 1 below).

### 3.1 Non-segmented markets

As a benchmark case, we examine here a setting where consumers all belong to a single group. The firm's set of pure advertising strategies becomes then $E_i = \{O, M\}$. The following result, due to Sharkey and Sibley (1993), shows that the advertising and pricing game described above has a unique symmetric equilibrium in which firms obtain zero profits.

**Proposition 1** *In the unique symmetric equilibrium of the game firms stay out of the market with probability* $\lambda_O = \frac{\phi_A+\phi_B}{R(p^m)}$ *and enter the entire market with probability* $\lambda_M = 1 - \lambda_O$ *in which case they advertise a price p randomly chosen from the set* $\sigma_M = \left[R^{-1}(\phi_A + \phi_B), p^m\right]$ *according to the price distribution* $F_M(p) = 1 - \frac{\phi_A+\phi_B}{R(p^m)-(\phi_A+\phi_B)} \left(\frac{R(p^m)-R(p)}{R(p)}\right)$. *This equilibrium exists always.*

We note that firms cannot be active in the market with probability one because competition would drive revenues down to zero. In equilibrium, firms must randomize between staying out of the market and advertising in the entire market, which yields zero-profits.

### 3.2 Segmented markets

We now move to consider the situation described in Section 2 where there are two consumer segments. In this case $E_i = \{O, A, B, M\}$. Following Varian (1980), we shall say that the equilibrium generates *partial (or temporary) segmentation* when the different firms direct their promotional efforts to distinct consumer groups with strictly positive probability. In that case, it happens from time to time that consumers in segment $A$ observe firm $i$'s offer only, while consumers in segment $B$ observe firm $j$'s offer only, $i \neq j$, $i, j = 1, 2$. The case of *full (or permanent) segmentation* refers to the situation where different consumer groups *always* observe the offer of distinct firms.

**Lemma 1** *A pure advertising-strategy cannot be part of an equilibrium; as a result, even though there are two segments of consumers in the market, the market equilibrium cannot exhibit permanent segmentation.*

Note that a market would be permanently segmented if one of the two firms sent its ads to segment $A$ while the other firm advertised in segment B. However, this would not be an equilibrium. Indeed, in such a case, firms would be charging the monopoly price and then a firm would strictly gain by deviating and advertising a price slightly lower than the monopoly price in the entire market. This result contrasts with Roy (2000), where the market is permanently segmented in equilibrium, with one firm serving a group of customers and the rival firm serving a different group. This difference in results stems from the modelling of advertising. While we consider here price advertising, which is predominantly short-run, Roy (2000) studies a model where firms first advertise to create awareness and then they compete in prices. Roy's modelling strategy implies that advertising has a long-run nature, which implies that by advertising to a particular segment, the firm is able to commit not to invade the consumers' segment addressed by the rival firm.

The main result of our paper is presented in the next Proposition. An equilibrium where the market is partially segmented always exists. Moreover, firms are able to obtain positive profits in equilibrium if there is sufficient variation in per-consumer advertising costs across segments.

**Proposition 2** *There always exists an equilibrium. In particular:*

*I. A positive-profits symmetric equilibrium exists if and only if* $\frac{\phi_A}{\mu_A} < \frac{\phi_B}{\mu_B}\left(\frac{\mu_B R(p^m)-\phi_B}{\mu_B R(p^m)}\right)$. *This equilibrium is unique and takes the following form: With probability* $\lambda_j \in (0,1)$ *firms send ads to segment* $j$*'s consumers and charge a price* $p$ *randomly chosen from a convex set* $\sigma_j = [\underline{p}_j, \overline{p}_j]$, *for every* $j = A, B, M$. *Furthermore,* $\underline{p}_A < \overline{p}_A = \underline{p}_M < \underline{p}_B < \overline{p}_B = \overline{p}_M = p^m$. *Firms obtain a profit* $E\pi = \lambda_A \mu_B R(p^m) - \phi_B > 0$.

*II. If* $\frac{\phi_A}{\mu_A} \geq \frac{\phi_B}{\mu_B}\left(\frac{\mu_B R(p^m)-\phi_B}{\mu_B R(p^m)}\right)$ *then there exists a zero-profits symmetric equilibrium which takes the following form: With probability* $\lambda_j \in (0,1)$ *firms advertise in segment* $j$ *and charge a price* $p$ *randomly chosen from a convex set* $\sigma_j = [\underline{p}_j, \overline{p}_j]$, *for every* $j = O, A, B, M$. *Furthermore,* $\underline{p}_A < \overline{p}_A = \underline{p}_M = \underline{p}_B < \overline{p}_B = \overline{p}_M = p^m$.[13]

*Any equilibrium generates partial segmentation, i.e., different firms direct their promotional efforts to distinct consumer groups with strictly positive probability.*

Proposition 2 shows that when a firm is able to tell groups of consumers apart and the advertising technology permits to address ads to the distinct groups of buyers, a positive profits equilibrium exists in what is otherwise a Bertrand-like environment. For this to happen, the per-consumer advertising costs across the segments must be sufficiently asymmetric.[14] This result is driven by

[13] *To be complete, we note that other zero-profit symmetric equilibria exist. If* $\phi_A/\mu_A > R(p^m) - \phi_B/\mu_B$, *then there exists an equilibrium where* $\lambda_O + \lambda_A + \lambda_B = 1$. *In addition, for any vector of parameters, there exists a continuum of equilibria where* $\lambda_O+\lambda_A+\lambda_B+\lambda_M = 1$; *this equilibrium differs from that in Proposition 2 in that* $\overline{p}_A = \overline{p}_B = \overline{p}_M = p^m$. *These equilibria are similar to the zero-profits equilibrium described in the Proposition and their derivations are omitted to save space. Details can be obtained from the authors upon request.*

[14] We note that a result analogous to that in Proposition 2 can be obtained in a setting in which the two consumer segments partially overlap, i.e. a fraction of consumers observe prices advertised in both segments. For details, see our working paper Galeotti and Moraga-González (2003).

two equilibrium properties, which we now summarize in the following proposition.

**Lemma 2** *In the equilibria described in Proposition 2, $F_B(p)$ (weakly) dominates $F_M(p)$ in a first-order stochastic sense and so does $F_M(p)$ with respect to $F_A(p)$. Moreover, firms send ads to segment A's consumers more frequently than they do to segment B's consumers.*

We start by noting that the market equilibrium will necessarily exhibit partial segmentation. This is because in any equilibrium, with positive probability consumers in segment $A$ are only aware of one firm, while consumers in segment $B$ are only aware of the rival firm. Even though segmentation is a source of economic profits, Proposition 2 also shows that it is not sufficient for firms to obtain positive rents; for this to happen, partial segmentation must occur often enough.

The second property of equilibria is that more profitable segments attract more advertising than less profitable segments, which results in lower prices being offered in the former than in the latter. Note that the ranking of prices and the intensities of advertising in Lemma 2 rely on the following inequality that compares the ex-ante profitability of different markets:

$$\frac{\phi_A}{\mu_A} \leq \frac{\phi_A + \phi_B}{\mu_A + \mu_B} \leq \frac{\phi_B}{\mu_B}. \tag{1}$$

Segment $A$ is, *ex ante*, the most attractive of the markets because the per-consumer advertising cost is the lowest. The opposite holds for segment $B$ and this makes it the least attractive of the options. Advertising in the entire market is more attractive than advertising in segment $B$ but less attractive than advertising in segment $A$. In equilibrium, the three advertising strategies must yield the same level of profits. As a result, firms compete more intensively for $A$-consumers than for $B$-consumers which, in turn, leads to higher prices being offered in segment $B$ than in segment $A$. When firms go for the entire market they advertise intermediate prices.[15]

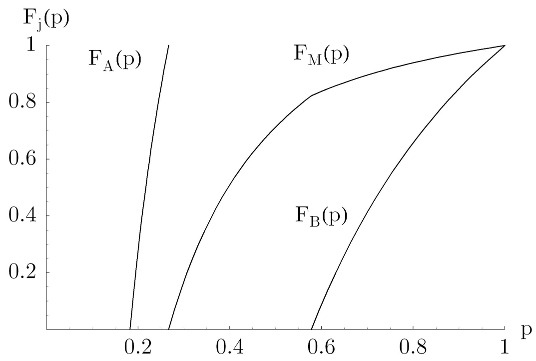
Substantial differences in profitability across segments cause significant differences in advertising intensity and prices across the two segments. This fact makes advertising to both groups of consumers at a time a relatively ineffective option as the firms have to offer the same price in both segments. As a result, partial segmentation occurs very frequently, which allows firms to extract positive rents. In contrast, when per-consumer advertising costs are similar across segments, firms compete *as if* the market were composed of a single group, which leads to zero-profits.

[15]Expression (1) is also useful to understand how our results would be modified if there were economies or diseconomies of scale in advertising. In the presence of strong economies of scale, the entire market would be the most profitable market while in the presence of diseconomies of scale it would be the least profitable market. The results above would then be modified accordingly.

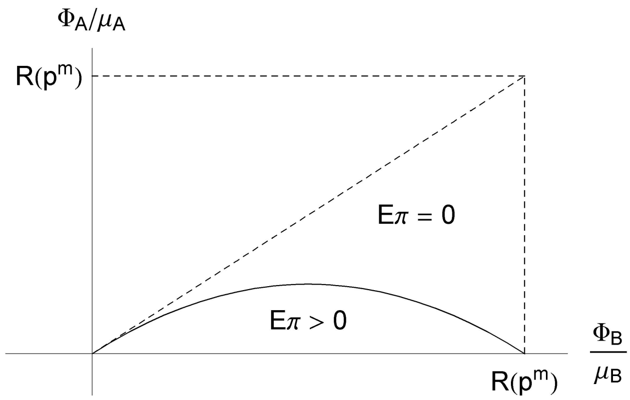
Figure 1a shows the distributions of prices advertised in segment $A$, in segment $B$ and in the entire market.[16] The existence conditions of the equilibria described in Proposition 2 can easily be represented in the space of per-consumer advertising costs. In Figure 1b, a positive-profits equilibrium exists in the region of parameters indicated by $E\pi > 0$; in the other region, firms obtain zero profits in equilibrium. It is worth noting that this condition is obtained by requiring that the expected profits to a firm who advertises in segment $B$ is positive. That is,

$$E\pi = \lambda_A \mu_B R(p^m) - \phi_B > 0 \tag{2}$$

which, indeed, it requires that consumers in segment $B$ observe only the offer of one firm sufficiently often, i.e., $\lambda_A$ must be sufficiently high.



(a) Equilibrium price distributions



(b) Existence regions of positive and zero-profits equilibria

Figure 1: Equilibria in the two-segment market

**Comparative statics:** In this model the likelihood of partial market segmentation is entirely determined by firms' advertising strategies and, as we have seen, this has a direct effect on expected profits and pricing behavior. As a result, it is interesting to study how changes in advertising costs shape segmentation, profits and pricing.

Assume, without loss of generality, $\phi_A = \gamma R(p^m)$, with $\gamma \in (0, \mu_A)$ and $\phi_B = \beta\phi_A$, with $\beta \in [\mu_B/\mu_A, \mu_B/\gamma)$. The following proposition focuses on the effects of a change in $\gamma$. Note that an increase in $\gamma$ increases the advertising costs in both segments but at the same time it increases the

[16]In Figure 1a, parameters are chosen such that firms obtain positive profits in equilibrium. In particular, segments are assumed to be of equal size, demand is assumed to be inelastic with monopoly price equal to 1, and advertising costs $\phi_A = 1/15$ and $\phi_B = 2/15$.

extent of asymmetry between segments.[17]

**Proposition 3** *I. In the positive-profits equilibrium described in Proposition 2,* $\frac{\partial \lambda_A}{\partial \gamma} > 0, \frac{\partial \lambda_B}{\partial \gamma} > 0$*, and* $\frac{\partial \lambda_M}{\partial \gamma} < 0$*. Further, an increase in* $\gamma$ *widens* $\sigma_A$ *and* $\sigma_B$ *and narrows* $\sigma_M$*. Furthermore, expected profits are increasing in* $\gamma$*, if* $\gamma$ *is low to begin with, while they are decreasing in* $\gamma$ *otherwise. Finally, as* $\gamma \to 0$*,* $\lambda_M \to 1$ *and* $F_M(p)$ *converges to a price distribution that is degenerate at the marginal cost.*

*II. In the zero-profits equilibrium described in Proposition 2,* $\frac{\partial \lambda_A}{\partial \gamma} = 0$*,* $\frac{\partial \lambda_B}{\partial \gamma} < 0$*,* $\frac{\partial \lambda_M}{\partial \gamma} < 0$ *and* $\frac{\partial \lambda_O}{\partial \gamma} > 0$*. Further, an increase in* $\gamma$ *does not alter* $\sigma_A$ *and narrows* $\sigma_B$ *and* $\sigma_M$*.*

An increase in the asymmetry across segments raises the likelihood of partial segmentation. That is, firms increase the frequency with which they go for the segments, while they advertise in the entire market less often. This has two implications. The first implication is that firms compete more often for the segments and less often for the entire market, which results in greater price dispersion at the segment level, and lower price dispersion at the market level.[18] The second implication is that overall price competition between firms weakens and, as a result, the gross revenues of the firms rise. When the costs of advertising are low to begin with, the revenue increase offsets the increase in the costs of advertising and firms obtain higher rents; otherwise, firms' profits go down with an increase in $\gamma$.

## 4 Conclusions

We have examined a strategic game of advertising and pricing between homogeneous product sellers. The objective has been to clarify the role of market segmentation. When sellers operate in a market with a single consumer segment, price competition drives profits down to zero. When the market has two consumer segments instead, and when there are enough differences in per-consumer advertising costs across segments, firms obtain positive profits in equilibrium. As a result, we conclude, market segmentation weakens firm competitiveness.

[17] An increase in $\gamma$ increases $\phi_j/\mu_j$, $j = A, B$ as well as the difference $(\phi_B/\mu_B) - (\phi_A/\mu_A)$. Note that there are other ways to increase the asymmetries across segments, e.g. by increasing the advertising costs of segment $B$ or by changing the distribution of consumers across segments. In our working paper we show that these changes yield similar results.

[18] We are using here the term price dispersion to refer to the range of prices (the difference between the minimum and the maximum of the support of a price distribution). Numerical simulations reveal that the variances of the price distributions $F_A(p)$ and $F_B(p)$ both increase as $\gamma$ increases; by contrast we have seen that the variance of $F_M(p)$ exhibits a non-monotonic relationship with respect to $\gamma$ first decreasing and then increasing.

There is empirical evidence which shows that market segmentation helps firms to extract rents, e.g., Raj (1982) and Chakravorty and Nauges (2005). Generally, in these studies products are not homogeneous and therefore it is difficult to distinguish whether these effects are due to market segmentation or product differentiation. Our paper shows that a minimal form of market segmentation induces price variation across segments and this variation is inherently linked to the relative profitability of various segments. One of the empirical challenges for future research is to distinguish price variation due to targeting and price variation due to product differentiation.

The analysis in this paper is also important for the study of the relationship between market competition and advertising in industries (for a recent study of the US PC industry see Sovinsky-Goeree, 2005). It suggests that not only the volume of advertising matters but also the firms' ability to target ads to distinct groups of consumers. Indeed, for fixed advertising volumes, one should expect greater price-cost margins in markets fairly segmented relative to more integrated markets.

Along the way we have assumed firms were not allowed to price-discriminate, i.e. firms could not send ads to distinct consumer groups with different prices and, as a result, they were forced to advertise a uniform price. In some settings price discrimination is certainly unfeasible. For instance, legal restraints typically imply that a firm cannot discriminate between persons of different sexual orientation, nor between men and women, French-speaking and Dutch-speaking individuals, etc. Sometimes price discrimination is legal but yet impractical. For example, when a shop has a single point-of-sale, advertised prices must equal on-the-shop prices; in addition, it does not seem common practice to charge consumers different prices in the shop just because they have seen different ads. However, there may be contexts where price discrimination is feasible. In our working paper Galeotti and Moraga-González (2003) we show that when firms can advertise different prices to the two groups of consumers at a time, they obtain zero profits in equilibrium. This is because price discrimination breaks the link across consumer segments and each sub-market can be considered separately. This result constitutes another instance showing that firms may benefit from bans on price discrimination. Other cases have been put forward by e.g. Holmes (1989) and Thisse and Vives (1988).[19]

Our model is also useful to analyze different economic problems such as strategic interaction between media firms as well as strategic competition in two-sided markets. Consider for example

[19] Armstrong and Vickers (2001) show that this result can be reversed if markets are sufficiently competitive. See Stole (2003) for a recent survey on price discrimination.

the following two-stage game of intermediary competition in two-sided markets. In the first stage intermediaries strategically choose firm advertising fees and consumer subscription fees. In the second stage, firms decide where to advertise and which price to charge and consumers decide where to subscribe. The analysis provided in this paper is a partial characterization of the second stage of this game. The complete analysis of this new model is left for further research.

# 5 Appendix

**Proof of Proposition 1:** It is easy to see that pure advertising-strategy profiles cannot be part of an equilibrium. Therefore, firms must randomize between sending ads to all consumers in the market and staying out, i.e., $\lambda_O + \lambda_M = 1$, $\lambda_j \in (0,1)$, $j = O, M$. Denote firm $i$'s strategy as $s^i = \{\lambda_O, (\lambda_M, F_M(p))\}$, $i = 1, 2$. It is easy to verify that $E\pi_i\left(\lambda_M = 1, p; s^{-i}\right) = R(p)\left[\lambda_O + \lambda_M(1 - F_M(p))\right] - \phi_A - \phi_B = 0$ for any $p \in \sigma_M$ only if $\lambda_O, \lambda_M, F_M(p)$ and $\sigma_M$ take the form specified in Proposition 1. Moreover, it is readily seen that firms do not have an incentive to deviate, that $\lambda_j \in (0,1)$, $j = O, M$ and that $F_M(p)$ is a well-behaved distribution function defined over the set $\sigma_M$ for any $\phi_j, \mu_j$, $j = A, B$. This completes the proof. ■

**Proof of Lemma 1:** We first prove that pure strategy equilibria do not exist. Obviously, any strategy profile in which either firm (or both) stays out of the market with probability one is not part of an equilibrium. Next, suppose that both firms advertise in segment $j$ with probability one, $j = A, B, M$. If this were an equilibrium, firms would advertise a price equal to marginal cost, in which case they would not cover advertising costs. Thus, this cannot be part of an equilibrium.

Suppose now that firms advertise in different segments, e.g., firm 1 advertises in $A$ while firm 2 advertises in $B$. If this were an equilibrium, firms would charge the monopoly price $p^m$ and obtain profits $\pi_1 = \mu_A R(p^m) - \phi_A$ and $\pi_2 = \mu_B R(p^m) - \phi_B$. But then firm 1 would find it profitable to advertise a price slightly lower than the rival's price in segment $B$. Now, assume that firm 1 advertises just in a single segment, say $A$, and the other firm goes for the entire market, i.e., $s^1 = \{\lambda_A = 1, F_A(p)\}$ and $s^2 = \{\lambda_M = 1, F_M(p)\}$. If this were an equilibrium, firms' profits would be given by:

$$
\begin{aligned}
E\pi_1(\lambda_A = 1, p; s^2) &= R(p)\mu_A(1 - F_M(p)) - \phi_A \\
E\pi_2(\lambda_M = 1, p; s^1) &= R(p)[\mu_A(1 - F_A(p)) + \mu_B] - \phi_A - \phi_B.
\end{aligned}
$$

It must be the case that $\overline{p}_A < \overline{p}_M$ because otherwise firm 1 advertising the upper bound $\overline{p}_A$ in $A$ would make negative profits. Since firm 2's profits must be constant for all prices in $\sigma_M$, it follows that $E\pi_2 = \mu_B R(\overline{p}_M) - \phi_A - \phi_B$. It is then obvious that firm 2 would gain by exiting segment $A$. The other pure entry-strategy profiles are ruled out analogously. Finally, we show that permanent segmentation cannot occur in equilibrium. Since a pure strategy equilibrium does not exist, we need to prove that $\lambda_O^1 + \lambda_A^1 = 1$ and $\lambda_O^2 + \lambda_B^2 = 1$, cannot be an equilibrium either. If it were an equilibrium, then firm 1 would charge $p^m$ in segment $A$ and obtain positive profits, which contradicts that firm 1 is indifferent between advertising to segment A's consumers and staying out of the market. ■

**Proof of Proposition 2:** We start by proving Part I of Proposition 2. The following proposition provides necessary conditions for an equilibrium with positive profits.

**Proposition 4** *If a positive-profits equilibrium exists, then (a)* $\lambda_j \in (0,1)$, $j = A, B, M$, *and* $\lambda_O = 0$, *(b)* $F_A(p)$, $F_B(p)$ *and* $F_M(p)$ *are atomless price distributions and (c)* $\underline{p}_A < \overline{p}_A = \underline{p}_M < \underline{p}_B < \overline{p}_B = \overline{p}_M = p^m$.

**Proof of Proposition 4:** (a) The result is proved by contradiction. By Lemma 1, firms must employ a random advertising-strategy. Obviously, for a positive-profits equilibrium $\lambda_O = 0$. Suppose, first, that $\lambda_A + \lambda_B = 1$, $\lambda_j > 0, j = A, B$. Denote firms' strategies as $s^i = \{(\lambda_A, F_A(p)), (\lambda_B, F_B(p))\}$, $i = 1, 2$. Then,

$$E\pi_i\left(\lambda_A = 1, p; s^{-i}\right) = R(p)\mu_A[\lambda_A(1 - F_A(p)) + \lambda_B] - \phi_A.$$

The upper bound of $F_A(p)$ cannot be lower than $p^m$ because otherwise a firm advertising such price in $A$ would gain by slightly raising its price, i.e., $\overline{p}_A = p^m$. For analogous reasons, $\overline{p}_B = p^m$. Thus if $s = (s^1, s^2)$ constitutes an equilibrium with positive profits then $E\pi_i\left(\lambda_A = 1, p^m; s^{-i}\right) = E\pi_i\left(\lambda_B = 1, p^m; s^{-i}\right) > 0$. But then a firm would double its profits by deviating and advertising $p^m$ in the entire market.

Second, suppose that $\lambda_A + \lambda_M = 1$, $\lambda_j > 0, j = A, M$. Let $s^i = \{(\lambda_A, F_A(p)), (\lambda_M, F_M(p))\}$, $i = 1, 2$ denote firm $i$'s strategy. Then,

$$E\pi_i\left(\lambda_A = 1, p; s^{-i}\right) = R(p)\mu_A[\lambda_A(1 - F_A(p)) + \lambda_M(1 - F_M(p))] - \phi_A,$$
$$E\pi_i\left(\lambda_M = 1, p; s^{-i}\right) = R(p)[\lambda_A(\mu_A(1 - F_A(p)) + \mu_B) + \lambda_M(1 - F_M(p))] - \phi_A - \phi_B.$$

As above we observe that $F_A(p)$ and $F_M(p)$ cannot have atoms. Further $\overline{p}_A < \overline{p}_M$; otherwise, if $\overline{p}_A \geq \overline{p}_M$ a firm advertising $\overline{p}_A$ in $A$ would always obtain negative profits. Observe next that $\overline{p}_M = p^m$ ; this is because there is a strictly positive probability that a firm advertising in the entire market is the only active firm in segment $B$; then, a firm advertising a different upper bound would gain by raising its price. Since firms must be indifferent between the distinct price and advertising strategies, equilibrium profits would be: $E\pi_i\left(\lambda_M = 1, p^m; s^{-i}\right) = \lambda_A R(p^m)\mu_B - \phi_A - \phi_B$. We now note that a firm can gain by advertising $p^m$ only in segment $B$. Indeed, profits to firm $i$ from such a deviation would be $E\pi_i^d\left(\lambda_B = 1, p^m; s^{-i}\right) = \lambda_A R(p^m)\mu_B - \phi_B$, clearly greater than equilibrium profits for all $\phi_A > 0$. The case in which $\lambda_B + \lambda_M = 1$, $\lambda_j \in (0,1)$ $j = B, M$ is ruled out similarly.

(b) We now prove that a symmetric positive-profits equilibrium exists only if $F_A(p)$, $F_B(p)$ and $F_M(p)$ are atomless. Let $s^i = \{(\lambda_A, F_A(p)), (\lambda_B, F_B(p)), (\lambda_M, F_M(p))\}$, $i = 1, 2$ denote firms' strategies. The expected payoffs to a firm from the different actions are:

$$E\pi_i\left(\lambda_A = 1, p; s^{-i}\right) = R(p)\mu_A[\lambda_A(1 - F_A(p)) + \lambda_B + \lambda_M(1 - F_M(p))] - \phi_A \quad (3)$$

$$E\pi_i\left(\lambda_B = 1, p; s^{-i}\right) = R(p)\mu_B[\lambda_A + \lambda_B(1 - F_B(p)) + \lambda_M(1 - F_M(p))] - \phi_B \quad (4)$$

$$E\pi_i\left(\lambda_M = 1, p; s^{-i}\right) = E\pi_i\left(\lambda_A = 1, p; s^{-i}\right) + E\pi_i\left(\lambda_B = 1, p; s^{-i}\right) \quad (5)$$

Inspection of these expressions reveals that atoms can be undercut profitably.

(c) Finally, we prove that a symmetric equilibrium exists only if $\underline{p}_A < \underline{p}_M = \overline{p}_A < \underline{p}_B < \overline{p}_M = \overline{p}_B = p^m$. The proof follows from a series of 4 claims.

**Claim 1:** If a positive-profits equilibrium exists then (a) $\nexists p \in \sigma_A \cap \sigma_B$ and (b) $\underline{p}_A < \underline{p}_B$.

**Proof.** We show this by contradiction. (a) Suppose $\exists p \in \sigma_A \cap \sigma_B$; then, if a firm were indifferent between advertising such a price in segment $A$ and in segment $B$, and that price yielded positive profits, the firm would gain by deviating and advertising it in the entire market, which yields a contradiction. (b) Suppose that $\underline{p}_A > \underline{p}_B$ (the case $\underline{p}_A = \underline{p}_B$ is ruled out by Claim 1). In equilibrium the expected profit to a firm advertising $\underline{p}_B$ in segment $B$ is $E\pi_i(\lambda_B = 1, \underline{p}_B; s^{-i}) = R(\underline{p}_B)\mu_B[\lambda_A + \lambda_B + \lambda_M(1 - F_M(\underline{p}_B))] - \phi_B$. However note that if a firm advertises the same price $\underline{p}_B$ in segment $A$ the expected profit is $E\pi_i(\lambda_A = 1, \underline{p}_B; s^{-i}) = R(\underline{p}_B)\mu_A[\lambda_A + \lambda_B + \lambda_M(1 - F_M(\underline{p}_B))] - \phi_A$. Since $\phi_A/\mu_A \leq \phi_B/\mu_B$ then $E\pi_i(\lambda_A = 1, \underline{p}_B; s^{-i}) \geq E\pi_i(\lambda_B = 1, \underline{p}_B; s^{-i}) > 0$. As a result, a firm setting $\underline{p}_B$ in segment $B$ would gain by deviating and advertising $\underline{p}_B$ in the entire market. ■

**Claim 2:** If a positive-profits equilibrium exists then (a) $\underline{p}_M \leq \overline{p}_A$, and (b) $\overline{p}_M > \underline{p}_B$.

**Proof.** (a) If $\underline{p}_M > \overline{p}_A$, then a firm advertising $\overline{p}_A$ in $A$ would gain by slightly increasing its price. (b) Suppose, on the contrary, that $\overline{p}_M \leq \underline{p}_B$. Consider first that $\overline{p}_M < \underline{p}_B$. We have two possibilities. One, $\overline{p}_A \leq \overline{p}_M < \underline{p}_B$; but then a firm advertising $\overline{p}_M$ in $M$ would strictly gain by increasing the price until $\underline{p}_B$. Two, $\overline{p}_M < \overline{p}_A < \underline{p}_B$; in this case a firm advertising $\overline{p}_A$ in $A$ would gain by raising its price until $p^m$, which contradicts part (a) of Claim 1. It remains to prove that $\overline{p}_M = \underline{p}_B$ cannot be part of an equilibrium. We start by noting that $\overline{p}_M = \underline{p}_B > \overline{p}_A$; for otherwise a firm advertising $\overline{p}_A$ in $A$ would gain by raising its price. Further, given $\overline{p}_M = \underline{p}_B > \overline{p}_A$ it follows that $\overline{p}_B = p^m$ and thus firms' expected profit in equilibrium is $\lambda_A R(p^m)\mu_B - \phi_B > 0$. Furthermore, in equilibrium $E\pi_i\left(\lambda_M = 1, \overline{p}_M; s^{-i}\right) = E\pi_i\left(\lambda_B = 1, \overline{p}_M; s^{-i}\right)$, which holds if and only if $\lambda_B R(\overline{p}_M)\mu_A - \phi_A = 0$. However we note that if a firm deviates by advertising $p = p^m > \overline{p}_M$ in $M$ then $E\pi_i^d\left(\lambda_M = 1, p^m; s^{-i}\right) = \lambda_A R(p^m)\mu_B - \phi_B + \lambda_B R(p^m)\mu_A - \phi_A > \lambda_A\mu_B R(p^m) - \phi_B$. ■

**Claim 3:** If a positive-profits equilibrium exists, $\overline{p}_M = \overline{p}_B = p^m$.

**Proof.** We prove this by contradiction. First, suppose that $\overline{p}_B < \overline{p}_M$. The support configuration would then be $\underline{p}_M \leq \overline{p}_A < \underline{p}_B < \overline{p}_B < \overline{p}_M = p^m$, where $\overline{p}_M = p^m$, for reasons discussed above. In equilibrium, $E\pi_i\left(\lambda_M = 1, p; s^{-i}\right) = E\pi_i\left(\lambda_M = 1, p^m; s^{-i}\right)$, $\forall p \in [\overline{p}_B, p^m]$, which, using (5), yields:

$$F_M(p) = 1 - \frac{\lambda_A\mu_B + \lambda_B\mu_A}{\lambda_M}\left(\frac{R(p^m) - R(p)}{R(p)}\right), \text{ for all } p \in [\overline{p}_B, p^m]. \tag{6}$$

Similarly, $E\pi_i\left(\lambda_M = 1, p; s^{-i}\right) = E\pi_i\left(\lambda_B = 1, p; s^{-i}\right)$, $\forall p \in [\underline{p}_B, \overline{p}_B]$, which using (4) and (5), yields:

$$F_M(p) = 1 - \frac{1}{\lambda_M}\left(\frac{\phi_A}{\mu_A R(p)} - \lambda_B\right) \text{ for all } p \in [\underline{p}_B, \overline{p}_B] \tag{7}$$

The price distributions (6) and (7) must be equal at $p = \overline{p}_B$. Imposing this condition we obtain $R(\overline{p}_B) = \frac{(\lambda_A\mu_B + \lambda_B\mu_A)R(p^m) - \frac{\phi_A}{\mu_A}}{(\lambda_A - \lambda_B)\mu_B}$.

A firm must be indifferent between advertising a price $p \in \sigma_B$ in segment B and advertising $p^m$ in the entire market, i.e., $E\pi_i\left(\lambda_B = 1, p; s^{-i}\right) = E\pi_i\left(\lambda_M = 1, p^m; s^{-i}\right)$. Using (7), this yields:

$$F_B(p) = \frac{\phi_A}{\mu_A\mu_B\lambda_B R(p)} + \frac{\lambda_A}{\lambda_B} - \frac{\lambda_A\mu_B + \lambda_B\mu_A}{\lambda_B\mu_B}\frac{R(p^m)}{R(p)}, \tag{8}$$

and by solving $F_B(\underline{p}_B) = 0$ in (8) we obtain $R(\underline{p}_B) = \frac{(\lambda_A\mu_B + \lambda_B\mu_A)R(p^m) - \frac{\phi_A}{\mu_A}}{\lambda_A\mu_B}$. Since $\underline{p}_B$ must be positive in equilibrium, it must be the case that $(\lambda_A\mu_B + \lambda_B\mu_A)R(p^m) - \frac{\phi_A}{\mu_A} > 0$. Since $\overline{p}_B$ must also be positive, this implies that $\lambda_A - \lambda_B > 0$. Now we can compare $\overline{p}_B$ and $p^m$. For $\overline{p}_B < p^m$ we need that $\lambda_B R(p^m)\mu_A - \phi_A < 0$; but then a firm charging $p^m$ in $M$ obtains profit equal to $\lambda_A R(p^m)\mu_B - \lambda_B R(p^m)\mu_A - \phi_A - \phi_B$, so the firm would gain by entering only segment $B$. This constitutes a contradiction and proves that $\overline{p}_B \geq \overline{p}_M$.

It remains to prove that $\overline{p}_B > \overline{p}_M$ cannot be part of an equilibrium. If this were so, then $\overline{p}_B = p^m$, and therefore the support configuration would be $\underline{p}_M \leq \overline{p}_A < \underline{p}_B < \overline{p}_M < \overline{p}_B = p^m$. Moreover, in equilibrium $E\pi_i\left(\lambda_B = 1, p^m; s^{-i}\right) = \lambda_A R(p^m)\mu_B - \phi_B$. We know that a firm which deviates by advertising $p^m$ in $M$ would obtain a profit equal to $E\pi_i^d\left(\lambda_M = 1, p^m; s^{-i}\right) = E\pi_i\left(\lambda_B = 1, p^m; s^{-i}\right) + \lambda_B R(p^m)\mu_A - \phi_A$. For this deviation to be unprofitable, $\lambda_B R(p^m)\mu_A - \phi_A \leq 0$. Furthermore, for any $p \in [\underline{p}_B, \overline{p}_M]$, $E\pi_i\left(\lambda_M = 1, p; s^{-i}\right) = E\pi_i\left(\lambda_B = 1, p; s^{-i}\right)$, which yields $F_M(p) = 1 - \frac{\phi_A}{\lambda_M \mu_A R(p)} + \frac{\lambda_B}{\lambda_M}$. The condition $F_M(\overline{p}_M) = 1$ yields $\lambda_B R(\overline{p}_M)\mu_A - \phi_A = 0$, which contradicts the condition above that $\lambda_B R(p^m)\mu_A - \phi_A \leq 0$. Therefore, if an equilibrium exists, then $\overline{p}_B = \overline{p}_M$, and by the usual arguments $\overline{p}_B = \overline{p}_M = p^m$. ■

**Claim 4:** If a positive-profits equilibrium exists then (a) $\underline{p}_A < \underline{p}_M$ and (b) $\underline{p}_M = \overline{p}_A$.

**Proof.** (a) Assume that $\underline{p}_A \geq \underline{p}_M$ , i.e., $\underline{p}_M \leq \underline{p}_A < \overline{p}_A < \underline{p}_B < \overline{p}_B = \overline{p}_M = p^m$. In equilibrium, for any $p \in \sigma_A$, $E\pi_i\left(\lambda_M = 1, p; s^{-i}\right) = E\pi_i\left(\lambda_A = 1, p; s^{-i}\right)$. This holds if and only if $R(p)\mu_B[\lambda_A + \lambda_B + \lambda_M(1 - F_M(p))] - \phi_B = 0$. This yields an expression for $F_M(p) = 1 - \frac{\phi_B}{\lambda_M \mu_B R(p)} + \frac{\lambda_A + \lambda_B}{\lambda_M}$. Further, in equilibrium $E\pi_i\left(\lambda_A = 1, p; s^{-i}\right) = \lambda_A \mu_B R(p^m) - \phi_B$. Using $F_M(p)$, this equality leads to $F_A(p) = \frac{\phi_B - \lambda_A \mu_B^2 R(p^m) - \phi_A \mu_B}{\lambda_A \mu_A \mu_B R(p)}$. Since $F_A(p) > 0$ for all $p \in \sigma_A$, it must be the case that $\phi_B - \lambda_A \mu_B^2 R(p^m) - \phi_A \mu_B > 0$. But then $F_A(p)$ would be strictly decreasing in $p$, which cannot happen in equilibrium. (b) The proof of this part is analogous to the proof of part (a) and therefore omitted. ■ This completes the proof of Proposition 4. ■

Next, we show that there exists a unique equilibrium which satisfies the conditions of Proposition 4. Consider the following strategy:

$$
\begin{aligned}
\lambda_A &= \frac{\phi_B - \phi_A \mu_B}{\mu_B^2 R(p^m) + \phi_B \mu_A} \text{ and } F_A(p) = 1 - \frac{\lambda_B + \lambda_M}{\lambda_A} \frac{R(\overline{p}_A) - R(p)}{R(p)}, \text{ for all } p \in \sigma_A = [\underline{p}_A, \overline{p}_A] \\
\lambda_B &= \phi_A / \mu_A R(p^m) \text{ and } F_B(p) = 1 - \frac{\lambda_A - \lambda_B}{\lambda_B}\left[\frac{R(p^m) - R(p)}{R(p)}\right], \text{ for all } p \in \sigma_B = [\underline{p}_B, p^m]; \\
\lambda_M &= 1 - \lambda_A - \lambda_B \text{ and } F_M(p) = \begin{cases} 1 - \frac{\lambda_A \mu_B (R(p^m) - R(p)) - \lambda_B R(p) + \phi_A}{\lambda_M R(p)} & \text{for all } p \in [\underline{p}_M, \underline{p}_B] \\ 1 - \frac{1}{\lambda_M}\left(\frac{\phi_A}{\mu_A R(p)} - \lambda_B\right) & \text{for all } p \in [\underline{p}_B, p^m] \end{cases} \\
\text{with } \underline{p}_A &= R^{-1}((1 - \lambda_A) R(\overline{p}_A)) < \overline{p}_A = \underline{p}_M = R^{-1}(\phi_B / \mu_B) < \\
\underline{p}_B &= R^{-1}(R(p^m)(\lambda_A - \lambda_B)/\lambda_A) < \overline{p}_B = \overline{p}_M = p^m.
\end{aligned}
$$

It is easy to check that the equilibrium condition $E\pi_i(\lambda_j = 1, p; s^{-i}) = \lambda_A \mu_B R(p^m) - \phi_B$ for any $p \in \sigma_j$, is satisfied if and only if $\lambda_j, F_j(p)$ and $\sigma_j$, $j = A, B, M$ take the form specified above.

To prove that this strategy indeed constitutes an equilibrium and to prove existence we need to show $(i)$ that firms do not have an incentive to deviate from the strategies prescribed and $(ii)$ that $\lambda_A, \lambda_B, \lambda_M \in (0,1)$, that $\lambda_A + \lambda_B + \lambda_M = 1$, that the lower and upper bounds of the supports of the price distributions satisfy the inequality given in Proposition 4, that the price distributions are well-behaved and finally that expected profits are strictly positive, whenever $\mu_A, \mu_B, \phi_A$ and $\phi_B$ satisfy the condition given in the Proposition.

We start by showing that firms cannot profitably deviate. There are various ways in which a firm can deviate. A firm may deviate by advertising a price $p^d \notin \sigma_j$ in segment $j$. Let us see that this is not profitable. First, suppose firm $i$ advertises $p^d \notin \sigma_A$ in $A$. We have two possibilities. One, let $p^d \in (\overline{p}_A, \underline{p}_B]$, then using (3) and the expression for $F_M(p)$ we have that $E\pi_i\left(\lambda_A = 1, p^d; s^{-i}\right) = \lambda_A \mu_A \mu_B \left(R(p^m) - R(p^d)\right) - \phi_A (1 - \mu_A)$, which is strictly decreasing in $p^d \in (\overline{p}_A, \underline{p}_B]$. Therefore, this deviation is not profitable. Two, let $p^d \in \sigma_B$; then using ( 3) and the expression for $F_M(p)$ when $p \in \sigma_B$ leads to $E\pi_i\left(\lambda_A = 1, p^d; s^{-i}\right) = 0$ so the deviation is not profitable. Second, let firm $i$ deviate by advertising a price $p^d \notin \sigma_B$ in segment $B$; again, we have two possibilities. One, let $p^d \in \sigma_A$; then, using (4), we observe that $E\pi_i\left(\lambda_B = 1, p^d; s^{-i}\right) = \mu_B R(p^d) - \phi_B$. Since this expression is strictly increasing in $p^d$, firm $i$ will set $p^d = \overline{p}_A = R^{-1}(\phi_B/\mu_B)$; however this yields zero profits. Thus, this deviation is not profitable. Two, let $p^d \in [\overline{p}_A, \underline{p}_B)$, then using (4) and the expression for $F_M(p)$ when $p \in [\overline{p}_A, \underline{p}_B)$, we have that $E\pi_i\left(\lambda_B = 1, p^d; s^{-i}\right) = \lambda_A \mu_B \mu_A R(p^d) + \lambda_A \mu_B R(p^m) + \phi_A \mu_B - \phi_B$. Since this expression is strictly increasing in $p^d$, firm $i$ does not deviate. Third, suppose firm $i$ deviates by advertising a price $p^d \notin \sigma_M$ in $M$. Then $p^d \in [p_A, \overline{p}_A)$. Using (5) and the expression for $F_A(p)$ we obtain that $E\pi_i\left(\lambda_M = 1, p^d; s^{-i}\right) = \mu_B R(p^d) + (1 - \lambda_A) \mu_A R(\overline{p}_A) - \phi_A - \phi_B$, which is strictly increasing in $p^d$. Hence, this deviation is not profitable. We now observe that a firm may also deviate by advertising a price $p^d \in \sigma_j$ in segment $j' \neq j$. This type of deviation is however ruled out by the cases above where a firm charges a price $p^d \notin \sigma_{j'}$ in $j'$. Finally, a firm may also deviate by advertising a price $p^d \notin \sigma_j$ in $j' \neq j$, but these deviations are also ruled out by the previous arguments. This completes the proof of $(i)$.

We now show $(ii)$. We start noting that $\lambda_A > 0$ since $\phi_B/\mu_B \geq \phi_A/\mu_A > \phi_A$. Moreover, $\lambda_A < 1$ if and only if $\phi_A/\mu_A > (\phi_B/\mu_B - R(p^m))/\mu_A$, which is always satisfied because $\phi_B/\mu_B - R(p^m) < 0$. It is readily seen that $0 < \lambda_B < 1$. We note that $\lambda_M$ is strictly positive if and only if

$$\frac{\phi_A}{\mu_A}(\mu_B R(p^m)(\mu_B - \mu_A) + \phi_B \mu_A) < \mu_B R(p^m)\left(\mu_B R(p^m) - \phi_B\right). \tag{9}$$

If the LHS of (9) is negative, the condition holds; otherwise, $\lambda_M > 0$ requires

$$\frac{\phi_A}{\mu_A} < \frac{\mu_B R(p^m)\left(\mu_B R(p^m) - \phi_B\right)}{\mu_B R(p^m)(\mu_B - \mu_A) + \phi_B \mu_A}. \tag{10}$$

We now observe that expected profit $E\pi = \lambda_A \mu_B R(p^m) - \phi_B$ is strictly positive if and only if

$$\frac{\phi_A}{\mu_A} < \frac{\phi_B}{\mu_B}\left(1 - \frac{\phi_B}{\mu_B R(p^m)}\right) \tag{11}$$

which is the condition in the Proposition. We now note that $\underline{p}_A < \underline{p}_M$ because $\lambda_A > 0$. Further, $\underline{p}_M < \underline{p}_B$ if and only if (11) holds. Furthermore, $\underline{p}_B > 0$ if and only if $\lambda_A > \lambda_B$, or

$$\frac{\phi_A}{\mu_A} < \frac{\phi_B}{\mu_B}\left(\frac{R(p^m)\mu_B}{\mu_B R(p^m) + \phi_B \mu_A}\right). \tag{12}$$

We now prove that if condition (11) holds, then (10) and (12) also hold. We prove this by showing that the RHS of (11) is lower than the RHS of (10) and (12). Consider first (10). We need to show that

$$\begin{aligned}
\frac{\phi_B}{\mu_B}\left(1 - \frac{\phi_B}{\mu_B R(p^m)}\right) &< \frac{\mu_B R(p^m)\left(\mu_B R(p^m) - \phi_B\right)}{\mu_B R(p^m)(\mu_B - \mu_A) + \phi_B \mu_A} \\
\frac{1}{\mu_B}\left(\frac{\phi_B}{\mu_B R(p^m)}\right) &< \frac{\mu_B R(p^m)}{\mu_B R(p^m)(\mu_B - \mu_A) + \phi_B \mu_A}
\end{aligned}$$

Since $\phi_B < \mu_B R(p^m)$, it is sufficient to prove that

$$\begin{aligned}
\frac{1}{\mu_B} &< \frac{\mu_B R(p^m)}{\mu_B R(p^m)(\mu_B - \mu_A) + \phi_B \mu_A} \\
\mu_B R(p^m)(\mu_B - \mu_A) + \phi_B \mu_A &< \mu_B^2 R(p^m) \\
-\mu_A(\mu_B R(p^m) - \phi_B) &< 0.
\end{aligned}$$

Similarly, consider (12). We need to show that

$$\begin{aligned}
\frac{\mu_B R(p^m) - \phi_B}{\mu_B R(p^m)} &< \frac{R(p^m)\mu_B}{\mu_B R(p^m) + \phi_B \mu_A} \\
-\mu_B R(p^m)\phi_B(1 - \mu_A) - \phi_B^2 \mu_A &< 0.
\end{aligned}$$

It remains to check that the price distributions are increasing in $p$. This is readily seen for $F_A(p)$ and $F_M(p)$; for $F_B(p)$, this follows from the fact that $\lambda_A > \lambda_B$. This completes the proof that this equilibrium exists whenever $\frac{\phi_A}{\mu_A}\mu_B R(p^m) < \frac{\phi_B}{\mu_B}(\mu_B R(p^m) - \phi_B)$. Proposition 4 implies that this is the unique positive-profits equilibrium. This completes the proof of the first part of Proposition 2.

We now prove Part II of Proposition 2. Let $\lambda_j \in (0,1)$, $j \in \{A, B, M\}$ and $\lambda_O \in [0,1)$ and let $\underline{p}_A < \overline{p}_A = \underline{p}_B = \underline{p}_M < \overline{p}_B = \overline{p}_M = p^m$. First, since $p^m \in \sigma_B = \sigma_M$ in equilibrium it must be the case that $E\pi_i(\lambda_B = 1, p^m; s^{-i}) = E\pi_i(\lambda_M = 1, p^m; s^{-i}) = 0$. Solving, we obtain $\lambda_O + \lambda_A = \phi_B/\mu_B R(p^m)$ and $\lambda_O + \lambda_B = \phi_A/\mu_A R(p^m)$. Similarly, since $\overline{p}_A \in \sigma_j$, $j = A, B, M$, in equilibrium $E\pi_i(\lambda_j = 1, \overline{p}_A; s^{-i}) = 0$, for any $j = A, B, M$. Solving these conditions we obtain $\lambda_A = (\phi_B\mu_A - \phi_A\mu_B)/\phi_B\mu_A$, and $\overline{p}_A = R^{-1}(\phi_B/\mu_B)$. Plugging $\lambda_A$ into the expressions above yields

$$\lambda_O = \frac{\phi_B^2\mu_A - \mu_B R(p^m)(\phi_B\mu_A - \phi_A\mu_B)}{\phi_B\mu_B\mu_A R(p^m)} \text{ and } \lambda_B = \frac{(\phi_B\mu_A - \phi_A\mu_B)(\mu_B R(p^m) - \phi_B)}{\phi_B\mu_A\mu_B R(p^m)}$$

and $\lambda_M$ simply follows from $\lambda_M = 1 - \lambda_O - \lambda_A - \lambda_B$. Second, in equilibrium it must be the case that $E\pi_i(\lambda_j = 1, p; s^{-i}) = 0$, for $j = B, M$, $\forall p \in \sigma_B = \sigma_M$. Solving these conditions yields

$$F_B(p) = F_M(p) = 1 - \frac{\phi_B}{R(p^m)\mu_B - \phi_B}\frac{R(p^m) - R(p)}{R(p)}.$$

Third, for any price $p \in \sigma_A$ it must be the case that $E\pi_i(\lambda_A = 1, p; s^{-i}) = 0$, which yields:

$$F_A(p) = 1 - \frac{1}{\lambda_A}\left(\frac{\phi_A}{R(p)\mu_A} - (1 - \lambda_A)\right),$$

and by solving $F_A(\underline{p}_A) = 0$, we obtain $\underline{p}_A = R^{-1}(\phi_A/\mu_A)$.

We now prove that this characterization constitutes an equilibrium. It is easy to check $(i)$ that firms do not have an incentive to deviate from the strategies prescribed in the Proposition, and $(ii)$ that the lower and upper bounds of the supports of the price distributions satisfy $\underline{p}_A < \overline{p}_A = \underline{p}_B = \underline{p}_M < \overline{p}_B = \overline{p}_M = p^m$ and that price distributions are well-behaved. Finally, note that it is readily seen that $\lambda_A, \lambda_B, \lambda_M \in (0,1)$ and $\lambda_O < 1$; moreover, inspection of the expression for $\lambda_O$ reveals that $\lambda_O \geq 0$ if and only if $\phi_A/\mu_A \geq (\phi_B/\mu_B)(1 - \phi_B/R(p^m)\mu_B)$. This completes the proof of part II of Proposition 2. ■

For later reference, let assume, without loss of generality, that $\phi_A = \gamma R(p^m)$, with $\gamma \in (0, \mu_A)$ and $\phi_B = \beta\phi_A$, with $\beta \in [\mu_B/\mu_A, \mu_B/\gamma)$. We note that the existence condition for the positive-profits equilibrium can be rewritten as $\gamma < \mu_B(\mu_A\beta - \mu_B)/\beta^2\mu_A$.

**Proof of Lemma 2:** Consider first the positive-profits equilibrium. Given that $\sigma_A, \sigma_B$ and $\sigma_M$ must satisfy Proposition 4, we only need to show that $F_M(p) > F_B(p)$ for all $p \in \sigma_B$. Using the expressions above, it suffices to show that $\lambda_A(1-\lambda_A) > \lambda_B$. Using the formulas for $\lambda_A$ and $\lambda_B$ given in Proposition 2, one gets that $\lambda_A(1-\lambda_A) > \lambda_B$ if and only if

$$\frac{\gamma(\beta-\mu_B)}{\mu_B^2+\gamma\beta\mu_A}\left(1-\frac{\gamma(\beta-\mu_B)}{\mu_B^2+\gamma\beta\mu_A}\right) \quad > \quad \frac{\gamma}{\mu_A}.$$

Isolating $\gamma$ we obtain

$$\gamma < \frac{\beta\mu_B-\mu_B^2(1+\beta)}{\beta^2\mu_A} = \frac{\mu_B(\beta-\mu_B(1+\beta))}{\beta^2\mu_A} = \frac{\mu_B(\beta\mu_A+\mu_B)}{\beta^2\mu_A},$$

which is always satisfied when the condition in Part I of Proposition 2 holds. For the zero-profits equilibrium, the results follow straightforwardly. ■

**Proof of Proposition 3.** Parti $I$. First, $\frac{\partial\lambda_A}{\partial\gamma} = \frac{\mu_B^2(\beta-\mu_B)}{(\mu_B^2+\mu_A\beta\gamma)^2}$ which is strictly positive given the equilibrium condition. Second, $\frac{\partial\lambda_B}{\partial\gamma} = \frac{\gamma}{\mu_A} > 0$; as a consequence $\frac{\partial\lambda_M}{\partial\gamma} < 0$. Third, we claim that an increase in $\gamma$ widens $\sigma_B$. To see this note that $R(\underline{p}_B) = R(p^m)\left(1-\frac{\lambda_B}{\lambda_A}\right)$ and $\frac{\lambda_B}{\lambda_A} = \frac{\mu_B^2+\mu_A\beta\gamma}{\mu_A(\beta-1)}$. Since an increase in $\gamma$ raises $\frac{\lambda_B}{\lambda_A}$, it follows that $\frac{\partial R(\underline{p}_B)}{\partial\gamma} < 0$; thus the claim follows. Fourth, an increase in $\gamma$ widens $\sigma_A$ if and only if $\frac{\partial\left(\frac{R(\overline{p}_A)}{R(\underline{p}_A)}\right)}{\partial\gamma} = \frac{\partial\left(\frac{1}{1-\lambda_A}\right)}{\partial\gamma} > 0$; since $\frac{\partial\lambda_A}{\partial\gamma} > 0$ this is always satisfied. Fifth, an increase in $\gamma$ narrows $\sigma_M$ because $\frac{\partial R(\underline{p}_M)}{\partial\gamma} = \frac{\beta R(p^m)}{\mu_B} > 0$. Sixth, equilibrium profits are $E\pi = R(p^m)(\lambda_A\mu_B - \beta\gamma)$ and in equilibrium $\gamma \in (0, \frac{\mu_B(\beta\mu_A-\mu_B)}{\beta^2\mu_A})$. Further, $\frac{\partial E\pi}{\partial\gamma} = R(p^m)(\frac{\partial\lambda_A}{\partial\gamma}-\beta)$ and using the expression for $\frac{\partial\lambda_A}{\partial\gamma}$ derived above, we note that $\frac{\partial E\pi}{\partial\gamma} > 0$ if and only if $\mu_B^3(\beta-\mu_B)-\beta(\mu_B^2+\mu_A\beta\gamma)^2 > 0$; otherwise $\frac{\partial E\pi}{\partial\gamma} < 0$. It is readily seen that the expression $\mu_B^3(\beta-\mu_B)-\beta(\mu_B^2+\mu_A\beta\gamma)^2$ is decreasing in $\gamma$ and strictly positive for $\gamma = 0$. Moreover, the expression $\mu_B^3(\beta-\mu_B)-\beta(\mu_B^2+\mu_A\beta\gamma)^2$ is negative for $\gamma = \frac{\mu_B(\beta\mu_A-\mu_B)}{\beta^2\mu_A}$ whenever $(\beta-\mu_B)(\mu_B-\beta\mu_A) < 0$, which is always satisfied in equilibrium. Finally, we note that as $\gamma \to 0$, $\lambda_A$ and $\lambda_B$ converge to zero which implies that $\lambda_M \to 1$. In addition $\underline{p}_M$ goes to zero and $F_M(p) \to 1$. This completes the proof of part $(I)$. We now prove Part II. First, it is readily seen that $\lambda_A$ is independent of $\gamma$. Second, note that in equilibrium $\lambda_B = \lambda_A - R(p^m)(\phi_B/\mu_B - \phi_A/\mu_A)$; since $(\phi_B/\mu_B - \phi_A/\mu_A) > 0$, it is increasing in $\gamma$ and $\lambda_A$ is constant in $\gamma$, it follows that $\lambda_B$ decreases in $\gamma$. Third, in equilibrium $\lambda_M = 1-\lambda_A-R(p^m)(\phi_A/\mu_A)$, and since $\phi_A$ is increasing in $\gamma$ it follows that $\lambda_M$ is decreasing in $\gamma$. These three remarks imply that $\lambda_O$ is increasing in $\gamma$. Finally, it is easy to very how the supports change with $\gamma$. This concludes the proof of part $II$ of the Proposition. ■

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