Strategic Intermediation in a Two-Sided Market

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Abstract

We examine a two-sided market where intermediaries compete to attract advertising from firms and audience from buyers. Firms sell homogeneous products, compete in prices and must advertise them in the intermediary platforms to attract consumers. Buyers must subscribe to the intermediaries to receive product and price information. We show that a monopolist intermediary fully internalizes the externalities between buyers and sellers, which produces an efficient outcome. By contrast, when the market for information is operated by two intermediaries, firms and consumers attempts to obtain surplus from participation lead to the emergence of coordination frictions, which yield an inefficient outcome.

Keywords: Two-sided markets, network externalities, intermediation, advertising

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1 Introduction

This paper presents an analysis of strategic intermediation in a two-sided market. Two-sided markets are characterized by the existence of two groups of agents which derive gains from conducting transactions with each other, like for example tenants and landlords, and the existence of intermediaries which facilitate these transactions, like real state agents. There are many more examples of two-sided markets, like videogame platforms, internet portals or dating services.[1] An important characteristic of two-sided markets is that the functioning of the market depends not only on the total level of transaction costs jointly faced by the two groups of agents (*price level*) but also on the particular allocation of those costs between them (*price structure*) (Rochet and Tirole, 2004). We examine pricing behavior in a two-sided market that is monopolized and compare it with the case of strategic intermediation. The novel aspect of our paper is that we model explicitly the interaction between the two groups of agents in the market; this enables us to better understand the implications of changes in the price level and in the price structure on market competitiveness and thus on social welfare. The details of our model are as follows.

There are three types of agents in the economy: intermediaries, firms and consumers. Intermediaries, who own information platforms without which firms and consumers cannot conduct transactions, compete to attract firm advertising and consumer audience. Firms market a homogeneous product and must advertise their prices in the intermediary platforms to reach consumers. Likewise, buyers must subscribe to the intermediaries to access price information. The interaction between these three types of agents is modelled via the following two-stage game. In the first stage, intermediaries simultaneously choose their advertising fees for the firms and their subscription charges for the consumers to maximize their profits. In the second stage, firms simultaneously choose where to place their advertisements and which price to charge, while consumers decide which intermediary to subscribe to. The market clears when each intermediary releases the price information it has obtained from firm advertising to its subscribing consumers and transactions take place. We focus on monopolistic intermediation and on strategic intermediation when firms and consumers employ single-homing strategies.[2]

We first examine the case of monopolistic intermediation. For any given pair of (strictly positive) advertising and subscription fees, there are two equilibria in the continuation game. One involves

[1]See Armstrong (2004), Evans (2003) and Rochet-Tirole (2003) for an extensive set of examples.

[2]In Section 4 we examine how our main results change if we allow for multi-homing strategies.

full consumer participation and the other only partial consumer participation; in both cases firms do not participate fully and there is price dispersion. In the equilibrium with partial consumer participation, firms advertise more frequently as the fraction of subscribing consumers increases; likewise, consumers' expected utility is larger the more frequently the firms advertise their prices in the intermediary. We find that partial consumer participation cannot be part of a subgame perfect equilibrium (SPE). This is because otherwise consumer elasticity of participation is positive, i.e., the fraction of subscribing consumers is increasing in the subscription charge. This is due to the indirect network externalities between buyers and sellers. If consumers do not participate fully, they must be indifferent between participating and not participating. An increase in the consumer subscription charge, ceteris paribus, makes buyer participation less profitable; to reestablish equilibrium firms must advertise more frequently, which can only occur if buyers participate more often. In the SPE of the game the monopoly intermediary charges nothing for firm advertising and extracts all the rents from consumers. In this way the intermediary fully internalizes the network externalities present in the market, which yields an efficient outcome. In spite of the monopoly power of the intermediary, and in spite of the market power of the firms, the equilibrium is efficient because it entails full firm and full consumer participation as well as marginal cost pricing.

We then turn to study the case of competing intermediation. In particular we examine a market for intermediation services operated by two intermediaries. Our first finding is that there are continuation game equilibria entailing full firm and full consumer participation. This implies that, for some advertising and subscription fees, both groups of agents, firms and consumers, obtain positive surplus. This differs from the monopoly case where firms do not participate fully and we now explain why this happens. In the monopoly case firm competition is so intense that if firms did participate in the market with probability one they would not cover their advertising costs. In the duopoly market, firms can mitigate competition by randomizing their advertising strategies in such a way that, even if they participate in the market with probability one, the chance that they compete for the same consumers is relatively low.[3]

Our second finding is that a unique symmetric outcome where both intermediaries are active in the market for information can be sustained as a subgame perfect equilibrium. Compared to the case of monopolistic intermediation, we see that the *price level* is lower and the *price structure* is

[3] See Galeotti and Moraga-González (2003) for a model of strategic targeted advertising which delivers this kind of result.

different. In particular, both sides of the market are equally treated and face identical positive fees. In equilibrium firms randomize between advertising in intermediary A and in intermediary B and so do consumers with respect to their subscription strategies. The market equilibrium exhibits price dispersion and firms and consumers are left with no surplus. In spite of full firm and consumer participation, the equilibrium is not efficient due to a coordination friction. This inefficiency cannot be eliminated if we allow for firm multi-homing strategies, nor can it be mitigated if firms and consumer multi-home and intermediaries' costs are large enough. Seen together, these results indicate that there may be excessive incentives to enter the market for intermediation services.

Our paper is a contribution to the literature on two-sided markets. These are markets where two or more groups of agents conduct transactions via intermediaries. In the classical literature on intermediation, intermediaries "make the market" by choosing input bid-prices and output ask-prices to maximize their profits (see Spulber (1999) and the references therein). Some authors have analyzed how intermediated exchange can arise in search markets (see e.g. Gehrig (1993) and Yavas (1994,1996)). More recent papers include Armstrong (2004), Armstrong and Wright (2004), Caillaud and Jullien (2001, 2003), Gabszewicz and Wauthy (2004) and Rochet and Tirole (2003, 2004).[4] These papers examine various kinds of two-sided markets, including monopolistic intermediation as well as competing intermediation, with single-homing and with multi-homing strategies. A common feature of these papers is that agents on one side of the market meet agents on the other side of the market according to an exogenously specified matching process. This formulation assumes away the conflicting interests agents on one side of the market typically have to attract agents on the other side of the market; in this sense, such formulation is inadequate for markets where one group of agents supplies rival goods to the other side of the market, like in housing and labor markets. Our paper models this interaction explicitly in a homogeneous product setting and this allows us to better understand how intermediation affects the competitiveness of the market. For example, while Armstrong (2004) and Rochet and Tirole (2003) obtain the outcome of monopolistic intermediation to be inefficient, we find that a monopolist can efficiently internalize the externalities between buyers and sellers. Further, while Armstrong and Wright (2004) competing intermediaries charge nothing to buyers and get the bulk of their profits from sellers under single-homing, our intermediaries treat the two sides of the market equally.

[4] See also Nocke, Peitz and Stahl (2004) for a study of how different ownership structures of a monopolistic platform affect its platform size.

The paper most closely related to ours is by Baye and Morgan (2001).[5] This paper examines a monopolist gatekeeper's decisions to attract advertising by homogeneous product firms and consumer subscriptions. An important difference with our paper is that in their model consumers and firms are located in segmented markets, with one firm per location, so in the absence of the intermediary firms and buyers can transact on their own. They show that the monopolist intermediary profit is maximized when there is full consumer participation but only partial firm participation. As a result, the market outcome is inefficient. Their result differs from ours and the reason is that in their model consumers can buy locally at a price that is equal to the price the local firm advertises in the monopolist's platform. Indeed, if there were full firm participation, the local firm would charge a price equal to the marginal cost and then consumers would prefer to buy locally rather than to pay subscription fees. A second important difference is that while their paper focuses on monopolistic intermediation, we also examine here how intermediary competition affects the competitiveness of homogeneous product markets.

Our paper is also related to a recent literature on advertising in commercial media markets. In these papers consumers derive utility from media consumption and the media bridge the gap between sellers and buyers. Anderson and Coate (2003) analyze the private and social provision of programming and advertising in a market where sellers offer new products. In a related paper Dukes (2003) studies the private and the social provision of advertising when sellers offer differentiated products. Gabszewicz, Laussel and Sonnac (2001) and Gal-Or and Dukes (2003) present spatial models of broadcasting competition and study the extent to which media providers differentiate their services between them. Finally, Rysman (2004) structurally estimates network effects in a model where yellow page directories compete to attract advertisements and readers.

The remainder of the paper is organized as follows. Section 2 presents the model we examine. Section 3.1 studies the case of monopolistic intermediation. Section 3.2 examines a market with two intermediaries when firms and consumers single-home. Section 4 discusses the possible implications of multi-homing. Section 5 concludes by summarizing our main findings. Some of the proofs have been relegated to an appendix to ease the reading of the paper.

[5] See also Baye and Morgan (2000).

2 The model

We examine a two-sided market where intermediaries compete to attract advertising from firms and audience from buyers. As an example, consider the case of newspapers or magazines as information intermediaries. Sellers can reach consumers by placing advertisements in different newspapers so a newspaper' readership is a valuable good that intermediaries offer to firms. Likewise, consumers get information about goods and their prices via the newspapers so advertising by firms is a valuable good that intermediaries offer to consumers. As a result, newspapers compete in advertising fees to attract firm advertisements and in subscription fees to attract readers. The real state market is another example where our model fits well. We now present the model formally.

Consider a market for information services operated by two competing intermediaries, labelled A and B. The two intermediaries compete to sell advertising space for firms by setting their advertising fees ϕ_j, $j = A, B$. Likewise, they compete to attract consumers by setting subscription fees κ_j, $j = A, B$. This pricing scheme involving lump-sum fees is reasonable in situations where monitoring transactions is quite costly.[6] An intermediary pure strategy is then a pair $\{\phi_j, \kappa_j\}$, $j = A, B$. We assume that intermediaries face an arbitrarily fixed cost of processing information $\varepsilon > 0$. This cost can also interpreted as an entry cost into the market for intermediation services.

On the supply side of the market there are $(N \geq)2$ firms producing a homogeneous product and competing in prices to sell their products to the consumers.[7] Sellers produce the good at constant returns to scale and their identical unit production cost is normalized to zero. We assume that firms cannot sell their goods in the absence of a market for information so they have to advertise their prices to be able to sell. We also assume that firms cannot advertise their products in both platforms, i.e., we are restricting the analysis to the case known in the literature as *single-homing*.[8] Everything else equal, firms prefer to advertise their prices in the intermediary with a larger audience. A firm i may thus decide to advertise her price either in intermediary A, or in intermediary B, or, finally, not to advertise at all and stay out of the market. We shall represent this set of pure advertising-strategies as $E_i = \{O, A, B\}$, where O indicates the decision staying out of the market. A firm i's advertising strategy is then a probability function over the set E_i.

[6]We follow Baye and Morgan (2001) in our formulation. See Armstrong (2004) and Rochet and Tirole (2003) for a discussion of different pricing mechanisms.

[7]For simplicity, we will set $N = 2$ in our computations but no result really depends on this.

[8]See Section 4 for a discussion of the implications of multi-homing strategies on our main results.

We will refer to λ_j^i as the probability with which a firm i chooses action $j \in E_i$. A firm i's pricing strategy is denoted by a distribution of prices $F_j^i(p)$ accompanying advertising decision $j \in E_i$. Let σ_j^i denote the support of $F_j^i(p)$ and let $\overline{p}_j^i$ and $\underline{p}_j^i$ denote the maximum and the minimum price in σ_j^i, respectively. A strategy for firm i is thus denoted by a collection of pairs $\{\lambda_j^i, F_j^i\}_{j \in E_i}$, $i = 1, 2$.

On the demand side of the market, there is a unit mass of consumers. All consumers are identical and are willing to pay $v > 0$ for one unit of the good. We assume that consumers initially ignore the existence and the price of the goods so they cannot buy unless they get information from the intermediaries. Again, we examine the case of single-homing, i.e., we assume that a consumer can only become member of one of the intermediary platforms. Everything else equal, consumers prefer to be members of the intermediary who attracts more advertising. A consumer may thus decide to subscribe to intermediary A, or to intermediary B, or, finally, decide to stay out of the market altogether. We shall represent this set of strategies as $R = \{O, A, B\}$. A consumer subscription strategy is then a probability function over the set R. We will refer to μ_k as the probability with which a consumer chooses action $k \in R$.

Intermediaries, firms and consumers play the following two-stage game. In the first stage, intermediaries simultaneously choose their advertising fees and their subscription charges to maximize their profits. In the second stage, firms and consumers observe intermediaries' fees and the former simultaneously choose where to place their ads and which price to charge while the latter decide which intermediary to subscribe to. The market clears when intermediaries release the price information they obtain from advertisers to their subscribing buyers and transactions take place. Our interest lies on the characterization of subgame perfect equilibria (SPE). To solve for a SPE, we proceed backwards.

It is known that the existence of network externalities often leads to multiple equilibria. In this connection, some important observations are in line. One, there is a trivial continuation game equilibrium that we ignore in the remainder of the paper: in this equilibrium firms and consumers exit the market after observing any profile of advertising and subscription fees. Two, there are strategies that can virtually sustain any outcome as a SPE; for example, to sustain a pair $\{\hat{\phi}, \hat{\kappa}\}$ as a symmetric SPE one can propose a continuation game strategy profile where a deviation from $\{\hat{\phi}, \hat{\kappa}\}$ triggers the exit of firms and consumers from the market. This type of strategies are not very interesting for several reasons: (i) they require quite a bit of coordination between agents with conflicting interests, (ii) they can be ruled out using standard trembling-hand type of arguments and (iii) they can be ruled out if intermediaries have already positive firm and consumer bases.

3 Analysis

Our objective is to analyze the effects of intermediary competition in two-sided markets. The analysis will also clarify the role played by information intermediaries in the competitiveness of homogeneous product markets. We start by examining the benchmark case of a market for information services operated by a monopolist. We show that, in spite of the monopoly power of the intermediary, and in spite of the market power of the product sellers, the unique SPE of the game is efficient. In equilibrium the intermediary subsidizes the firms to attract advertising and to increase competition between sellers; this gives consumers large incentives to subscribe to the intermediary, who in turn expropriates them by charging large subscription fees.

3.1 Monopoly intermediation

As a benchmark case, we examine here a setting where the market for information is operated by a monopolist intermediary, denoted M. In this situation firms can either advertise in the monopoly platform or stay out of the market, i.e., $E_i = \{M, O\}$. Likewise, consumers can decide to subscribe to the intermediary or not at all, i.e., $R = \{M, O\}$.

Continuation game equilibria. We start by describing the second stage of the game. Let $\{\phi, \kappa\}$ denote the monopoly intermediary strategy, $0 \leq \phi \leq v$, $0 \leq \kappa \leq v$. Then:

Proposition 1 *There are two symmetric equilibria: one with full consumer participation and the other with partial consumer participation. (i) The full consumer participation equilibrium takes the following form: with probability $\lambda_M = (v - \phi)/v$, firms advertise in the monopoly platform a price p randomly chosen from the set $\sigma_M = [\phi, v]$ according to the price distribution $F_M(p) = 1 - \frac{\phi}{v-\phi}\frac{v-p}{p}$; with the remaining probability $\lambda_O = 1 - \lambda_M$, firms stay out of the market. Consumers subscribe to the intermediary with probability 1. (ii) The partial consumer participation equilibrium takes the following form: with probability $\lambda_M = (\mu_M v - \phi)/\mu_M v$, firms advertise in the monopoly platform a price p randomly chosen from the set $\sigma_M = \left[\frac{\phi}{\mu_M}, v\right]$ according to the price distribution $F_M(p) = 1 - \frac{\phi}{\mu_M v-\phi}\frac{v-p}{p}$; with the remaining probability $\lambda_O = 1 - \lambda_M$, firms stay out of the market.*

Consumers subscribe to the intermediary with probability $\mu_M = \frac{\phi}{v(1-\sqrt{\kappa/v})}$. *These equilibria exist for all* $0 \leq \phi < v$ *and* $0 \leq \kappa \leq (v-\phi)^2/v$.

Proof. The proof is in the Appendix.

This Proposition shows that equilibria in the continuation game are characterized by partial firm participation and either full consumer participation or partial consumer participation. In the first case, consumers obtain positive utility while in the second case they are indifferent between subscribing to the intermediary and exiting the market. An interesting feature of these equilibria is that they exhibit random advertising and random pricing: for any $\phi > 0$, firms advertise their prices with probability less than 1 because otherwise competition would drive prices down to marginal costs and they would not cover the advertising fee. As a result, a firm is alone in the market with strictly positive probability, which implies that a pure strategy pricing equilibrium does not exist. The product market thus exhibits price dispersion. In equilibrium a firm is indifferent between advertising and not advertising at all so it obtains zero profits.

It is interesting to pay some attention to the (indirect) network externalities that arise in this setting. The (expected) payoff to an advertising firm equals $E\pi = (1-\lambda_M)\mu_M v - \phi$, while the (expected) utility a subscribing consumer obtains is $Eu = \lambda_M^2 v - \kappa$ (see the proof of Proposition 1 in the Appendix). Thus, everything else equal, firm profits increase as the fraction of consumers who subscribes to the monopolist intermediary rises; likewise, everything else equal, the utility a consumer obtains from subscribing to the intermediary rises as firms advertise more frequently. These two implications are typically imposed by assumption in the literature on two-sided markets. We see, however, that an explicit model of market interaction leads to a different result. In particular, in the continuation game equilibrium, an increase in the number of subscribing buyers has no effect on the profits of the firms. This is because an increase in the fraction of consumers who subscribe to the intermediary boosts firm incentives to advertise, which increases firm competitiveness and in turn decreases profits. When we look at the buyer side of the market instead, a different result appears. In the equilibrium entailing full consumer participation, a buyer's utility increases as more firms are on the other side of the market. The gains to the consumers are related to two facts. One, the more frequently firms are present on the other side of the market the higher the probability a buyer gets served; two, the more frequently firms advertise in the market, the lower the prices they charge.

Subgame perfect equilibrium. We now move to the first stage of the game. The monopoly intermediary, anticipating the equilibria in the continuation game, chooses the pair of advertising and subscription fees $\{\phi, \kappa\}$ to maximize its profits. The profits of the intermediary are:

$$\Pi(\phi,\kappa) = \phi \sum_{i=1}^{2} \binom{2}{i} \lambda_M^i (1-\lambda_M)^{2-i} + \kappa\mu_M - \varepsilon = 2\lambda_M \phi + \kappa\mu_M - \varepsilon \tag{1}$$

We first show that a SPE with partial consumer participation does not exist. Suppose consumers stay out of the market with strictly positive probability in the continuation game equilibrium. Then, by Proposition 1, the equilibrium fraction of consumers who subscribe to the intermediary is $\mu_M = \frac{\phi}{v(1-\sqrt{\kappa/v})}$. We note that this fraction is increasing in the subscription fee κ, i.e., the elasticity of consumer demand for participation in the platform is strictly positive! This is somewhat surprising and we explain now how this result relates to network externalities. In the continuation game equilibrium, buyers are indifferent between subscribing to the intermediary and staying out of the market; therefore, it must be the case that $Eu = \lambda_M^2 v - \kappa = 0$. Ceteris paribus, an increase in κ yields negative utility to consumers. To reestablish equilibrium firms must advertise more frequently, which can only occur if the fraction of subscribing consumers increases ($\lambda_M = 1 - \phi/\mu_M v$). We also note that the fraction of consumers who subscribe to the intermediary is increasing in the advertising fee ϕ. The reason for this result is somewhat related. Ceteris paribus, an increase in ϕ decreases the frequency with which firms advertise their product, which yields negative utility to the consumers. To reestablish equilibrium firms must increase their propensity to advertise, which again can only happen if the fraction of subscribing consumers rises. Plugging the expression for μ_M into (1), the profits of the monopoly intermediary are equal to

$$\Pi(\phi,\kappa) = 2(1 - \frac{\phi}{\mu_M v})\phi + \kappa\mu_M - \varepsilon = 2(\sqrt{\kappa/v})\phi + \kappa\frac{\phi}{v(1-\sqrt{\kappa/v})} - \varepsilon$$

We note now that this payoff is strictly increasing in κ, for all ϕ, which follows straightforwardly from the observations above: an increase in κ increases buyers' demand for accessing price information μ_M and also sellers' demand for advertising λ_M. As a result a SPE must involve full consumer participation.

Consider then that $\mu_M = 1$. In this case the problem of the monopolist intermediary is

$$\max_{\phi,\kappa}\{\Pi(\phi,\kappa) = 2(1 - \frac{\phi}{v})\phi + \kappa - \varepsilon\} \tag{2}$$

$$\text{subject to } \kappa \leq \frac{(v-\phi)^2}{v}$$

Since the consumer demand for information is inelastic, the intermediary has an incentive to continue increasing its charge κ till the constraint is binding. As a result, the problem in (2) reduces to

$$\max_{\phi}\{\Pi(\phi) = 2(1-\frac{\phi}{v})\phi + \frac{(v-\phi)^2}{v} - \varepsilon\} \tag{3}$$

and setting $\kappa = \frac{(v-\phi)^2}{v}$. Note that the subscription fee charged to the consumers decreases as ϕ increases. This is because an increase in ϕ reduces the frequency with which firms advertise their prices in the monopoly intermediary, and this decreases consumer willingness to pay for information. Note also that the elasticity of firm demand for advertising is negative. The first order condition of the problem in (3) is

$$\frac{d\Pi}{d\phi} = 2(1-\frac{2\phi}{v}) - \frac{2(v-\phi)}{v} = -\frac{2\phi}{v} < 0,$$

which implies that the intermediary's profits are strictly decreasing in ϕ. As a result:

Theorem 1 *In a monopolized market for information the unique outcome which can be sustained as a SPE takes the following form: The monopolist intermediary sets an advertising fee* $\phi^* = 0$ *and a subscription fee* $\kappa^* = v$. *Firms enter the market with probability 1, advertise a price equal to the marginal cost and obtain zero profits. Consumers subscribe to the intermediary with probability 1, buy a product surely and obtain no utility. In equilibrium the monopoly intermediary obtains a profit* $\Pi = v - \varepsilon$ *and the market outcome is efficient.*

We would like to mention three issues here. First, this result extends straightforwardly to the case of N firms and consumers holding elastic demands.[9] Second, the efficiency result contrasts with the results in Armstrong (2004), Rochet and Tirole (2003) and Baye and Morgan (2001). In our model the elasticity of consumer demand for participation is positive while the elasticity of firm demand for advertising is negative. As a result, the monopolist can efficiently internalize the externalities between buyers and sellers by decreasing the advertising fee and increasing the subscription charge, which maximizes participation and thus welfare. Third, the fact that monopolistic intermediation yields an efficient outcome is something that can only arise due to the two-sided nature of the market. To see this, suppose that consumer behavior were exogenous. Then the monopoly intermediary would maximize profits by setting an advertising fee $\phi = v/2$. As a result, the product market would be characterized by random advertising and price dispersion, which implies that consumers would not buy with strictly positive probability and a dead-weight loss would result.[10]

[9] The proof of this result is available from the authors upon request.

[10] For details of these derivations see Galeotti and Moraga-González (2004).

3.2 Competing intermediation.

Consider now that the two intermediaries A and B operate in the market for information and compete to attract firm advertisements and consumer audience. We assume that firms and consumers use single-homing strategies, i.e., a firm advertises its price in a single intermediary platform and consumers subscribe to just one of the intermediaries. Later in Section 4 we discuss the case of multi-homing. Of course firms and consumers will choose the most attractive intermediary to conduct transactions. In what follows we concentrate on describing equilibria where both intermediaries are active. It is obvious that there also exist two equilibria where only one of the intermediaries is active; these equilibria are characterized as in Theorem 1.

Continuation game equilibria. In what follows we characterize firms' equilibrium advertising strategies and buyers' subscription decisions for any given profile of intermediaries' advertising and subscription fees. It turns out that the payoff function of each intermediary does not depend on firms' pricing behavior, so we will only provide the equilibrium firms' advertising strategies in the main text and relegate the details about firms' pricing behavior to the appendix.

For the purpose of this paper, it is enough to characterize equilibria for the region of parameters where the inequality $\phi_A/\mu_A \leq \phi_B/\mu_B$ holds. The equilibria for the complementary region follow by symmetry. The next Proposition shows that there are four equilibria: one with full firm and consumer participation, one where firms and consumers participate only partially, one with full firm but partial consumer participation and one where consumers participate fully and firms only partially. An interesting feature of the continuation game equilibria is that firms advertise some times in intermediary A, some times in intermediary B, and randomize their prices in all cases. In particular, firms advertise more (less) frequently and charge lower (higher) prices in the, ceteris paribus, more (less) profitable intermediary.

Proposition 2 *(I) There exists a single symmetric equilibrium where* $\lambda_O + \lambda_A + \lambda_B = 1$ *and* $\mu_O + \mu_A + \mu_B = 1$*; in particular with probability* $\lambda_j = 1 - \frac{\phi_j}{\mu_j v}$ *firms advertise in intermediary* j *a price randomly chosen from an atomless price distribution* $F_j(p), j \in \{A, B\}$*, and consumers subscribe to intermediary* j *with probability* $\mu_j = \frac{\phi_j}{v - \sqrt{v\kappa_j}}, j \in \{A, B\}$*. For this equilibrium to exist the advertising and subscription fees* $\{(\phi_A, \kappa_A), (\phi_B, \kappa_B)\}$ *must be such that the following conditions hold: (i)* $\phi_A/\mu_A \geq v - \phi_B/\mu_B$ $(\lambda_O \geq 0)$*; (ii)* $(\phi_A/(v - \sqrt{v\kappa_A})) + (\phi_B/(v - \sqrt{v\kappa_B})) \leq 1$ $(\mu_O \geq 0)$

(II) There exists a continuum of symmetric equilibria where $\lambda_A + \lambda_B = 1$ *and* $\mu_O + \mu_A + \mu_B = 1$*;*

in particular with probability $\lambda_j = \frac{\mu_j v - \phi_j + \phi_{j'}}{(1-\mu_O)v}$ *firms advertise in intermediary* j *a price randomly chosen from an atomless price distribution* $F_j(p), j, j' \in \{A, B\}$, $j \neq j'$, *and consumers subscribe to intermediary* B *with an arbitrarily chosen probability* μ_B, *to intermediary* A *with probability* $\mu_A = \frac{(\mu_B v - \phi_B + \phi_A)\sqrt{\kappa_A}}{\sqrt{\kappa_B} v} + \frac{\phi_A - \phi_B}{v}$ *and stay out of the market with probability* $\mu_O = 1 - \mu_A - \mu_B$. *For this equilibrium to exist the advertising and subscription fees* $\{(\phi_A, \kappa_A), (\phi_B, \kappa_B)\}$ *must be such that the following conditions hold: (i)* $\phi_A/\mu_A \leq v - \phi_B/\mu_B$ $(E\pi^* \geq 0)$; *(ii)* $(\sqrt{\kappa_B} + \sqrt{\kappa_A})(\mu_B v - \phi_B + \phi_A) \leq v\sqrt{\kappa_B}$ $(\mu_O \geq 0)$.

(III) There exists a single symmetric equilibrium in which $\lambda_A + \lambda_B = 1$ *and* $\mu_A + \mu_B = 1$; *in particular with probability* $\lambda_j = \frac{\mu_j v - \phi_j + \phi_{j'}}{v}$ *firms advertise in intermediary* j *a price randomly chosen from an atomless price distribution* $F_j(p), j, j' \in \{A, B\}$, $j \neq j'$, *and consumers subscribe to intermediary* j *with probability* $\mu_j = \frac{v - k_{j'} + k_j}{2v} + \frac{\phi_j - \phi_{j'}}{v}, j, j' \in \{A, B\}$, $j \neq j'$. *For this equilibrium to exist the advertising and subscription fees* $\{(\phi_A, \kappa_A), (\phi_B, \kappa_B)\}$ *must be such that the following conditions hold: (i)* $\phi_A/\mu_A \leq v - \phi_B/\mu_B$ $(E\pi^* \geq 0)$; *(ii)* $\lambda_j^2 v - \kappa_j \geq 0, j \in \{A, B\}$, $(Eu^* \geq 0)$.

(IV) There exists a single symmetric equilibrium where $\lambda_O + \lambda_A + \lambda_B = 1$ *and* $\mu_A + \mu_B = 1$; *in particular with probability* $\lambda_j = 1 - \frac{\phi_j}{\mu_j v}$ *firms advertise in intermediary* j *a price randomly chosen from an atomless price distribution* $F_j(p), j \in \{A, B\}$, *and consumers subscribe to intermediary* A *with a probability* μ_A *solution to*

$$\kappa_B - \kappa_A - \frac{(\phi_A \mu_B - \phi_B \mu_A)(\mu_B(\mu_A v - \phi_A) + \mu_A(\mu_B v - \phi_B))}{\mu_A^2 \mu_B^2 v} = 0 \tag{4}$$

For this equilibrium to exist the advertising and subscription fees $\{(\phi_A, \kappa_A), (\phi_B, \kappa_B)\}$ *must be such that the following conditions hold: (i)* $\phi_A/\mu_A \geq v - \phi_B/\mu_B$ $(\lambda_O \geq 0)$; *(ii)* $\lambda_j^2 v - \kappa_j \geq 0, j \in \{A, B\}$, $(Eu^* \geq 0)$.

Proof. The proof is in the Appendix.

The proof of this proposition proceeds as follows. We first rule out all pure advertising strategy profiles, symmetric and asymmetric. We then note that a firm equilibrium requires sellers to allocate strictly positive probability to advertising in intermediary A and in intermediary B. Moreover, we see that if the per consumer advertising fees are relatively high, firms cannot obtain positive profits in equilibrium and thus allocate positive probability to staying out of the market. These firm equilibria are accompanied by either full consumer participation or partial consumer participation. The existence conditions reported in Proposition 2 are those that are useful in the analysis that follows; the rest of the conditions for existence of each type of equilibrium are relegated to the appendix.

We would like to highlight the fact that, in the presence of competing intermediaries, firms can obtain positive profits in the continuation game, as opposed to the monopoly intermediary case where firms just break even. The reason for this result is that firms, by advertising in different intermediaries, can target their ads to different sets of customers; this mitigates competition in the product market because the probability firms coincide at selling to the same consumers is small.[11]

For each of the continuation game equilibria described in Proposition 2, the (expected) payoff to a firm can be written as $E\pi_i = (1-\lambda_j)\mu_j v - \phi_j$, and consumers' (expected) utility as $Eu = \lambda_j^2 v - \kappa_j$, $j \in \{A, B\}$. Once again we see that the nature of market interaction is important to understand the impact of the (indirect) network externalities. Consider a firm advertising in intermediary j; an increase in the fraction of consumers who subscribe to j does result in greater profits for the firm only if firms obtain positive profits in the continuation game equilibrium, otherwise its profits remain constant. That is, we see that the payoff to an advertising firm is not necessarily increasing in the fraction of subscribing consumers.

Subgame perfect equilibrium. We now examine intermediaries' decisions. Each intermediary, anticipating firms' and consumers equilibrium behavior in the continuation game, takes the rival intermediary's strategy as given and chooses its advertising fee and its subscription charge to maximize its profits. The (expected) payoff to intermediary $j = A, B$ is given by:

$$E\Pi_j(\phi_j, \kappa_j; \phi_{-j}, \kappa_{-j}) = \phi_j \sum_{i=1}^{2} \binom{2}{i} 2^{i-1} \lambda_j^i(\cdot)(1-\lambda_j(\cdot))^{2-i} + \kappa_j \mu_j(\cdot) - \varepsilon = 2\lambda_j(\cdot)\phi_j + \kappa_j\mu_j(\cdot) - \varepsilon$$

Our first result in this Section is that for intermediaries to maximize their profits, it must be the case that all consumers participate in the market.

Proposition 3 *A subgame perfect equilibrium with partial consumer participation does not exist.*

Proof. We prove this result by contradiction. First, suppose that the strategies $\{(\phi_A, \kappa_A), (\phi_B, \kappa_B)\}$ are part of a SPE where in the continuation game firms and consumers play strategies satisfying $\lambda_O + \lambda_A + \lambda_B = 1$ and $\mu_O + \mu_A + \mu_B = 1$, $\lambda_j, \mu_j > 0$, $j = A, B, O$. From Proposition 2 it follows that $\mu_O = 1 - \phi_A/(v - \sqrt{\kappa_A v}) - \phi_B/(v - \sqrt{\kappa_B v})$, which is monotonically decreasing in ϕ_A and in κ_A; moreover, the equilibrium payoff to, say, intermediary A can be written as

$$\Pi_A = 2\phi_A \frac{\sqrt{\kappa_A v}}{v} + \frac{\phi_A \kappa_A}{v - \sqrt{\kappa_A v}} - \varepsilon$$

[11]This observation is robust to consumer and firm multi-homing; see Galeotti and Moraga-González (2003) for a model of strategic targeted advertising.

This payoff is strictly increasing in ϕ_A and κ_A, which implies that the condition $\mu_O \geq 0$ is eventually violated.

Consider now that the strategies $\{(\phi_A, \kappa_A), (\phi_B, \kappa_B)\}$ are part of a SPE where firms obtain positive profits and consumers do not participate fully, i.e., $\lambda_A + \lambda_B = 1$, $\lambda_j > 0$, $j = A, B$, and $\mu_O + \mu_A + \mu_B = 1$, $\mu_j > 0$, $j = A, B, O$. In this case, Proposition 2 tells us that there is a continuum of equilibria in the continuation game. For a given μ_B the probability with which consumers stay out of the market is $\mu_O = 1 - \mu_B - (\mu_B\sqrt{\kappa_A v} - \phi_B + \phi_A)/(v - \sqrt{\kappa_A v})$, which is monotonically decreasing in ϕ_A; moreover, the payoff to, say, intermediary A equals

$$\Pi_A = 2\phi_A \frac{\sqrt{\kappa_A v}}{v} + \frac{\mu_B\sqrt{\kappa_A v} - \phi_B + \phi_A}{v - \sqrt{\kappa_A v}}\kappa_A - \varepsilon$$

Since this payoff is strictly increasing in ϕ_A, the condition $\mu_O \geq 0$ is eventually violated. ■

We now present our main result in this Section:

Theorem 2 *In a duopolistic market for information the unique symmetric outcome which can be sustained as a SPE with two active intermediaries is a follows: Intermediaries set advertising fees* $\phi_A^* = \phi_B^* = v/4$ *and subscription fees* $\kappa_A^* = \kappa_B^* = v/4$*. With probability* $1/2$*, firms advertise a price randomly chosen from the set* $[v/2, v]$ *according to the price distribution* $F(p) = 1 - (v-p)/p$ *in intermediary* A*, and with the remaining probability they do so in intermediary* B*. Consumers subscribe to intermediary* A *with probability* $1/2$ *and to intermediary* B *with probability* $1/2$*. In equilibrium each intermediary obtains a profit* $\Pi = 3v/8 - \varepsilon$*, firms obtain zero profits and consumers obtain no utility. The market outcome is inefficient and the dead-weight loss is equal to* $v/4$*.*

This result shows that there is a single symmetric outcome where both intermediaries are active in the market that can be sustained as a SPE.[12] As mentioned above, there are two other monopoly equilibria, one where all transactions take place in intermediary A and one where firms and consumers all go to platform B. Monopoly equilibria require firms and consumers all to coordinate their actions and advertise in and subscribe to the same intermediary. In our view, this is not a likely outcome in the presence of competing intermediaries. The reason is that firms do indeed prefer not to advertise where other firms do advertise, in this way relaxing competition between them.

[12]This outcome is 'unique' in the sense explained above in Section 2. Many other strategy profiles can be sustained as a SPE with strategies prescribing firms and consumers to exit the market after deviations; however, those equilibria would easily be ruled out if, for example, intermediaries had existing firm and consumer bases.

This attempt by the firms to relax competitiveness is exploited by the competing intermediaries to survive in the market.[13]

In the equilibrium with both intermediaries active, firms and consumers are equally treated and both groups pay positive fees. They participate in the market with probability one, but their rents are fully extracted by the competing intermediaries. A feature of the outcome is that it is inefficient. The inefficiency arises out of a coordination problem. Even though both groups of participants enter the market with probability one, the fact that they randomize their advertising and their subscription decisions implies that there is a strictly positive probability that firms and consumers do not meet to conduct transactions. This inefficiency result follows straightforwardly from the fact that neither firms nor consumers can employ multi-homing strategies. In what follows we argue that market inefficiency is a likely outcome also with multi-homing possibilities.

4 Discussion: multi-homing

In the analysis above we have assumed that neither firms nor consumers can multi-home. It is clear that the inefficient outcome we have obtained in the presence of competing intermediation is intimately linked to this assumption and we would like to explore how robust this result is to multi-homing. We would first like to point out that, given intermediaries strategies in Theorem 2, neither firms nor consumers wish to deviate by multi-homing. The reason is that both groups of participants are indeed indifferent between the two intermediaries and they derive zero payoff in equilibrium; as a result, multi-homing gives no greater payoff to the deviant group.

Our second observation is more important since it states simple conditions under which there cannot be efficient equilibria even if we allow for multi-homing strategies.

Proposition 4 *An efficient symmetric outcome where both intermediaries are active in the market cannot be sustained as a subgame perfect equilibrium (i) if only firms can use multi-homing strategies, (ii) or if both groups of participants can multi-home and* $\varepsilon > v/4$.

Proof. We prove this result by contradiction. We first note that for an efficient equilibrium two requirements are needed: one, both groups of agents must participate fully, and two, at least one group of participants must multi-home with probability one. We have then three cases. (a) Suppose

[13] Monopoly equilibria are also subject to the criticism in footnote 12.

first that both groups of agents multi-home. Unless consumer subscription fees are zero, this cannot be an equilibrium because given that, say, all firms multi-home, a consumer has no incentive to multi-home. The same reasoning applies to the firm side of the market. As a result, only if all advertising and subscription fees are zero, both groups of participants will multi-home; however, this cannot be sustained as an equilibrium since intermediaries must cover their operational cost $\varepsilon > 0$. (b) Suppose now that firms multi-home and consumers do not. In this case, it is readily seen that firm behavior must involve marginal cost pricing. As a result, advertising fees must be zero because otherwise firms would not multi-home with probability one. Since consumers do not care which intermediary to subscribe to, intermediaries must engage in Bertrand competition for consumers. As a result, consumers subscription charges go down to zero and the reasoning in part (a) applies here too. (c) Finally, assume that consumers multi-home with probability one. Again, it is easy to see that firm pricing must be competitive and thus advertising fees must be zero. In the continuation game, firms may play one of the following strategies: (i) advertise in one of the intermediaries with probability one, (ii) randomize between advertising their prices in A, in B and in both at the same time. It is clear that (i) is not compatible with consumer multi-homing unless subscription fees are zero, which leads us again to point (a) above. Suppose firms play strategy (ii). Denoting the probability with which a firm takes action j as λ_j, this strategy must satisfy $\lambda_A + \lambda_B + \lambda_M = 1$ (where $\lambda_M \geq 0$ is the probability with which a firm advertises in both intermediaries). The payoff to a consumer who multi-homes is simply $v - \kappa_A - \kappa_B$ since he buys with probability one and pays the marginal cost. The payoff if the consumer deviates by subscribing only to intermediary j is $(1-(\lambda_{-j})^2)v - \kappa_j$, which reduces to $3v/4 - \kappa_A$ in symmetric equilibrium. This implies that a consumer finds it profitable to deviate for fees κ_A and κ_B above $v/4$. Since the payoff to the intermediaries would be $v/4 - \varepsilon$, the result follows. ■

Proposition 4 shows that the market failure that arises out of coordination frictions between the two groups of participants continues to hold if only firms employ multi-homing strategies and advertise their prices in both intermediaries. The reason can be explained in terms of a free-rider problem: given that all firms multi-home, a consumer has no reason to multi-home since he can access the same information by subscribing to a single intermediary. This leads to a situation where intermediaries undercut each other subscription fee to attract consumers, which cannot be sustained in equilibrium.

A necessary condition for an efficient outcome is then that consumers multi-home. In that case, firms must charge prices equal to marginal cost but this cannot be sustained as an equilibrium provided that the intermediaries operational costs are sufficiently large. If one interprets these costs as entry costs, our results point to excessive incentives to enter the market for intermediation services.

A complete analysis of the game described above allowing for multi-homing strategies is beyond the scope of the present paper. We expect firms and consumers to randomize across different advertising and subscription strategies, including multi-homing. Preliminary work that we have done indicates that the set of continuation game equilibria expands considerably when allowing for multi-homing. Interestingly, for given advertising and subscription fees consumer multi-homing gives firms more possibilities to obtain positive profits in the continuation game equilibria. The main question is whether intermediaries will continue to extract all rents generated in the market. This is work that we expect to complete in the near future.

5 Conclusions

This paper has examined a two-sided market where intermediaries compete to attract advertising from firms and audience from buyers. Firms sold homogeneous products, competed in prices and had to advertise in the intermediary platforms to attract consumers. Consumers had to subscribe to the intermediaries to receive product and price information. In the benchmark case of monopoly intermediation, we saw that the elasticity of firm demand for advertising was negative while the elasticity of consumer demand for information was positive; as a result the intermediary maximized profits by not charging anything to the firms for advertising their prices in the platform while charging positive fees to the consumers for accessing price information. In SPE all firms charged the competitive price and the intermediary extracted all the rents from the buyers. Monopolistic intermediation thus yielded an efficient outcome. By contrast, in the presence of rival intermediaries, we saw how competition meant an equal treatment of both sides of the market. Intermediary competition led firms and consumers to allocate themselves across intermediaries randomly, which gave rise to a coordination friction in the market and a market failure with its associated fall in social welfare. Even though the analysis was done assuming that neither firms nor consumers could multi-home, the market inefficiency is likely to remain when allowing for multi-homing strategies. Our results thus point towards excessive incentives to enter in the intermediation market.

6 Appendix

Proof of Proposition 1. Suppose firms believe that a fraction $\mu_M \in (0,1]$ of consumers are active in the market. Given $\phi \in [0,v)$, the characterization of firms' equilibrium behavior presented in Proposition 1 is due to Sharkey and Sibley (1993). We now turn to the consumers' side. Given firms' behavior, for an equilibrium either (i) $\mu_M = 1$ or (ii) $\mu_O + \mu_M = 1$, with $\mu_M \in (0,1)$. We start by considering the full consumer participation case, i.e. $\mu_M = 1$. The expected utility to a consumer from subscribing to the intermediary is:

$$Eu_i\left(\mu_M = 1\right) = \lambda_M^2\left(v - E\left[\min p_1, p_2\right]\right) - 2\lambda_O\lambda_M\left(v - E\left[p\right]\right) - \kappa$$

where E denotes the expectation operator. Using the optimal firms' pricing behavior described in the Proposition 1, we obtain that:

$$Eu\left(\mu_M = 1\right) = \lambda_M^2 v - \kappa = \frac{\left(v - \phi\right)^2}{v} - \kappa$$

In equilibrium a consumer must obtain (weakly) positive utility, i.e. $\kappa \leq \frac{(v-\phi)^2}{v}$.

We now turn to the case of partial consumer participation, i.e. $\mu_O + \mu_M = 1$, with $\mu_M \in (0,1)$. Similar computations as those for the previous case reveal that if a consumer subscribes to the monopolist intermediary he obtains an expected utility:

$$Eu\left(\mu_M = 1\right) = \lambda_M^2 v - \kappa = \frac{\left(\mu_M v - \phi\right)^2}{\mu_M^2 v} - \kappa$$

Since consumers stay out of the market with positive probability, for an equilibrium it must be the case that:

$$\frac{\left(\mu_M v - \phi\right)^2}{\mu_M^2 v} - \kappa = 0$$

Solving this equation for μ_M and imposing the condition $\mu_M \geq 0$ yields $\mu_M^* = \phi/v\left(1 - \sqrt{\kappa/v}\right)$. Finally to establish existence of this equilibrium we must check that $\mu_M^* \leq 1$, which holds if and only if $\kappa \leq \frac{(v-\phi)^2}{v}$. This completes the proof of the Proposition.■

Proof of Proposition 2 Let ϕ_A and ϕ_B be given and assume, without loss of generality, that $\phi_A/\mu_A \leq \phi_B/\mu_B$. We focus on symmetric equilibria where consumers subscribe to each of the intermediaries with strictly positive probability, i.e. $\mu_O + \mu_A + \mu_B = 1$, where $\mu_A, \mu_B \in (0,1)$ and

$\mu_O \in [0,1)$.[14] Our first observation is that an equilibrium where firms use symmetric pure advertising strategies does not exist. Indeed, suppose firms advertised with probability 1 in intermediary j, i.e. $\lambda_j = 1$, $j = A, B$. Then a standard undercutting argument implies that firms would set a price equal to the marginal cost and therefore they would obtain negative profits. An equilibrium where firms stay out of the market with probability 1, i.e. $\lambda_O = 1$, does not exist for obvious reasons.[15]

Second, we examine symmetric mixed advertising strategy profiles. We claim that in equilibrium $\lambda_j > 0$, $j = A, B$. Suppose, on the contrary, that $\lambda_O + \lambda_j = 1$, $\lambda_j, \lambda_O > 0$, for some $j = A, B$. If this were an equilibrium, firms would obtain zero profits and then a firm would gain by deviating and advertising the monopoly price in intermediary $j' \neq j$. This implies that in equilibrium $\lambda_A + \lambda_B + \lambda_O = 1$, with $\lambda_A, \lambda_B > 0$ and $\lambda_O \geq 0$.

Thus, we have four equilibrium candidates: (I) $\lambda_O + \lambda_A + \lambda_B = 1$ and $\mu_O + \mu_A + \mu_B = 1$, (II) $\lambda_A + \lambda_B = 1$ and $\mu_O + \mu_A + \mu_B = 1$, (III) $\lambda_A + \lambda_B = 1$ and $\mu_A + \mu_B = 1$, (IV) $\lambda_O + \lambda_A + \lambda_B = 1$ and $\mu_A + \mu_B = 1$. When analyzing each of these candidates we first keep consumers' behavior as fixed and analyze the optimal firms' behavior. In the second step we endogenize consumers' behavior. Finally, we determine the equilibrium existence conditions.

(I) Suppose that $\lambda_O + \lambda_A + \lambda_B = 1$ and $\mu_O + \mu_A + \mu_B = 1$. Denote a firm's strategy as $s^i = \{(\lambda_j, F_j(p))\}_{j \in \{A,B,O\}}$, where λ_A, λ_B, λ_O, are given in Proposition (2,I), and $F_j(p)$ and σ_j, $j = A, B$ take the following form:

$$\begin{aligned} F_j(p) &= 1 - \frac{1-\lambda_j}{\lambda_j}\frac{v-p}{p} \\ \sigma_j &= [\phi_j/\mu_j, v] \end{aligned} \tag{5}$$

In equilibrium it must be the case that $E\pi_i(\lambda_j, p; s^{-i}) = 0$ for any $p \in \sigma_j$, $j = A, B$; it is easy to check that these conditions hold if and only if firms use the strategy profile s^i specified above.

[14] We note that if either $\mu_A = 1$ or $\mu_B = 1$ there exists a symmetric continuation Nash equilibrium which takes the same form of the full consumer participation equilibrium presented for the monopoly case in Proposition 1. Similarly, if either $\mu_0 + \mu_A = 1$ or $\mu_0 + \mu_B = 1$ there exists a symmetric continuation equilibrium which is the same as the partial consumer participation equilibrium presented in Proposition 1.

[15] We also note that asymmetric pure advertising strategies cannot be part of an equilibrium. To see this note that a strategy profile where one firm stays out of the market with probability one cannot be part of an equilibrium. Therefore, the only possibility is that $\lambda_A^1 = 1$ and $\lambda_B^2 = 1$ (or $\lambda_B^1 = 1$ and $\lambda_A^2 = 1$). If this were an equilibrium, each firm would advertise the monopoly price, i.e. $p_A^1 = p_B^2 = v$. Giving that $\phi_j < \mu_j v$, $j = A, B$, firm 2 gains by deviating and advertising a price slightly lower than its rival's price in intermediary A.

Further, since firms randomize over all possible actions, it follows that no firm wants to deviate from the strategy.

We now endogenize the consumers side. The utilities to a consumer who subscribes either to A or B are respectively,

$$Eu(\mu_A = 1) = \lambda_A^2 (v - E_{F_A}[\min\{p_1, p_2\}]) + 2\lambda_A (1 - \lambda_A)(v - E_{F_A}(p)) - \kappa_A \quad (6)$$

$$Eu(\mu_B = 1) = \lambda_B^2 (v - E_{F_B}[\min\{p_1, p_2\}]) + 2\lambda_B (1 - \lambda_B)(v - E_{F_B}(p)) - \kappa_B \quad (7)$$

Using the expression (5) we easily obtain that for any $j = A, B$:

$$E_{F_j}(p) = v\frac{1-\lambda_j}{\lambda_j}\ln\frac{1}{1-\lambda_j} \quad (8)$$

$$E_{F_j}[\min\{p_1, p_2\}] = 2v\frac{1-\lambda_j}{\lambda_j}\left(1 - \frac{1-\lambda_j}{\lambda_j}\ln\frac{1}{1-\lambda_j}\right) \quad (9)$$

where $E_{F_j}[\min\{p_1, p_2\}]$ is the expected minimum price or a random sample of size 2 from F_j. We can then rewrite expressions (6) and (7) as follows:

$$Eu(\mu_A = 1) = v\lambda_A^2 - \kappa_A$$

$$Eu(\mu_B = 1) = v\lambda_B^2 - \kappa_B$$

Since consumers stay out with some positive probability, for an equilibrium to exist it must be the case that $Eu(\mu_j = 1) = 0$ for any $j = A, B$. Solving this system and imposing the additional conditions that $\mu_j < 1$, for any $j = A, B$, we obtain that:

$$\mu_j = \frac{\phi_j}{v - \sqrt{v\kappa_j}},\ j = A, B$$

We now analyze the parameter region $\{\phi_j, \kappa_j\}_{j=A,B}$ for which this equilibrium exists. This amounts to determine $\{\phi_j, \kappa_j\}_{j=A,B}$ for which $\lambda_j \in (0,1)$, $\mu_j \in (0,1)$ and $F_j(p)$ is well behaved for any $j = A, B, O$. It is easy to check that all these conditions are satisfied if and only if: (i) $\lambda_O \geq 0 \iff \phi_A/\mu_A \geq v - \phi_B/\mu_B$ and (ii) $\mu_O \geq 0 \iff (\phi_A/(v - \sqrt{v\kappa_A})) + (\phi_B/(v - \sqrt{v\kappa_B})) \leq 1$.

(II) Assume that $\lambda_A + \lambda_B = 1$ and $\mu_O + \mu_A + \mu_B = 1$. Denote as $s^i = \{(\lambda_j, F_j(p))\}_{j\in\{A,B\}}$, where λ_A, λ_B are given Proposition (2,II), and $F_j(p)$ and σ_j, $j = A, B$ take the following form:

$$F_j(p) = 1 - \frac{1-\lambda_j}{\lambda_j}\frac{v-p}{p} \quad (10)$$

$$\sigma_j = [\lambda_{j'}v, v],\ j' = A, B \text{ and } j' \neq j$$

In equilibrium it must be the case that $E\pi_i(\lambda_j, p; s^{-i}) = \lambda_B \mu_A v - \phi_B$ for any $p \in \sigma_j$, $j = A, B$; it is easy to check that these conditions hold if and only if firms use the strategy profile s^i specified above.

We now endogenize the consumers side. The utilities to a consumer who subscribes either to A or B take the same form as expressions (6) and (7), respectively; further, the expressions for $E_{F_j}(p)$ and $E_{F_j}[\min\{p_1, p_2\}]$ are also like those in (8) and (9). Thus, it follows that:

$$\begin{aligned} Eu(\mu_A = 1) &= v\lambda_A^2 - \kappa_A \\ Eu(\mu_B = 1) &= v\lambda_B^2 - \kappa_B \end{aligned}$$

Similarly to case (I), since consumers stay out of the market with strictly positive probability for an equilibrium we require that $Eu(\mu_j = 1) = 0$ for $j = A, B$. An examination of this system of equations reveals that there are infinite solutions and that, fixing μ_B, $Eu(\mu_j = 1) = 0$ for $j = A, B$, if and only if:

$$\mu_A = \frac{(\mu_B v - \phi_B + \phi_A)\sqrt{\kappa_A}}{v\sqrt{\kappa_B}} + \frac{\phi_A - \phi_B}{v}$$

We now analyze the parameter region $\{\phi_j, \kappa_j\}_{j=A,B}$ for which this equilibrium exists. It is easy to check that there are two relevant conditions: (i) $E\pi^* \geq 0 \iff \phi_A/\mu_A \leq v - \phi_B/\mu_B$ and (ii) $\mu_O \geq 0 \iff (\sqrt{\kappa_B} + \sqrt{\kappa_A})(\mu_B v - \phi_B + \phi_A) \leq v\sqrt{\kappa_B}$

(III) Assume that $\lambda_A + \lambda_B = 1$ and $\mu_A + \mu_B = 1$. Denote as $s^i = \{(\lambda_j, F_j(p))\}_{j \in \{A,B\}}$, where λ_A, λ_B are given in Proposition (2,III), and $F_j(p)$ and σ_j, $j = A, B$ take the following form:

$$\begin{aligned} F_j(p) &= 1 - \frac{1-\lambda_j}{\lambda_j}\frac{v-p}{p} \\ \sigma_j &= [\lambda_{j'} v, v], \ j' = A, B \text{ and } j' \neq j \end{aligned} \tag{11}$$

In equilibrium it must be the case that $E\pi_i(\lambda_j, p; s^{-i}) = \lambda_B \mu_A v - \phi_B$ for any $p \in \sigma_j$, $j = A, B$. These conditions hold if and only if firms use the strategy profile s^i specified above.

We now endogenize the consumers side. Similarly to cases (I) and (II) just analyzed, the expected utilities to a consumer who subscribes either to A or B are respectively:

$$\begin{aligned} Eu(\mu_A = 1) &= v\lambda_A^2 - \kappa_A \\ Eu(\mu_B = 1) &= v\lambda_B^2 - \kappa_B \end{aligned}$$

For an equilibrium, it must be the case that a consumer is indifferent between the two subscribing actions, i.e. $Eu(\mu_A = 1) = Eu(\mu_B = 1)$. That is,

$$v\lambda_A^2 - \kappa_A = v\lambda_B^2 - \kappa_B$$

This condition is satisfied if and only if

$$\mu_j = \frac{v - \kappa_{j'} + \kappa_j}{2v} + \frac{\phi_j - \phi_{j'}}{v},\ j, j' = A, B,\ j \neq j'$$

The relevant conditions for existence are: (i) $E\pi^* \geq 0 \iff \phi_A/\mu_A \leq v - \phi_B/\mu_B$ and (ii) $Eu^* \geq 0 \iff \lambda_j^2 v - \kappa_j \geq 0$.

(IV) Finally let us assume that $\lambda_O+\lambda_A+\lambda_B = 1$ and $\mu_A+\mu_B = 1$. Denote as $s^i = \{(\lambda_j, F_j(p))\}_{j\in\{A,B\}}$, where λ_O, λ_A, λ_B are given in Proposition (2,IV), and $F_j(p)$ and σ_j, $j = A, B$ take the following form:

$$\begin{aligned} F_j(p) &= 1 - \frac{1-\lambda_j}{\lambda_j}\frac{v-p}{p} \\ \sigma_j &= [\lambda_{j'}v, v],\ j' = A, B \text{ and } j' \neq j \end{aligned} \quad (12)$$

In equilibrium it must be the case that $E\pi_i(\lambda_j, p; s^{-i}) = 0$ for any $p \in \sigma_j$, $j = A, B$. These conditions hold if and only if firms use the strategy profile s^i specified above. Further, since firms randomize over all possible actions they cannot deviate profitably.

We now endogenize consumers side. Again here we obtain that,

$$\begin{aligned} Eu(\mu_A = 1) &= v\lambda_A^2 - \kappa_A \\ Eu(\mu_B = 1) &= v\lambda_B^2 - \kappa_B \end{aligned}$$

and for an equilibrium

$$v\lambda_A^2 - \kappa_A = v\lambda_B^2 - \kappa_B$$

The solution of this equation is a value $\mu_A \in (0,1)$ such that condition (4) holds. It is easy to see that under some parameter restrictions there is a solution to this condition. For example when $\kappa_A = \kappa_B$ the unique $\mu_A \in (0,1)$ which solves condition (4) is $\mu_A = \phi_A/(\phi_A + \phi_B)$. Similarly to the previous cases, the relevant conditions for existence are: (i) $\lambda_O \geq 0 \iff \phi_A/\mu_A \geq v - \phi_B/\mu_B$ and (ii) $Eu^* \geq 0 \iff v\lambda_j^2 - \kappa_j \geq 0$, $j = A, B$. The proof of Proposition 2 is now complete.■

Proof of Theorem 2. This proof is structured in two parts. We first prove that the symmetric outcome given in the Theorem can be sustained as a SPE. Then we prove uniqueness in the sense explained in the main text of the paper.

We start by proving that the strategies provided in the Theorem are a SPE. Our first remark is that Proposition 2 shows that, for equal advertising and subscription fees, the above firms and consumers strategies constitute a mutual best-response; therefore, firms and consumers do not want to deviate. In what follows, we show that no intermediary has an incentive to deviate from the above strategy profile. To show this we examine the continuation game equilibrium a given deviation leads to and its consequences for the payoff of the deviant. In proving this Theorem we will make use of Proposition 2 repeatedly; we will refer to the different continuation game equilibria given in Proposition 2 as equilibria I, II, III and IV. If a deviation by one player does not lead to any of these continuation game equilibria, we assume that consumers and firms stay out of the market altogether.

One, consider that intermediary A deviates by decreasing ϕ_A. Then it can easily be seen that, in continuation game equilibria I, II and III, the profits of the deviant decrease as ϕ_A falls. For example in I the payoff to the deviant is $\Pi_A^d = 2\phi_A\sqrt{\kappa_A v}/v + \phi_A\kappa_A/(v - \sqrt{\kappa_A v})$, which falls if ϕ_A decreases; and similarly for II and III. Consider now continuation game equilibrium IV; we note that this equilibrium cannot be reached after the deviation. This follows from noting, first, that $\phi_A/\mu_A = v - \phi_B/\mu_B$ for the proposed strategies, and, second, that, since the consumer subscription fees are equal, $\mu_A = \phi_A/(\phi_A + \phi_B)$ and thus μ_A decreases (and so μ_B increases) as ϕ_A falls. As a result, both ratios ϕ_A/μ_A and ϕ_B/μ_B decrease, which implies that the condition $\phi_A/\mu_A \geq v - \phi_B/\mu_B$ gets violated. This proves that the deviant does not gain by decreasing ϕ_A.

Two, consider that intermediary A deviates by increasing ϕ_A. This deviation cannot be followed by the continuation game equilibrium I because the condition $\mu_O \geq 0$ would be violated. This follows from noting, first, that for the equilibrium candidate $\mu_O = 0$ and, second, that μ_A increases as ϕ_A rises while μ_B remains constant if firms and consumers play I. A similar consideration applies to the continuation equilibrium II. Further, firms and consumers cannot play equilibrium III because firms would not be indifferent between advertising in intermediary A and in intermediary B. This follows from noting that an increase in ϕ_A increases μ_A (and so decreases μ_B) while leaves λ_A and λ_B unchanged so the equilibrium condition $E\pi^* = (1 - \lambda_A)\mu_A v - \phi_A = (1 - \lambda_B)\mu_B v - \phi_B$ would be violated. Finally, consider that firms and consumers contemplate playing equilibrium IV after the deviation; in this case, since an increase in ϕ_A reduces λ_A and since at the equilibrium outcome candidate $Eu^* = \lambda_A^2 v - \kappa_A = 0$, then consumers would face negative utility. As a result, if the deviant increases ϕ_A firms and consumers stay out of the market altogether and the deviation is not profitable.

Three, consider that the deviant decreases κ_A. Using Proposition 2 it is not difficult to show that in all equilibria of the continuation game the payoff to the deviant falls as κ_A decreases. For example consider continuation equilibrium III. The payoff to the deviant would be $\Pi_A^d = (2\phi_A + \kappa_A)(v - \kappa_B + \kappa_A)/2v + (\phi_A - \phi_B)\kappa_A/v$, which decreases as κ_A falls.

Four, suppose now that the deviant increases κ_A. This deviation can neither be followed by continuation equilibrium I nor by II because the condition $\mu_O \geq 0$ would be violated. Suppose now that firms and consumers contemplate playing either equilibrium III or IV after the deviation. In both cases consumers must be indifferent between subscribing to intermediary A and to B so the condition $Eu^* = \lambda_A^2 v - \kappa_A = \lambda_B^2 v - \kappa_B \geq 0$ must hold. Note that for the proposed strategies, $\lambda_B^2 v - \kappa_B = 0$. Since an increase in κ_A decreases λ_B, it follows that consumers would get negative utility. As a result, intermediary A cannot gain by increasing its subscription fee κ_A.

Since the strategy of the intermediaries is twofold, we must also check that no intermediary has an incentive to deviate by changing its advertising fee and its subscription fee at the same time. So, to complete the proof, we now examine what happens if the deviant decreases both ϕ_A and κ_A. In the continuation equilibrium I, the payoff to the deviant is $\Pi_A^d = 2\phi_A\sqrt{\kappa_A v}/v + \phi_A\kappa_A/(v - \sqrt{\kappa_A v})$ so a decrease in both ϕ_A and κ_A is clearly not profitable. The same argument holds in the continuation game equilibria II and III. Take now the continuation game equilibrium IV. In this case, equation (4) reveals that if κ_A falls, it must be the case that $\phi_A\mu_B - \phi_B\mu_A > 0$ for the consumers to be indifferent between intermediaries, which is impossible.

Consider next that the deviant decreases ϕ_A and increases κ_A at the same time. For the proposed equilibrium strategies $\phi_A/\mu_A = v - \phi_B/\mu_B$. If this deviation is followed by the continuation game equilibrium I, since players strategies satisfy $\phi_j/\mu_j = v - \sqrt{\kappa_j v}, j \in \{A, B\}$, the condition $\phi_A/\mu_A \geq v - \phi_B/\mu_B$ would be violated. If the deviation is followed by the continuation equilibrium II or III, we note that λ_B would decrease and since at the equilibrium candidate $Eu^* = \lambda_B^2 v - \kappa_B = 0$, consumers would obtain negative utility. We now take up the case where the continuation game equilibrium is IV. Here we note that an increase in κ_A tends to increase λ_A and decrease λ_B so the utility condition is violated; however, a decrease in ϕ_A increases λ_A and increases λ_B, which can reestablish the utility condition. Since decreasing ϕ_A decreases the profits of the deviant (this follows from examining the derivative $d\Pi_A^d/d\phi_A$ for $\lambda_A > 1/2$), it follows that the best deviation consists of decreasing ϕ_A and increasing κ_A in such a way that $Eu = 0$. We now claim that this deviation violates the condition $\phi_A/\mu_A \geq v - \phi_B/\mu_B$, or $\lambda_A + \lambda_B \leq 1$. Suppose, on the contrary, and that this inequality holds. Then it must be the case that

$$\frac{d\lambda_A}{d\kappa_A} - \frac{d\lambda_A}{d\phi_A} \leq \frac{d\lambda_B}{d\phi_A} - \frac{d\lambda_B}{d\kappa_A} \tag{13}$$

Since $Eu = 0$ must hold, we get

$$\begin{aligned} 2\lambda_B v \left(\frac{d\lambda_B}{d\kappa_A} - \frac{d\lambda_B}{d\phi_A}\right) &= 0 \\ 2\lambda_A v \left(\frac{d\lambda_A}{d\kappa_A} - \frac{d\lambda_A}{d\phi_A}\right) &= 1 \end{aligned}$$

If these two conditions hold, then condition (13) is violated. As a result, if a player deviates by decreasing ϕ_A and increasing κ_A, firms and consumers would stay out altogether.

We now examine what happens if the deviant increases both ϕ_A and κ_A. This deviation cannot lead to continuation game equilibria I or II because the condition $\mu_O > 0$ would be violated. Consider now the continuation game equilibrium III. Since an increase in ϕ_A leaves λ_A and λ_B unchanged, while an increase in κ_A increases λ_A and decreases λ_B, again the utility condition would be violated. The same applies in the continuation game equilibrium IV. As a result, this deviation leads to the continuation game where firms and consumers stay out with probability one.

Finally, suppose the deviant increases ϕ_A and decreases κ_A. In equilibrium I, $\phi_j/\mu_j = v - \sqrt{v\kappa_j}$, $j \in \{A, B\}$, which implies that the condition $\phi_A/\mu_A \leq \phi_B/\mu_B$ would be violated. A similar argument rules out equilibrium II after the deviation. Consider now continuation game equilibrium III. As shown above, the indifference condition for the firms would be violated. Finally considering IV we can use again equation (4) to argue that if κ_A falls, it must be the case that $\phi_A\mu_B - \phi_B\mu_A > 0$ for the consumers to be indifferent between intermediaries, which is impossible. Thus, also in this case consumers and firms get out of the market after the deviation. Since we have shown that any possible deviation is not profitable, the first part of the proof is now complete.

We now turn to show that this is the unique symmetric outcome which can be sustained as a SPE in the sense explained above. Consider any $\phi_A = \phi_B = \phi \neq v/4$ and $\kappa_A = \kappa_B = \kappa \neq v/4$. Our first remark is that if $\{\phi, \kappa\}$ is an interior point of the regions labelled above as I, II, III or IV, then intermediary A gains by increasing slightly either ϕ_A or κ_A. This follows from the arguments presented in the first part of the proof. The second observation is that if $\{\phi, \kappa\}$ are such that the existence conditions (i) and (ii) given in each continuation equilibrium I, II, III or IV bind, then $\phi = \kappa = v/4$, which constitutes a contradiction. For example, assume that ϕ and κ are such that firms and consumers play continuation equilibrium (I); suppose also that both conditions (i) and (ii) bind fro the proposed ϕ and κ. If this were the case then $\mu_A = \mu_B = \phi/\left(v - \sqrt{v\kappa}\right)$ and the fact that conditions (i) and (ii) bind would imply that $\phi = \kappa = v/4$. A similar argument holds for the other continuation game equilibria. To complete the proof, we now examine whether pairs of strategies $\{\phi, \kappa\}$ for which one of the two existence conditions in the different continuation game equilibria I, II, III or IV is binding.

First, assume ϕ and κ are such that we are in equilibrium I; we have two possibilities. (I_a) Suppose condition (i) binds while condition (ii) does not. It is easy to see that a marginal increase in ϕ_A increases μ_A and leaves λ_A unaltered. This implies that the payoff to intermediary A increases as ϕ_A increases. To complete the argument, we note that an increase in ϕ_A leaves condition (i) unaltered because in equilibrium $\phi_j/\mu_j = v - \sqrt{v\kappa_j}$, $j = A, B$. Thus, there exists a profitable deviation for intermediary A. (I_b) Assume that condition (i) does not bind, while condition (ii) binds. We note that if intermediary A slightly increases κ_A, both μ_A and λ_A increase, which implies that the profit of intermediary A goes up. We also note that as κ_A increases condition (ii) no longer binds.

Second, assume ϕ and κ are such that firms and consumers play the strategies prescribed by continuation game equilibrium II. We have again two cases. (II_a) Suppose that (i) binds, while (ii) does not. An increase in κ_A leads to an increase in μ_A, while λ_A remains constant; thus, intermediary A gains from such a deviation. We now observe that if κ_A increases marginally condition (i) does not bind anymore. (II_b) Assume now that condition (i) does not bind, while condition (ii) binds. We note that if κ_B increases then μ_B stays constant and λ_B increases; thus, intermediary B gains from this deviation. To complete we note that since $\phi_A = \phi_B = \phi$, condition (ii) can be written as $\sqrt{\kappa_A}\mu_B v - \sqrt{\kappa_B} v \left(1 - \mu_B\right) = 0$; thus if κ_B increases then condition (ii) no longer binds.

Third, assume ϕ and κ are such that we are in equilibrium III. Two cases must be distinguished. (III_a) Suppose that (i) binds, while (ii) does not bind. We make two observations here: one, that the profit of intermediary A is increasing in ϕ_A; two, that as ϕ_A increases then both conditions

(i) and (ii) are unaltered. The latter follows by noting that as ϕ_A increases then λ_A and λ_B stay constant; the former follows by noting that in equilibrium $\phi_A/\mu_A = \lambda_B v - (\lambda_A v \mu_B)/\mu_A + \phi_B/\mu_A$ (this follows from the fact that firms must be indifferent between advertising in A and in B), which implies that as ϕ_A varies $\phi_B/\mu_B + \phi_A/\mu_A$ remains unaltered. (III_b) Suppose now that (i) does not bind, while (ii) binds. It is readily seen that the argument presented in (III_a) applies also here.

Fourth, assume ϕ and κ are such that firms and consumers play equilibrium IV. Again there are two cases to consider. (IV_a) Suppose that (i) binds, while (ii) does not bind. We start by noting that if $\kappa_A = \kappa_B = \kappa$ then the unique solution to condition (4) is $\mu_A = \phi_A/\left(\phi_A + \phi_B\right)$. Therefore the expected profit of intermediary A is $\Pi_A = \frac{2}{v}\left(v - \phi_A - \phi_B\right)\phi_A + \kappa_A \phi_A/\left(\phi_A + \phi_B\right)$. Taking the derivative with respect to ϕ_A yields

$$\partial \Pi_A/\partial \phi_A = -4\phi_A/v + 2 - 2\phi_B/v + \kappa_A \phi_B/\left(\phi_A + \phi_B\right)^2$$

Since condition (i) binds it must be the case that $\phi_A = v/2 - \phi_B$. Using this fact it follows that:

$$\partial \Pi_A/\partial \phi_A = 2\phi_B/v + 4\kappa_A \phi_B/v^2 > 0$$

Thus, intermediary A gains by slightly increasing ϕ_A. We now observe that since in equilibrium $\mu_A = \phi_A/\left(\phi_A + \phi_B\right)$, if ϕ_A goes up ϕ_A/μ_A and ϕ_B/μ_B also increase, which implies that condition (i) no longer binds. (IV_b) Suppose now that (i) does not bind, while (ii) binds. Also in this case if $\kappa_A = \kappa_B = \kappa$ the unique solution to (4) is $\mu_A = \phi_A/\left(\phi_A + \phi_B\right)$ and therefore

$$\Pi_A = \frac{2}{v}\left(v - \phi_A - \phi_B\right)\phi_A + \kappa_A \frac{\phi_A}{\left(\phi_A + \phi_B\right)}$$

Further, since condition (ii) binds, it follows that in equilibrium

$$\kappa_A = \left(v - \phi_A - \phi_B\right)^2/v$$

Which implies that:

$$\begin{aligned} \Pi_A &= \frac{\phi_A}{\left(\phi_A + \phi_B\right)v}\left(v - \phi_A - \phi_B\right)\left(v + \phi_A + \phi_B\right) \\ &= \frac{\phi_A v}{\left(\phi_A + \phi_B\right)} - \frac{\phi_A\left(\phi_A + \phi_B\right)}{v} \end{aligned}$$

Taking the derivative with respect to ϕ_A yields

$$\frac{\partial \Pi_A}{\partial \phi_A} = v\frac{\phi_B}{\left(\phi_A + \phi_B\right)^2} - \frac{\left(2\phi_A + \phi_B\right)}{v}$$

and if we evaluate this derivative at the proposed equilibrium, i.e. $\phi_A = \phi_B = \phi$, we obtain:

$$\frac{\partial \Pi_A}{\partial \phi_A}(\phi) = \frac{v^2 - 12\phi^2}{4\phi v} < 0$$

where the inequality follows by noting that since condition (i) does not bind then $\phi > v/4$. This implies that decreasing ϕ_A increases the profit of intermediary A. It is now enough to note that as ϕ_A decreases both λ_A and λ_B increase, which implies that condition (ii) no longer binds. The proof of the Theorem is now complete. .■

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