Illiquidity Contagion and Liquidity Crashes

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SoFiE Conference
Plan

1. Introduction
2. Model
3. Benchmark: Segmented Markets
4. Partial Integration with fully price informed dealers
5. Partial Integration with imperfectly price informed dealers
6. Full Integration: the role of cross-market arbitrageurs
The Flash Crash

- The flash crash of May 6, 2010: Very large price declines in thousands of U.S. stocks, ETFs and index futures without apparent changes in fundamentals. Very quick reversal.
The Flash Crash

“Many of the almost 8,000 individual equity securities and exchange traded funds ("ETFs") traded that day suffered similar price declines and reversals within a short period of time, falling 5%, 10% or even 15% before recovering most, if not all, of their losses. However, some equities experienced even more severe price moves, both up and down. Over 20,000 trades across more than 300 securities were executed at prices more than 60% away from their values just moments before.” CFTC-SEC (2011)

- **Proximate cause:** a sell order for 75,000 contracts in the CME e-mini S&P500 futures ($4.1 billion of notional amount). Large?
- **Many aspects of the Flash Crash remain mysterious.**
Contagious Illiquidity and Liquidity Crashes

- A market wide evaporation of liquidity.

![Graph showing comparison of buy-side market depth for E-Mini (all quotes), and SPY and Aggregate S&P 500 (within 500 basis points of mid-quote).]
- Consolidated depth (green: buy side, blue: sell side) for Procter and Gamble from 9:00 a.m to 4:00 p.m.
Illiquidity crashes for close substitutes

“Many of the securities experiencing the most severe price dislocations on May 6 were equity-based ETFs [...]” (CFTC-SEC (2011), page 39)
Introduction

Some puzzles

- **Illiquidity Contagion:** How can a large sell order in one security trigger an evaporation of liquidity in hundreds of securities?
  
  1. Arbitrage? But then why an evaporation on both the sell and buy sides of limit order books? Why not an inflow of buy limit and buy market orders in the e.mini futures?

- **Causality:** Did the liquidity crash caused the price crash or vice versa?

- **Market Integration:** Why have Exchange Traded Funds (ETFs) been more affected?

- **Market Structure:** Is the current market structure intrinsically more fragile? Why? HFTs or something else?
Our Hypothesis

- **Important change in market structure in recent years:** information on prices is more widely and quickly disseminated than ever.

- **Example:**

<table>
<thead>
<tr>
<th>Date</th>
<th>Markets</th>
<th>Technology</th>
<th>Reduction in delay</th>
</tr>
</thead>
<tbody>
<tr>
<td>1846</td>
<td>NYSE-Philadelphia</td>
<td>Telegraph</td>
<td>One day to a few hours</td>
</tr>
<tr>
<td>1975</td>
<td>NYSE-Regionals</td>
<td>Consolidated Tape</td>
<td>5-10 mns to 1-2 mns</td>
</tr>
<tr>
<td>1980</td>
<td>NYSE</td>
<td>Upgrades on floor</td>
<td>2 mns to 20 sds</td>
</tr>
<tr>
<td>Circa 2005</td>
<td>Electronic.</td>
<td>Co-location</td>
<td>sd to nano sds</td>
</tr>
</tbody>
</table>

- **Source:** Garbade and Silber (1978) and Easley, Hendershott and Ramadorai (2009)
Liquidity providers in one asset class increasingly rely on the information contained in the prices of other asset classes to set their quotes.

The liquidity levels of various markets have become more interconnected.

Why? Prices are more informative when liquidity is higher and vice versa.
Contagion and Illiquidity Multiplier

- Plausible? → Theory.

Figure 2: Cross-asset learning and liquidity spillovers.
Modeling approach

- A rational expectations model of trading with two risky securities (two "asset classes").

- Main assumptions:
  1. CARA Gaussian framework
  2. Payoffs have a factor structure: two risk factors
  3. Two types of traders:
     - **Specialized dealers**: provide liquidity in only one asset class; specialization brings information on one risk factor.
     - **Cross-market arbitrageurs**: can trade in both markets; no private information;

- Dealers in one asset class use the price of the other asset as a source of information on the risk factor on which they have no expertise.
Main properties of the model

- **Contagion and fragility**
  1. **Interconnected liquidity**: the illiquidity levels of each asset are positively inter-connected → an increase in illiquidity for one asset affects all other assets.
  2. **Illiquidity multiplier**: Small shocks to the liquidity of one asset can ultimately have large effects on the liquidity of all assets.
  3. **Fragility**: Multiple rational expectations equilibria with high or low levels of illiquidity in all securities → Liquidity crash: A jump from a low to a high illiquidity equilibrium

- **The channels for market integration** (cross-asset learning and cross-market trading) break down in the high illiquidity equilibrium, especially if assets are close substitutes.
Literature

  1. These models focus on the source of **sharp price declines and the propagation of these declines**.
  2. We focus on discrete changes in illiquidity and magnification of **illiquidity shocks**, which then can be associated with large price declines.

Plan

1. Introduction
2. Model
The Model

- **Two assets** $D$ and $F$
- **Securities Payoffs at date $t=2$:**

  \[
  v_D = \delta_D + d \times \delta_F + \eta, \\
  v_F = f \times \delta_D + \delta_F + \epsilon
  \]

  with $\delta_j \sim N(0, 1)$ for $j \in \{D, F\}$, $\eta \sim N(0, \sigma^2_{\eta})$, $\delta_D$, $\delta_F$ and $\eta$ are independent.

- For parsimony we assume $d \geq 0$, $\epsilon = 0$ and $f = 1$

- **Hence both payoffs are positively correlated and the correlation increases in** $d$ **and decreases in** $\sigma^2_{\eta}$. 
Market Participants

- Trades at date $t=1$. In each market:
  1. A continuum of risk averse dealers with CARA utility functions;
  2. A continuum of risk averse “cross-market arbitrageurs” with CARA utility functions;
  3. "Liquidity demand shocks": $u_j \sim N(0, \sigma^2_{u_j})$ for $j \in \{D, F\}$. These demands are independent from each other and independent of other variables.

- Both arbitrageurs and dealers supply liquidity to liquidity traders

- Dealers and arbitrageurs have different "business models"
  1. Dealers specialize in assets in which they are well informed (as suggested by Schultz (2003)). They have expertise in assessing one risk factor $\Rightarrow$ Dealers specialized in different assets have different information.
  2. Arbitrageurs have no information on risk factors but engage in cross-market hedging to reduce the risk of their portfolios.
Market Participants

- **Dealers in security** $j$:
  1. Know $\delta_j$
  2. A fraction $\mu_j$ observes the price in the other market ($p_{-j}$)
  3. Choose their demand function $x_j^k(p_j)$ to maximize:

\[
E \left[ U((v_j - p_j)x_j^k) | \delta_j, \text{Price Information} \right] \quad \text{for } k \in \{D, F\}
\]

4. **Price Information** = $\{p_F, p_D\}$ if a dealer is a pricewatcher and **Price Information** = $\{p_j\}$, otherwise.

- **Cross-market arbitrageurs**:
  1. They have no private information
  2. They choose their arbitrage portfolio ($x_D^H(p_D, p_F), x_F^H(p_D, p_F)$) to maximize

\[
E \left[ U((v_D - p_D)x_D^H + (v_F - p_F)x_F^H) | \{p_F, p_D\} \right]
\]
Market Clearing

- **Equilibrium at date t=1: Supply=Demand in market j:**

\[ \mu_j x_j^W (\delta_j, p_j, p_{-j}) + (1 - \mu_j) x_j^I (\delta_j, p_j) + \lambda x_j^H (p_j, p_{-j}) + u_j = 0, \quad (1) \]

- \( \lambda = \text{Index of arbitrage capital.} \)

- **Dealers and arbitrageurs have rational expectations.**

- **Various Cases:**
  1. \( \mu_D = \mu_F = 0 \) and \( \lambda = 0 \): markets are segmented.
  2. \( \mu_D > 0 \) and \( \mu_F > 0 \) and \( \lambda = 0 \): markets are integrated through cross-asset learning.
  3. \( \mu_D = \mu_F = 0 \) and \( \lambda > 0 \): markets are integrated through cross-asset trading.
  4. \( \mu_D = \mu_F = 1 \) and \( \lambda > 0 \): full integration.
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3. Benchmark: Segmented Markets
Benchmark

- **Segmented markets:** dealers do not watch prices/No arbitrageurs \( (\mu_D = \mu_F = 0 \text{ and } \lambda = 0) \)
- The equilibrium is unique.
- Prices:

\[
p_j = \underbrace{\delta_j} + \underbrace{B_{j0}} \times \underbrace{u_j} \quad \text{for } j \in \{D, F\}
\]

\[
B_{D0} = \frac{\text{Var}(v_D | \delta_D)}{\gamma_D} = \frac{\sigma_{\eta}^2 + d}{\gamma_D} \quad \text{and} \quad B_{F0} = \frac{\text{Var}(v_F | \delta_F)}{\gamma_F} = \frac{1}{\gamma_F}
\]

- \( B_{j0} \) = Sensitivity of price to liquidity demand = "Illiquidity of security \( j \)."
- Parameters \( \{\sigma_{\eta}^2, d, \gamma_D\} \) are the "liquidity fundamentals" of security \( D \) (similar interpretation for \( \gamma_F \))
- Shocks to liquidity fundamentals in one market do not affect the other market \( \rightarrow \) No liquidity spillovers if no cross-asset learning
Price informativeness and liquidity

- The price of security $j$ is a noisy signal about factor $\delta_j$:
  \[ p_j = \delta_j + B_{j0} \times u_j \quad \text{for } j \in \{ D, F \} \]

  - Illiquidity
  - Noise

- The informativeness of security $D$ for dealers in security $F$ depends on their belief regarding the liquidity of security $D$ and vice versa.

- These beliefs are part of the equilibrium if markets are not fully segmented. When $\mu_\delta > 0$, we focus on linear Rational Expectations Equilibria.
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4. Partial Integration with fully price informed dealers
No arbitrageurs, All Dealers are Pricewatchers

- **When** $\mu_D = \mu_F = 1$, there always exists at least one rational expectations equilibrium. In such an equilibrium:

  $$ p_j = \delta_j + A_{j1} (\delta_{-j} + B_{-j1} u_{-j}) + B_{j1} \times u_j $$

  New: Information from the price of security $-j$

  with

  $$ B_{j1} = B_{j0} (1 - \rho_{j1}^2) $$

  Illiquidity with wide dissemination of price info

- **Variable** $\rho_{j1}^2$ measures the informativeness of the price of security $-j$ about the payoff of security $j$ for dealers in security $j$. 
Self-reinforcing illiquidity 1/2

- **Illiquidity of security** $F \rightarrow$ **Informativeness of the price of security** $F \rightarrow$ **Illiquidity of security** $D$ and vice versa

- Formally:

  $$\rho^2_{D1} = \frac{d^2}{(\sigma^2_\eta + d^2)(1 + B^2_{F1}\sigma^2_uF)}$$

  $$\rho^2_{F1} = \frac{1}{(1 + B^2_{D1}\sigma^2_uD)}.$$

- **Solving for the equilibrium** $\iff$ Find solutions to:

  $$B_{D1} = f_1(B_{F1}; \gamma_D, \sigma^2_\eta, d, \sigma^2_uF)$$

  $$B_{F1} = g_1(B_{D1}; \gamma_F, \sigma^2_uD).$$

- $\implies$ **Interconnected liquidity/Positive Liquidity spillovers**
Suppose that the risk tolerance of dealers in security $D$, $\gamma_D$, decreases:

1. Security $D$ becomes less liquid since dealers in this asset class have less risk appetite: $B_{D1} \uparrow$
2. Hence the price of security $D$ is less informative for dealers in security $F$: $\rho_{F1} \downarrow$
3. Thus, inventory risk is higher for dealers in security $F$ and security $F$ becomes less liquid: $B_{F1} \uparrow$.

More generally, a shock to the liquidity fundamental of one security induces a change in the same direction for the liquidity of securities $D$ and $F \implies$ positive co-movements in liquidity.
Equilibrium multiplicity 1/2

- Only the extreme equilibria: $L$ and $H$ are stable
Equilibrium Multiplicity 2/2

- **Corollary:** If $\sigma^2_\eta > 4d^2$, the rational expectations equilibrium is unique.

- Otherwise, multiple equilibria

- Multiple equilibria are more likely if assets’ payoffs are more correlated.
Illiquidity Crashes

- A switch from L to H generates a discrete market-wide increase in illiquidity: an illiquidity crash.

Vicious illiquidity loop
Illiquidity crashes
Illiquidity Crashes

- **Illiquidity crash + large sell order** $\implies$ **Price Crash**
- **Example:** Suppose that $\gamma_j = d = 1$, $\sigma_\eta = 0.2$ and $\sigma_{u_j} = 2$. **Now consider the arrival of a sell order in security F equal to** $u_F = -5$ (likelihood less than 1%).

<table>
<thead>
<tr>
<th>Equilibrium Type</th>
<th>L</th>
<th>H</th>
<th>Ratio H/L</th>
</tr>
</thead>
<tbody>
<tr>
<td>Illiq F ($\times 10^2$)</td>
<td>0.6</td>
<td>63</td>
<td>105</td>
</tr>
<tr>
<td>Illiq D ($\times 10^2$)</td>
<td>4</td>
<td>65</td>
<td>16</td>
</tr>
<tr>
<td>$\Delta p_F$</td>
<td>$-2.1%$</td>
<td>$-223%$</td>
<td>106</td>
</tr>
<tr>
<td>$\Delta p_D$</td>
<td>$-2.1%$</td>
<td>$-99%$</td>
<td>47.14</td>
</tr>
</tbody>
</table>

Changes in prices are expressed as % of price volatility in the low illiquidity equilibrium.
The illiquidity multiplier

- Even in a given equilibrium, **liquidity is fragile**: a small shock to the illiquidity of one market can trigger large changes in the liquidity of both markets.

- Consider again a change in $\gamma_D$.

  1. **The total effect** of a change in dealers’ risk appetite in security $D$ is given by:

     \[
     \frac{dB_{D1}}{d\gamma_D} = \kappa \times \frac{\partial f_1}{\partial \gamma_D} < 0
     \]

     \[
     \frac{dB_{F1}}{d\gamma_D} = \kappa \times \frac{\partial g_1}{\partial B_{D1}} \times \frac{\partial f_1}{\partial \gamma_D} < 0
     \]

  2. $\kappa = \text{“illiquidity multiplier”}$

  3. $\kappa$ is greater than one in the extreme equilibria and can be large
Example

\[ \gamma_F = \frac{1}{24}; \gamma_D = 1.8; \quad d = 0.5; \quad \sigma_{u_F} = 0.1; \quad \sigma_{u_D} = 1.6 \]
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Market integration as a source of fragility

- **Our hypothesis:** *increase in* the scope of access to price information has made liquidity more interconnected and liquidity crashes more likely.

- We explore this proposition by analyzing the effect of varying the fraction of dealers with price information.

- $\mu_j =$ Fraction of pricewatchers in security $j$. 
Equilibrium with inattentive dealers

- Two types of dealers in each security: pricewatchers and inattentive dealers. Example: Security $F$

<table>
<thead>
<tr>
<th>Dealers’ Type</th>
<th>Information</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pricewatchers</td>
<td>$\delta_F, p_F, p_D$</td>
</tr>
<tr>
<td>Inattentive</td>
<td>$\delta_F, p_F$</td>
</tr>
</tbody>
</table>

- Inattentive dealers are less informed than pricewatchers

1. $\implies$ Differential access to price information by a fraction of dealers is a source of adverse selection.

2. $\implies$ Analysis is similar but more complex.
Findings

- The equilibrium is unique if $\mu_D$ or $\mu_F$ is small $\implies$ liquidity crashes arise only when the fraction of pricewatchers become large enough.

- In a given equilibrium, if dealers’ risk bearing capacity is not too high:
  1. Liquidity spillovers are positive and there is an illiquidity multiplier as in the baseline model.
  2. An increase in the fraction of pricewatchers makes all effects stronger.
  3. Liquidity is maximal when the scope of dissemination for price information is maximal ($\mu_D = \mu_F = 1$) $\implies$ Trade-off between liquidity and fragility.
Evolution of the illiquidity multiplier $\kappa$ as a function of $\mu_D$ (the fraction of pricewatchers in security $D$).
Co-movements in liquidity and price information

- **Example:** numerical values: $d = 0.9; \sigma_{u_j} = 1/2; \sigma_\eta = 2; \gamma_F = 1/2; \mu_D = 0.1$ or $\mu_D = 0.9$. We compute the covariance in the illiquidity of both markets for each value of $\mu_F$ assuming that $\gamma_D$ is uniformly distributed over $[0.5, 1]$.

- **Testable prediction:** co-movements in liquidity become stronger when price information is more widely disseminated.
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6. Full Integration
Cross-market arbitrageurs

- Cross-market traders seem to have played an important role during the Flash Crash.

"cross-market arbitrageurs transferred this sell pressure to the equities markets by opportunistically buying E-Mini contracts and simultaneously selling products like SPY, or selling individual equities in the S&P 500 Index."

"Cross-market strategies primarily focus on the contemporaneous trading of securities-related products [...] to capture temporary price differences between any two related products, but with limited or no exposure to subsequent price moves in those products [...] Some firms focus on “one-way” strategies by acting as a liquidity provider (i.e., trading passively by submitting non-marketable resting orders) primarily in one product, and then hedging by trading another product." (CFTC-SEC(2010))
Cross-market arbitrageurs and market integration

- **Cross-market arbitrageurs do exactly this in our model:** if liquidity traders sell one asset, they will provide liquidity by buying the asset and hedge their positions by selling the other asset.

- **BUT:** Securities $D$ and $F$ are imperfect substitutes since $\sigma_\eta > 0 \implies$ arbitrage is risky in our model. Hedging reduces the risk of long/short portfolios but cannot fully eliminate it.

- The cost of hedging depends on dealers’ supply of liquidity: if arbitrageurs respond to a sell order in security $F$, they will hedge by selling security $D$ to ... DEALERS.

- As a result, cross-market arbitrageurs and dealers complement each other in integrating markets.
Market integration: cross-asset learning and cross-asset trading

- **Example**: Suppose that $\lambda = 1$, $\gamma_j = d = 1$, $\sigma_\eta = 0.2$ and $\sigma_{uj} = 2$.

<table>
<thead>
<tr>
<th>Market Integration = $\text{Var}(p_D - p_F)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arbitrageurs Only</td>
</tr>
<tr>
<td>-------------------</td>
</tr>
<tr>
<td>$\text{Iliq F (} \times 10^2\text{)}$</td>
</tr>
<tr>
<td>$\text{Iliq D (} \times 10^2\text{)}$</td>
</tr>
<tr>
<td>$\text{Var}(p_D - p_F)$</td>
</tr>
</tbody>
</table>

- Markets are integrated both through the actions of dealers and arbitrageurs.
Fragility with cross-market arbitrageurs

- Results are qualitatively unchanged with arbitrageurs. In particular:
  1. The informativeness of prices is not affected (arbitrageurs have no information and their flows are known once prices are known).
  2. The parameters for which multiple equilibria are obtained are exactly the same with and without cross-market arbitrageurs.
  3. The presence of arbitrageurs dampen the illiquidity multiplier and the size of illiquidity co-movements but do not suppress it.
Co-movement and Arbitrage Capital

**Example:** numerical values: \( d = 0.9; \sigma_{uj} = 1/2; \sigma_\eta = 2; \gamma_F = 1/2; \mu_D = 0.1 \) or \( \mu_D = 0.9 \). We compute the covariance in the illiquidity in both markets for each value of \( \lambda \) assuming that \( \sigma_\eta \) is uniformly distributed over \([0.1, 2.5]\).

![Graphs](image-url)
Implication: illiquidity crashes should be relatively stronger for close substitutes

- **Close substitutes**: $d = 1$ and $\sigma_\eta$ small $\implies$ high correlation in the payoffs of the assets.

- **When securities are close substitutes, prices are very informative in the low illiquidity equilibrium**
  
  1. Dealers’ demand function is very elastic (a small drop in price is sufficient to induce dealers to buy a large number of shares)
  2. Arbitrageurs’ demand function is also very elastic because hedging costs are small.
  3. Liquidity is very high and assets look very integrated

- **A switch from L to H triggers a very sharp drop in price informativeness**
  
  1. Sharp increase in illiquidity in relative terms
  2. Sharp drop in integration in relative terms.
Example

- **Suppose that** \( \lambda = \mu_D = \mu_F = 1; \gamma_j = d = 1, \) and \( \sigma_{u_j} = 2. \)

<table>
<thead>
<tr>
<th>( \eta )</th>
<th>Corr</th>
<th>( \text{Illiqu}^H_D / \text{Illiqu}^L_D )</th>
<th>( \text{Illiqu}^H_F / \text{Illiqu}^L_F )</th>
<th>( \text{Var}^H(\Delta p) / \text{Var}^L(\Delta p) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>97%</td>
<td>37.93</td>
<td>943</td>
<td>( 8.7 \times 10^8 )</td>
</tr>
<tr>
<td>0.2</td>
<td>95%</td>
<td>10.28</td>
<td>63</td>
<td>226,705</td>
</tr>
<tr>
<td>0.25</td>
<td>94%</td>
<td>6.81</td>
<td>26</td>
<td>15280</td>
</tr>
</tbody>
</table>
Illiquidity crashes for close substitutes

“Many of the securities experiencing the most severe price dislocations on May 6 were equity-based ETFs […]” (CFTC-SEC (2011), page 39)
Conclusions

Thanks!