Advertising to Status-Conscious Consumers

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Abstract

This paper develops a simple, social theory of advertising, in a setting where consumers value social status. Consumers differ in their wealth, which is unobservable, and all consumers want others to believe they are wealthy. A monopolist advertises and sells a conspicuous good that allows consumers to signal their wealth through their purchases. Advertising is purely informative: consumers who receive an ad are able to buy the conspicuous good and also to recognize it when others buy.

I show that in equilibrium, the firm can use advertising to exploit consumer status concerns by increasing the stigma of those who don’t buy and promoting widespread recognition of those who do. High levels of advertising can induce consumers to behave as if they have a preference for conformity, small changes in advertising levels can have large effects on demand, and the firm may advertise to consumers it knows are unwilling to buy.

1. Introduction

It has long been recognized that advertising can influence the behavior of status-conscious consumers. Advertising can create symbolic value for a brand, by presenting desirable imagery to consumers, and then associating this imagery with the brand (Meenaghan, 1995). Brand image will matter to the many consumers who care about the image they project of themselves through their purchases (Aaker (1997), Kapferer and

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Bastien (2009)). For this reason, advertising that associates a product with a particular image, such as exclusivity or prestige, can affect willingness to pay by influencing the social status consumers receive from their peers.

This paper examines the link between advertising and social status, and its implications for firm and consumer behavior. It develops a simple, social theory of advertising, where the status associated with buying a good depends on the type of consumers who are expected to buy, and where advertising increases the social pressure to visibly consume.

By exploring the social role of advertising, this paper makes a number of contributions. First, it identifies a novel mechanism through which advertising affects social status: by increasing the stigma of poor consumers who don’t buy and promoting widespread recognition of wealthy consumers who do. Second, it demonstrates how informative advertising can increase willingness to pay, without transmitting information about product characteristics. Third, it shows that small changes in advertising levels can have large effects on demand, and that the way consumers respond to a change in price depends on the level of advertising. Fourth, it helps explain the broad advertising of high-end goods to people who are unlikely to buy, and the importance of assuring status-conscious consumers that others are also informed.

Specifically, I consider a monopolist that faces a market of consumers who differ in their wealth, where wealth is unobservable. All consumers want others to believe they are wealthy. The firm produces an observable conspicuous good, and chooses both price and the level of advertising. A consumer only becomes informed about the conspicuous good if he receives an ad. His willingness to pay will depend on the difference in social status associated with buying and not buying, which in turn depends on what other consumers will believe about his wealth, conditional on his purchase.

I first assume that advertising simply informs consumers of the conspicuous good’s existence, making it possible for them to buy. I show that in equilibrium, an increase in advertising levels always increases willingness to pay, even though consumption externalities can be negative as poor consumers begin to buy. The relationship between advertising levels and aggregate demand can also be discontinuous. Moreover, high levels of advertising tend to make consumers behave as if they have a preference for conformity, so that a drop in price that makes the conspicuous good less exclusive increases willingness to pay. I show that the firm’s optimal price will vary with advertising costs, that the equilibrium level of advertising is socially excessive, and that the welfare effects of an advertising tax can differ from a sales tax.

Advertising increases willingness to pay through its impact on stigma, by reducing the social status of poor consumers who don’t buy. Consumers who don’t buy the conspicuous good may have one of two reasons: they are either unwilling to buy because they are relatively poor, or they are unable to buy because
they don’t receive an ad. High levels of advertising decrease the size of the latter group, so that not buying sends a clearer signal of being poor. In this way, advertising increases willingness to pay not by making buying more attractive, but by making not buying less attractive.

I then assume that advertising informs consumers in an additional way, allowing them to recognize the conspicuous good when it is bought by others. Advertising’s role in promoting recognition provides another channel through which advertising increases willingness to pay. I show that it may also cause the firm to advertise broadly, even if it is possible to target ads directly on potential demand. Ads that inform consumers who don’t buy are not wasted; they ensure consumers who do buy can signal their wealth through their purchases. The firm may want to advertise in multiple media, but its ability to do so will be limited by problems of commitment, as it must convince consumers who do buy that those who don’t are also informed.

The analysis here shows that purely informative advertising can affect willingness to pay without transmitting information about product characteristics. Advertising expenditure does not serve as a signal of quality, as in Kihlstrom and Riordan (1984) and Milgrom and Roberts (1986), and ads do not reveal a consumer’s match value with the product, as in Anderson and Renault (2006). Instead, advertising shapes consumer beliefs about who is likely to buy the good, and who is likely to recognize it.

These results can be seen as a rationale for what Bagwell (2007) terms the complementary view of advertising, which models prestige effects by placing advertising levels directly into the utility function (Stigler and Becker (1977), Becker and Murphy (1993)). The results also offer some support for the persuasive approach employed by Buehler and Halbheer (2011), where brand image depends directly on the level of advertising. The benefit of studying the underlying mechanism by which advertising affects social status lies in the specific predictions it generates about consumer behavior, firm behavior and welfare.

By establishing a link between stigma and advertising, this paper helps explain how the social pressure certain consumers feel to buy widely-known brands, such as Nike running shoes, can depend on the low status associated with not buying (Elliott and Leonard, 2004). Consumers know that others are informed about these brands, so that not buying sends a clear, negative signal. Corneo and Jeanne (1997), Benabou and Tirole (2006), and Benabou and Tirole (2012) also show that people may take a particular action so as to avoid low status, but this paper shows that stigma in consumer choice depends crucially on the level of advertising.

This paper can also explain why firms selling high-end goods sometimes advertise broadly, instead of targeting ads directly at wealthy consumers. Wide-circulation magazines such as The Economist and GQ consistently feature ads for luxury products that the vast majority of readers would never buy. Examples from autumn 2012 include the Signature Zirconium cellular phone from Vertu, at a price of $9,000, and the
Annual Calendar Chronograph watch from Patek Philippe, at a price of $60,000.¹ Similarly, Audi advertised its $100,000 A8 model during the broadcast of the 2011 Super Bowl. The analysis here shows that firms may advertise broadly to ensure that poor consumers who don’t buy can recognize wealthy consumers who do. Krahmer (2006) also considers the link between advertising and recognition, but not how it affects the choice between broad and targeted advertising.

The results also suggest why firms that do engage in targeted advertising of high-end goods often do so in specialized magazines, rather than online.² The mechanism by which advertising increases the social pressure to consume relies on consumers knowing the ads they see are also seen by others. In practice, their level of confidence may well depend on the media through which firms advertise. With online advertising, different consumers visiting the same website will receive different ads, depending on their browsing history. This means that a consumer who views an ad online can infer relatively little about the ads viewed by others. In contrast, with print advertising, consumers know that others reading the same magazine are likely to come across the same ads.³

The idea that consumers should know that others are informed also plays a role in the literature on advertising and network goods (Bagwell and Ramey (1994), Chwe (2001), Pastine and Pastine (2002), Clark and Horstmann (2005), Sahuguet (2011)). However, this literature has little to say about advertising and status concerns, where consumption externalities can be negative. It also cannot explain why firms intentionally advertise to consumers who are unlikely to buy.

This paper adds to a recent literature on how firm communications can influence status-conscious consumers. The focus on informative advertising, stigma and recognition differs from Buehler and Halbheer (2012), Kuksov et al. (2012), and Yoganarasimhan (2012), which instead consider persuasion, cheap talk, and information disclosure about product characteristics. Many other papers also adopt a signaling approach to social status, but do not consider the role of advertising (see, e.g., Bernheim (1994), Ireland (1994), Pesendorfer (1995), Bagwell and Bernheim (1996), Corneo and Jeanne (1997)).

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 explores how advertising affects stigma, and its implications for consumers, the firm and welfare. Section 4 looks at advertising’s impact on recognition, including the issue of targeting. Section 5 then concludes. All proofs

²Information on specialized luxury magazines can be found at www.luxurysociety.com.
³A number of articles discuss print and online advertising for luxury goods, without exploring what consumers can infer about the ads received by others. See “How do you market exclusivity and elite-ness to the superrich?” by Margaret Johnson, Warc Exclusive, April 2008, and “Quality insights to solve a luxury problem - marketing to affluent consumers in the digital age” by Beth Uyenco, Olivier Goulet and Alex Charlton, ESOMAR Worldwide Multi Media Measurement, 2008.
can be found in the appendix.

2. The Model

This section sets out a model of status-driven consumption, where consumer behavior is based on Corneo and Jeanne (1997). The main innovation here is the introduction of advertising. A monopolist produces a conspicuous good at zero marginal cost, and chooses both the price and how much advertising to undertake. Consumers can only buy the conspicuous good if they receive an ad.

Specifically, the firm chooses \( p \geq 0 \), and \( \phi \in [0, 1] \), where \( \phi \) is the probability that each consumer receives an ad. Throughout most of the analysis, I will assume that advertising is random, so that technological or informational constraints make it impossible to target ads at specific groups of consumers. The cost of advertising is \( KA(\phi) \), where \( K \geq 0 \) is a shift parameter associated with changes to advertising technology. Costs are increasing and convex in the advertising level: \( A(0) = 0 \), \( A' > 0 \), and \( A'' > 0 \).

Consumers in this market are ordered according to their wealth. Wealth \( w \) is distributed on an interval \( W \subset \mathbb{R}^+ \), according to CDF \( F \) and pdf \( f \), which is common knowledge, where \( F \) and \( f \) are continuously differentiable. The relationship between wealth \( w \) and rank \( r \) in the wealth distribution is \( r = 1 - F(w) \). The total mass of consumers is \( M \), which I normalize to one.

Consumers have unit demand for the conspicuous good but are only able to buy if they receive an ad. In contrast, all consumers can buy a positive quantity of a numeraire good, which is competitively supplied at unit price. Purchase of the conspicuous good is observable, but wealth and consumption of the numeraire good are not, as described in more detail below.

Consumers experience intrinsic and status utility, both of which depend on their purchases. Intrinsic utility depends only on consumption of the numeraire good, \( c_r \geq 0 \), while status utility depends on the purchase of the conspicuous good, \( b_r \in \{0, 1\} \). Specifically, the utility of consumer \( r \) is

\[
U_r = u(c_r) + b_r s_1 + (1 - b_r) s_0.
\]

Intrinsic utility is given by \( u(c_r) \), where \( u(c) \) is continuously differentiable, \( u'(c) > 0 \) and \( u''(c) < 0 \). Status utility is either equal to \( s_1 \) if consumer \( r \) purchases the conspicuous good, \( b_r = 1 \), or \( s_0 \) if he does not, \( b_r = 0 \). The values of \( s_1 \) and \( s_0 \) will depend on consumer beliefs and hence on equilibrium strategies.

The timing of the game is as follows. First, the firm chooses \((p, \phi)\), which is observed by all consumers who receive an ad. Each consumer \( r \) then makes a purchase decision: \((c_r, b_r)\) if he receives an ad and \( c_r \) if he does not. I assume throughout Section 3 that \((p, \phi)\) and \( b_r \) are then publicly revealed, while I assume in
Section 4 that they are only revealed to consumers who themselves received an ad. Consumers update their beliefs about each other’s rank, pay-offs are realized and the game ends.

Let \( a(r) \) denote the status utility from being precisely identified as rank \( r \) in the wealth distribution. I assume \( a'(r) < 0 \), so high status is associated with high wealth and low rank. Let the probability distribution \( \mu_{r'}(\cdot|b_r) \) denote the posterior beliefs of another consumer \( r' \) about the rank of consumer \( r \), conditional on his purchase. Consumer \( r \)’s status utility, conditional on his purchase, is equal to the expectation of \( a \), given these posterior beliefs, averaged over all \( r' \in [0,1] \):

\[
s_i = \int_0^1 \int_0^1 a(x)\mu_{r'}(x|b_r = i)dxdr',
\]

for \( i \in \{0,1\} \). In the case where \( a(r) \) is linear, a consumer’s status utility just depends on the average belief about his rank.

The firm’s strategy is a pair \((p, \phi)\), which it chooses in order to maximize expected profits given the strategies of consumers. The strategy of consumer \( r \) is a choice of \((c_r, b_r)\) if he receives an ad, for each pair \((p, \phi)\), and of \( c_r \) if he does not. Each consumer maximizes utility, given his budget constraint \( c_r + pb_r \leq w_r \), the equilibrium strategies of other consumers, and beliefs about his type, where I assume a consumer who is indifferent about buying will choose \( b_r = 1 \). Beliefs \( \mu_{r'}(\cdot|b_r) \) are consistent with equilibrium strategies, in the sense of following from Bayes’ rule whenever possible. If \( b_r \) is publicly revealed, then all consumers will hold the same beliefs, \( \mu(\cdot|b_r) \).

\(^4\) The model of Corneo and Jeanne (1997) is recovered if \( K = 0 \) and \( \phi = 1 \), so if advertising is costless and all consumers are informed with probability one.

I conclude this section by discussing two assumptions about observability. First, consumers who receive an ad can observe the firm’s chosen advertising level. This assumption is reminiscent of signaling models in which consumers observe advertising expenditure (see, e.g., Kihlstrom and Riordan (1984), Bagwell and Ramey (1994)). It is plausible if firms advertise in a restricted set of media, such as specific magazines, newspapers or television channels, so that consumers can gauge the scale of an advertising campaign by the medium through which they receive an ad.

Second, I assume throughout Section 3 that \((p, \phi)\) is revealed to all consumers, even those who don’t receive an ad. One interpretation is that all consumers eventually become informed, but a fraction \( \phi \) of consumers come across the ads first and can buy before others do. In this sense, \( \mu(\cdot|b_r) \) represents beliefs at an interim stage, after the remaining fraction \( 1 - \phi \) of consumers observe the ads but before they are able to buy.

\(^4\)Note that an individual consumer cannot influence \( s_1 \) and \( s_0 \) by his own actions. Hence, for any \((p, \phi)\), beliefs \( \mu(\cdot|b_r) \) reflect the actual distribution of rank whenever Bayes’ rule can be applied, both in equilibrium and after any unilateral deviation.
That being said, this assumption is above all made for technical reasons. Otherwise, willingness to pay would depend on the expectation of \((p, \phi)\), rather than just on its realized value. The firm might then want to deviate from the expected \((p, \phi)\) so as to manipulate beliefs, with the deviation observed by some consumers but not by others. I touch on this issue in Section 4, but a more detailed analysis is beyond the scope of this paper.

3. Analysis

I denote the signaling value of the conspicuous good by \(S\), defined as the difference in status utility between buying and not buying: \(s_1 - s_0\), with \(s_i\) given by (1), \(i \in \{0,1\}\). Throughout this section, I assume that price, advertising level, and purchase of the conspicuous good are all publicly revealed, so that all consumers hold the same beliefs. For given \(S > 0\), a consumer \(r\) with wealth \(w_r\) is willing to buy at price \(p\) if

\[ u(w_r - p) + S \geq u(w_r). \]

Denote this consumer’s willingness to pay by \(V(r, S)\), which is the value of \(p\) for which \(S = u(w_r) - u(w_r - p)\), if such a solution exists, and \(w_r\), if it does not:

\[
V(r, S) = \begin{cases} 
  w_r - u^{-1}(u(w_r) - S), & S < u(w_r) - u(0) \\
  w_r, & S \geq u(w_r) - u(0).
\end{cases}
\]

(2)

Willingness to pay is increasing in wealth and in the signaling value, but at a decreasing rate. Wealthy consumers are also willing to pay more for a marginal increase in signaling value: \(V_1 < 0, V_2 \geq 0, V_{11} \leq 0, V_{22} \leq 0\) and \(V_{12} \leq 0\), where \(V_i\) denotes the derivative of (2) with respect to its \(i\)th argument. These inequalities follow directly from \(u'(w) > 0\) and \(u''(w) < 0\) and are strict for all consumers for whom the budget constraint does not bind, \(V(r, S) < w_r\).

Since willingness to pay is increasing in wealth, consumers will demand the conspicuous good if and only if their rank is below a certain cut-off, \(r_0 \in [0,1]\), whose precise value depends on \(\phi\) and \(p\). A low value of \(r_0\) means that only the wealthiest consumers demand the conspicuous good. I will therefore interpret \(r_0\) as a measure of exclusivity.

For given \(r_0\) and \(\phi\), the signaling value will depend on how rank is distributed within three groups of consumers: those who buy the conspicuous good, those who want to buy but who don’t receive an ad, and those who do not want to buy, whether or not they receive an ad.

Consumers who buy the conspicuous good must have rank \(r \leq r_0\) and must also receive an ad. Random
advertising then implies that quantity sold is $Q = \phi r_0$, and that consumers who buy have rank independently drawn from a uniform distribution on $[0, r_0]$. Since beliefs follow from equilibrium strategies, (1) implies that the status from buying is

$$s_1(r_0) = \frac{\int_0^{r_0} a(r)dr}{r_0},$$

for any $r_0 > 0$. Define $s_1(0) = a(0)$, which is the limit of (3) as $r_0$ tends to zero. This means that a consumer who buys when nobody else does is believed to have the highest possible wealth.

The remaining mass $1 - \phi r_0$ of consumers don’t buy the conspicuous good, of whom $1 - r_0$ have rank on $(r_0, 1]$ and $(1 - \phi) r_0$ have rank on $[0, r_0]$. The former group does not want to buy because willingness to pay is too low, and the latter group is not able to buy because consumers don’t receive an ad. It follows from (1) that the status utility from not buying is

$$s_0(r_0, \phi) = \frac{(1 - \phi) \int_0^{r_0} a(r)dr + \int_{r_0}^{1} a(r)dr}{(1 - \phi) r_0 + (1 - r_0)},$$

for any $(r_0, \phi) \neq (1, 1)$. Define $s_0(1, 1) = a(1)$, which is the limit of (4) evaluated at $\phi = 1$, as $r_0$ tends to 1. Thus, a consumer who does not buy when everyone else does is believed to have the lowest possible wealth.

Figure 1 illustrates these three groups of consumers, for the case where $r_0 = 0.4$ and $\phi = 1/3$.

The horizontal dimension depicts rank $r$, and the vertical dimension depicts the probability that each consumer receives an ad, $\phi$. The dark blue circles represent consumers who buy, the light blue circles represent consumers who don’t buy because their willingness to pay is too low, while the medium blue circles represent consumers who don’t buy because they don’t receive an ad. In this sense, $s_1$ depends on the average horizontal position of the dark blue circles, and $s_0$ depends on the average horizontal position of the remaining circles.
By definition, the signaling value is

\[ S(r_0, \phi) = s_1(r_0) - s_0(r_0, \phi), \quad (5) \]

with \( s_1(r_0) \) given by (3) and \( s_0(r_0, \phi) \) given by (4). The signaling value is always positive, \( S(r_0, \phi) > 0 \) for all \( r_0 < 1 \), since consumers who buy have higher wealth on average than consumers who don’t.

I will refer to \( S_B(r_0) \equiv S(r_0, 1) \) as the baseline signaling value, given by (5) evaluated at \( \phi = 1 \). This is the signaling value if all consumers were able to buy the conspicuous good regardless of advertising, so precisely the signaling value from Corneo and Jeanne (1997). It is also the signaling value if \( K \) were sufficiently small for the firm to choose the maximum level of advertising, informing all consumers with probability one.

Given price \( p \), the cut-off \( r_0 \) follows from (2) and (5). It is the value of \( r \) for which a consumer of this rank has willingness to pay equal to the price, given a signaling value consistent with him being the cut-off consumer. That is, \( r_0 \) is defined implicitly by \( p = V(r_0, S(r_0, \phi)) \). Equivalently, from (2), I can write

\[ r_0 = D(S(r_0, \phi), p), \quad (6) \]

with \( D_1 > 0 \) and \( D_2 < 0 \).

I first consider the demand side of the market, and examine how consumer behavior depends on the advertising level. I then turn to the supply side to explore how the firm’s optimal \( \phi \) and \( p \), and therefore \( r_0 \), depend on advertising costs.

3.1. Advertising and Consumer Behavior

For a given advertising level \( \phi \) and cut-off \( r_0 \), quantity sold is \( Q = \phi r_0 \). A marginal increase in advertising then yields

\[ \frac{dQ}{d\phi} = r_0 + \phi \frac{dr_0}{d\phi}, \]

Here I distinguish between advertising’s direct impact on demand, \( r_0 \), and its indirect impact on demand, \( \phi \frac{dr_0}{d\phi} \). The direct impact on demand is the familiar one of informative advertising: for a given cut-off \( r_0 \), advertising increases sales by informing consumers whose willingness to pay exceeds the price. This direct impact is always positive, \( r_0 > 0 \).

The indirect impact on demand results from the interaction between advertising and social status. Keeping \( r_0 \) constant, advertising’s direct impact allows more consumers to buy, influencing the signaling value
through (5), and affecting willingness to pay through (2). Consumers then reevaluate whether they want to buy the conspicuous good, resulting in a new equilibrium cut-off \( r_0 \).

The analysis will focus on this novel second effect of informative advertising. A first issue is whether the indirect impact is positive, \( \frac{dr_0}{d\phi} > 0 \), so whether advertising increases the equilibrium cut-off. If it does, then advertising’s indirect impact will reinforce its direct impact, further increasing revenues as a broader range of consumers decide to buy. A second issue is identifying when the indirect impact tends to be large.

I begin by differentiating (6) with respect to \( \phi \) and \( p \) and rearranging to obtain

\[
\frac{dr_0}{d\phi} = \left( \frac{1}{1 - D_1 \frac{\partial S}{\partial r_0}} \right) D_1 \frac{\partial S}{\partial \phi},
\]

(7)

and

\[
\frac{dr_0}{dp} = \left( \frac{1}{1 - D_1 \frac{\partial S}{\partial r_0}} \right) D_2.
\]

(8)

I will focus on situations where demand is locally downwards sloping, \( \frac{dr_0}{dp} < 0 \). This is always the case at the optimal cut-off if \( r_0 < 1 \), since otherwise the firm could increase sales by marginally increasing the price. Comparing (7) and (8) then shows that \( \frac{dr_0}{d\phi} \) has the same sign as \( \frac{\partial S}{\partial \phi} \). That is, advertising’s indirect impact is positive if and only if the sales resulting from its direct impact increase the signaling value.

**Proposition 1.** Suppose \( \phi < 1 \). Then for any \( r_0 \in (0, 1) \), advertising’s indirect impact on demand is positive: \( \frac{\partial S}{\partial \phi} > 0 \) and \( \frac{\partial^2 S}{\partial \phi^2} > 0 \), where \( \frac{\partial S}{\partial \phi} \big|_{r_0=0} = \frac{\partial S}{\partial \phi} \big|_{r_0=1} = 0 \). Moreover, for any \( r_0 \in (0, 1] \), the difference in signaling value with the baseline is strictly positive and decreasing in exclusivity: \( S_B - S > 0, \frac{\partial (S_B - S)}{\partial r_0} > 0 \).

There are a number of points to take from Proposition 1, which are explained in more detail below. First, advertising’s indirect impact on demand always reinforces its direct impact, even though consumption externalities can be negative in this setting. Second, advertising increases willingness to pay for the conspicuous good not by increasing the status from buying, but by increasing the stigma from not buying. Third, the extent to which advertising can increase stigma depends on the value of \( r_0 \), so on whether the conspicuous good is exclusive or not. Fourth, small changes in advertising levels can have large effects, particularly when advertising levels are already high.

Advertising’s indirect impact on demand depends on how its direct impact changes the signaling value. Proposition 1 shows that the signaling value always increases, \( \frac{\partial S}{\partial \phi} > 0 \), resulting in higher willingness to pay and increased demand. This is the case even though consumption externalities can sometimes be negative when consumers value social status, depending on the identity of consumers who buy. For example, selling a product to poor consumers may decrease the willingness to pay of wealthy consumers, who are no longer
able to signal their wealth through their purchases. Burberry faced this concern in the 1990’s when lower class consumers began to buy their products, which threatened to hurt their brand image (Kapferer and Bastien, 2009). The difference here is that advertising’s direct impact only increases sales from the “right” type of consumers. Advertising informs all consumers with equal probability but only the wealthy choose to buy, which in turn makes buying more attractive for all other consumers.

That being said, advertising only makes the conspicuous good more attractive in a relative sense. By definition, the signaling value is the difference in status utility between buying and not buying, $S(r_0, \phi) = s_1 - s_0$, where (3) shows that $s_1$ is independent of $\phi$: $\frac{\partial S}{\partial \phi} = -\frac{\partial s_0}{\partial \phi} > 0$. It follows that advertising’s indirect impact on demand works only through increasing the stigma of poor consumers who don’t buy. Advertising increases the social pressure to buy the conspicuous good not by making buying any better, but by making not buying worse.

To see how advertising increases stigma, recall that consumers who don’t buy belong to one of two groups: poor consumers who don’t want to buy, and wealthy consumers who don’t receive an ad. An increase in advertising decreases the size of the latter group, so that not buying sends a clearer signal of being poor. Consumers who don’t buy would like to claim they are wealthy but ignorant, but high levels of advertising mean that ignorance is no longer an excuse.

Figure 2 illustrates advertising’s indirect impact where $r_0 = 0.4$, when $\phi$ is increased from $1/3$ to $2/3$.

The increased number of dark blue circles compared with Figure 1 shows advertising’s direct impact on demand. The average horizontal position of these circles is unchanged, but the average position of the remaining circles has shifted to the right, representing a drop in status for consumers who don’t buy. This drop in status then increases willingness to pay, and the equilibrium cut-off increases to $r_0 = 0.6$.

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5Benabou and Tirole (2012) make a similar point in the context of intrinsic motivation, showing that extrinsic incentives can reduce the social incentive to take a particular action by making it a weaker signal of intrinsic motivation.

6The change in equilibrium cut-off will induce an additional change in $S = s_1 - s_0$, as described later in expression (9). However, the resulting value of $S$ will always be higher than before the increase in advertising.
Although advertising’s indirect impact is always positive, its magnitude will depend on the extent to which advertising can increase stigma. Proposition 1 shows that this extent is decreasing in exclusivity. Recall that a small value of $r_0$ is interpreted as high exclusivity, so that $s_0(r_0, \phi) - s_0(r_0, 1)$ is increasing in $r_0$.

This difference in stigma is precisely equal to the difference in signaling value, $S_B(r_0) - S(r_0, 1)$.

The intuition is that stigma is low whenever sales are also low, regardless of consumers’ reasons for not buying. If the conspicuous good is very exclusive, then most consumers won’t buy whether or not they receive an ad, so that stigma varies little with advertising levels. In contrast, if the conspicuous good is not exclusive, then sales may increase dramatically as consumers become informed, which identifies those who don’t buy as being poor.
Figure 4 shows advertising’s impact on the signaling value, rather than on aggregate demand. However, advertising’s impact on the inverse demand curve will look similar to Figure 4 whenever utility is close to linear in the numeraire good. Using the terminology of Johnson and Myatt (2006), advertising then has elements in common with both real information and hype. Advertising effectively rotates the inverse demand curve, as would real information. However, this rotation occurs around its vertical intercept and increases quantity demanded at any price, similar to hype.

Finally, Proposition 1 shows that small changes in advertising levels can have large effects when many consumers demand the conspicuous good and when advertising levels are already high. Note that two elements of Proposition 1 are in apparent contradiction: \( \frac{\partial S}{\partial \phi} \bigg|_{r_0=1} = 0 \) and \( \frac{\partial (S_B - S)}{\partial r_0} > 0 \). This means that when many consumers demand the conspicuous good, the marginal indirect impact of advertising is low, but the extent to which advertising can increase stigma is high.

To see how these two elements can be reconciled, consider some \( r_0 \) fixed close to 1 and suppose \( \phi \) is allowed to vary. Proposition 1 then shows that the indirect impact of advertising is negligible for most values of \( \phi \), but that it becomes large after the advertising level exceeds a certain threshold.

Figure 5 illustrates how \( \frac{\partial S}{\partial \phi} \) varies with \( r_0 \) for two values of \( \phi \), where one curve lies above the other by \( \frac{\partial^2 S}{\partial \phi^2} > 0 \). As \( \phi \) tends to 1, the value of \( r_0 \) at which \( \frac{\partial S}{\partial \phi} \) attains its maximum will tend to 1 as well, and the maximum itself will increase without bound, even though \( \frac{\partial S}{\partial \phi} \bigg|_{r_0=1} = 0 \).

![Figure 5](image)

It follows that when \( r_0 = 1 \), the relationship between advertising and demand is discontinuous. If all consumers demand the conspicuous good, then willingness to pay is zero for any advertising level \( \phi < 1 \), as consumers who don’t buy are just as wealthy on average as consumers who do. Willingness to pay then becomes strictly positive when \( \phi = 1 \), as a consumer who doesn’t buy is believed to be the poorest type.
The poorest consumer always has a lower incentive to buy than any other type, so these particular out-of-equilibrium beliefs are reasonable in the sense of the D1 Criterion (Cho and Kreps, 1987). Moreover, any other out-of-equilibrium beliefs would generate a discontinuity in the baseline when \( r_0 \) takes on a value of 1. The formal point is that Bayes’ rule implies different beliefs when \( r_0 = 1 \) and \( \phi \) tends to 1, compared to when \( \phi = 1 \) and \( r_0 \) tends to 1. Thus, when many consumers already buy the conspicuous good, small changes in the firm’s strategy must have large effects.

The magnitude of advertising’s indirect impact depends on \( \frac{\partial S}{\partial \phi} \), but (7) shows that it also depends on how the signaling value varies with exclusivity, \( \frac{\partial S}{\partial r_0} \). I will use the following terminology, introduced by Corneo and Jeanne (1997).

**Definition 1.** For particular values of \( r_0 \) and \( \phi \), consumption is snobbish if \( \frac{\partial S}{\partial r_0} < 0 \), and consumption is conformist if \( \frac{\partial S}{\partial r_0} > 0 \).

If consumption is snobbish, then the signaling value is increasing with exclusivity, which tends to limit the indirect impact of advertising. Advertising’s direct impact on demand increases willingness to pay so that poorer consumers begin to buy. However, poorer consumers anticipate that the good will become less exclusive, which dampens the initial change in willingness to pay and the extent to which the cut-off increases. The situation is reversed if consumption is conformist, in which case the anticipated drop in exclusivity further increases willingness to pay.

In this way, the sign of \( \frac{\partial S}{\partial r_0} \) determines how the partial effect on the signaling value compares to the total effect:

\[
\frac{dS}{d\phi} = \frac{\partial S}{\partial \phi} + \frac{\partial S}{\partial r_0} \frac{dr_0}{d\phi}.
\]

(9)

Proposition 1 shows that advertising’s direct impact on demand always increases the signaling value, so the partial effect is always positive. The indirect impact also increases the signaling value when consumption is conformist, in which case the total effect exceeds the partial effect. In contrast, the indirect impact decreases the signaling value when consumption is snobbish, so the partial effect exceeds the total effect.

The distinction between conformist and snobbish consumption is also important as to whether demand is upwards or downwards sloping. Looking at (8), a necessary condition for demand to be upwards sloping is that consumption be sufficiently conformist. The intuition is that at any given price, an increase in \( r_0 \) always yields a marginal consumer with lower wealth, who is willing to pay less for any given signaling value.
Demand can only be upwards sloping if the change in $r_0$ generates a sufficiently large increase in the signaling value for the firm to increase its price.

In the baseline, $\phi = 1$, consumers who don’t buy all have lower wealth than consumers who do, so the status from buying and from not buying are both increasing with exclusivity. If $s_1$ varies more quickly than $s_0$, then consumption will be snobbish,

![Figure 6](image)

whereas if $s_0$ varies more quickly than $s_1$, then consumption will be conformist.

![Figure 7](image)

In the baseline, whether consumption is snobbish or conformist depends only on the shape of the rank utility function $a(r)$. In contrast, when $\phi < 1$, it also depends crucially on the level of advertising. I begin with the following lemma.
Lemma 1. For any $\phi < 1$, the status utility from not buying is non-monotonic in exclusivity. That is, there exists $\theta^* \in (0, 1)$, such that $\frac{\partial s}{\partial r_0} < 0$ for all $r_0 \in [0, \theta^*)$ and $\frac{\partial s}{\partial r_0} > 0$ for all $r_0 \in (\theta^*, 1]$, with $\frac{\partial s}{\partial r_0}|_{r_0=\theta^*} = 0$. Moreover, $\theta^*$ is unique and increasing in $\phi$.

When some consumers are uninformed, the status associated with not buying may actually increase as the conspicuous good becomes less exclusive. Lemma 1 shows this will be the case whenever exclusivity is already low.

The intuition is that consumers who don’t buy cannot have higher wealth on average than consumers who do, since willingness to pay is increasing in wealth. Not buying provides the highest status when consumers who don’t buy are similar on average to consumers in the entire population. This is precisely the case when exclusivity is high, because the majority of consumers don’t buy. But this is also the case when exclusivity is low, as then the main reason for not buying is not receiving an ad, and ads are sent randomly throughout the population. The status from not buying must therefore be non-monotonic in exclusivity, as illustrated earlier in Figure 3.

I now use Lemma 1 to show that high levels of advertising tend to make consumption more conformist.

Proposition 2. If consumption is snobbish when $\phi = 1$, then it is also snobbish for all $\phi < 1$. That is, at any $r_0$ for which $\frac{\partial S}{\partial r_0} < 0$, then $\frac{\partial S}{\partial r_0} < 0$ holds as well.

For any $\phi < 1$, there are certain values of $r_0$ for which consumption is snobbish. That is, there exists $\theta^* \in [0, 1)$ such that $\frac{\partial S}{\partial r_0} < 0$ for all $[\theta^*, 1]$. Moreover, when $\phi$ is sufficiently small, $\theta^* = 0$.

Consumption will always be snobbish, regardless of the equilibrium cut-off, unless the advertising level exceeds a certain threshold. This is the case even if consumption is conformist for all $r_0$ in the baseline. The intuition is that a drop in exclusivity does not affect stigma when sales are already very low, but it does reduce the status from buying.

Proposition 2 suggests that when advertising levels are low, consumers will react to a change in price as if they have a taste for exclusivity, where a product becomes less attractive when a broader range of consumers decide to buy. When advertising levels are high, consumers may instead appear to have a taste for conformity, where increasing the range of consumers who buy makes buying more attractive. The underlying mechanism is the same in both cases, as consumers are simply trying to signal their wealth through their purchases.

This result gives another reason why small changes in advertising levels can have large effects, particularly when the advertising level is already high. Low levels of advertising imply snobbish consumption, which limits advertising’s marginal indirect impact on demand. High levels of advertising may generate conformist consumption, which increases the size of this marginal impact.
Proposition 2 also implies that sufficiently high levels of advertising are necessary for demand to be upwards sloping. Demand can only be locally upwards sloping if consumption is conformist at that particular value of \( r_0 \), so if the advertising level exceeds some threshold. Moreover, demand can only be upwards sloping for all \( r_0 \) if all consumers are informed, \( \phi = 1 \), since otherwise willingness to pay drops to zero at \( r_0 = 1 \).

3.2. Advertising Cost and Firm Behavior

I now turn to the supply side of the market to address how the firm’s choice of advertising and price, and therefore of exclusivity, depend on the cost of advertising. I will assume throughout that demand is downwards sloping, so there is a unique value of \( r_0 \) consistent with each pair \((p, \phi)\). Following (8) and Proposition 2, a sufficient condition for downwards sloping demand is that consumption in the baseline not be too conformist, i.e. that \( \frac{\partial S_B}{\partial r_0} \) not be too large.

I begin by showing how the equilibrium advertising level depends on advertising costs.

**Lemma 2.** The firm’s optimal choice of \( \phi \) is decreasing in \( K \).

This result is not surprising, but it is useful in terms of comparative statics. It means that to show how the firm’s optimal \( p \) and \( r_0 \) depend on \( K \), it is sufficient to show how they depend on \( \phi \).

Proposition 1 showed that increased advertising leads to an increase in \( r_0 \), but without taking into account how the firm optimally adjusts its price. If the firm reduces its price as it advertises more heavily, then exclusivity will drop by even more than suggested in Section 3.1. If the firm instead increases its price, then exclusivity will drop by less, and could conceivably rise.

For a given advertising level, \( p = V(r_0, S(r_0, \phi)) \) implies that profits can be written either in terms of \( p \) or \( r_0 \). However, it is more straightforward to analyze how profits vary with \( r_0 \), because the signaling value depends directly on exclusivity rather than price.

**Proposition 3.** Suppose that either (i) utility is sufficiently close to linear in the numeraire good, \( \frac{d^2u(c)}{dc^2} < \epsilon \) for all \( i \geq 2 \), for some \( \epsilon > 0 \) sufficiently small, or (ii) consumption is sufficiently snobbish in the baseline, \( f(w_{r_0})\frac{\partial S_B}{\partial r_0} \leq -u'(w_{r_0}) \) for all \( r_0 \in [0, 1] \). Then the optimal \( r_0 \) is strictly higher at \( \phi = 1 \) than at \( \phi < 1 \), whenever the marginal consumer’s budget constraint does not bind.

Proposition 3 shows that in equilibrium, low advertising costs, and hence high advertising levels, are associated with low levels of exclusivity. Recall from Figure 4 that an increase in \( \phi \) rotates \( S(r_0, \phi) \) around its intercept on the vertical axis. If willingness to pay varies little across consumers, then the inverse demand curve rotates in a similar way, marginal revenue increases for every \( r_0 \), and the optimal \( r_0 \) increases as well.
This is what occurs when utility is close to linear in consumption of the numeraire good, since it is the concavity of \( u(c) \) that drives differences in willingness to pay.

Otherwise, the firm faces a trade-off when deciding how to adjust exclusivity. If \( r_0 \) increases, then more consumers demand the conspicuous good and increased advertising has a larger impact on stigma. However, if \( r_0 \) decreases, then the marginal consumer becomes wealthier and is willing to pay more for any given change in the signaling value. Proposition 3 shows that this first effect dominates when consumption is sufficiently snobbish.

Simulations suggest that this relationship between exclusivity and advertising levels, and hence between exclusivity and advertising cost, holds more generally. I assume rank utility is quadratic, \( a(r) = a_0 + a_1 r + a_2 r^2 \), where \( a_0, a_1 \) and \( a_2 \) are constants, wealth \( w \) is uniformly distributed on an interval \([\underline{w}, \bar{w}]\), advertising costs are \( K\phi^\beta \) for \( \beta > 1 \), and consumers exhibit constant relative risk aversion towards the numeraire good, \( u(c) = \frac{c^{1-\alpha}}{1-\alpha} \) for \( \alpha > 0 \), where \( u = \ln(c) \) for \( \alpha = 1 \). For given \( K \), I solve for the firm’s profit-maximizing choice of \( \phi \) and \( r_0 \), and I then let \( K \) vary.

All simulation results show that the profit-maximizing value of \( r_0 \) is decreasing in \( K \), regardless of whether consumption is snobbish or conformist. Figure 8 illustrates the case when \( a_0 = 1, a_1 = 0, a_2 = -1, \underline{w} = 0, \bar{w} = 10, \beta = 2 \) and \( \alpha = 1 \).

In this particular case, exclusivity also varies discontinuously with advertising costs, because the optimal \( \phi \) drops when \( K \) exceeds a certain threshold. This discontinuity arises because advertising’s indirect impact on demand causes revenues to be convex in the advertising level.

These results also help shed light on why advertising levels can be low for highly exclusive goods. For example, very high-end watch brands like Audemars Piguet and Ulysse Nardin are advertised less heavily.
than Rolex and Breitling, and Rolls Royce and Maserati are advertised less heavily than BMW and Mercedes. It might simply be that advertising’s direct impact is small for highly exclusive goods, as most consumers would be unwilling to buy even if they received an ad. However, this explanation is complementary to one involving social status. A small direct impact translates into a small indirect impact, with little change in stigma or willingness to pay.

While a reduction in advertising costs tends to result in lower exclusivity, it has an ambiguous impact on the firm’s optimal price. The relationship between $\phi$ and $p$ is difficult to analyze analytically because the signaling value depends only indirectly on price. For this reason, I turn again to simulations, which show the firm may either increase or decrease its price in response to changes in costs. Figure 9 shows a case where the optimal price is decreasing in $K$, for the same parameter values as above. Figure 10 shows another case where the optimal price is increasing in $K$.

Figure 9

Price as a Function of Advertising Cost, $a(r) = 1 - r^2$, $\alpha=1$

Figure 10

Price as a Function of Advertising Cost, $a(r) = 1 - 4r + r^2$, $\alpha=2$
Although the precise nature of the relationship is ambiguous, the optimal price clearly depends on the level of advertising. This is unlike the standard analysis of informative advertising under monopoly with constant marginal production costs, where the optimal price and advertising level are independent of one another (see, e.g., Bagwell (2007)).

The simulations also suggest that any price increase accompanying higher levels of advertising will tend to be moderate. If the price increase were large, then the number of consumers willing to buy would drop, which would be inconsistent with lower exclusivity. The firm may in fact prefer to reduce its price, to convince more consumers to buy and increase advertising’s impact on stigma.

3.3. Welfare

The fact that advertising works by increasing stigma suggests that limiting advertising may be welfare improving. The following result echoes Corneo and Jeanne (1997), who consider prohibiting sale of the conspicuous good.

**Proposition 4.** Suppose $\phi > 0$. Then for any $r_0 > 0$, the sum of individual utilities is strictly lower than when $\phi = 0$. The utility of every consumer is strictly lower than when $\phi = 0$ if

$$u(w_0) - u(w_0 - V(r_0, S(r_0, \phi))) > \int_{r_0}^{1} \frac{a(r) dr}{r_0} - \int_{0}^{1} a(r) dr.$$

From the perspective of consumer welfare, sale of the conspicuous good amounts to pure waste, since aggregate status utility is constant. Selling the conspicuous good simply redistributes status from poor consumers who don’t buy to wealthy consumers who do. Wealthy consumers also consume less of the numeraire good, which reduces the sum of individual utilities. Wealthy consumers may be willing to pay such a high price to avoid stigma that banning the sale of the conspicuous good, or equivalently a ban on advertising, leaves all consumers better off.

Corno and Jeanne interpret the sum of individual utilities as social welfare if the conspicuous good is provided competitively, so if the firm earns zero profits. With this interpretation, advertising is socially excessive, as setting $\phi = 0$ is welfare improving. This stands in contrast to the standard result that a monopolist always underprovides informative advertising (Shapiro, 1980).\footnote{A possible critique is that sale of the conspicuous good might still increase each consumer’s utility if it is paired with an appropriate set of compensating transfers, since wealthy consumer have higher willingness to pay for status. However, advertising is certainly socially excessive when $\phi = 0$ constitutes a Pareto improvement.}

Corno and Jeanne also consider the imposition of a per unit luxury tax on the conspicuous good. They again assume price is fixed due to competitive pressures, and that a tax of $t > 0$ then brings the price to
The tax is redistributed in a lump sum way to all consumers who buy. They show that a marginal increase in the tax will increase the utility of all consumers if it decreases quantity sold, which occurs if and only if demand is locally downwards sloping.

To address these issues here, I can interpret any $K > 0$ as implicitly including an advertising tax. Keeping the price of the conspicuous good fixed, an increase in the tax then corresponds to an increase in advertising costs, which has the following impact on consumer welfare.

**Proposition 5.** Fix $p$ and $K$, and suppose the firm chooses the optimal $\phi < 1$, with corresponding $r_0 \in (0, 1)$. Identify an increase in $K$ with an increase in advertising tax. Then if demand is locally downwards sloping, $\frac{dr_0}{dp} < 0$, a marginal tax increase will increase the sum of individual utilities. If demand is locally upwards sloping, $\frac{dr_0}{dp} > 0$, then it will increase the sum of individual utilities if $\phi$ is sufficiently small, or if $r_0$ is sufficiently close to zero.

The impact of an advertising tax is similar to a luxury tax when demand is locally downwards sloping. An advertising tax causes advertising levels to drop, consumers respond by reducing demand, and they instead consume more of the numeraire good. One difference is that an advertising tax does not benefit all consumers, since wealthy consumers who would like to buy may no longer receive an ad. Another difference is that an advertising tax can also be welfare improving when demand is locally upwards sloping. Consumers then increase their demand when advertising levels drop, but sales $Q = \phi r_0$ may still decrease.

### 4. Advertising and Recognition

Up until now, I have assumed that advertising simply allows consumers to buy the conspicuous good. But a good deal of advertising also informs consumers by promoting recognition. Advertising can help familiarize consumers with particular brands or products, so that they can recognize and distinguish between these products when displayed by others.

The importance of recognition is echoed in the marketing literature on brand image: a brand can be thought of as an idea, where that idea is more powerful if widely shared. More people should therefore be familiar with the brand than just the consumers who buy (Kotler and Keller, 2008). For example, it is precisely because everyone knows BMW and what it stands for, even those who will never buy a BMW, that the brand has so much power (Kapferer, 2008).

To explore the relationship between advertising, recognition, and social status, I now assume that ads transmit two types of information. They allow consumers to buy the conspicuous good, and also to recognize it when others buy.
Recall that consumer $r$’s status utility depends on what other consumers believe about his rank, conditional on his purchase. Until now, I assumed that $b_r$ was publicly revealed, so that all consumers held the same beliefs about one another. I incorporate recognition into the analysis by assuming that a consumer $r’ \neq r$ only observes $b_r$ if he himself received an ad. This means that only a fraction $\phi$ of consumers are able to update their beliefs about each other’s rank from the prior. The signaling value is therefore $\phi S(r_0, \phi)$, with $S$ given by (5).

With this approach to recognition, high levels of advertising help ensure that the conspicuous good is effectively visible. Physical visibility is not enough for consumer purchases to influence beliefs. For example, seeing a new high-end smartphone may suggest little about its owner unless one can distinguish the phone from other lower-end models. Goods can only be truly conspicuous if they can be recognized, which creates another channel through which informative advertising influences willingness to pay.

Taking into account recognition means that willingness to pay is lower than in Section 3, where the signaling value was $S(r_0, \phi)$, since buying the conspicuous good no longer influences the beliefs of all consumers. Willingness to pay is still increasing in the advertising level, since

$$
\frac{\partial}{\partial \phi} (\phi S(r_0, \phi)) = (s_1 - s_0) + \phi \frac{\partial s_0}{\partial \phi},
$$

where both terms on the right-hand side of (10) are positive. The first term captures the relationship between advertising and recognition. An increase in $\phi$ allows more consumers to recognize the conspicuous good, so that buying has a larger effect on social status. The second term reflects advertising’s impact on stigma. Both terms in (10) are increasing in $\phi$, so the marginal impact of advertising is again increasing in the advertising level.

Looking at (10) suggests that incorporating recognition into the analysis does not dramatically change the firm’s incentives. As in Section 3, advertising has a direct impact on demand, and an indirect impact brought about by changes to the signaling value. The only difference appears to be that advertising now affects the signaling value through two channels rather than one. Despite this appearance, recognition turns out to be crucial when the firm can use targeted advertising.

The analysis so far considered a single advertising technology, where ads were sent randomly across all consumers. In practice, however, firms may be able to target ads at specific groups who are more likely to buy. Firms often do just that, putting great effort into selecting which of distinct audiences to reach via specialized cable television, satellite radio, and magazines (Esteban et al., 2006). Targeting is also becoming easier as technology improves (Johnson 2011, Esteves and Resende 2011).

To explore the issue of targeting, I now assume the firm can choose between various media that differ
in how closely they target wealthy consumers. Specifically, the firm chooses targeting $t \in [0, 1]$, where all consumers on $[0, t]$ then receive an ad with probability $\phi$. Setting $t = 1$ corresponds to random advertising across all consumers.

I also place more structure on the model by assuming a constant reach, independent readership advertising technology (Grossman and Shapiro, 1984). Rather than explicitly setting $\phi$, the firm chooses $t$ and $n$, where $n$ is the number of ads. Each ad reaches a mass $z > 0$ of consumers randomly drawn from $[0, t]$, at cost $Kz$, where $K > 0$ and $z$ is small. The reach and the cost of ads are independent of $t$, and so do not directly influence the optimal choice of targeting.

Deriving the relationship between $t$, $n$, $\phi$ and advertising costs is now straightforward. A firm that sends $n$ ads on $[0, t]$ will inform a fraction $\phi = 1 - (1 - \frac{z}{t})^n$ of these consumers. This implies

$$n(\phi, t, z) = \frac{\ln(1 - \phi)}{\ln(1 - \frac{z}{t})}.$$  

where the cost of these ads is $n(\phi, t, z)Kz$. Taking the limit as $z$ tends to zero, the cost of informing a fraction $\phi$ of consumers on $[0, t]$ is

$$C(\phi, t) = -K\ln \left( \frac{(1 - \phi)^t}{1} \right).$$  \hspace{1cm} (11)

The cost of informing a total of $\Phi$ consumers on any $[0, t']$, by advertising on $[0, t]$ is therefore

$$C(\Phi, t, t') = -K\ln \left( \frac{(1 - \frac{\Phi}{\min(t, t')}^t)}{1} \right).$$  \hspace{1cm} (12)

Given $\phi$, $t$ and $r_0$, quantity sold is $Q = \phi \min(r_0, t)$. It follows from (5) that when advertising does not promote recognition, the signaling value is

$$S(r_0, \phi, t) = s_1(\min(r_0, t)) - s_0(\min(r_0, t), \phi),$$  \hspace{1cm} (13)

with $s_1$ and $s_0$ given by (3) and (4), and $r_0$ defined by $p = V(r_0, S(r_0, \phi, t))$. If advertising does not promote recognition, then the signaling value is $\phi t S(r_0, \phi, t)$, where $\phi t$ consumers are informed, with $r_0$ defined by $p = V(r_0, \phi t S(r_0, \phi, t))$.

**Proposition 6.** Suppose that advertising does not promote recognition, with $S(r_0, \phi, t)$ given by (13). Then for any $K > 0$, the firm targets ads precisely on potential demand, $t = r_0$. 

---

8This particular targeting technology is also used in Hernandez-Garcia (1997), Esteban et al. (2001) and Esteban et al. (2006), and amounts to assuming the firm can target consumers with high valuation.
When recognition is not an issue, the firm only wants to inform consumers who are willing to buy. Informing other consumers has no direct impact on sales, and hence no indirect impact through increased stigma. The firm therefore chooses \( t \geq r_0 \) to minimize \( C(\Phi, t, r_0) \), given by (12), which is achieved by setting \( t = r_0 \). If the firm advertised more broadly, \( t > r_0 \), then more ads would be needed to reach any given number of consumers willing to buy.\(^9\) If instead \( t < r_0 \), then the firm could increase its price without sacrificing sales. Proposition 6 echoes Hernandez-Garcia (1997), Esteban et al. (2001) and Esteban et al. (2006), who find a monopolist will use targeted advertising whenever it is the least costly way to inform potential demand.

I now show that the firm’s strategy may be quite different when taking into account recognition.

**Proposition 7.** Suppose that advertising promotes recognition, with signaling value \( \phi tS(r_0, \phi, t) \). Then in the limit as \( K \) tends to zero, the firm advertises as broadly as possible, \( t = 1 \).

When recognition is important, ads received by consumers who don’t buy are no longer wasted. These ads ensure wealthy consumers who do buy can be recognized, which increases their willingness to pay.

It follows that the firm faces a trade-off in its choice of targeting. On the one hand, targeting ads directly on potential demand is the most efficient way to reach consumers who would like to buy. On the other hand, broad advertising is the most efficient way to generate recognition, since it minimizes the probability that any consumer receives multiple ads: \( C(\Phi, t, 1) \) is decreasing in \( t \). The firm chooses \( t = 1 \) when advertising costs are small, since reaching \( \Phi \geq t \) consumers for any \( t < 1 \) would still be infinitely costly.

This result suggests that firms may still want to advertise broadly, even though targeting technology is available, if recognition is important for strengthening brand image. Miller (2009) expresses this idea in the context of luxury goods:

>> The luxury brands with the highest brand equity ... advertise in Vogue and GQ not so much to inform rich potential consumers that they exist, but to reassure rich potential consumers that poorer Vogue and GQ readers will recognize and respect these brands when they see them displayed by others. (Miller 126)

Kapferer and Bastien (2009) make a similar point, espousing what they call an anti-law of marketing for luxury brands. They argue that more people should be familiar with a brand than those likely to buy, and that traditional advertising campaigns may be ineffective if they focus only on the target market.

Proposition 7 shows that broad advertising is optimal whenever costs are sufficiently low. However, this may no longer be the case when advertising costs are high. If \( K \) is sufficiently high, then any ads the firm does send should be targeted precisely on potential demand, \( t = r_0 \). Consider the firm’s incentive to

\(^9\)By \( p = V(r_0, S(r_0, \phi, t)) \) and (13), the value of \( r_0 \) does not depend on \( t \) over this interval.
target when $\Phi$ is small, so when it sends out very few ads. Differentiating (12) with respect to $\Phi$ shows that marginal costs, evaluated at $\Phi = 0$, are independent of $t$. This means that targeting on $[0, r_0]$ is just as effective in promoting recognition as advertising more broadly, as the first few ads will always reach uninformed consumers.

If $K$ takes on an intermediate value, then the firm must balance the need to inform its potential demand with its desire to achieve broad recognition. Clearly, however, the firm has a higher incentive to inform wealthy consumers on $[0, r_0]$ than to inform poor consumers on $(r_0, 1]$. Informing poor consumers only increases recognition, while informing wealthy consumers directly increases both recognition and sales. This suggests the firm might want to use multiple media, to advertise more heavily on $[0, r_0]$ than on $(r_0, 1]$, even if it were constrained to offer a single price.

The problem with using multiple media is one of commitment. Suppose the firm has access to two different media, where $m_1$ reaches consumers on $[0, r_0]$ and $m_2$ reaches consumers on $(r_0, 1]$. Suppose furthermore that $\phi_1 > 0$ and $\phi_2 > 0$, where a consumer observes the value of $\phi_i$ if he receives an ad through $m_i$. Then sales only depend on the behavior of consumers reached through $m_1$. However, willingness to pay depends on their beliefs about $\phi_2$, which they don’t observe. The firm could always save on advertising costs by deviating to $\phi_2 = 0$, and fool wealthy consumers into believing that poor consumers will recognize their purchase.

For this reason, the firm can only credibly promise to advertise in media that reach a sufficient number of consumers willing to buy. For example, if $m_1$ reached consumers on $[0, r_0]$ but $m_2$ reached consumers on $[0, 1]$, then deviating to $\phi_2 = 0$ would be less attractive as it would reduce sales.

More generally, lack of commitment will limit advertising levels whenever the firm uses multiple media. In the last example, marginally reducing $\phi_2$ from its equilibrium level reduces revenues, but by less than if $\phi_2$ were publicly revealed. The reason is that consumers who only receive ads through $m_1$ will not detect this deviation. Their willingness to pay remains unchanged, giving the firm a lower incentive to advertise than under full observability.

This issue of commitment explains the emphasis in the quote from Miller (2009), that ads in wide-circulation magazines not only inform poor readers, but also demonstrate to wealthy readers that poor readers are informed. Chwe (2001) makes a similar point in the context of network goods, arguing that ads placed in the mass media create common knowledge that many consumers are likely to buy. The analysis here suggests how advertising in a single, broad medium can promote recognition while avoiding problems of commitment: all informed consumers are likely to notice a firm’s deviation, since they all become informed in the same way.
5. Conclusion

This paper explores the social side of informative advertising, where consumers care about the image they project about themselves through their purchases. Consumers differ in their wealth, which is unobservable, and a firm produces a conspicuous good that can allow consumers to signal their wealth through their purchases. Advertising informs consumers by allowing them to buy the conspicuous good, and also to recognize it when others buy. In this setting, advertising helps the firm to exploit consumer status concerns, by increasing the stigma of consumers who don’t buy and promoting widespread recognition of those who do.

Taking advertising’s social role into account can help shed light on a variety of issues, such as how advertising relates to conformist behavior, the link between information and persuasion, the broad advertising of high-end goods, and the need to reassure consumers that others are informed. While the issues are different, the analysis suggests they are linked by a common thread: how advertising affects the social pressure to consume.

Appendix

Proof of Proposition 1. From (5), write \( S_B - S = s_0(r_0, \phi) - s_0(r_0, 1) \). Substitute for \( s_0 \) using (4) and rearrange to obtain

\[
S_B - S = \frac{(1 - \phi)r_0}{1 - \phi r_0} \left( \int_0^{r_0} a(r) dr - \int_{r_0}^{1} a(r) dr \right),
\]

where the expression in large brackets is just \( S_B > 0 \). The partial derivative with respect to \( r_0 \) is then

\[
\frac{\partial(S_B - S)}{\partial r_0} = \left( \frac{(1 - \phi)(1 - \phi r_0) + \phi(1 - \phi) r_0}{(1 - \phi r_0)^2} \right) S_B + \left( \frac{(1 - \phi)r_0}{1 - \phi r_0} \right) \frac{\partial S_B}{\partial r_0}.
\]

That is,

\[
\frac{\partial(S_B - S)}{\partial r_0} = \left( \frac{1 - \phi}{(1 - \phi r_0)^2} \right) S_B + \left( \frac{(1 - \phi)r_0}{1 - \phi r_0} \right) \frac{\partial S_B}{\partial r_0},
\]

which is positive if and only if

\[
S_B + r_0(1 - \phi r_0) \frac{\partial S_B}{\partial r_0} > 0.
\]
Since $S_B > 0$, a sufficient condition for condition (15) to hold is $\frac{\partial}{\partial r_0}(r_0S_B) > 0$. Using (3), (4) and (5), write

$$r_0S_B = \int_0^{r_0} a(r)dr - \frac{r_0}{1 - r_0} \int_{r_0}^1 a(r)dr,$$

where

$$\frac{\partial(r_0S_B)}{\partial r_0} = a(r_0) + \frac{r_0}{1 - r_0} a(r_0) - \frac{1}{(1 - r_0)^2} \int_{r_0}^1 a(r)dr,$$

$$= \frac{1}{1 - r_0} \left(a(r_0) - \frac{\int_{r_0}^1 a(r)dr}{1 - r_0}\right),$$

which is strictly positive by $a'(r) < 0$. Returning to (14), the right-hand side is proportional to $S_B$, so that

$$S = \left(\frac{1 - r_0}{1 - \phi r_0}\right)S_B.$$ 

By definition, $S_B$ is independent of $\phi$. Thus,

$$\frac{\partial S}{\partial \phi} = \frac{r_0(1 - r_0)}{(1 - \phi r_0)^2} S_B.$$ 

By $S_B > 0$, it follows that $\frac{\partial S}{\partial \phi} > 0$ and $\frac{\partial^2 S}{\partial \phi^2} > 0$ for all $r_0 \in (0, 1)$, and that $\frac{\partial S}{\partial \phi}|_{r_0=0} = \frac{\partial S}{\partial \phi}|_{r_0=1} = 0$.

**Proof of Lemma 1.** From (4), write

$$s_0 = \frac{(1 - \phi) \int_0^{r_0} a(r)dr + \int_{r_0}^1 a(r)dr}{1 - \phi r_0}.$$ 

The partial derivative with respect to $r_0$ is

$$\frac{\partial s_0}{\partial r_0} = \frac{(1 - \phi)a(r_0) - a(r_0)}{(1 - \phi r_0) \left(1 - \phi \int_0^{r_0} a(r)dr + \int_{r_0}^1 a(r)dr\right)}$$

which is positive if and only if

$$-a(r_0)(1 - \phi r_0) + (1 - \phi) \int_0^{r_0} a(r)dr + \int_{r_0}^1 a(r)dr \geq 0.$$ 

This condition is equivalent to

$$\int_0^1 a(r)dr - \phi \int_0^{r_0} a(r)dr - a(r_0)(1 - \phi r_0) \geq 0.$$
Condition (17) is violated at \( r_0 = 0 \) and is strictly satisfied at \( r_0 = 1 \), since \( a'(r) < 0 \) implies
\[
a(1) < \int_0^1 a(r)dr < a(0).
\]
Moreover, the left-hand side of (17) is increasing with \( r_0 \), since
\[
\frac{\partial}{\partial r_0} \left( \int_0^1 a(r)dr - \phi \int_0^{r_0} a(r)dr - a(r_0)(1 - \phi r_0) \right) = -a'(r_0)(1 - \phi r_0),
\]
which is strictly positive by \( a'(r) < 0 \). Hence there exists a unique \( \theta^* \in (0, 1) \) such that (17) is violated for all \( r_0 \in [0, \theta^*) \), satisfied with equality for \( r_0 = \theta^* \), and strictly satisfied for all \( r_0 \in (\theta^*, 1] \).

To show that \( \theta^* \) is increasing in \( \phi \), it is sufficient to show that the left-hand side of (17) is decreasing in \( \phi \). That is,
\[
\frac{\partial}{\partial \phi} \left( \int_0^1 a(r)dr - \phi \int_0^{r_0} a(r)dr - a(r_0)(1 - \phi r_0) \right) = r_0 \left( a(r_0) - \frac{\int_0^{r_0} a(r)dr}{r_0} \right) < 0,
\]
which holds by \( a'(r) < 0 \).

**Proof of Proposition 2.** Proposition 1 showed that \( \frac{\partial}{\partial r_0} (S_B - S) > 0 \), so that \( \frac{\partial S_B}{\partial r_0} < 0 \) implies \( \frac{\partial S}{\partial r_0} < 0 \). Using (5), the partial derivative of \( S \) with respect to \( r_0 \) is
\[
\frac{\partial S}{\partial r_0} = \frac{\partial s_1}{\partial r_0} - \frac{\partial s_0}{\partial r_0},
\]
where \( \frac{\partial s_1}{\partial r_0} < 0 \) follows from (3) and \( a'(r) < 0 \). Moreover, by Lemma 1, there exists \( \theta^* \in (0, 1) \) such that \( \frac{\partial s_0}{\partial r_0} > 0 \) for all \( r_0 \in (\theta^*, 1] \). This implies \( \frac{\partial s}{\partial r_0} < 0 \) for all \( r_0 \in [\theta^*, 1] \).

By (16), \( \lim_{\phi \to 0} \frac{\partial s_0}{\partial r_0} = 0 \), whereas \( \frac{\partial s_1}{\partial r_0} < 0 \) is independent of \( \phi \). It follows that when \( \phi \) is sufficiently small, \( \frac{\partial S}{\partial r_0} < 0 \) for all \( r_0 \in [0, 1] \).

**Proof of Lemma 2.** For given \( r_0 \) and \( \phi \), price is \( p = V(r_0, S(r_0, \phi)) \) and quantity sold is \( \phi r_0 \). Profits are therefore
\[
\pi = V(r_0, S(r_0, \phi))\phi r_0 - KA(\phi).
\]
Define \( (r_0^*, \phi^*) = \arg\max_{r_0, \phi} \pi(r_0, \phi) \), and suppose first that \( (r_0^*, \phi^*) \) is unique. If \( \frac{\partial \pi}{\partial r_0} = 0 \) and \( \frac{\partial \pi}{\partial \phi} = 0 \) both hold at \( (r_0^*, \phi^*) \), then taking the differential of each first order condition gives
\[
\frac{\partial^2 \pi}{\partial r_0^2} dr_0^* + \frac{\partial^2 \pi}{\partial r_0 \partial \phi} d\phi^* = 0,
\]
where
\[
\frac{\partial^2 \pi}{\partial r_0 \partial \phi} \, dr_0^* + \frac{\partial^2 \pi}{\partial \phi^2} \, d\phi^* = A'(\phi).
\]
(18)

Solving this system of equations yields

\[
\frac{dr_0^*}{dK} = \left( \frac{-1}{\frac{\partial^2 \pi}{\partial r_0^2} - \frac{\partial^2 \pi}{\partial r_0 \partial \phi}} \right) \frac{\partial^2 \pi}{\partial r_0 \partial \phi} A'(\phi),
\]
and

\[
\frac{d\phi^*}{dK} = \left( \frac{1}{\frac{\partial^2 \pi}{\partial r_0^2} - \frac{\partial^2 \pi}{\partial r_0 \partial \phi}} \right) \frac{\partial^2 \pi}{\partial r_0 \partial \phi} A'(\phi),
\]

where \(A'(\phi) > 0\). The second order condition implies \(\frac{\partial^2 \pi}{\partial r_0^2} < 0\) and \(\frac{\partial^2 \pi}{\partial r_0 \partial \phi} - \frac{\partial^2 \pi}{\partial r_0 \partial \phi} > 0\), so that \(\frac{d\phi^*}{dK} < 0\).

If instead \(\frac{\partial \pi}{\partial \phi} > 0\) at \((r_0^*, \phi^*)\), it follows that \(\phi^* = 1\). Moreover, \(\frac{\partial \pi}{\partial \phi} > 0\) continues to hold after a marginal change in \(K\), so that the optimal level of exclusivity remains \(r_0^* = 1\).

If instead \(\frac{\partial \pi}{\partial \phi} = 0\) at \((r_0^*, \phi^*)\) but \(\frac{\partial \pi}{\partial r_0} > 0\), it follows that \(r_0^* = 1\). Again by continuity, \(\frac{\partial \pi}{\partial r_0} > 0\) continues to hold after a marginal change in \(K\), so the optimal level of exclusivity remains \(r_0^* = 1\). Plugging \(\frac{dr_0^*}{dK} = 0\) into (18) yields \(\frac{d\phi^*}{dK} = A'(\phi) \frac{\partial^2 \pi}{\partial \phi^2} \), which is negative by the second order condition.

Finally, suppose \((r_0^*, \phi^*)\) is not unique. Index these pairs by \(n \in \mathbb{N}\), where without loss of generality \(\phi_n\) is increasing in \(n\). Let \(\pi_n^*\) denote profits evaluated at pair \(n\); by assumption, these profits are independent of \(n\), at this particular value of \(K\). Now consider a marginal increase in \(K\). By the Envelope Theorem, \(\frac{d\pi_n}{dK} = \pi_n = -A(\phi_n) < 0\), which is increasing in magnitude with \(n\). Hence, \(\phi_n^*\) is no longer optimal for any \(n \geq 2\), and advertising levels must decrease.

**Proof of Proposition 3.** For given \(\phi\) and \(r_0\), revenues are

\[
R(r_0, \phi) = V(r_0, S(r_0, \phi))\phi r_0.
\]

The partial derivative of \(R\) with respect to \(r_0\) is therefore

\[
\frac{\partial R}{\partial r_0} = \phi \left( V_1 + V_2 \frac{\partial S}{\partial r_0} \right) r_0 + \phi V(r_0, S),
\]
(19)

where \(V_i\) denotes the partial derivative of \(V\) with respect to its \(i\)th argument. Looking at the right-hand side of (19), Proposition 1 implies \(S < S_B\), so that \(V(r_0, S) < V(r_0, S_B)\). Moreover, (2) implies

\[
V(r, S) = w_r - u^{-1}(u(w_r) - S(r, \phi)),
\]
for any consumer \( r \) for who the budget constraint does not bind. Differentiating \( V \) with respect to its first argument and using \( dw/dr = -1/f(w) \) yields

\[
V_1 = \frac{1}{f(w_r)} \left( -1 + \frac{u'(w_r)}{w'(u^{-1}(u(w_r) - S(r_0, \phi)))} \right) < 0.
\]

Differentiating \( V \) with respect to its second argument yields

\[
V_2(r, S(r_0, \phi)) = \frac{1}{w'(u^{-1}(u(w_r) - S(r_0, \phi)))} > 0.
\]

Thus, the expression in large brackets in (19) is equal to

\[
\frac{1}{f(w_{r_0})} \left( -1 + \frac{f(w_{r_0})}{w'(u^{-1}(u(w_r) - S(r_0, \phi)))} \right).
\]

(20)

If \( u(c) \) is linear, then write \( u(c) = \alpha \) for some constant \( \alpha > 0 \). Hence when \( \phi = 1 \), (2) then implies \( V(r_0, S_B) = S_B/\alpha \), and (20) simplifies to \( \frac{\partial R}{\partial r_0} = \frac{\phi}{\alpha} \). It follows that (19) can be written as

\[
\frac{\partial R}{\partial r_0} = \frac{\partial (rS_B)}{\partial r_0} \frac{\phi}{\alpha},
\]

which was shown in the proof of Proposition 1 to be strictly positive, for all \( r_0 \in [0, 1] \). By continuity, it will continue to be strictly positive if \( \frac{\partial u(c)}{\partial c} < \epsilon \) for all \( i \geq 2 \), when \( \epsilon > 0 \) is sufficiently small. The optimal value of \( r_0 \) in the baseline is therefore \( r_0 = 1 \). This is strictly greater than the optimal value of \( r_0 \) when \( \phi < 1 \), since \( R(1, \phi) = 0 \).

If instead \( f(w_{r_0}) \frac{\partial S}{\partial r_0} < -u'(w_{r_0}) \), then the numerator of (20) is negative at \( \phi = 1 \). The magnitude of the numerator is also larger at \( \phi < 1 \) than at \( \phi = 1 \), since \( \frac{\partial S}{\partial r_0} < \frac{\partial S}{\partial r_0} \). The denominator is positive and smaller at \( \phi < 1 \) than at \( \phi = 1 \), since \( S < S_B \). Taken together, (20) is negative, and smaller in magnitude when \( \phi = 1 \) than when \( \phi < 1 \). It then follows from (19) that \( \frac{\partial R(r_0, \phi)}{\partial r_0} < \frac{\partial R(r_0, 1)}{\partial r_0} \), for all \( r_0 \in (0, 1] \), so the optimal \( r_0 \) when \( \phi < 1 \) is strictly lower than in the baseline.

**Proof of Proposition 4.** For given \( p, \phi \), and corresponding \( r_0 \), the sum of individual utilities is

\[
\phi \int_0^{r_0} \left( u(w_r - p) + s_1 \right) dr + (1 - \phi) \int_0^{r_0} \left( u(w_r) + s_0 \right) dr + \int_{r_0}^{1} \left( u(w_r) + s_0 \right) dr.
\]

Substituting for \( s_1 \) and \( s_0 \) using (3) and (4) gives

\[
\phi \int_0^{r_0} u(w_r - p) dr + (1 - \phi) \int_0^{r_0} u(w_r) dr + \int_{r_0}^{1} u(w_r) dr + \int_{r_0}^{1} a(r) dr,
\]

(21)

which is strictly greater at \( \phi = 0 \) than at \( \phi > 0 \), since \( p = V(r_0, S(r_0, \phi)) > 0 \).
A consumer who does not buy when $\phi > 0$ is always better off when $\phi = 0$. His status utility increases from $s_0$, given by (4), to $\int_0^{r_0} a(r)dr$. For a consumer with wealth $w_r$ who does buy, his status utility decreases from $s_1$, given by (3), to $\int_0^1 a(r)dr$. He will be better off if $u(w_r - V(r_0, S(r_0, \phi))) \geq \int_0^{r_0} a(r)dr$, where the left-hand side is increasing in $r$ by $u''(w) < 0$.

Proof of Proposition 5. For given $p$, $\phi$ and corresponding $r_0$, the sum of individual utilities is given by (21). Lemma 2 showed that a marginal increase in $K$ causes the optimal $\phi$ to drop, whenever $\phi < 1$. The rate of change of (21) with respect to $\phi$ is

$$\int_0^{r_0} u(w_r - p) - u(w_r)dr + \phi \left( u(w_{r_0} - p) - u(w_{r_0}) \right) \frac{dr_0}{d\phi}.$$ (22)

Looking at (22), both the integral and the expression in large brackets are negative, while comparing (7) to (8) shows that $\frac{dr_0}{dp}$ and $\frac{dr_0}{d\phi}$ have the opposite sign. It follows that (22) is negative if $\frac{dr_0}{dp} < 0$, so if demand is locally downwards sloping. It is also negative if $\frac{dr_0}{dp} > 0$, so if demand is locally upwards sloping, as long as $\phi$ or $\frac{dr_0}{d\phi}$ are sufficiently small. By (7), $\frac{dr_0}{dp}$ is proportional to $\frac{dS}{dp}$, where $\frac{dS}{dp}|_{r_0=0}$ by Proposition 1.

Proof of Proposition 6. Consider a candidate equilibrium with $p$, $\phi$ and $t$, where $r_0$ is defined by $p = V(r_0, S(r_0, \phi, t))$, with $S(r_0, \phi, t)$ given by (13). Suppose first that $t < r_0$. Then a fraction $\phi$ of consumers on $[0, t]$ buy, giving quantity sold $\phi t$. The price is $p = V(r_0, S(t, \phi))$, with $S(t, \phi)$ given by (5). By $V_1 < 0$, the price is strictly lower than the willingness to pay of consumer $t$. The firm can therefore increase its price to $p = V(t, S(t, \phi))$, with no effect on quantity sold. Hence $t < r_0$ cannot be optimal.

Suppose instead that $t \geq r_0$. Then quantity sold is just the mass of consumers informed on $[0, r_0]$. By (12), the cost of informing $\Phi$ consumers on $[0, r_0]$ is $C(\Phi, t, r_0) = -K \ln \left(1 - \frac{\Phi}{r_0}\right)$, which is increasing in $t$. Moreover, $p = V(r_0, S(r_0, \phi, t))$ and (13) imply that $r_0$ is independent of $t$ for all $t \geq r_0$. The firm therefore minimizes its costs by setting $t = r_0$.

Proof of Proposition 7. As shown in the proof of Proposition 6, the firm will always set $t \geq r_0$, so that (13) implies $S(r_0, \phi, t)$ is independent of $t$. I can write $p = V(r_0, \phi t S(r_0, \phi))$, with $S(r_0, \phi)$ given by (5), where quantity sold is $\phi r_0$. Profits are therefore

$$\pi = V(r_0, \phi t S(r_0, \phi)) \phi r_0 - C(\phi, t),$$
with $C(\phi, t)$ given by (11). For any $t \geq r_0$ and $\phi > 0$, $\phi t S(r_0, \phi)$ is strictly increasing in $t$. If $t < 1$, then $V_2 > 0$ implies that setting $t = 1$ will strictly increase revenues, by an amount that is independent of $K$. It will also increase costs, but by an amount that is proportional to $K$. It follows that $t = 1$ must be optimal for $K$ sufficiently small.

References


