Capacity Constraints and Naive Consumers' Beliefs About Demand (Draft)

Nick Vikander*

January 2010

Abstract

This paper looks at how a firm can use a capacity constraint to manipulate consumer beliefs about demand. Willingness to pay is increasing in beliefs about aggregate quantity demanded, consumers can observe quantity sold but not quantity demanded, and some naive consumers do not understand the firm may directly signal through its price.

A capacity constrained firm may then set a low price and sell out to fool naive consumers into believing demand is higher than it actually is. Doing so increases demand in future periods. An unconstrained firm will impose a capacity constraint on itself and sell out if demand is sufficiently low, and firm uncertainty about demand pushes the optimal price down despite the increased probability of excess demand.

1 Introduction

Consider a person walking by a restaurant who glances in to gage its popularity. He is more interested in dining there if he believes others want to dine there too.

His inference can be quite precise if he sees free tables inside. Given the price, the number of people who want to dine there must be close to the number actually sitting in the restaurant. Otherwise more customers would have arrived to fill the free tables.

His beliefs may be very different, and more positive, if the restaurant is completely full. He can no longer infer the exact number of customers who want to dine there, since some may have been turned away for lack of space. He knows their number is at least equal to restaurant's capacity, but it might also be larger.

This paper looks at how such a restaurant can exploit the discontinuity between beliefs and quantity sold when it sells at capacity. It considers a context

*Tinbergen Institute and Erasmus University Rotterdam, vikander@tinbergen.nl, www.tinbergen.nl/∼vikander

where some consumers are boundedly rational, making it possible for the firm to manipulate their beliefs.

A monopolist faces a market of consumers in a repeated setting. Each consumer's willingness to pay for the good is increasing in his beliefs about aggregate quantity demanded. I view this as a pure social effect, modeled directly in the utility function, where consumers want to do the "in" thing.[1]

A fraction of consumers are uncertain about aggregate demand. There is a stochastic term in the demand function, of which they know the distribution but not the realization. These consumers are also naive, in that they do not consider the firm might signal through its actions. Specifically, they do not directly infer anything about the stochastic term in demand from seeing the firm's price.

Consumers can observe quantity sold in each period but cannot directly observe quantity demanded. If the firm sells at capacity, naive consumers are unable to infer quantity demanded by say looking at the size of queues. Instead, they update their beliefs by considering what values of the stochastic term are consistent with the observed quantity sold, given the firm's price.

In equilibrium, the firm may strategically use a capacity constraint and its price to manipulate the beliefs of naive consumers. If the firm is not capacity constrained, then it will sometimes impose such a constraint on itself and set a price so as just to sell out. Doing so makes naive consumers believe demand is higher than it actually is, which itself increases demand in future periods.

The firm will impose a capacity constraint if and only if demand is sufficiently low, and it will do so for many values of the stochastic term even as the fraction of naive consumers tends to zero. Naive consumers never learn that their beliefs are wrong, since a firm that sells out in one period also sells out in all others.

If the firm has an exogenous capacity constraint, then it will sometimes also set its price so as to sell out even though that does not maximize immediate profits. For any given capacity constraint and almost every level of demand, it will sell out if the strength of social effects is sufficiently high. However, keeping the strength of social effects for naive consumers constant, the firm will not sell out when demand is sufficiently high unless doing so maximizes immediate profits.

The mechanism at work pushes the firm to sell out, but does not push it to have strictly positive excess demand. That changes if the firm is also somewhat uncertain about demand, and the price which maximizes immediate profits yields excess demand with positive probability. Then the firm will reduce its price, to be more likely to sell out and manipulate naive consumers, even though the low price makes excess demand more likely. As the amount of uncertainty tends to zero, the probability that excess demand is strictly positive tends to one.

I then explore this issue from another perspective, by looking for an equilibrium where beliefs are correct. I change the model by assuming consumers can directly observe quantity demanded, but not all consumers observe the price.

[1] Alternatively, consumers might be uncertain about quality and learn something by seeing if others buy. I say more about this informational perspective in the literature review.

A typical restaurant will have many different prices. Somebody walking by and peering in may have an expectation of prices which are relevant for them, but it may be difficult or time consuming to confirm that by looking through the posted menus.

The firm is then tempted to drop the price below its expected level. The higher sales fool these consumers into thinking demand is higher than it actually is, which increases future demand. A capacity constraint makes such a deviation less attractive, helping the firm commit to a higher price and increasing equilibrium profits.

As far as I know, this is the first paper to explore how a firm may try to use a capacity constraint to influence consumer beliefs about demand. Various papers have looked at other reasons a capacity constrained firm may set a low price to generate excess demand, or why rationing production may be optimal.

The most similar paper is Becker (1991), where consumers' willingness to pay is also increasing in aggregate quantity demanded. He shows that a capacity constrained firm may not raise its price despite facing excess demand, but the mechanism driving his results is quite different. There is no demand uncertainty, and so no question of influencing consumer beliefs. Becker shows that if the aggregate inverse demand curve is upward sloping then the optimal price can create excess demand, and that sufficiently strong social effects can yield such aggregate demand. Marginally increasing the price can then cause quantity demanded to collapse to zero. In contrast, upward slopping inverse demand plays no role in this paper.

DeSerpa and Faith (1996) put forward another argument based on upward sloping aggregate inverse demand, for goods where part of the enjoyment is jointly produced during consumption. Consumers may enjoy hearing quiet chatter in a restaurant, or cheering at a concert. If low valuation consumers generate more of this "noise" than others, a capacity constrained firm may choose to have excess demand. A random rationing rule then allows some low valuation consumers to be served, which increases the willingness to pay of others.

Other papers explore how a firm can benefit by committing to ration future production. DeGraba (1995) shows that if ex-ante identical consumers learn their precise valuations over time, the firm may want to sell early when willingness to pay is more uniform. Consumers will buy early if they expect they may be rationed later on. Nocke and Peitz (2007) look at clearance sales, where the firm charges a high initial price followed by a lower price with a threat of rationing. That can induce high valuation consumers to separate and purchase before low valuation consumers. Gilbert and Klemperer (2000) show rationing can encourage low valuation consumers to make sunk investments since they are then more likely to be served.

Denicolo and Garella (1999) looks at excess demand in a durable goods monopoly, where the firm cannot commit to rationing. They show excess demand can be optimal if the rationing rule is not efficient. Rationing then effectively shifts some high valuation consumers from earlier to later periods, smoothing demand over time and helping the firm commit to a higher price.

The literature on how social effects can generate positive consumption externalities goes back many years, including Leibenstein (1950) on bandwagon effects and Becker (1974) on social interactions. An important difference between the current paper and most others, including those on network externalities, is consumers do not care about whether others actually buy the good, but whether they want to buy it.

Bikhchandani, Hirshleifer, and Welch (1992) and Banerjee (1992) give an alternative explanation for bandwagon or herd behavior, based on uncertainty about quality. If each consumer has independent information about quality, consumers who see others buy will be more willing to buy themselves because they infer quality is high. Becker (1991) argues including a term for social effects directly in the utility function can also be interpreted as a reduced form approach to this idea. The interpretation is then that a small increase in quantity demanded provides a small amount of positive information about quality, and thus a small increase in willingness to pay.

The informational approach becomes different if consumers fully ignore their own information to imitate each other, causing informational cascades. Seeing a restaurant filled through such a cascade may not provide any more information than if is half empty. Most consumers are not following their own signals, so information is no longer aggregated.

The paper is also related to the recent literature in industrial organization on how firms may try to hide information from boundedly rational consumers. Gabaix and Laibson (2006) show firms may shroud add-on prices to fool naive consumers. Spiegler (2006) considers consumers who evaluate a product by only sampling quality in one dimension, and shows firm may obfuscate by randomizing over quality in other dimensions. Chioveanu and Zhou (2009) and Piccione and Spiegler (2009) look at price competition with framing. Firms randomize over both prices and the frames in which prices are presented, to prevent consumers from making price comparisons.

2 A Simple Model

A monopolist produces a homogeneous good at zero marginal cost, which it can sell in each of periods $t = 1, 2, 3, \ldots$. I explore two assumptions regarding capacity constraints. The firm may have an exogenous capacity constraint, so it cannot sell more than quantity Q_0 in each period. Or it may not be capacity constrained, but can decide to impose such a constraint on itself before the start of period 1. If imposed, the capacity constraint holds for all periods.

A subscript on a function or variable will refer to its value in a given period. The discount factor is δ.

The firm faces a mass B/A of consumers, each with unit demand in each period. A consumer's intrinsic willingness to pay for the good is an IID draw from the uniform distribution on $[0, A]$, with $A > 0$. The uniform distribution will yield linear aggregate demand, but none of the paper's results depend on linearity.

A consumer's willingness to pay is his intrinsic willingness to pay plus an extra term, $CE[\epsilon]$. That is a constant $C > 0$ times the consumer's expectation of a stochastic term ϵ, where the expectation is conditional on the information at his disposal. The term ϵ is a draw from the uniform distribution on $[-\Delta, \Delta]$, with $\Delta > 0$.

Aggregate demand in each period is equal to the quantity demanded of all consumers plus the stochastic term ϵ. That gives the following aggregate demand and aggregate inverse demand:

$$Q(p) = \frac{1}{B}[A - p + CE[\epsilon] + B\epsilon]$$

$$p(Q) = A - B(Q - \epsilon) + CE[\epsilon]$$

The term $CE[\epsilon]$ is a simple way to capture the idea that consumers are willing to pay more for a good if they believe it is popular.[2]

The firm and a fraction $(1-\alpha)$ of consumers are informed, and know the realization of ϵ. A fraction α of consumers are uniformed and only know its distribution. They are also naive, in that they do not directly infer anything about ϵ from the firm's choice of price. All consumers can observe quantity sold in each period but cannot directly observe quantity demanded.

Assuming only naive consumers are uninformed simplifies the problem, because it means the firm cannot signal ϵ through pricing alone. The only consumers who need to update their beliefs about demand are naive consumers, and they only do so through the mechanism I am interested in: by observing price, quantity sold, and then making an inference based on their prior beliefs about ϵ.

The timing of the game is as followed. Before period 1, the stochastic term ϵ is realized, and both the firm and a fraction $(1-\alpha)$ of consumers learn its value. If the capacity constraint is not exogenous, the firm can choose a capacity constraint Q_0 such that it cannot sell more than Q_0 in any period.

In any period t, the firm sets price p_t, consumers observe the price and decide whether to buy. If quantity demanded exceeds capacity, then quantity Q_0 is allocated to consumers according to some rationing rule. Consumers observe quantity sold q_t, and naive consumers update their beliefs about ϵ using Bayes' rule.

3 Analysis

3.1 Endogenous capacity constraint

I first look at the case where the firm is not capacity constraint, but can decide to impose such a constraint on itself before the start of period 1. The firm's optimal strategy is as follows:

[2]Multiplying C by $E[Q(p)]$ would not change the analysis for most parameter values, though it would imply upwards sloping inverse demand for C sufficiently large. That would present another rationale for excess demand, as analyzed in Becker (1991).

Theorem 1. *There exists a critical value ϵ_0 with $0 < \epsilon_0 < \Delta$, such that the following holds. For $\epsilon > \epsilon_0$, the firm does not impose a capacity constraint and sets price*

$$p_1 = \frac{A + B\epsilon + (1-\alpha)C\epsilon}{2}$$

$$p_t = \frac{A + B\epsilon + C\epsilon}{2}$$

for all $t \geq 2$.

For $\epsilon \leq \epsilon_0$ the firm chooses the following capacity constraint

$$Q_0 = \frac{1}{2B}[A + B\epsilon + (1-\alpha)C\epsilon + \delta\alpha C(\frac{\epsilon + \Delta}{2})]$$

and sets prices so that $q_t = Q_0$, that is

$$p_1 = \frac{A + B\epsilon + (1-\alpha)C\epsilon}{2} - \delta\frac{\alpha C}{2}(\frac{\epsilon + \Delta}{2})$$

$$p_t = \frac{A + B\epsilon + C\epsilon}{2} + \frac{\alpha C}{2}[\Delta - \delta(\frac{\epsilon + \Delta}{2})]$$

for all $t \geq 2$

Proof. See appendix □

The firm will sometimes use a capacity constraint to manipulate the beliefs of naive consumers. If it does so, it will set a price in each period so that demand exactly equals capacity. Naive consumer believe excess demand is strictly positive, though in fact it is zero.

Three aspects of the result are of particular interest. First, the firm sets a capacity constraint and sells out if and only if demand is sufficiently low. Second, the firm does so for a large range of ϵ, even as the fraction of naive consumers α or the strength of social effects C tends to zero. Third, naive consumers who are fooled about ϵ never observe $q_t < Q_0$ and so never discover their beliefs are incorrect.

The first two aspects both come from the trade-off the firm faces when deciding whether to set a capacity constraint. Compared to profits when the firm is unconstrained, imposing a capacity constraint generates certain benefits but also certain costs.

The benefit of a capacity constraint is that it allows the firm to increase demand in future periods by manipulating the beliefs of naive consumers when the constraint binds. For all $\epsilon < \Delta$, selling out in period 1 leads naive consumers to believe ϵ is higher than it actually is.

The cost of a capacity constraint is that it limits the quantity the firm can sell in future periods. If demand is lower than expected, $\epsilon < 0$, then this is not a problem. Without a capacity constraint, consumers infer the value of ϵ after period 1 and reduce their demand for future periods. The firm then sets a price so that quantity sold decreases after period 1. Even if there was a capacity constraint at $Q_0 = q_1$, it would no longer bind.

If demand is higher than expected, $\epsilon > 0$, then a capacity constraint imposes strictly positive costs. Without a capacity constraint, demand increases after period 1 as naive consumers update their beliefs. The firm then sets a price such that quantity sold also increases. Imposing a capacity constraint means demand as of period 2 is even higher, as naive consumers are fooled. But the firm cannot meet that increased demand in an optimal way by increasing both price and quantity, but instead can only increase price.

The benefit of a capacity constraint is strictly decreasing in ϵ. The extent to which naive consumers overestimate demand is higher when demand is low, and can be written as $(\Delta - \epsilon)/2$. For any $\epsilon < \Delta$, the benefit is strictly positive. When $\epsilon = \Delta$, naive consumers are not fooled and the benefits are zero.

The cost of a capacity constraint is zero for all $\epsilon \leq 0$, and is strictly increasing in ϵ for all $\epsilon \geq 0$. The larger the value of ϵ, the more the firm would increase quantity sold as of period 2 if it were not capacity constrained.

It follows that the firm only imposes a capacity constraint and sells out if demand is sufficiently low, so if ϵ is below a threshold.

That threshold is strictly positive, so for any $\alpha > 0$ and $C > 0$, however small, the ex-ante probability the firm will impose a capacity constraint exceeds one half. That is because both the benefits and the costs of a capacity constraint result from the behavior of naive consumers. When α or C is small, the benefits become small, but so do the costs.

Prices and quantities sold are nonetheless continuous in α and C. As the fraction of naive consumers or the strength of social effects goes to zero, the firm's price under a capacity constraint tends to the optimal price without a capacity constraint. Q_0 tends to the quantity that would be sold without a capacity constraint, which is then the same in each period.

In the proof, I derive an explicit expression for the critical value ϵ_0. I do not yet have analytic results for how ϵ_0 varies with the model parameters. But simulations suggest that with linear demand, ϵ_0 is decreasing in both C and α. That is the firm is more likely to use a capacity constraint if there are few naive consumers or if social effects are weak. This may seem surprising, since the only purpose of using a capacity constraint is to manipulate naive consumers and exploit the social effects of demand. If such a result holds, it will be because the costs of a capacity constraint are increasing in C and α faster than the benefits.

Another feature of the result is that naive consumers are never confronted with evidence that their beliefs are incorrect. If the firm imposes a capacity constraint and sells out in period 1, then it also sells out in all future periods. Naive consumers therefore never observe quantity sold being less than capacity, which would reveal the true value of ϵ.[3] Demand increases after naive consumers are first fooled, giving the firm an incentive to actually increase quantity sold in later periods. It cannot do so because of its capacity constraint, and so instead

[3] Naive consumers might wonder why they are never rationed, even though they believe there is excess demand.

increases the price just enough so that it sells out again.

The firm will always increase its price after period 1. As well, the firm's optimal period 1 price after imposing a capacity constraint is lower than what it would have been for the same value of ϵ had it not imposed the constraint. That suggests informed consumers may benefit from the presence of naive consumers, at least those who are only willing to buy at the lower period 1 price.

The presence of naive consumers does not necessarily increase profits. If the firm is impatient enough, expected profits are increasing in the number of informed consumers. If δ is small, firm profits approximately equal its period 1 profits without a capacity constraint

$$\pi_1^u = \frac{1}{4B}[A + B\epsilon + C(1-\alpha)\epsilon]^2$$

where u stands for unconstrained. The function is strictly convex in ϵ, and its expectation is decreasing in α.

More informed consumers yield more variation in period 1 demand. For example, if demand is above average, informed consumers realize this and adjust their willingness to pay upwards, which increases demand still further. The firm takes advantage of the variation by tailoring its price to the state of demand, increasing it when demand is high and decreasing it when demand is low.

So far, a firm that imposes a capacity constraint will always sell out, but the desire to influence naive consumers does not push it towards strictly positive excess demand. This is no longer the case if the firm is somewhat uncertain about demand.

Instead of receiving a perfectly accurate signal about ϵ, I now assume the firm and a fraction $(1-\alpha)$ of consumers receive a signal with a small amount of noise. They all receive the signal $\epsilon + X$ where X is a single draw from a uniform distribution on $[-X_0, X_0]$ with $X_0 > 0$. I consider values of X_0 that are small, so I can neglect the probability they receive a signal that is not in $[-\Delta, \Delta]$.

Result 2. *Let $\epsilon < \epsilon_0$ from Theorem 1, so a firm that knew ϵ would impose a capacity constraint and sell out. Let p_1^c be the corresponding optimal period 1 price.*

As X_0 tends to zero, the firm's optimal strategy tends to that from Theorem 1. The optimal period 1 price tends to p_1^c from below, and the probability that there is strictly positive excess demand tends to 1.

Proof. See appendix □

The only element here that is discontinuous is how selling out in period 1 influences beliefs. A price such that $q_1 = Q_0$ generates strictly higher future profits than the limiting price such that q_1 tends to Q_0 from below.

When the uncertainty facing the firm is small, its incentives are similar to those in Theorem 1. Here, it will set a capacity constraint and a price in the hope that period 1 demand is just high enough to sell out.

The discontinuity at Q_0 implies the firm would much rather have quantity demanded slightly above capacity than slightly below. Uncertainty about demand is small, and the firm only loses a small amount of profits if quantity demanded slightly exceeds capacity. In contrast, it suffers a discrete drop in profits if quantity demanded is slightly below capacity because naive consumers are not fooled.

As uncertainty decreases, the firm sets a price such that it is increasingly likely to give small but strictly positive excess demand. Demand uncertainty is therefore associated with a lower price. This is in sharp contrast to what one would usually expect from a capacity constrained firm with zero marginal costs. Such a firm would increase its price if faced with demand uncertainty. Uncertainty increases the probability it will have strictly positive excess demand, in which case a price increase would not cost any sales.

3.2 Exogenous capacity constraint

I now assume Q_0 is exogenous, and return to the assumption that the firm and informed consumers know the exact realization of ϵ.

The firm now faces a different trade-off when deciding whether to sell-out. The benefit of selling out is the same as before, fooling naive consumers. It is again strictly positive for all $\epsilon < \Delta$ and is decreasing in ϵ. The cost of selling out when Q_0 is exogenous is that the firm may have to charge a lower initial price than that which maximizes period 1 profits to make quantity demanded sufficiently high.

The cost is zero if quantity demanded at the price that maximizes period 1 profits is exactly Q_0. Otherwise, the cost is strictly positive. The quantity sold that maximizes period 1 profits is decreasing in demand, so, for a given Q_0, the cost of selling out is decreasing in ϵ.

The net effect of ϵ on the firm's incentive to sell out is ambiguous, since both the costs and benefits are decreasing in ϵ. Clearly, if is ϵ sufficiently large so that selling out maximizes period 1 profits, then the firm should also sell out here. Doing so has the added benefit of increasing future profits by fooling naive consumers.

I can also say the following.

Result 3. *If $q_1 = Q_0$, then $q_t = Q_0$ for all $t \geq 2$. Furthermore, for*

$$\frac{1}{2B}[A + B\Delta + C(1-\alpha)\Delta] \leq Q_0 \leq \frac{1}{2B}[A + B\Delta + C(1-\alpha)\Delta]$$

there exist values of ϵ such that $q_1 = Q_0$ even though this does not maximize period 1 profits.

Proof. See Appendix □

If the firm sells out in period 1, it is optimal to sell out in all further periods as in Theorem 1. Selling out increase future demand, so the firm will not want to decrease quantity sold. Once again, naive consumer are never confronted with evidence that their beliefs are incorrect.

The condition on Q_0 means there are large values of ϵ such that selling out maximizes period 1 profits, and small values of ϵ such that it does not.

If the condition holds, there is an intermediate value of ϵ such that the price that maximizes period 1 profits without a capacity constraint gives quantity demanded of exactly Q_0. For ϵ just below this intermediate value, selling out gives slightly lower period 1 profits than if the firm set a slightly higher price. Selling out is nevertheless profitable because of how it affects the beliefs of naive consumers. As ϵ approaches this intermediate value from below, the cost of selling out approaches zero while the benefit remains strictly positive.

I want to look at the firm's strategy as the benefits from fooling naive consumers increase. I now allow naive consumers to have a different value of C than informed consumers. I denote this value of naive consumers by C^n, and I look at results when C^n is large.

Having C^n differ from C allows me to isolate what happens when the firm's incentive to manipulate naive consumer beliefs increases. If C became large for all consumers, that would also change the willingness to pay of informed consumers. For example, for $\epsilon > 0$, the firm would always want to sell out just to serve the informed consumers, which is not the effect I am interested in.

Result 4. *Say* $Q_0 \leq \frac{1}{2B}[A + B\Delta + C(1-\alpha)\Delta]$. *Then for* C^n *sufficiently large,* $q_t = Q_0$ *for all* t*, for all values of* ϵ.

Say $Q_0 > \frac{1}{2B}[A + B\Delta + C(1-\alpha)\Delta]$. *Then for* C^n *sufficiently large but fixed, there exists* $\epsilon_0 < \Delta$ *such that* $\epsilon \leq \epsilon_0$ *implies* $q_t = Q_0$ *and* $\epsilon > \epsilon_0$ *implies* $q_1 < Q_0$ *for all* t.

As C^n *tends to infinity,* ϵ_0 *tends to* Δ.

Proof. See appendix □

The first inequality means that for sufficiently high values of $\epsilon \leq \Delta$, the firm would like to sell out even if doing so did not affect the beliefs of naive consumers.

$C^n > 0$ means the firm now has an added benefit from selling out, and this is increasing without bound in C^n. The firm has a cost of selling out for low values of ϵ but which is independent of C^n. For C^n sufficiently large, the firm will sell out for all values of ϵ.

The situation is different when Q_0 is large enough to violate the inequality. For such a large Q_0, the cost of selling out is strictly positive for all ϵ. In contrast, the benefit from selling out decreases to zero as ϵ approaches Δ. That implies for any fixed value of C^n, the firm will not sell out for values of ϵ sufficiently close to Δ.

As C^n increases, so does the incentive to sell out for any value of ϵ. For C^n large but fixed, the firm will want to sell out for all values of ϵ that are below a certain threshold. The reason is again that the gain of selling out is large while the cost of doing so is independent of C^n. As in Theorem 1, the firm sells out if and only if demand is sufficiently low.

As C^n continues to increase, the range of ϵ for which selling out is profitable increases as well. In the limit, the critical value then tends to Δ.

3.3 Imperfectly observing price

The results so far depend on some consumers being fooled. That may seem unattractive in an equilibrium, and excludes the possibility consumers can infer demand by looking at the length of queues.

I now explore another way a capacity constraint can increase firm profits, where equilibrium beliefs about demand are correct. In contrast to the previous analysis, the capacity constraint benefits the firm by helping it commit *not* to fool consumers into thinking demand is larger than it is.

The idea is consumers may be able to observe quantity demanded, but may not always observe the price. If not, they have an expectation of the price which turns out to be correct in equilibrium. For a consumer walking by a restaurant but who does not immediately want to eat, it may in fact be more difficult to confirm the price than to get an idea about demand. Glancing in the window suggests something about demand, while he must stop and go through the menu to confirm the difference prices being charged.

The firm may then be tempted to drop its price below its expected level to generate more sales and fool these consumers into believing demand is higher than it actually is. Consumers are then willing to pay a higher price when they return in the future.

To illustrate this point in a simple setting, I make a number of changes to the model. I look at a game of only two periods. In period 1, consumers decide whether to buy, just as in the original model. After period 1, these consumers are replaced by an ex-ante identical group of consumers who decide whether to buy in period 2.

Consumers in period 2 all directly observed quantity demanded in period 1. A fraction β of them do not observe the period 1 price, but have an expectation that is correct in equilibrium. All consumers in period 2 observe the period 2 price and then decide whether to buy.

Nothing in this mechanism depends on uninformed consumers being naive, since they are not fooled in equilibrium. Still, I maintain the assumption for convenience. It allows me to neglect how the firm may directly signal to uninformed consumers through its price. The assumptions about the length of the game and replacing consumers after period 1 simplify the analysis but do not seem to alter the basic mechanisms at work.

Result 5. *Consider the following changes to the model: there are two periods, and all consumers are replaced after period 1 by an ex-ante identical group of new consumers. All consumers in period 2 observe quantity demanded in period 1, but a fraction β of them do not observe the period 1 price. Then the firm sets*

$$p_1 = min\left\{\frac{1}{2}[A + B\epsilon + (1-\alpha)C\epsilon], (1 - \frac{\alpha\beta C}{2B})[A + B\epsilon + (1-\alpha)C\epsilon]\right\}$$

and a capacity constraint:

$$Q_0 = q_1 = \frac{1}{B}[A + B\epsilon + (1-\alpha)C\epsilon - p_1]$$

It sets

$$p_2 = \frac{1}{2}[A + B\epsilon + (1-\alpha)C\epsilon]$$

so that

$$q_2 = \frac{1}{2B}[A + B\epsilon + (1-\alpha)C\epsilon] < Q_0$$

Proof. See appendix □

The result shows it is always profitable for the firm to impose a capacity constraint on itself. The capacity constraint serves as a commitment device.

If all consumers could observe the period 1 price, then the firm could achieve first best profits without a capacity constraint. It would just charge the same profit maximizing price $[A + B\epsilon + (1-\alpha)C\epsilon]/2$ in both periods.

Here, the firm knows it can increase period 2 demand by dropping the period 1 price below its expected level. Doing so fools a fraction β of uninformed period 2 consumers into believing ϵ is higher than it actually is. Period 1 profits decrease at an increasing rate as the firm reduces its price. To have no incentive to drops it price, the firm must therefore charge a price well below the first best level. It would like to charge a higher price but cannot commit to doing so.

Imposing a binding capacity constraint decreases the firm's incentive to reduces the period 1 price. The capacity constraint means the firm cannot serve any of the extra period 1 demand generated by reducing its price, which makes the deviation less attractive.

The firm would prefer if all consumers were able to observe the price, since then it would not have a commitment problem. That may be one reason why restaurants may take great efforts to post their prices for consumers walking by to see.[4]

If $\delta\alpha\beta C \leq 4B$, then a capacity constraint allows the firm to achieve the same profits as if all consumers observed the period 1 price. In particular, the inequality will hold if either the fraction of naive consumers, the fraction of consumers who do not observe the price, or the strength of social effects is small.

4 Conclusion

This paper examines how a firm can manipulate consumer beliefs about demand in a repeated setting by strategically using a capacity constraint. There are

[4] There may of course be other reasons to clearly post prices, such as price competition or as a way to commit not to raise the price above its expected level once consumers enter the store

social effects to consumption, consumers can observe quantity sold in each period but not quantity demanded, and naive consumers do not understand the firm may directly signal through its price.

A firm with an exogenous capacity constraint may set a low price so as just to sell out, so that naive consumer believe demand is higher than it actually is. A firm without a capacity constraint may impose one on itself to achieve the same effect, increasing future demand through the social effects of consumption. It will impose a capacity constraint and sell out if demand is sufficiently low, and does so for many parameter values even as the fraction of naive consumers tends to zero. When the firm is also somewhat uncertain about demand, the above mechanism pushes the optimal price down, increasing the probability of excess demand. If I require that even naive consumers have correct beliefs in equilibrium, a capacity constraint can still help as a commitment device if some consumers cannot directly observe the price.

An avenue for future research would be to work further with the idea that consumers can observe quantity sold but not quantity demanded, and explore the impact of capacity constraints on informational cascades. A binding capacity constraint cuts consumers off from private information others may have about quality, as they cannot observe exactly how many others wanted to buy before them. A firm may prefer being capacity constrained if that makes it easier to start a positive cascade, or more difficult to reverse one that has already occurred.

Appendix

Proof of Theorem 1. I use the superscripts u and c to denote constrained and unconstrained values. If the firm does not impose a capacity constraint, it knows all consumers will learn ϵ after period 1. Inverse demand in period 1 and periods $t \geq 2$ is therefore $A - B(Q - \epsilon) + C(1 - \alpha)\epsilon$ and $A - B(Q - \epsilon) + C\epsilon$ respectively. Simple calculations give optimal prices

$$p_1^u = \frac{A + B\epsilon + C(1 - \alpha)\epsilon}{2}$$

$$p_t^u = \frac{A + B\epsilon + C(1 - \alpha)\epsilon}{2}$$

and quantities sold

$$q_1^u = \frac{A + B\epsilon + C(1 - \alpha)\epsilon}{2B}$$

$$q_t^u = \frac{A + B\epsilon + C\epsilon}{2B}$$

for $t \geq 2$.

Per period profits are

$$\pi_1^u = \frac{1}{4B}[A + B\epsilon + C(1 - \alpha)\epsilon]^2$$

$$\pi_t^u = \frac{1}{4B}[A + B\epsilon + C\epsilon]^2$$

for $t \geq 2$.

Total discounted profits are

$$\pi^u = \frac{1}{4B}[A + B\epsilon + C(1-\alpha)\epsilon]^2 + \frac{\delta}{1-\delta}\frac{1}{4B}[A + B\epsilon + C\epsilon]^2$$

Say instead the firm imposes a capacity constraint Q_0. Doing so can only be profitable if it influences naive consumers' beliefs, which can only happen if quantity demanded in period 1 is greater or equal to Q_0.

Naive consumers then update their beliefs to $\epsilon' + \Delta/2$, where ϵ' is the minimum value of ϵ such that period 1 quantity demanded satisfied $D_1 \geq Q_0$, conditional on p_1. Since $\epsilon' \geq -\Delta$, demand in periods $t \geq 2$ will be higher than demand in period 1. That implies the firm will set $Q_0 \geq q_1^u$.

If $D_1 > Q_0$, the firm could marginally increase p_1 without affecting quantity sold. That would also increase ϵ', giving naive consumers more favorable beliefs and thus increasing demand in future periods. The firm therefore sets p_1 such that $q_1 = Q_0$.

Note that if $Q_0 > q_1^u$, then marginally decreasing Q_0 will increase π_1^c. The reason is the firm can then charge a price closer to the unconstrained first period optimum.

Say $q_t < Q_0$ for $t \geq 2$ so the capacity constraint does not bind at the optimal p_t. The firm must then be charging the same p_t as would be optimal if it were unconstrained, given the new beliefs of naive consumers. This price is strictly greater than p_1^c, since demand in period 2 is higher than demand in period 1. That implies $Q_0 > q_1^u$. Marginally decreasing Q_0 would allow the firm to charge a lower price p_1, closer to p_1^u, which strictly increases π_1^c but leaves π_2^c unaffected. That implies $q_t = Q_0$ for all t.

The firm's total discounted profits are

$$\pi = Q_0[A + B\epsilon - BQ_0 + C(1-\alpha)\epsilon] + \frac{\delta}{1-\delta}Q_0[A + B\epsilon - BQ_0 + C(1-\alpha)\epsilon + C\alpha(\frac{\Delta+\epsilon}{2})]$$

Optimizing with respect to Q_0 gives

$$Q_0 = \frac{A + B\epsilon + (1-\alpha)C\epsilon}{2B} + \delta\frac{\alpha C}{2B}(\frac{\Delta+\epsilon}{2})$$

$$p_1^c = \frac{1}{2}[A + B\epsilon + (1-\alpha)C\epsilon] - \delta\frac{\alpha C}{2}(\frac{\Delta+\epsilon}{2})$$

$$p_2^c = \frac{1}{2}[A + B\epsilon + C\epsilon] + \frac{\alpha C}{2}[\Delta - \delta(\frac{\Delta+\epsilon}{2})]$$

I can write:

$$Q_0 = q_1^u + \delta\frac{\alpha C}{2B}(\frac{\Delta+\epsilon}{2})$$

$$Q_0 = q_2^u - \frac{\alpha C}{2B}[\epsilon - \delta(\frac{\Delta+\epsilon}{2})]$$

Note that $q_1^u < q_1^c$, $p_1^c < p_1^u$ and $p_2^c > p_2^u$. With a little bit of algebra, period 1 profits can be written as

$$\pi_1^c = \pi_1^u - \frac{1}{4B}[\delta\alpha C(\frac{\Delta+\epsilon}{2})]^2$$

The discounted sum of profits as of period 2 can be written as

$$\frac{\delta}{1-\delta}\pi_2^c = \frac{\delta}{1-\delta}\pi_2^u + \frac{\alpha C}{4B}(A+B\epsilon+C\epsilon)(\Delta-\epsilon) - \frac{\alpha^2C^2}{4B}[\epsilon-\delta(\frac{\Delta+\epsilon}{2})][\Delta-\frac{\delta}{1+\delta}(\frac{\Delta+\epsilon}{2})]$$

where $\pi_1^c < \pi_1^u$ and $\pi_2^c > \pi_2^u$. That implies

$$(\frac{\delta}{1-\delta})(\pi^u - \pi^c) = \frac{\alpha C}{4B}\left\{\alpha C[\frac{\delta}{1+\delta}(\frac{\Delta+\epsilon}{2})]^2 - \delta(A+B\epsilon+C\epsilon)(\Delta-\epsilon) + \delta\alpha C[\epsilon - \frac{\delta}{1+\delta}(\frac{\Delta+\epsilon}{2})][\Delta-\frac{\delta}{1+\delta}(\frac{\Delta+\epsilon}{2})]\right\}$$

The firm imposes a capacity constraint if and only if

$$\alpha C[\delta(\frac{\Delta+\epsilon}{2})]^2 - \delta(A+B\epsilon+C\epsilon)(\Delta-\epsilon) + \delta\alpha C[\epsilon-\delta(\frac{\Delta+\epsilon}{2})][\Delta-\delta(\frac{\Delta+\epsilon}{2})] < 0$$

A quick check shows that the inequality holds for $\epsilon = -\Delta$, but does not hold for $\epsilon = \Delta$. Rearranging the LHS and doing some algebra gives a quadratic expression in ϵ, where the coefficient for ϵ^2 is

$$\frac{\delta}{1-\delta}[(B+C) - \delta\frac{\alpha C}{4}]$$

The coefficient for ϵ is

$$\frac{\delta}{1-\delta}[\frac{\alpha C\Delta}{2}(2-\delta) + A - \Delta(B+C)]$$

The constant term is

$$-\frac{\delta}{1-\delta}[\frac{\alpha C}{4}\delta + \Delta A]$$

The coefficient for ϵ^2 and the constant term are both positive. That implies the existence of a critical value $0 < \epsilon_0 < \Delta$, such that the firm imposes a capacity constraint if and only if $\epsilon < \epsilon_0$.

Labeling these expressions A', B' and C' respectively, the quadratic formula gives $\epsilon_0 = [-B' + \sqrt{B'^2 - 4A'C'}]/2A'$

Sketch of Proof of Result 2. From Theorem 1, optimally choosing Q_0 and selling out when $\epsilon < \epsilon_0$ yields higher profits as of period 2 compared to not imposing a constraint. Denote the difference in these total future profits by $Y > 0$.

By continuity, as X_0 tends to zero, the firm will want to impose a capacity constraint Q_0 that tends to the value from Theorem 1. The expected difference in total profits as of period 2 from imposing a capacity constraint will tend to Y.

The firm will not set p_1 such that $D_1 > q_0$ for all values of ϵ consistent with X, since otherwise marginally increasing p_1 would increase profits.

Say the firm sets p_1 such that $q_1 = q_0$ for $X = -X_0$. Marginally increasing p_1 by some small amount $\epsilon^{'}$ would decrease quantity demanded by $\epsilon^{'}/B$, and to order $\epsilon^{'}$ that would increase expected period 1 profits by $Q_0\epsilon^{'}$.

There is then a probability $\epsilon^{'}/2X_0B$ that the firm does not sell out, which would decrease future profits by some amount $Y(X_0)$ which might depend on X_0.

The firm will not decrease the price if

$$Q_0\epsilon^{'} < Y(X_0)\frac{\epsilon^{'}}{2X_0B}$$

As X_0 becomes small, the inequality tends to

$$Q_0 < \frac{Y}{2X_0B}$$

which holds.

If the firm instead set p_1 such that $q_1 < q_0$ for $X = -X_0$, then increasing p_1 as above would be even less attractive. In this case, the expected increase in period 1 profits would be strictly less than $Q_0\epsilon^{'}$. The firm would instead want to increase p_1 until $q_1 = q_0$ for $X = -X_0$.

Sketch of Proof of Result 3. First, show that if the firm finds it profitable to sell out in period 1, then it will also find it profitable to sell out in period 2 (and therefore for all $t \geq 2$). I can assume that after selling out in period 1, the firm's period 2 price that maximizes period 2 profits gives a quantity strictly less than Q_0 (otherwise there is nothing to check). So I can assume

$$Q_0 \geq \frac{1}{2B}[A + B\epsilon + C(1-\alpha)\epsilon + C\alpha(\frac{\Delta+\epsilon}{2})$$

I want to show the firm will not want to sell out in period 2 if it does not sell out in period 1. The condition reads

$$Q_0[A+B\epsilon+C(1-\alpha)\epsilon-BQ_0]+\frac{\delta}{1-\delta}Q_0[A+B\epsilon+C(1-\alpha)\epsilon+C\alpha(\frac{\Delta+\epsilon}{2})-BQ_0] \geq$$

$$\frac{1}{4B}[A+B\epsilon+C(1-\alpha)\epsilon]^2+\frac{\delta}{1-\delta}\frac{1}{4B}[A+B\epsilon+C\epsilon]^2$$

implies

$$Q_0[A+B\epsilon+C(1-\alpha)\epsilon+C\alpha(\frac{\Delta+\epsilon}{2})-BQ_0]+\frac{\delta}{1-\delta}Q_0[A+B\epsilon+C(1-\alpha)\epsilon+C\alpha(\frac{\Delta+\epsilon}{2})-BQ_0] \geq$$

$$\frac{1}{4B}[A+B\epsilon+C(1-\alpha)\epsilon+C\alpha(\frac{\Delta+\epsilon}{2})]^2+\frac{\delta}{1-\delta}\frac{1}{4B}[A+B\epsilon+C\epsilon]^2$$

The condition can be rewritten as

$$\frac{\delta}{1-\delta}Q_0[A+B\epsilon+C(1-\alpha)\epsilon+C\alpha(\frac{\Delta+\epsilon}{2})-BQ_0] \geq$$

$$\frac{1}{4B}[A+B\epsilon+C(1-\alpha)\epsilon]^2+\frac{\delta}{1-\delta}\frac{1}{4B}[A+B\epsilon+C\epsilon]^2-Q_0[A+B\epsilon+C(1-\alpha)\epsilon-BQ_0]$$

implies

$$\frac{\delta}{1-\delta}Q_0[A+B\epsilon+C(1-\alpha)\epsilon+C\alpha(\frac{\Delta+\epsilon}{2})-BQ_0] \geq$$

$$\frac{1}{4B}[A+B\epsilon+C(1-\alpha)\epsilon+C\alpha(\frac{\Delta+\epsilon}{2})]^2+\frac{\delta}{1-\delta}\frac{1}{4B}[A+B\epsilon+C\epsilon]^2-Q_0[A+B\epsilon+C(1-\alpha)\epsilon+C\alpha(\frac{\Delta+\epsilon}{2})-BQ_0]$$

Some algebra shows this is the case if

$$Q_0 \geq \frac{1}{2B}[A+B\epsilon+C(1-\alpha)\epsilon+\frac{1}{2}[C\alpha(\frac{\Delta+\epsilon}{2})]]$$

This is implied by

$$Q_0 \geq \frac{1}{2B}[A+B\epsilon+C(1-\alpha)\epsilon+C\alpha(\frac{\Delta+\epsilon}{2})$$

Now say the following holds

$$\frac{1}{2B}[A+B\Delta+C(1-\alpha)\Delta] \leq Q_0 \leq \frac{1}{2B}[A+B\Delta+C(1-\alpha)\Delta]$$

Then there exists some e^* such the price maximizing period 1 profits yields $D_1 = Q_0$. Denote the total profits as of period 2 after imposing a capacity constraint and selling out minus these profits without selling out by $Y(\epsilon)$. As ϵ tends to e^* from below, $\pi_1^u - \pi_1^c$ tends to zero but $Y(\epsilon)$ tends to $Y > 0$. The firm sacrifices period 1 profits by selling out, but by doing so earns higher total profits.

Sketch of Proof of Result 4. The previous result means that if the firm sells out in period 1 and then acts optimally in all future periods (selling out there), profits are

$$\pi^c = Q_0[A+B\epsilon+C(1-\alpha)\epsilon-BQ_0]+\frac{\delta}{1-\delta}Q_0[A+B\epsilon+C(1-\alpha)\epsilon+C^n\alpha(\frac{\Delta+\epsilon}{2})-BQ_0]$$

If the firm does not sell out in period 1, $\epsilon > 0$ and C^n is large, then the firm will want to sell out in all future periods. Profits are then

$$\pi^u = \frac{1}{2B}[A+B\epsilon+C(1-\alpha)\epsilon]+\frac{\delta}{1-\delta}Q_0[A+B\epsilon+(C(1-\alpha)+C^n\alpha)\epsilon-BQ_0]$$

Say $Q_0 < \frac{1}{2B}[A+B\epsilon+C(1-\alpha)\epsilon]$. For $\epsilon < \Delta$ but large enough, selling out in period 1 maximizes period 1 profits so it must maximizes total profits as well.

For smaller ϵ, selling out in period 1 does not maximizes period 1 profits but by an amount independent of C^n. Selling out increases the discounted stream of future profits by

$$\frac{\delta}{1-\delta}Q_0C^n\alpha\epsilon(\frac{\Delta-\epsilon}{2})$$

For C^n sufficiently large, it is therefore profitable to sell out in period 1.

Say $Q_0 > \frac{1}{2B}[A+B\epsilon+C(1-\alpha)\epsilon]$. Then the above reasoning holds for any $\epsilon < \Delta$. For any fixed $\epsilon < \Delta$, there exists C^n sufficiently large such that the firm will sell out in period 1 for that value of ϵ. Also, for any fixed C^m however large, there exist values of ϵ close to Δ such that the firm will not sell out in period 1.

Sketch of Proof of Result 5. Say the firm does not impose a capacity constraint. Then profits are

$$\pi_1 = p_1 \frac{1}{B}[A + B\epsilon + C(1-\epsilon) - p_1]$$

$$\pi_2 = p_2 \frac{1}{B}[A + B\epsilon + C(1-\epsilon) + (p_1^* - p_1)\frac{\alpha\beta C}{B} - p_2]$$

If the firm charges p_1 a certain amount below its expected level, a fraction $\alpha\beta$ of period 2 consumers believe ϵ is larger than it actually is, by an amount divided by B.

Differentiating $\pi_1 + \delta\pi_2$ with respect to p_1 and p_2 and setting them equal to zero gives

$$p_2 = \frac{1}{2}[A + B\epsilon + (1-\alpha)C\epsilon + (p_1^* - p_1)\frac{\alpha\beta C}{B}]$$

$$p_1 = \frac{1}{2}[A + B\epsilon + (1-\alpha)C\epsilon - \delta\alpha\beta C p_2]$$

or

$$p_2 = \frac{1}{2B}[A + B\epsilon + (1-\alpha)C\epsilon]$$

$$p_1 = \frac{1}{2B}[A + B\epsilon + (1-\alpha)C\epsilon](1 - \frac{\delta\alpha\beta C}{2B})$$

Imposing a capacity constraint will lead to higher profits if it permits the same p_2 as above, and a higher p_1 that is closer to the first best (which equals p_2).

If the firm imposes a capacity constraint, it will have to bind in period 1 to have any effect. The commitments problem implies the firm will charge $p_1 \leq p_2$, so it does not have to worry about excess demand in period 2. Profits are therefore:

$$\pi = p_1 Q_0 + p_2 \frac{\delta}{B}[A + B\epsilon + C(1-\epsilon) + (p_1^* - p_1)\frac{\alpha\beta C}{B} - p_2]$$

where

$$Q_0 = \frac{1}{B}[A + B\epsilon + (1-\alpha)C\epsilon - p_1]$$

The derivative of π with respect to p_2 is the same as above, giving again

$$p_2 = \frac{1}{2B}[A + B\epsilon + (1-\alpha)C\epsilon]$$

The derivative with respect to p_1 is discontinuous at p_1^*. A marginal increase has the following effects on profits:

$$\frac{1}{B}[A + B\epsilon + (1-\alpha)C\epsilon - 2p_1] - p_2 \frac{\delta}{B} - \frac{\alpha\beta C}{B}$$

A marginal decrease has the following effects on profits

$$-\frac{1}{B}[A + B\epsilon + (1-\alpha)C\epsilon - p_1] + p_2 \frac{\delta}{B}\frac{\alpha\beta C}{B}$$

The firm has no incentive to deviate from p_1 if and only if

$$\frac{1}{2}[A + B\epsilon + (1-\alpha)C\epsilon](1 - \frac{\alpha\beta C}{B}) \leq p_1 \leq [A + B\epsilon + (1-\alpha)C\epsilon](1 - \frac{\alpha\beta C}{2B})$$

Clearly, the firm does not want to charge a p_1 that is too low to satisfy the inequality. The only condition the firm must worry about is that p_1 not be too high as noted above, which gives the result.

References

Banerjee, A. V. (1992, August). A simple model of herd behavior. *The Quarterly Journal of Economics 107*(3), 797–817.

Becker, G. S. (1974, Nov.-Dec.). A theory of social interactions. *Journal of Political Economy 82*(6), 1063–93.

Becker, G. S. (1991, October). A note on restaurant pricing and other examples of social influences on price. *Journal of Political Economy 99*(5), 1109–16.

Bikhchandani, S., D. Hirshleifer, and I. Welch (1992, October). A theory of fads, fashion, custom, and cultural change in informational cascades. *Journal of Political Economy 100*(5), 992–1026.

Chioveanu, I. and J. Zhou (2009, September). Price competition and consumer confusion. MPRA Paper 17340, University Library of Munich, Germany.

DeGraba, P. (1995, Summer). Buying frenzies and seller-induced excess demand. *RAND Journal of Economics 26*(2), 331–342.

Denicolo, V. and P. Garella (1999, Spring). Rationing in a durable goods monopoly. *RAND Journal of Economics 30*(1), 44–55.

DeSerpa, A. C. and R. L. Faith (1996, October). "bru-u-u-uce": The simple economics of mob goods. *Public Choice 89*(1-2), 77–91.

Gabaix, X. and D. Laibson (2006, May). Shrouded attributes, consumer myopia, and information suppression in competitive markets. *The Quarterly Journal of Economics 121*(2), 505–540.

Gilbert, R. J. and P. Klemperer (2000, Spring). An equilibrium theory of rationing. *RAND Journal of Economics 31*(1), 1–21.

Leibenstein, H. (1950, May). Bandwagon, snob, and veblen effects in the theory of consumers demand. *The Quarterly Journal of Economics 64*(2), 183–207.

Nocke, V. and M. Peitz (2007, 07). A theory of clearance sales. *Economic Journal 117*(522), 964–990.

Piccione, M. and R. Spiegler (2009, October). Price differentiation under limited comparability. *Working paper*.

Spiegler, R. (2006, June). Competition over agents with boundedly rational expectations. *Theoretical Economics 1*(2), 207–231.